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OPTIMAL CONTROL OVER UNSTABLE MACROECONOMIC SYSTEMS

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ОПТИМАЛЬНОЕ УПРАВЛЕНИЕ НЕУСТОЙЧИВЫМИ МАКРОЭКОНОМИЧЕСКИМИ СИСТЕМАМИ

The article presents a mathematical description of the process of an optimal control over an unstable macroeconomic system based on the Leontief's input-output model. The optimal equation allows setting a balanced growth rate for a macroeconomic system. It is the main problem in the current development of regional and national economies. The methods of an optimal control are generally applicable to stable systems. This article shows that a developing macroeconomic system is unstable and therefore an optimal control over it has its peculiarities. An unstable macrosystem is divided into two subsystems: a stable multidimensional and an unstable one-dimensional. The stable system is optimized via standard methods, where a single growing exponent sets the growth rate of the entire system from the second unstable system. In order to divide the system, the author suggests using a homothetic transformation. To calculate the parameters of an optimal control a Riccati equation is used. The results of solving a matrix of factors determine the cost of restructuring unstable macroeconomic systems with a balanced growth rate. The knowledge of the cost of an optimal control and restructuring creates prerequisites for a more effective process to manage socio-economic politics in the region and the whole country. These results play a vital role in decision-making processes of management and administrative bodies concerning statistical analyses and managing the economic situation. The results are based on the hypothesis that the dynamic models of macroeconomic systems are linear. In practice, actual economic systems are subject to various effects like synergy and self-organization. They cannot be described under the linearity hypothesis. Our future research requires the elaboration upon the problems of an optimal control over nonlinear and unstable economic systems.

MACROECONOMIC SYSTEMS; ECONOMIC GROWTH; MATHEMATICAL MODELING; CONSUMPTION; GROSS OUTPUT; OPTIMAL CONTROL, RESTRUCTURING.

Представлено математическое описание процесса оптимального управления неустойчивой макроэкономической системой на основе модели Леонтьева. Оптимальное управление позволяет перевести макроэкономическую систему на сбалансированные темпы развития, что является основной проблемой развития региональных и страновых экономик. Методы оптимального управления в основном применимы для устойчивых систем. Показано, что развивающаяся макроэкономическая система является неустойчивой и поэтому оптимальное управление в ней имеет особенности. Неустойчивая макросистема делится на две подсистемы: устойчивую многомерную и неустойчивую одномерную. Далее устойчивая система оптимизируется стандартными методами, причем темп роста всей системы контролируется единственной растущей экспонентой от второй неустойчивой системы. Для разделения системы используется преобразование подобия. Расчет параметров оптимального управления основан на решении уравнения Риккати. Полученные в результате решения матрицы коэффициентов определяют стоимость реструктуризации неустойчивых макроэкономических систем со сбалансированным темпом роста. Знание стоимостных данных затрат на оптимальное управление и реструктуризацию создает предпосылки для более эффективного ведения процесса управления социально-экономической политикой внутри региона и страны в целом. Область применения результатов распространяется на управленческие административные органы, принимающие решения по вопросам статистического анализа и управления макроэкономической ситуацией. В основе полученных результатов лежит гипотеза о том, что динамические модели макроэкономических систем являются линейными. На практике в реальных экономических системах наблюдаются различные эффекты, например, синергия и самоорганизация, которые невозможно описать в рамках линейных предположений. Раскрытие вопросов оптимального управления в нелинейных и неустойчивых экономических системах является продолжением данных исследований.

МАКРОЭКОНОМИЧЕСКИЕ СИСТЕМЫ; ЭКОНОМИЧЕСКИЙ РОСТ; МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ; ПОТРЕБЛЕНИЕ; ВАЛОВОЙ ВЫПУСК; ОПТИМАЛЬНОЕ УПРАВЛЕНИЕ, РЕСТРУКТУРИЗАЦИЯ.

Problem statement. To ensure an optimal control over macroeconomic systems is a serious problem. Its solution will allow the radical restructuring [1, 2] of the economy at a macro- and meso-level and at minimal cost. This restructuring is essential to achieve a balanced growth of gross output, GDP and other macroeconomic indicators [3–6].

It is widely known that an optimal control can be achieved in stable systems. That is why the economic community frequently raises a question about the stability of certain economic systems, e. g. a firm or a country. There is a variety of methods to achieve a stable growth [7]. However, there is a contradiction in the definition of a «stable growth» itself.

On the one hand, if a system is growing, then its parameters increase, i. e. grow. It is desirable for economic systems to be constantly growing. However, on the other hand, systems with an indefinite increase in any parameter are unstable. Hence a macroeconomic system with constantly growing parameters is unstable as well. To achieve a stable growth in an unstable system is rather difficult but possible. This article presents an approach to solve this problem.

The process of restructuring the macroeconomic system is based on an optimal control. It should follow a certain plan in order to set a balanced growth rate of the system while maintaining a certain proportion of material [8], capital, labour and other costs. Any system operates to achieve some sort of goal. That is why for a further discussion we should introduce a concept of an ideal macroeconomic system where all cost proportions are balanced. Let us call a system with balanced development trajectories an ideal model. This article presents the method of forming ideal trajectories, to which every macroeconomic system should aspire in order to achieve desired growth rates and proportions. It is necessary to create an optimal criterion for control signifying an actual optimal control over an economic system. Both ideal and growing systems are unstable.

Problem solution. Regarding theoretical grounds of the article, we should mention that modern theories on economic growth are based on two sources: the neoclassical theory conceived by J.B. Say and fully formed in the works of J.B. Clark (1847-1938) and the Keynesian theory of macroeconomic equilibrium [9].

We will point out the Neumann model and the Leontief's model in dynamic contrast among all the models of economic dynamics. Most of them are able to demonstrate the transient processes and control over them, the structural shifts and statistical stability more complete [10, 11]. One of the most useful properties of these models is their ability to be presented in a form of differential equations that describe the dynamic economic systems.

Balanced trajectories with the maximum growth rate are called turnpikes. The term was proposed by a Nobel Prize winner, Paul Samuelson. John von Neumann created the first turnpike model was in the 1930s. His model of an expanding economy had a deep impact on the making of mathematical economics [12]. The theoretical principles of the turnpike were summarized in the Gale model. The Leontief model is a special case of it, as it is shown in [13].

A dynamic variation of the Leontief model [14] is a system of inhomogeneous linear differential equations:

$$X(t) = AX(t) + B\dot{X}(t) + Y(t).$$

$$\text{or } \dot{X}(t) = B^{-1}(E - A)X(t) - B^{-1}Y(t). \qquad (1)$$

The formal solution to the system (1) has two parts – a free $X_{св}(t)$ and a forced $X_{вын}(t)$:

$$X(t) - X_{св}(t) + X_{вын}(t) \qquad (2)$$

$$\text{or } X(t) = \mathrm{e}^{B^{-1}(E-A)t}X(0) - - e^{B^{-1}(E-A)t}\int_0^t e^{-B^{-1}(E-A)\tau}B^{-1}Y(\tau)d\tau. \qquad (3)$$

where $e^{B^{-1}(E-A)t}$ is a matrix exponent.

The equation (3) is greatly simplified, if you assume that there is a connection between the end product and the gross output by introducing a norms of consumption matrix Q:

$$Y(t) = QX(t). \qquad (4)$$

This assumption can be considered valid because, the gross output for consumption will be constant for rather large intervals of time. The simplified system will have a consumption loop and will look like this:

$$\dot{X}(t) = GX(t). \qquad (5)$$

The matrix $G = B^{-1}(E - A - Q)$ is a homogenous matrix. The solution to this matrix will no longer be so complex. In fact, it will be quite compact:

$$X(t) = e^{Gt}X(0), \qquad (6)$$

where $X(0)$ are the starting values of the system representing the level of gross output for the current year.

Using a classic method of calculating transient processes, we get a solution to (5) that looks like this:

$$X(t) = C_1e^{\lambda_1 t} + C_2e^{\lambda_2 t} + ... + C_n\, e^{\lambda_n t}, \qquad (7)$$

where C_1, C_2, ..., C_n are integration constants; λ_n are eigenvalues of matrix G, that define the unique dynamic properties (UDP) of a socio-economic system [15].

In accordance with the system of national accounts, production records in Russia is kept for 17 types of economic activities. In order to predict the growth of gross output we need to solve a system of differential equations with a degree of 17. The best way to solve such a multi-dimensional problem is to do it through a matrix using a homothetic transformation. In this case, we can present our model as a state space model:

$$\dot{X}(t) = \bar{A}X(t) + \bar{B}Y(t), \quad (8)$$

where $\bar{A} = B^{-1}(E - A)$ is the main matrix, and $\bar{B} = -B^{-1}$ is a matrix of external influences.

Solving the system of differential equations (8) will allow us to determine the expected values of the gross output of a country or its regions. Obviously, disregarding the effects of external influences from the government and an ineffective production will make the resulting values unbalanced. It points out an important issue to balance the main macroeconomic factors for all types of economic activities. Thus, we need to establish such level of socio-economic consumption that would let the system stay in a constant and balanced expansion. Classical economists call this his expansion a turnpike development or Neumann ray [16]. This problem is solved by using Pontryagin's maximum principle from his optimal control theory.

The problem for decision makers is that they need to know not only the expected gross output values but also the optimal level of social consumption that takes into account all socio-economic capacity. The problem boils down to defining matrix Z that connects the end product Y with the gross output:

$$Y(t) = ZX(t). \quad (9)$$

The statement (9) lets us present the model with a consumption loop like this:

$$\dot{X}(t) = (\bar{A} + \bar{B}Z)X(t). \quad (10)$$

The information about an optimal control over the system consists in matrix $\bar{B}Z$. This is the value at which we have to change the coefficients of matrix $\bar{A}$ to achieve the balanced function of the macrosystem as a result of an optimal control.

Now we have the system with positive feedback. Systems with positive feedback are unstable. Methods for determining matrix Z, which contains the information about socio-economic norms and costs, are developed for stable systems. Now the problem of separating the generally unstable system (10) into subsystems arises. One of which would be stable and multidimensional and the other would be unstable and one-dimensional. Such division can be achieved by using a homothetic transformation that would outline n of new phase variables $\tilde{X}_h$ by using:

$$X_i = \sum_{h=1}^{n} t_{ih}\tilde{X}_h \quad \text{or} \quad X = T\tilde{X}. \quad (11)$$

As a result, system

$$\left.\begin{array}{ll} \dot{\tilde{X}}(t) = \tilde{G}\tilde{X}(t), & \tilde{X}(0) = \tilde{X}_0 \\ \text{where } \tilde{G} \equiv T^{-1}GT, & \tilde{X}_0 \equiv T^{-1}X_0 \end{array}\right\} \quad (12)$$

will contain matrix $\tilde{G}$, the structure of which is far simpler than the initial one. If there is a possibility to use a homothetic transformation (11) to transform matrix G into a diagonal matrix, then the initial system can be transformed to a system with separated variables by using coefficients $\tilde{X}_h$:

$$\frac{d\tilde{X}_h}{dt} = l_h\tilde{X}_h. \quad (13)$$

The solution to such a system will look like this:

$$\tilde{X}_h = \tilde{X}_{h0}e^{l_h t} \quad (h = 1, 2, ..., n). \quad (14)$$

The final solution to the system using the homothetic transformation method will contain a diagonal matrix $diag(e^{\lambda t})$:

$$X(t) = T \cdot diag(e^{\lambda t})T^{-1}X(0), \quad (15)$$

where λ and T are eigenvalues and eigenvector of matrix G.

Using a homothetic transformation lets us transform the matrix into a diagonal one where it can be divided into subsystems. These systems can be connected parallel. The body of mathematics for parallel system connection has been developed in control engineering and it is widely known. A homothetic transformation is applicable not only to closed-loop systems but also to open-loop ones. In this case, we need to do the following action on the transformed (converted) matrix:

$$\tilde{A} = T^{-1}\bar{A}T, \quad \tilde{B} = T^{-1}\bar{B}. \quad (16)$$

The dynamic properties of the converted system and that of the initial system are identical, because they have the same spectrum of eigenvalues. The main matrix of the converted system is diagonal. Thus it can be divided into parallel subsystems. In order to do that we use the Perron—Frobenius theorem. It states that in a model of a macroeconomic balance system, among positive eigenvalues there will surely be a minimal number, which would correspond to the entire positive eigenvector. To find the subsystem with the lowest eigenvalue is not a difficult task. It will be one-dimensional and the presence of a positive number in the index of an exponent will signify a constant growth, which in its turn would make it one of the unstable systems. The other subsystem will be stable and it is possible to synthesize an optimal control for it.

Le us present the converted system in the following form:

$$\begin{pmatrix} \dot{\tilde{X}}1 \\ \dot{\tilde{X}}2 \end{pmatrix} = \begin{pmatrix} \tilde{A}1 & \tilde{A}2 \\ \tilde{A}3 & \tilde{A}4 \end{pmatrix} \begin{pmatrix} \tilde{X}1 \\ \tilde{X}2 \end{pmatrix} + \begin{pmatrix} \tilde{B}1 & \tilde{B}2 \\ \tilde{B}3 & \tilde{B}4 \end{pmatrix} \begin{pmatrix} \tilde{Y}1 \\ \tilde{Y}2 \end{pmatrix}; \quad (17)$$

$$\tilde{X} = \begin{pmatrix} \tilde{X}1 \\ \tilde{X}2 \end{pmatrix}, \quad \tilde{A} = \begin{pmatrix} \tilde{A}1 & \tilde{A}2 \\ \tilde{A}3 & \tilde{A}4 \end{pmatrix}, \quad \tilde{B} = \begin{pmatrix} \tilde{B}1 & \tilde{B}2 \\ \tilde{B}3 & \tilde{B}4 \end{pmatrix}.$$

This would let us divide the matrices and vectors of the initial system into subparts by these dimensions:

$$\tilde{X}1[1], \tilde{X}2[n-1], \tilde{A}1[1], \tilde{A}2[1, n-1],$$

$$\tilde{A}3[n-1, 1], \ \tilde{A}4[n-1, n-1].$$

The dimensions of the submatrices in the matrix $\tilde{A}$ and $\tilde{B}$ are identical. The matrix of the converted system is diagonal. It means that the coefficients of submatrices $\tilde{A}2$ and $\tilde{A}3$ contain zeros that would let us present system (17) as a parallel connection of two subsystems:

$$\dot{\tilde{X}}1(t) = \tilde{A}1\tilde{X}1(t) + \tilde{B}2\tilde{Y}1(t); \quad (18)$$

$$\dot{\tilde{X}}2(t) = \tilde{A}4\tilde{X}2(t) + \tilde{B}4\tilde{Y}2(t). \quad (19)$$

Fig. 1 shows this connection graphically.

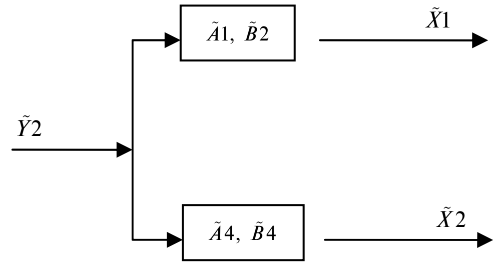

Fig. 1. Parallel connection of two subsystems

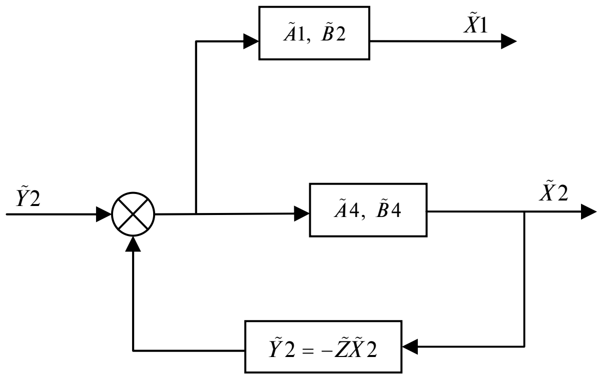

Fig. 2. Connecting subsystems with feedback

The entrance $\tilde{Y}2$ has an effect on both subsystems. It can be optimized by the optimal synthesis of the linear-quadratic regulator. Based on the structure of the system (17) the same entrance will influence an unstable system. Of course, this effect will be suboptimal. However, as a whole the system will perform more effectively because one of its subsystems would be optimized. Fig. 2 shows this situation graphically. The second system is controlled by the feedback from the linear-quadratic regulator. Therefore it can be considered optimal.

In order to determine $\tilde{Z}$ in the chain of a negative feedback $\tilde{Y}2 = -\tilde{Z}\tilde{X}2$ we need to minimize the square functional:

$$J(X) = \int_0^\infty (\tilde{X}2^T Q \tilde{X}2 + \tilde{Y}2^T R \tilde{Y}2) dt. \quad (20)$$

Here Q and R are matrices of the weight coefficient. These matrices set the ratio of the quality of the economic process management to the cost of management.

The functional (20) let us optimize the management in the system while spending minimum of effort to manage the dynamics of exit $\tilde{X}2$ by means of entrance $\tilde{Y}2$. To solve the minimization problem of (20) we will use the classic method of the calculus of variations. To do that let us introduce an auxiliary functional:

$$J(X) = \int_0^{\infty} [(\tilde{X}2^T R\tilde{X}2 + \tilde{Y}2^T Q\tilde{Y}2) - - 2\lambda^T (\dot{\tilde{X}}2 - \tilde{A}4\tilde{X}2 - \tilde{B}4\tilde{Y}2)]dt, \quad (21)$$

where λ – (n-1) is a dimensional vector of Lagrange multipliers.

The solution of the minimization problem (21) for subsystem (19) yields the following system:

$$\begin{cases} \dot{\tilde{X}}2 = \tilde{A}4\tilde{X}2 + \tilde{B}4\tilde{Y}2; \\ \dot{\lambda} = -Q\tilde{X}2 - \tilde{A}4^T \lambda; \\ \tilde{Y}2 = -R^{-1}\tilde{B}4^T \lambda. \end{cases} \quad (22)$$

By substituting value $\tilde{Y}2$ into the first equation of system (22), we get:

$$\begin{cases} \dot{\tilde{X}}2 = \tilde{A}4\tilde{X}2 - \tilde{B}4R^{-1}\tilde{B}4^T \lambda; \\ \dot{\lambda} = -Q\tilde{X}2 - \tilde{A}4^T \lambda. \end{cases} \quad (23)$$

In order to solve this system we need to substitute the corresponding variables:

$$\lambda = P\tilde{Y}2. \quad (24)$$

Multiplying the left part of the first equation in system (23) by matrix P and subtracting from it the second equation of the system will lead us to:

$$P\tilde{A}4 + \tilde{A}4^T P - P\tilde{B}4R^{-1}\tilde{B}4^T P + Q = 0. \quad (25)$$

The equation (25) is the Riccati algebraic matrix equation [17], which comes as a result of the Riccati differential equation being set in conditions of $t \to \infty$. To solve this equation is a difficult task. However, it is standardized and it has solutions in some cases. It allows us to determine the coefficients of matrix P. Having substituted the statement (24) into the last equation of system (23), we get the desired equation of optimal control:

$$\tilde{Y}2 = -R^{-1}(\tilde{B}4)^T P\tilde{X}2 = -\tilde{Z}\tilde{X}2, \quad \tilde{Z} = R^{-1}(\tilde{B}4)^T P. \quad (26)$$

The closed-loop matrix of the second subsystem with the linear-quadratic regulator $\tilde{Z}$ will be determined by the formula:

$$\tilde{G}4 = \tilde{A}4 - \tilde{B}4 \cdot \tilde{Z}. \quad (27)$$

Then the converted (and already optimized) system (17) will look like this:

$$\begin{pmatrix} \dot{\tilde{X}}1 \\ \dot{\tilde{X}}2 \end{pmatrix} = \begin{pmatrix} \tilde{A}1 & \tilde{A}2 \\ \tilde{A}3 & \tilde{G}4 \end{pmatrix} \begin{pmatrix} \tilde{X}1 \\ \tilde{X}2 \end{pmatrix}. \quad (28)$$

Or in its condensed form:

$$\dot{\tilde{X}}(t) = \tilde{A}onm\tilde{X}(t), \quad (29)$$

where $\tilde{A}onm$ is the matrix of optimized closed-loop converted system coefficients.

Results. Determining the close-loop matrix of the macrosystem's coefficients is achieved by means of an inverse homothetic transformation:

$$\bar{A}onm = T\tilde{A}onmT^{-1}. \quad (30)$$

This matrix is necessary to calculate the addition to the coefficients of that first unstable system. So we can get the optimal equation:

$$\bar{B}Z = \bar{A} - \bar{A}onm. \quad (31)$$

The equation (12) can help evaluate the optimal level of the end product accounting for the costs from socio-economic transformations of the macrosystem.

Conclusion. As we can see, dividing an unstable macroeconomic system into subsystems makes it possible to determine the optimal level of expenses for the system. It creates prerequisites for a more effective management of socio-economic policies inside a region or an entire country.

Directions for future research. The results are based on the hypothesis that the dynamic models of macroeconomic systems are linear. In practice, actual economic systems are subject to various effects like synergy and self-organization. [18, 19] They cannot be described under the linearity hypothesis. Our future research requires the elaboration upon the problems of an optimal control over nonlinear and unstable economic systems.

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