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COMPUTER SIMULATION OF VIBRATIONS OF TIBIA FIXED BY THE PLANE FRAME

In medical practice of treatment of fractures, the external fixators (both plane frames and numerous types of the Ilizarov apparatus) are widely spread. They fasten the bone parts together very hard and form the united biomechanical system “bone-fixator”. The osteosynthesis process (i.e. the bone consolidation) takes up such a long period of time and demands permanent control for stiffness of the bone callosity. The existing X-rays devices are not always able to trace the fine tissue changes in fracture zone and have strong restrictions in application frequency. A patent method of active resonance vibrational diagnostics (RVD) has been developed in the Laboratory of Biomechanics (ISPEU, Russia) for non-destructive testing mechanical defects of soft and hard tissues of human shank [1]. This approach permits a researcher to define a physiological state by means of analysis of investigated system response to the local vibrational effort [2].

Thus, for further development of the method and hardware it is necessary to receive clear patterns of amplitude-frequency characteristics of the idealised biomechanical system, to determine frequencies and forms of oscillations, to reveal correlation between the physical and mechanical characteristics of the bone in fracture region and changes of the system resonance properties.

For solving the problem, a three-dimensional finite element model of tibia fixed by plane frame used in traumatic surgery was developed on basis of direct geometrical measurements of a selected sample (Fig.1). The boundary conditions were taken as hinge supports at the ends.

Tibia material was considered to be piece-homogeneous isotropic continuous medium with the following physical-mechanical parameters for compact tissue: elasticity module $E = 18700 \text{ MPa}$, shear module $G = 6680 \text{ MPa}$, viscosity $\eta = 0.05 \text{ MPa sec}$, density $\rho = 1850 \text{ kg/m}^3$; for spongy tissue: elasticity module $E = 500 \text{ MPa}$, shear module $G = 200 \text{ MPa}$, density $\rho = 800 \text{ kg/m}^3$ [3]. The fixator material is characterized by elasticity module $E = 206000 \text{ MPa}$, shear module $G = 80000 \text{ MPa}$, density $\rho = 7900 \text{ kg/m}^3$ [3]. Thus, the epiphyseal sections or heads of tibia bone physiologically consisting of porous spongy tissue coated with a thin cortical layer were completely modelled by finite elements having the physical-mechanical characteristics of spongy tissue. The diaphyseal (middle) section of the bone was considered to be a hollow cylinder of a complex three-dimensional shape formed by compact tissue without the spongy tissue stratum and the marrow filling the inside of the bone.

For investigating the process of bone integrity regeneration in fracture region, a narrow layer of 4 mm wide in the bone middle cross-section was simulated by finite elements with different variants of the mechanical parameters of the regenerated tissue. The following medical cases have been studied: I - filling up the space of fracture with soft gel-like tissue; this probably occurs in the initial time period of neogenesis of a bone or at poor compression by means of a fixator; II - forming cartilaginous tissue that is typical for the so-called “false joints”, III - forming spongy tissue [3]. As a rule, parameter types shown above describe pathological cases.
of osteosynthesis such as slow consolidation, “false joints” or instability of the connection the recognition of which is difficult even when X-ray equipment is available. All data are summarised in Tabl. 1.

Table 1. Variants of mechanical characteristics of the regenerating osseous tissue in the fracture zone and values of the lowest resonance frequencies of the “bone - fixator” system

<table>
<thead>
<tr>
<th>Variants</th>
<th>Young module $E_r$ MPa</th>
<th>Shear module $G_r$ MPa</th>
<th>Density $\rho_r$ kg/m$^3$</th>
<th>Viscosity $\eta_r$ MPa⋅sec$^{-1}$</th>
<th>1st resonance frequency Hz</th>
<th>2nd resonance frequency Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.005</td>
<td>0.00167</td>
<td>1000</td>
<td>$5 \times 10^{-5}$</td>
<td>50</td>
<td>173</td>
</tr>
<tr>
<td>II a</td>
<td>3.87</td>
<td>1.3</td>
<td>900</td>
<td>$5.4 \times 10^{-4}$</td>
<td>98</td>
<td>214</td>
</tr>
<tr>
<td>II b</td>
<td>50</td>
<td>17</td>
<td>850</td>
<td>$1.93 \times 10^{-3}$</td>
<td>131</td>
<td>229</td>
</tr>
<tr>
<td>III</td>
<td>500</td>
<td>200</td>
<td>800</td>
<td>$7.4 \times 10^{-3}$</td>
<td>149</td>
<td>250</td>
</tr>
<tr>
<td>Normal</td>
<td>18700</td>
<td>6678</td>
<td>1850</td>
<td>$5 \times 10^{-2}$</td>
<td>161</td>
<td>276</td>
</tr>
</tbody>
</table>

The model described was analysed by the author’s program code MechanicsFE3D_VEO that allows calculating static stress state and forced vibrations of mechanical structures.

Analysis of amplitude-frequency characteristics of the hinge-supported tibia bone has shown the presence of the lowest resonance frequency equal to 210 Hz. The first vibration mode coincides with the static bend of bone and corresponds to transversal oscillations of the bone in the sagittal plane XY. Applying the external fixative apparatus to the bone reduces the first frequency to the value of 161 Hz and causes the appearance of the second frequency equal to 276 Hz (Fig.2). The first mode keeps its shape as a whole but the second one is close to torsion vibrations of the system.

Considerable reduction of the regenerative tissue elasticity module $E_r$ up to elasticity module of spongy tissue equal to 500 MPa leads to considerable decrease of both resonant frequencies. Not only essential decreasing of the first and second resonance frequencies and increasing the second resonance amplitude but also the qualitative change of the resonance lines take place in cases of lower elasticity modules: $E_r = 5$ kPa and $E_r = 3.87$ MPa. In case $E_r = 5$ kPa the additional sub-resonance peaks arise in the basic resonance zone which is typical for soft biological tissues. The shape of resonance lines in the case of regenerative tissue elasticity module $E_r = 500$ MPa is approximating that of the non-defective system “bone-fixator”.

Mathematical simulation of the biomechanical system composed of the tibia bone and external fixative apparatus made it possible to define basic frequencies and forms of the system oscillations. The obtained outcomes can be the theoretical basis for development of vibrational methods of diagnostics of the osseous tissue physiological state in the fracture region.

REFERENCES: