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THE MOTION OF A CHARGED PARTICLE IN THE FIELD OF AN ELECTROMAGNETIC WAVE AND IN THE CONSTANT MAGNETIC FIELD

We have done an analysis of the results on the motion of a charged particle in an external field of plane and arbitrarily polarized electromagnetic waves of high intensity in the presence of an external uniform static magnetic field. A point of interest was a solution of the equation of the motion of a charged particle in the field of the electromagnetic wave and a uniform constant magnetic field. We investigated the interaction of high intensity laser pulses with solid targets in relation to the practical development of multi-frequency lasers and the technology development of laser modulation. The problem in question is topical because of the wide practical application of high-temperature plasma forming on the surface of the target, and the search for new modes of laser-plasma interaction. The formulae for the average kinetic energy of a relativistic particle, depending on the initial data, for the amplitude of the electromagnetic wave and for the wave intensity and its polarization parameter were obtained. The dependence of the average kinetic energy on the intensity of the electromagnetic wave in the uniform constant magnetic field was derived.

PLANE ELECTROMAGNETIC WAVE, AVERAGE KINETIC ENERGY, CHARGED PARTICLE, ULTRASHORT LASER PULSE, STATIC MAGNETIC FIELD.

I. Introduction

A current problem of practical and theoretical interest is that of a charged particle accelerated by ultra-short laser pulses of high-intensive plasma [1 – 5]. An effective method for obtaining high-energy particles is targeting the front surface of a thin foil target with high-power laser pulses. The interaction of charged particles with ultra-short femtosecond laser pulses with intensities of emission up to 10^{22} W/cm^2 is one of the main areas of laser physics at the moment [6 – 8].

In this paper we consider the dynamics of a charged particle in an intensive electromagnetic field of elliptical polarization. The article [9] discussed the consistent derivation of the average kinetic energy of a particle. That was averaged over a period of motion in the field of the monochromatic electromagnetic wave for linear and circular polarized waves. A more general formula for the average kinetic energy of an elliptical polarized wave was found in [10]. The generalization of the results of articles [9, 10] in the presence of a uniform external field is of great practical interest; since this case corresponds to the phenomenon of cyclotron auto-resonance [11, 12] on the basis of which such systems operate as a free-electron laser and an undulator [13].

The problem of the motion of a charged particle in the field of a plane monochromatic electromagnetic wave was formulated and solved for linear and circular polarization of the wave [14], but the interest in this topic has appeared presently in connection with the development of high-power lasers.

The aim of this work is to analyze the motion of a particle in the external field of arbitrarily polarized electromagnetic waves of high intensity and in the uniform constant magnetic field and to derive the average kinetic energy of a particle over the oscillation period of the field.

II. Problem Statement

The equation of motion of a particle of mass m and charge q placed in an external field of a plane monochromatic wave and in a uniform constant magnetic field $\mathbf{H}_0$ has the form (see, for example [14], paragraph 17)

$$\frac{d\mathbf{p}}{dt} = q\mathbf{E} + \frac{q}{c}[\mathbf{v} \times \mathbf{H}_\Sigma]. \qquad (1)$$

In Eq. (1) $\mathbf{H}_\Sigma = \mathbf{H}_0 + \mathbf{H}$ is a superposition of the constant uniform magnetic field $\mathbf{H}_0$ and the magnetic field $\mathbf{H}$ of the plane monochromatic electromagnetic wave.

The particle momentum $\mathbf{p}$ and velocity $\mathbf{v}$ are related by equality ([14], paragraph 9)

$$\mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1 - \frac{v^2}{c^2}}}. \qquad (2)$$

The change in the particle energy

$$\varepsilon = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = \sqrt{m^2c^4 + p^2c^2} \qquad (3)$$

is determined by the equation

$$\frac{d\varepsilon}{dt} = q\mathbf{E}\mathbf{v}. \qquad (4)$$

It follows from (2) and (3) that the energy ε, momentum $\mathbf{p}$, and velocity $\mathbf{v}$ of the particle are related by equations

$$\mathbf{p} = \frac{\varepsilon\mathbf{v}}{c^2}, \quad \mathbf{v} = \frac{c^2\mathbf{p}}{\varepsilon}. \qquad (5)$$

We assume a uniform constant magnetic field $\mathbf{H}_0 = \mathbf{k}H_0$ ($\mathbf{k}$ – basis vector) and the plane wave propagates along the axis z. In the case of Fig. 1, the vector components of electric and magnetic fields of the plane monochromatic electromagnetic wave are given by:

$$\begin{cases} E_x = H_y = b_x \exp(-i\Phi); \\ E_y = -H_x = fb_y \exp(-i\Phi); \\ E_z = H_z = 0, \end{cases} \qquad (6)$$

where

$$\Phi = \omega\xi + \psi + \varphi; \quad \xi = t - z/c;$$

ω is the carrier frequency of the wave; ψ is a wave polarization parameter; φ is an angle of inclination of the axes of the ellipse Ox to the axis of the coordinate system; the x and y axes coincide with the b_x and b_y axes of the polarization ellipse of the wave and $b_x \geq b_y \geq 0$; $f = \pm 1$ is a polarization parameter (the upper and lower signs in the expression for E_y correspond to the right and left polarization [15]).

III. Solution of the Equation of the Charge Motion

The solution of Eqs. (1) and (4) with $\mathbf{E}$ and $\mathbf{H}$ from (6) has the form:

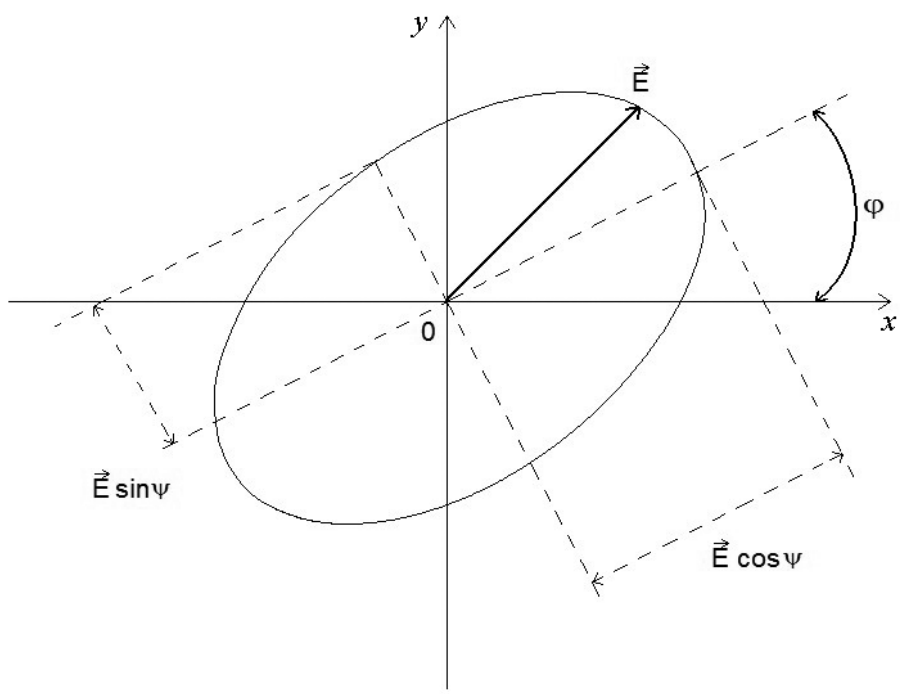

Fig. 1. The ellipse of polarization (see the text)

$$p_x = \frac{qb_x}{\omega}\sin\Phi + \frac{q}{c}H_0 y + \chi_x;$$
$$p_y = \frac{fqb_y}{\omega}\sin\Phi - \frac{q}{c}H_0 x + \chi_y. \quad (7)$$

The equations in (7) show the differentiation with respect to ξ :

$$\dot{x} = \frac{qb_x c}{\omega\gamma}\sin\Phi + \omega_c y + \frac{c}{\gamma}\chi_x;$$
$$\dot{y} = \frac{fqb_y c}{\omega\gamma}\sin\Phi - \omega_c x + \frac{c}{\gamma}\chi_y, \quad (8)$$

where $\omega_c = qH_0/\gamma$ is the cyclotron frequency.

Through the constants χ_x, χ_y, and γ determined by the initial phase of the wave

$$\Phi_0 = -kz_0 + \varphi + \psi$$

and the initial velocity of the particle $\mathbf{\upsilon}_0 = 0$; of (3) and (7) we find

$$\chi_x = \frac{m\upsilon_{x0}}{\sqrt{1-\frac{\upsilon_0^2}{c^2}}} - \frac{qb_x}{\omega}\sin\Phi_0 - \frac{q}{c}H_0 y_0;$$
$$\chi_y = \frac{m\upsilon_{y0}}{\sqrt{1-\frac{\upsilon_0^2}{c^2}}} - \frac{fqb_y}{\omega}\sin\Phi_0 + \frac{q}{c}H_0 x_0; \quad (9)$$
$$\gamma = \frac{mc\left(1-\frac{\upsilon_{z0}}{c}\right)}{\sqrt{1-\frac{\upsilon_0^2}{c^2}}}.$$

By transforming the system of differential Eqs. (8), we obtain

$$\ddot{x} + \omega_c^2 x = \frac{qcb_x}{\gamma}\cos\Phi + \frac{fq\omega_c b_y}{\gamma k}\sin\Phi + \frac{\omega_c c}{\gamma}\chi_y;$$
$$\ddot{y} + \omega_c^2 y = \frac{fqcb_y}{\gamma}\cos\Phi - \frac{q\omega_c b_x}{\gamma k}\sin\Phi - \frac{\omega_c c}{\gamma}\chi_x. \quad (10)$$

The solution of the differential equations of the second order (10) is determined in the form of a sum of solutions of the homogeneous equation and the particular solution of the inhomogeneous equation with the initial conditions. We obtain the following solutions for the *x* and *y* coordinates:

$$x = \left(\frac{qb_x\omega}{\gamma k(\omega^2-\omega_c^2)} - \frac{\omega\chi_y}{\omega_c\gamma k}\right)\cos\Phi_c + \frac{fqb_y\omega}{\gamma k(\omega^2-\omega_c^2)}\sin\Phi_c + \frac{\omega\chi_y}{\omega_c\gamma k} - \frac{qb_x\omega}{\gamma k(\omega^2-\omega_c^2)}\cos\Phi - \frac{fqb_y\omega_c}{\gamma k(\omega^2-\omega_c^2)}\sin\Phi;$$
$$y = \left(\frac{\omega\chi_x}{\omega_c\gamma k} + \frac{fqb_y\omega}{\gamma k(\omega^2-\omega_c^2)}\right)\cos\Phi_c - \frac{qb_x\omega}{\gamma k(\omega^2-\omega_c^2)}\sin\Phi_c - \frac{\omega\chi_x}{\omega_c\gamma k} - \frac{fqb_y\omega}{\gamma k(\omega^2-\omega_c^2)}\cos\Phi + \frac{qb_x\omega_c}{\gamma k(\omega^2-\omega_c^2)}\sin\Phi, \quad (11)$$

where $k = \omega/c$; $\Phi_c = \omega_c t$.

Using (7) and (11), we obtain expressions for the p_x and p_y components of the momentum of the particle:

$$p_x = A\sin\Phi + B\cos\Phi_c + C\sin\Phi_c + D\cos\Phi;$$
$$p_y = E\sin\Phi + F\cos\Phi_c + G\sin\Phi_c + I\cos\Phi. \quad (12)$$

Therefore,

$$A = \frac{qb_x}{\omega}\left(\frac{\omega^2}{\omega^2-\omega_c^2}\right);\ B = \chi_x + \frac{fqb_y\omega_c}{\omega^2-\omega_c^2};$$
$$C = -\frac{qb_x\omega_c}{\omega^2-\omega_c^2};\ D = -\frac{fqb_y\omega_c}{\omega^2-\omega_c^2};$$
$$E = \frac{fqb_y}{\omega}\left(\frac{\omega^2}{\omega^2-\omega_c^2}\right);\ F = \chi_y - \frac{qb_x\omega_c}{\omega^2-\omega_c^2};$$
$$G = -\frac{fqb_y\omega_c}{\omega^2-\omega_c^2};\ I = \frac{qb_x\omega_c}{\omega^2-\omega_c^2}. \quad (13)$$

From the formulae (3) and (4) we find the p_z component of the momentum of the particle:

$$p_z = \gamma g; \quad (14)$$

consequently,

$$g = h - \frac{q^2(b_x^2+b_y^2)}{4\gamma^2(\omega^2-\omega_c^2)}\cos(2\Phi) + \frac{1}{\gamma^2}(AB+EF)\sin\Phi\cos\Phi_c + \quad (15)$$

$$+\frac{1}{\gamma^2}(AC+EG)\sin\Phi\sin\Phi_c+$$
$$+\frac{1}{4\gamma^2}(B^2+F^2-(C^2+G^2))\cos 2\Phi_c+ \quad (15)$$
$$+\frac{1}{2\gamma^2}(BC+FG)\sin(2\Phi_c)+$$
$$\frac{1}{\gamma^2}(FI+BD)\cos\Phi_c\cos\Phi;$$

$$h=\frac{1}{2}\left[\frac{m^2c^2}{\gamma^2}-1+\frac{1}{2\gamma^2}(A^2+B^2+C^2+ D^2+E^2+F^2+G^2+I^2)\right]. \quad (16)$$

From (3) and (4) we find the expression for the energy of the particle:

$$\varepsilon=c\gamma(1+g). \quad (17)$$

From (5), (12) and (14) we obtain a parametric representation of the particle velocity in the parameter ξ:

$$\upsilon_x=\frac{dx}{dt}=\frac{c}{(1+g)\gamma}(A\sin\Phi+B\cos\Phi_c+ C\sin\Phi_c+D\cos\Phi),$$
$$\upsilon_y=\frac{dy}{dt}=\frac{c}{(1+g)\gamma}(E\sin\Phi+F\cos\Phi_c+ G\sin\Phi_c+I\cos\Phi), \quad (18)$$
$$\upsilon_z=\frac{dz}{dt}=\frac{cg}{1+g}.$$

From (11) and (18) we determine that the motion of a particle in the external field of the plane monochromatic electromagnetic wave in a constant uniform magnetic field is the imposition of movement with the constant velocity $\boldsymbol{\upsilon}$ and vibrational motion with the frequency $\tilde{\omega}=2\pi/\tilde{T}$ different from the frequency of the field ω and the cyclotron frequency ω_c. Then integrating (14), we obtain the equation of motion along the axis z:

$$z(t)=\tilde{z}+\tilde{\upsilon}_z t+\theta(t)+\eta(t), \quad (19)$$

where $\tilde{z}$ is constant, and

$$\theta(t+\tilde{T})=\theta(t),\ \ \eta(t+\tilde{T}_c)=\eta(t) \quad (20)$$

are periodic functions.

In formula (18)

$$\tilde{\upsilon}_z=\frac{ch}{1+h}. \quad (21)$$

It follows from (19) that $g(\xi)$ from (15) is a periodic function defined by the periods $\tilde{T}$ and $\tilde{T}_c$. The period $\tilde{T}$ of oscillation of the particle in the field of a plane monochromatic wave and the period $\tilde{T}_c$ of cyclotron oscillations are determined from the formulae

$$\Phi(t+\tilde{T})=\Phi(t)+2\pi,\ \ \Phi_c(t+\tilde{T}_c)=\Phi_c(t)+2\pi;$$

from which, taking into account (6), (19) and (20), it follows that

$$\tilde{T}=\frac{2\pi}{\omega}\left(\frac{1}{1-\tilde{\upsilon}_z/c}\right)=\frac{2\pi}{\omega}(1+h);\ \ \tilde{T}_c=\frac{2\pi}{\omega_c}. \quad (22)$$

Thus, the motion of a particle is a superposition of two kinds of periodic motion; they are $\tilde{T}$ and $\tilde{T}_c$.

IV. The Motion of a Particle Averaged over an Oscillation Period

In this section we will perform the averaging of the momentum $\mathbf{p}$ and the energy ε of the particles of the period of its oscillations (22) with (12), (14) and (17) in the field of an electromagnetic wave and the constant magnetic field.

Consider a new variable of the integration $\xi'=\xi(t')$, then

$$\Phi'=\Phi(t');$$
$$dt'=\frac{d\Phi'}{\omega}\frac{1}{1-v_z(t)/c}=\frac{1+g}{\omega}d\Phi'; \quad (23)$$
$$\Phi'_c=\Phi_c(t');dt'=\frac{d\Phi'_c}{\omega_c}.$$

Since the motion of a particle is a superposition of two kinds of periodic motion with frequencies ω and ω_c, averaging will be carried out according to the formula

$$\bar{f}(t)=\frac{1}{2\pi}\int\limits_{\Phi(t)}^{\Phi(\tilde{t})}\frac{1}{\tilde{T}}\int\limits_{\Phi_c(t)}^{\Phi_c(\tilde{t})}f(t')\frac{1+g}{\omega}d\Phi'd\Phi'_c, \quad (24)$$

where $f(t')$ is an arbitrary function taking into account (6), (19) and (22).

Averaging the components (18) of the particle velocity, we obtain:

$$\bar{\upsilon}_x=0;\ \ \bar{\upsilon}_y=0;\ \ \bar{\upsilon}_z=\frac{ch}{1+h}. \quad (25)$$

As might be expected, the speed of the particle $\bar{\upsilon}_z$ in (25) corresponds $\tilde{\upsilon}_z$ with (21).

It follows from (25) that the average transverse momentum component of the particle is zero. From the average value of the longitudinal component of the particle momentum, we obtain the expression:

$$\bar{p}_z = \frac{\gamma}{1+h}\left(h + h^2 + \frac{q^4(b_x^2+b_y^2)^2}{32\gamma^4(\omega^2-\omega_c^2)^2} + \right.$$
$$+\frac{q^2}{4\gamma^4}\left(\frac{\omega}{\omega^2-\omega_c^2}\right)^2 (b_x\chi_x + fb_y\chi_y)^2 +$$
$$+\frac{q^4(b_x^2+b_y^2)^2\omega_c^2\omega^2}{4\gamma^4(\omega^2-\omega_c^2)^4} + \frac{1}{2}\left(\frac{\chi_x^2+\chi_y^2}{4\gamma^2} - \right.$$
$$\left.-\frac{q\omega_c}{2\gamma^2(\omega^2-\omega_c^2)}(b_x\chi_y - fb_y\chi_x)\right)^2 + \qquad (26)$$
$$+\frac{q^2\omega_c^2}{8\gamma^4(\omega^2-\omega_c^2)^2}(b_x\chi_x + fb_y\chi_y)^2 +$$
$$+\frac{1}{4}\left(\frac{q\omega_c}{\gamma^2(\omega^2-\omega_c^2)}(b_x\chi_y - fb_y\chi_x) - \right.$$
$$\left.\left.-\frac{q^2(b_x^2+b_y^2)\omega_c^2}{\gamma^2(\omega^2-\omega_c^2)^2}\right)^2\right).$$

The average energy $\bar{\varepsilon}$ of the particles is determined by the formula:

$$\bar{\varepsilon} = \frac{c\gamma}{1+h}\left((1+h)^2 + \frac{q^4(b_x^2+b_y^2)^2}{32\gamma^4(\omega^2-\omega_c^2)^2} + \right.$$
$$+\frac{q^2}{4\gamma^4}\left(\frac{\omega}{\omega^2-\omega_c^2}\right)^2 (b_x\chi_x + fb_y\chi_y)^2 +$$
$$+\frac{q^4(b_x^2+b_y^2)^2\omega_c^2\omega^2}{4\gamma^4(\omega^2-\omega_c^2)^4} + \frac{1}{2}\left(\frac{\chi_x^2+\chi_y^2}{4\gamma^2} - \right.$$
$$\left.-\frac{q\omega_c}{2\gamma^2(\omega^2-\omega_c^2)}(b_x\chi_y - fb_y\chi_x)\right)^2 + \qquad (27)$$
$$+\frac{q^2\omega_c^2}{8\gamma^4(\omega^2-\omega_c^2)^2}(b_x\chi_x + fb_y\chi_y)^2 +$$
$$+\frac{1}{4}\left(\frac{q\omega_c}{\gamma^2(\omega^2-\omega_c^2)}(b_x\chi_y - fb_y\chi_x) - \right.$$
$$\left.\left.-\frac{q^2(b_x^2+b_y^2)\omega_c^2}{\gamma^2(\omega^2-\omega_c^2)^2}\right)^2\right).$$

It is obvious that (27) depends on the intensity of the wave from the polarization of the initial phase and the initial velocity.

V. The Case of an Arbitrary Polarization for a Particle Being Initially at Rest

Consider the case when the particle is initially at rest ($\boldsymbol{\upsilon}_0 = 0$). Formulae (9) and (12) express χ_x, χ_y, γ, and, taking into account that

$$\Phi(0) = \Phi_0 = -kz_0 + \varphi + \psi, \;\; \Phi_c(0) = \Phi_{c0} = 0,$$

we obtain

$$\chi_x = -\frac{qb_x}{\omega}\left(\frac{\omega^2}{\omega^2-\omega_c^2}\right)\sin\Phi_0 -$$
$$-\frac{fqb_y\omega_c}{\omega^2-\omega_c^2}(1-\cos\Phi_0);$$
$$\chi_y = -\frac{fqb_y}{\omega}\left(\frac{\omega^2}{\omega^2-\omega_c^2}\right)\sin\Phi_0 + \qquad (28)$$
$$+\frac{qb_x\omega_c}{\omega^2-\omega_c^2}(1-\cos\Phi_0).$$

For a wave with an arbitrary polarization [16]

$$b_x^2 \pm b_y^2 = \rho^2 b^2, \qquad (29)$$

where ρ is the ellipticity parameter ($\rho = \pm 1$ corresponds to the linear polarization and $\rho = \pm 1/\sqrt{2}$ does to the circular one).

In other cases, the value ρ corresponds to an elliptical polarization $(0 \le |\rho| \le 1)$, in which:

$$\chi_x^2 + \chi_y^2 = \frac{q^2\rho^2 b^2}{\omega^2}\left(\frac{\omega^2}{\omega^2-\omega_c^2}\right)^2 \sin^2\Phi_0 +$$
$$+\frac{q^2\rho^2 b^2\omega_c^2}{(\omega^2-\omega_c^2)^2}(1-\cos\Phi_0)^2; \qquad (30)$$

$$(b_x\chi_x + fb_y\chi_y)^2 =$$
$$= \frac{q^2\rho^4 b^4}{\omega^2}\left(\frac{\omega^2}{\omega^2-\omega_c^2}\right)^2 \sin^2\Phi_0; \qquad (31)$$

$$b_x\chi_y - fb_y\chi_x = \frac{q\rho^2 b^2\omega_c}{\omega^2-\omega_c^2}(1-\cos\Phi_0). \qquad (32)$$

From (15) we obtain the value of h at the initial time:

$$h = \frac{1}{2}\left[\frac{q^2\rho^2 b^2}{2m^2c^2(\omega^2-\omega_c^2)}\sin^2\Phi_0 + \right. \qquad (33)$$
$$\left. + \frac{q^2\rho^2 b^2(\omega^2+3\omega_c^2)}{2m^2c^2(\omega^2-\omega_c^2)^2}\right].$$

Let

$$\omega_c = n\omega, \qquad (34)$$

where n is a frequency ratio between ω_c and ω, and besides $n \in [0;1)$, then

$$h = \frac{\mu}{4}\left[\frac{1}{(1-n^2)}\sin^2\Phi_0 + \frac{(1+3n^2)}{(1-n^2)^2}\right]; \qquad (35)$$

and, according to (16),

$$\mu = \frac{q^2\rho^2 b^2}{m^2c^2\omega^2} = \frac{2q^2}{\pi m^2 c^5} I\lambda^2, \qquad (36)$$

where $I = c\rho^2 b^2 / 4\pi$ is the intensity of the elliptically polarized electromagnetic wave, and $\lambda = 2\pi c/\omega$ is the wavelength.

By substituting (28) – (36) in (27), we obtain the average energy of a particle at rest in the initial wave of arbitrary polarization:

$$\bar{\varepsilon} - mc^2 = \frac{mc^2\mu}{8Q^2}\times \qquad (37)$$
$$\times\left(2S + 2N + \frac{\mu(N^2+T+S^2)}{4Q^2+\mu N+\mu S}\right), \qquad (37)$$

where

$$S = (1-n^2)\sin^2\Phi_0;\ T = 4(2+n^2-2n^4)\sin^2\Phi_0;$$
$$N = 1+3n^2;\ Q = 1-n^2.$$

As can be seen from (37), the average energy of a particle depends on the intensity of the wave polarization parameter ψ, on the angle of inclination φ of the ellipse to the axis Ox of the coordinate system, on the initial phase, on the frequency ratio of the parameter n.

The energy $\langle\bar{\varepsilon}\rangle$ of the charged particle, being further averaged over the initial phase Φ_0, in the plane monochromatic arbitrarily polarized wave is given by

$$\langle\bar{\varepsilon}\rangle - mc^2 = \frac{mc^2\mu}{4(n^2-1)^2}\times$$
$$\times\left(\frac{\sqrt{2}J^3}{4\mu K} + \frac{\mu(1-n^2)^2}{2\sqrt{2}JK} - \frac{\sqrt{2}(2n^4-n^2-2)J}{(n^2-1)K} + \right. \qquad (38)$$
$$\left. + \frac{19n^4-6n^2-21}{4(n^2-1)} - \frac{2(n^2-1)^2}{\mu}\right),$$

where

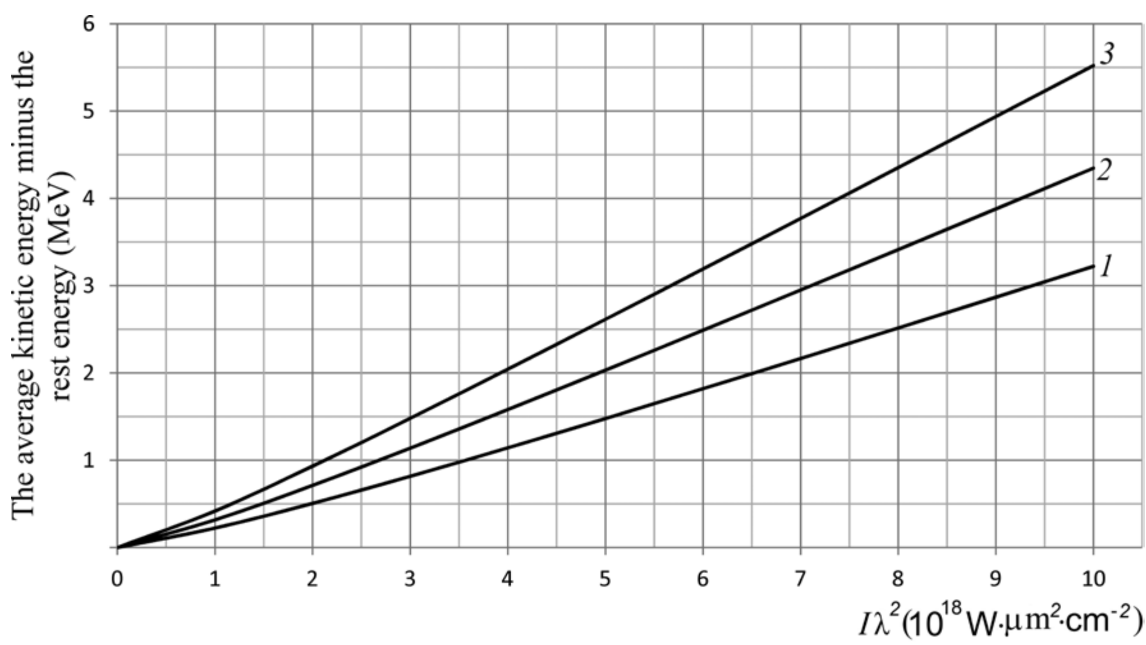

Fig. 2. The dependence of the average kinetic energy of the electron on the intensity of the plane monochromatic electromagnetic waves with elliptical polarization for $\rho = \sqrt{0.9}$, with different values for n: 0 (*1*), 0.2 (*2*), 0.4 (*3*)

$$J = \sqrt{3\mu n^2 + \mu + 4(n^2 - 1)^2};$$

$$K = \sqrt{\mu(n^2 + 1) + 2(n^2 - 1)^2}.$$

Fig. 2 shows the dependence of the average kinetic energy of electrons on the intensity of the monochromatic plane electromagnetic waves with elliptical polarization [10] for $\rho = \sqrt{0.9}$, $n = 0$, $n = 0.2$ and $n = 0.4$. In the absence of the constant magnetic field, for the cases of linear and circular polarization of (38) we obtain the formula for the average energy of a particle characteristics for the case of linear [9, 10] and circular [10] polarizations.

By substituting the values of the parameters

$$n = 0;\ \omega\xi_0 = \pi / 2,\ 3\pi / 2;\ \varphi = \pi / 2,$$

$$3\pi / 2;\ \psi = \pi / 2,\ 3\pi / 2;\ \rho = \pm 1 / \sqrt{2}$$

in (38), we obtain the kinetic energy of the particles for the circular polarization [9].

As can be seen from Fig. 2, the particle energy increases with increasing the values of

$$I\lambda^2 = 10 \times 10^{18} W \cdot \mu m^2 cm^{-2}$$

and of the parameter n. For elliptical polarization with $\rho = \sqrt{0.9}$; and with $n = 0.2$ the energy increases by 1.5 MeV and with $n = 0.4$ it does by 2.2 MeV.

VI. Conclusions

This article offers the exact solution of the equations of a charged particle motion in the external field for elliptically polarized electromagnetic waves and the uniform constant magnetic field. It indicates the dependence of the electron velocity on the intensity of the monochromatic plane wave in a uniform constant magnetic field for the cases of elliptical polarization which are, therefore, the cases of different initial conditions of the charged particle motion and wave polarization.

The values of the momentum and energy of the particle, averaged over the period of vibration, were calculated. It was shown that motion of the particle is the superposition of motion at a constant velocity and vibrational motion with the frequency of the electromagnetic field and the cyclotron frequency different from the field frequency. In the absence of a constant uniform field, all the formulae go to the appropriate formulae given in [10]. The solutions obtained are presented in the explicit dependence on the initial data, the amplitude of the electromagnetic wave, the wave intensity and its polarization parameter that allows everyone to apply the solutions in practice. The results of our investigation will be useful for the interpretation of experiments with plasma placed in a homogeneous constant magnetic field.

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Копытов Г.Ф., Мартынов А.А., Акинцов Н.С. ДВИЖЕНИЕ ЗАРЯЖЕННОЙ ЧАСТИЦЫ В ПОЛЕ ЭЛЕКТРОМАГНИТНОЙ ВОЛНЫ И В ПОСТОЯННОМ МАГНИТНОМ ПОЛЕ.

Решение уравнения движения заряженной частицы в поле электромагнитной волны и постоянном однородном магнитном поле представляет интерес для исследования взаимодействия лазерных импульсов большой интенсивности с твердыми мишенями, в связи с практической разработкой многочастотных лазеров и развитием техники модуляции лазерного излучения. Актуальность проблемы обусловлена широким практическим применением высокотемпературной плазмы, образующейся на поверхности мишени и поисками новых режимов взаимодействия лазер – плазма. Проведен анализ задачи о движении заряженной частицы во внешнем поле плоской, произвольно поляризованной электромагнитной волны большой интенсивности при наличии внешнего однородного постоянного магнитного поля. Получены формулы для средней кинетической энергии релятивистской частицы в зависимости от начальных данных, амплитуды электромагнитной волны, интенсивности волны и ее параметра поляризации. Исследованы различные случаи начальных условий движения заряженной частицы и поляризации волны. Получена зависимость средней кинетической энергии от интенсивности электромагнитной волны в постоянном магнитном поле.

ПЛОСКАЯ ЭЛЕКТРОМАГНИТНАЯ ВОЛНА, СРЕДНЯЯ КИНЕТИЧЕСКАЯ ЭНЕРГИЯ, УЛЬТРАКОРОТКИЙ ЛАЗЕРНЫЙ ИМПУЛЬС, ЗАРЯЖЕННАЯ ЧАСТИЦА, ПОСТОЯННОЕ МАГНИТНОЕ ПОЛЕ.

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