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AN ITERATIVE PROCEDURE OF CONSUMER RESEARCH IN A NARROW MARKET SEGMENT

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ОБ ОДНОЙ ИТЕРАЦИОННОЙ ПРОЦЕДУРЕ ИССЛЕДОВАНИЯ ПОТРЕБИТЕЛЬСКОГО СПРОСА НА УЗКОМ РЫНОЧНОМ СЕГМЕНТЕ

The article analyzes the conditions, peculiarities and application techniques of the step-by-step method of iteration to the equations simple regression which are not reduced to any linear forms of their own parameters and variables. Such a practical task arises often enough during the investigation of the class of Tornquist functions. In practice, such tasks arise in economics during research and analysis of consumer demand on a narrow market segment. In particular, the 2nd equation of this class successfully describes a Russian family's cost for the purchase of relatively expensive tourist products for foreign travels. The method of progressive approximation regards the plotting of the normal equations set with application of the Jordan–Gauß process and the expansion into Taylor's infinite series of the function of the corrections for parameters of the desired equation. The amendments themselves are allowing to calculate the parameters of the Tornquist functions (2nd equation) and accomplish the forecast estimations of budget charges for purchase of the expensive tourist's products. The considerations expressed in this article lead to three local outputs on the applicability of the procedure in question in dealing with similar ones on the content of economic and statistical research problems of supply and demand a high level of tourist products: in the epistemological and methodological aspects of the formulated problem of estimating the parameters in a typical nonlinear this technique (sequential iterative procedure) does not give a final decision, i. e. the results are not complete and not statistically pure; because of the inherent rounding even reliable and accurate measures of economic parameters are approximate. When using iterative procedures, approximation error is added to the actual implementation of the method, and its effectiveness depends on the more or less successful choice of initial conditions approximation and the convergence rate of the iteration process; finally, a common approach to stochastic approximation of a nonlinear function is understood by the author through the use of Taylor series and polynomials search for suitable forms. In particular, the linearization of the equations of simple regression with typical nonlinearity must be carried out by the power series expansion with all members of the first order of smallness eliminated from further calculations.

APPROXIMATION APPROACH; LINEAR FORM; REGRESSION EQUATION; PROBABILISTIC DISTRIBUTION; TOURIST'S PRODUCT; SEGMENT OF THE MARKET; CONSUMER DEMAND.

Статья посвящена анализу условий, особенностям и техникам применения метода последовательного приближения к уравнениям простой регрессии, которые не сводятся к каким-либо линейным формам в отношении собственных параметров и переменных. Такая практическая задача возникает достаточно часто при исследовании класса функций Торнквиста. Как показывает практика, такие задачи возникают в экономике при исследовании и анализе потребительского спроса на узком рыночном сегменте. В частности, второе уравнение этого класса удачно описывает расходы российских семей на приобретение сравнительно дорогих туристских продуктов для зарубежных путешествий. Итерационная процедура предусматривает построение системы нормальных уравнений с использованием способа Жордана–Гаусса и разложение в бесконечный ряд Тейлора функции поправок к параметрам искомого уравнения. Сами поправки позволяют рассчитать параметры функции Торнквиста (второе уравнение) и сделать по ним прогнозные оценки бюджетных расходов семей на дорогие турпродукты. Представленные результаты в части применимости рассматриваемой процедуры для исследования спроса и предложения на примере дорогих туристических продуктов позволяют сделать ряд выводов: для сформулированной проблемы методологических аспектов оценки параметров в типичном нелинейном формате последовательная повторяющаяся процедура не дает окончательного решения; из-за округления даже при использовании надежных исходных данных получаемые оценки экономических параметров приблизительны. Когда использование повторяющейся ошибки приближения процедур добавлено к фактической реализации метода, эффективность зависит от успешного выбора начального приближения условий и темпа сходимости итеративного процесса; общий подход к стохастическому приближению нелинейной функции использован автором с помощью ряда Тейлора и поиска полиномов подходящих форм.

ПОСЛЕДОВАТЕЛЬНОЕ ПРИБЛИЖЕНИЕ; ЛИНЕЙНАЯ ФОРМА; УРАВНЕНИЕ РЕГРЕССИИ; ВЕРОЯТНОСТНОЕ РАСПРЕДЕЛЕНИЕ; СИСТЕМА НОРМАЛЬНЫХ УРАВНЕНИЙ; СЕГМЕНТ РЫНКА; ПОТРЕБИТЕЛЬСКИЙ СПРОС.

Introduction. A sample produced according to the rules of statistical observation of the population of socio-economic nature is not, ultimately, necessarily representative. Even after the homogenization procedure, sample homogeneity of private units together may be questionable. For the same correlation values, the essential features of these sampling units can have a quasi-random character. If, moreover, even regression, based on such dubious information database designed to describe and identify patterns (pre-theoretical basis), expressed by a nonlinear equation can not be reduced to a linear form, then, an analyst tends to group separate issues to be detailed solutions. Some of the problems identified are addressed in this article.

The previously widespread orthodox (in the best semantic sense of the term, i. e., consistently and rigorously adhering to any existing views and eminent statisticians) viewpoint that the units (elements) of public and private statistical universe must have «... really good, uniform type of a general Bernoulli population, with which we have to deal in a particular activity ...» [1.20; 9], has recently been drawing increased serious but not always justified criticism.

For example, such critical constructions assumed that modern statistical science (!? — *A.B.&A.Ts.*) comes from the fact that the specifics of a complex set of features is not exhausted by its homogeneous parts, and is mainly in the structure of the population, and especially the nature of links and the relationship between the parts [2.17—26]. These authors suggest that a systematic approach is needed for a comprehensive study of complex aggregates, including both homogeneous and heterogeneous parts, from the standpoint of system analysis — the same approach that has received a powerful development in various fields of science, including the social and the humanitarian sectors. Heterogeneity of the study population in this case cannot be taken into account and it is not an obstacle to the application of the mathematical apparatus.

Review. However, the solution covers a sufficiently wide range of problems of social and economic nature relating to the statistical analysis of two-dimensional distributions with fixed because the phenomenon is often reduced to the construction of the equations of simple regression, non-linear with respect to its parameters and variables.

Suppose there is a regression equation resultant in the variable of the type

$$y_j^{(i)} = f(\vec{a}_k, x_i) + \varepsilon_i = f(a_o, a_1, ..., a_m, x_i) + \varepsilon_i. \quad (1)$$

In the formula of (1) f — is generally a nonlinear function of the simple regression from one unaccounted causal factor x_i n observations on aggregate, $i = \overline{1, n}$, with a vector $\vec{a}_k$ of unknown parameters $k = \overline{0, m}$; with ε_i, or $\vec{\varepsilon}$ — the random component, it is also a random component, or the residual value, or \residual vector\ and other synonymous concepts found in the statistical literature and describing the discrepancy between empirical and theoretical levels, as well as meeting the following mandatory requirements of the general theory of statistics:

$$\text{а) } E[\varepsilon_i] = 0; \text{ б) } \operatorname{cov}(\varepsilon_i, \varepsilon_i') = 0; \ - \exists,$$

$$\text{в) } \mathrm{M}\{\varepsilon_i\} \sim N(0, \sigma_\varepsilon^2), \quad (2)$$

i. e. the vanishing of the expectation (E) of the random component ε_i, the lack of statistical association (covariance — *cov*) and normal (N) mathematical distribution set (**M**) of random deviations $\{\varepsilon_i\}$. The sign «~» indicates the fact that the random variable under study is a probabilistic distribution.

Of course, conducting a regression analysis of any dimension is preceded by the presence of statistical association between productive and formative (or forming) features, such as estimates of the relationship between the cost of guided tours of a family and its income. Determination of the probability of the fact that the statistical relationship between the studied traits factor is random is called the *test of statistical significance.* In the course of this evaluation procedure, usually in the fully functional analytical system package *SPSS*-19.00 *Statistics for Windows*, the value of the selected statistical test is set, along with the number of degrees of freedom, *p*-rate statistics et al.

If the probability measure, i. e. the *p*-criterion, is below table level of 0.05, the relationship variables are recognized as statistically significant; if it is above, then such a connection cannot be considered significant. Naturally, the statistical significance of a possible link between variables depends on the sample size. It follows from a sample from a population of not less than the volume of a large sample, i. e., $n \geq 101$ [3.254].

Estimating the parameters of a simple regression equation (2) can be difficult, which is mainly related to the possibility (or rather impossibility) to linearize the function f with respect to its parameters and variable x_i with the known methods for subsequent evaluation using the *method of least squares* (*OLS*) which is a special case of the method of *maximum likelihood* (MLH developed by K. Gauss, P-C. Laplace and R. Fischer). Therefore, for such cases there is a separate procedure, since the OLS estimation of the parameters of the nonlinear form is unaccesptable, as all kinds of *OLS* estimates are determined based on the constructed system of normal equations.

It is known that the parameters of the regression equations, which are measured by the method of least squares, must be contained in the linear forms with respect to the independent variables and the resultant variable. However, not for all of the equations, both simple and multiple regressions can be kept to a desired linear form. By a similar equation the 2nd equation Tornquist[20] is applied.

Suppose we have information according to budget and sociological surveys of household expenditure on the *j*-th goods and services of a certain quality for the year y_j depending on the per capita income of the *i*-th family – x_i. This initial information grouped by income groups successfully *approximated* the 2nd Tornquist equation belonging to the group of remarkable nonlinear equations.

Approximation (*Approximation approach, method of approximation*) is an independent task belonging to a class of problems that are studied branches of mathematics – approximation theory and numerical analysis techniques. To calculate the values of complex functions, the values of the interval variation series are often calculated by an approximating function. Thus, the described approximation refers to a table or the mathematical function dependence.

The initial data for a particular purpose is usually a table of observations – a set of values of the independent variables and the corresponding values of the response function. The number of rows (nodes) of a tabulated function $\{n\}$ is called the *volume* of available sample of the target population N, $n \in N$, that coincides in meaning with regulated procedures that predicate development of statistical tables.

In processing the experimental or field sampling data two typical cases are usually considered. In the first case, the approximating function is limited to a range of given points and only serves as an interpolating function. In the second the approximating function acts as economic and statistical regularities, such as functions or equations of *Tornquist* (*Tornkuist functions*), and this function is an extrapolation of effective forecasting of variable elements. Here is full-scale data serving as reference points for identifying patterns of change, say, $y = f(x_i)$ with known boundary conditions and the resultant variable y and the independent variable is formed on the basis of any significant signs – x_i.

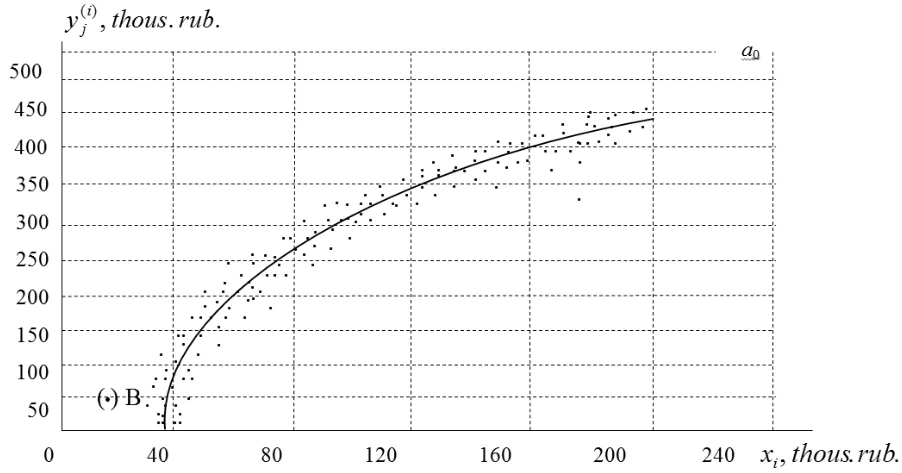
The shape of the equation is selected by the researcher in accordance with the behavior of the approximated function in the relevant range of the independent variables. The result of solving the problem of approximation are the parameter estimates (coefficients) of this equation. Obviously, the coefficients of the equation should be selected so that the values calculated by the equation of the response function are as close as the observations in the original table processed or measured according to the required specifications.

The second Tornquist equation as a function of demand for goods necessary second (less urgent) in relation to the empirical level of demand for the *j*-th good is as follows:

$$y_j^{(i)} = y_j(x_i) + \varepsilon_i = \frac{a_0(x_i - a_2)}{x_i + a_1} + \varepsilon_i, \quad a_0, a_1, a_2 > 0. \tag{3}$$

Configuration equations and laws describing them as acceptable to describe the formation of demand for individual *j*-th household services repair and restoration, relatively expensive tourist products and consumer goods of the second row (less urgent), high-quality food products (including premium) and so forth. The dependence has the following appearance, shown in Fig. 1.

[1] The group of equations is named for L. Tornquist, a famous Scandinavian econometrics expert in the field of statistics of supply and demand [10].



at (•) B: $y_j(x_i) = 0$, which is numerically equal to $x_i^{(0)} = a_2$

Fig. 1. The general behavior of the 2nd Tornquist equation, the demand for the *j*-th type of tourism products, depending on family income of the *i*-th income group in the field of empirical data

This feature has its limit a_0, but a higher level than in the first Tornquist equation; while the demand for this group of products occurs only after income of the consumer a_2 that is cut off on the x-axis reaches the corresponding income segment $x_i^{(0)} = a_2$. Graph of the same function is a concave curve in Fig. 1, i. e. a concave function for each arc of the curve is not lower than its chord chart converted concave downward (convex upward).

The equation of the expression (1) of auxiliary transformations, such as logarithms, the change of variables and substitution, to linear form is not given. Therefore, it is difficult to estimate the so-called 'true' values of the parameters $\hat{a}_0, \hat{a}_1, \hat{a}_2$ of this equation. Applying the same procedure, the *OLS*, with a single step to form a statistical estimation

$$y(x_i)x_i = -a_0a_2 - a_1y(x_i) + a_0x_i, \qquad (4)$$

supposedly linear with respect to parameters and variables of the equation, as does N.M. Světlov [4], is not correct, since the form of the expression (4 in the original source of the original recording with an asterisk*) cannot be divided on the dependent and independent variables on the right and the left side of the form.

But the value of the 2nd Tornquist equation for analysis and prediction is high, as the parameters of bread need to be known, and in such cases it may be recommended to use the method of successive approximation of the iterative type of task stochastic approximation, which is carried out in several stages (steps).

The initial (approximate) values of the parameters of the equation of the expression (3) for the three pairs of values from the Tab. 1 can be estimated in any way. These initial values generally do not coincide with the true and can be found by solving the system of nonlinear equations consisting of Tornquist functions and with a unique root.

$$\begin{cases} y_j^{(1)} = \dfrac{a_0^{(0)}(x_1 - a_2^{(0)})}{x_1 + a_1^{(0)}}; \\ y_j^{(2)} = \dfrac{a_0^{(0)}(x_2 - a_2^{(0)})}{x_2 + a_1^{(0)}}; \\ y_j^{(3)} = \dfrac{a_0^{(0)}(x_3 - a_2^{(0)})}{x_3 + a_1^{(0)}}, \end{cases} \qquad (5)$$

where $y_j^{(1)}, y_j^{(2)}, y_j^{(3)}, x_1, x_2, x_3$ are the empirical values of sample data, for example, three pairs of data from Tab. 1 of revenue and expenditure groups of families taken from each third of the span variation on the grounds x_i.

1. The first stage involves the solution of a system of equations of the expression (5), the exact location of its roots with respect to the initial values of the parameters $a_0^{(0)}, a_1^{(0)}, a_2^{(0)}$ is carried out by the Gauss-Jordan method. The approximate values are found this way, as a set of numbers-roots $a_0 = a_0^{(0)}, a_1 = a_1^{(0)}, a_2 = a_2^{(0)}$, and they differ from the true values of the parameters $\hat{a}_0, \hat{a}_1, \hat{a}_2$ by the value of the relevant correction α, β, γ[3].

Table 1

Background information for the approximate estimation of parameters of the 2nd Tornquist equation by the incremental iteration method

Number family group p/p	Average family income per month for the group, thous. rub.	Average costs for *j*-th tourist product*, thous. rub.
$i := \overline{1, n}$	$\overline{x}_i$	$\overline{y}_j^{(i)}$
1	**2**	**3**
1	45.0	100.0
2	80.0	300.0
3	120.0	400.0
4	160.0	450.0
5	200.0	490.0

* Calculated by the author for the relatively expensive tours to Mexico, South America and South Africa, based on a panel survey of consumers' budgets in St. Petersburg in 2012—2013. Grouped on a *large sample* volume of 142 units (households), as shown in Fig. 1, in five income groups from Tab. 1[2].

2. The second stage involves a statistical evaluation of these amendments, which allows to set the so-called true parameters

$$\hat{a}_0 = a_0^{(0)} + \alpha,\ \hat{a}_1 = a_1^{(0)} + \beta,\ \hat{a}_2 = a_2^{(0)} + \gamma. \quad (6)$$

To this end, let us substitute a separate 2nd Tornquist equation from the expression (3), as a function with yet unknown true parameters $\hat{a}_0, \hat{a}_1, \hat{a}_2$, taking into account the amendments of the expression (6)

$$y_j(x_i) = f(\hat{a}_0, \hat{a}_1, \hat{a}_2, x_i) = = f(a_0^{(0)} + \alpha,\ a_1^{(0)} + \beta,\ a_2^{(0)} + \gamma,\ x_i). \quad (7)$$

Let us expand the function of the expression (7), regarding the arguments $a_0^{(0)}, a_1^{(0)}, a_2^{(0)}$ and corrections α, β, γ as increments in the vicinity of the vector $\vec{a}_k^{(0)} = (a_0^{(0)}, a_1^{(0)}, a_2^{(0)})$ as a point estimate and the coordinates of the point A in Euclidean space E^n; $A \in E^n$, in an infinite Taylor series. As is known, the Taylor series is an expansion of a given analytic function in an infinite amount of power characteristics with respect to a nodal point with coordinates at point A of a three-dimensional space. The function is infinitely differentiable around (•) A, when $a \in [a_0^{(0)}, \hat{a}_0]$; $\beta \in [a_1^{(0)}, \hat{a}_1]$; $\gamma \in [a_2^{(0)}, \hat{a}_2]$.

The specified number is converging on the surface $\hat{a}_0, \hat{a}_1, \hat{a}_2$, as shown in Fig. 2, where the first positive octant with the surface on the positive corrections also designated as α, β, γ is depicted schematically for ease of interpretation, and it looks like a continuation of the power series expansion formula of the expression (7) for the partial derivatives of the initial parameters $a_0^{(0)}, a_1^{(0)}, a_2^{(0)}$

$$y_j(x_i) = f(a_0^{(0)}, a_1^{(0)}, a_2^{(0)}, x_i) + + \alpha \frac{\partial f(x_i)}{\partial a_0^{(0)}} + \beta \frac{\partial f(x_i)}{\partial a_1^{(0)}} + \gamma \frac{\partial f(x_i)}{\partial a_2^{(0)}} + R_s(\alpha, \beta, \gamma), \quad (8)$$

where $R_s(\alpha, \beta, \gamma)$ is the remaining term of the expansion of the original series.

Component $R_s(\alpha, \beta, \gamma)$ is a part (residual term of the expansion-type form of the *Lagrange or Peano*) consisting of partial derivatives of higher order than the first, in combination with the corresponding infinitesimal quantities (*ISQ*) α, β, γ of the second order and above, for example

$$\alpha\beta \frac{\partial^2 f(x_i)}{\partial a_0^{(0)} \partial a_1^{(0)}};\ \beta\gamma \frac{\partial^2 f(x_i)}{\partial a_1^{(0)} \partial a_2^{(0)}};\ \alpha\gamma \frac{\partial^2 f(x_i)}{\partial a_0^{(0)} \partial a_2^{(0)}}; \alpha^2 \frac{\partial^2 f(x_i)}{(\partial a_0^{(0)})^2};\ \beta^2 \frac{\partial^2 f(x_i)}{(\partial a_1^{(0)})^2};\ \gamma^2 \frac{\partial^2 f(x_i)}{(\partial a_2^{(0)})^2}. \quad (9)$$

All members of the decomposition, shown in equation (9), are infinitely small quantities of higher order of smallness, which can be neglected according to the theorem about the properties of infinitesimals (the algebraic sum of a finite number of infinitesimal is an infinitesimal). Therefore, members should be limited to no more than the expansion of the second order, $\because R_s(\alpha, \beta, \gamma) \to 0$, with the volume of observations $n \to \infty$.

[2] Since the distribution of family members per capita income is not significant, the average potential error sample average μ_y cost indicators for typological sample by the method of mechanical selection calculated by the formula $\mu_y = \sqrt{\frac{\sigma_y^2}{n}\left(\frac{N-n}{N-1}\right)}$.

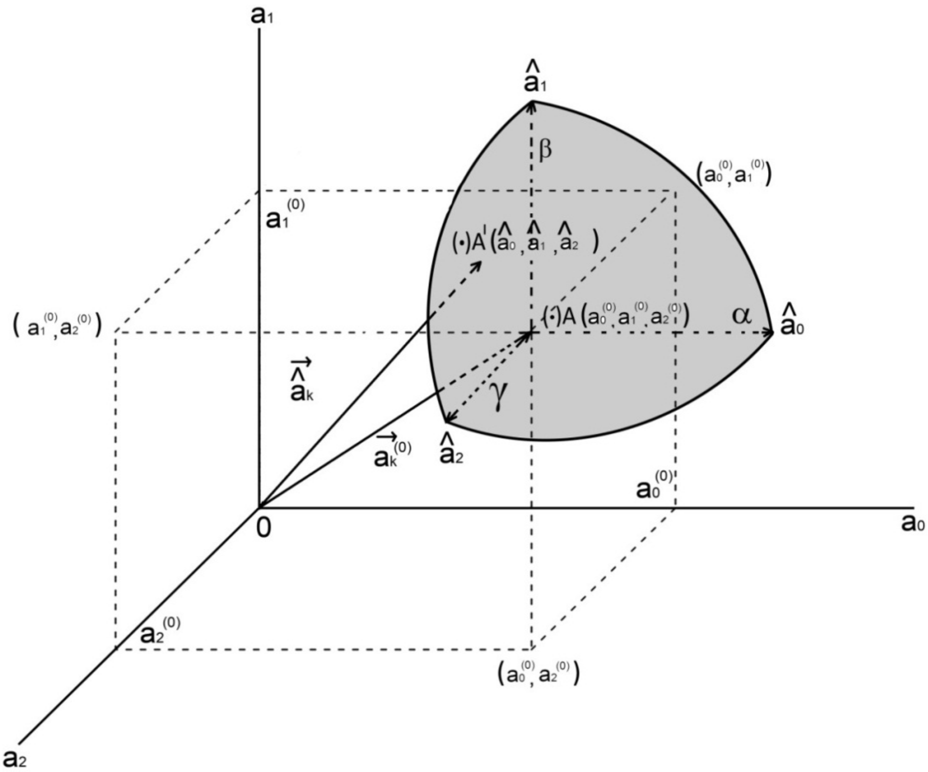


Fig. 2. Geometric interpretation of the implementation of the estimation procedure of the parameters of the 2nd equation by the Tornquist approximation method of successive approximations

3. Finally, the third stage is the last step of statistical estimation of the parameters of a simple regression. Let us designate the first partial derivatives of the formula in expression (8) through respective new variables

$$\frac{\partial f(x_i)}{\partial a_0^{(0)}} = z_1^{(i)}; \quad \frac{\partial f(x_i)}{\partial a_1^{(0)}} = z_2^{(i)}; \quad \frac{\partial f(x_i)}{\partial a_2^{(0)}} = z_3^{(i)}. \quad (10)$$

Keeping in mind that the difference between the empirical and theoretical values of the function of demand for tourist products of the j-th species for $\forall x_i \in [x_1, x_n]$ is the residual vector $\vec{\varepsilon} = \vec{Y} - \vec{Y}(X)$, the expression of the formula (1) can be visually simplified by writing it in the form of multiple regression equations of linear type with new declared variables $z_k^{(i)}$ and amendments in the form of parametric regression coefficients. The latter, in turn, are subject to statistical estimation regression intermediate residual vector $\varepsilon_i^{(0)}$

$$\varepsilon_i^{(0)} = \alpha\, z_1^{(i)} + \beta\, z_2^{(i)} + \gamma\, z_3^{(i)}, \quad (11)$$

where $\varepsilon_i^{(0)}$ characterizes the random component of the vector of demand $\vec{Y}$ and theoretical levels of demand, calculated by the function with the initial parameters $y_j(a_0^{(0)}, a_1^{(0)}, a_2^{(0)}, x_i)$. Component $\varepsilon_i^{(0)}$ obviously does not generally coincide with a random component, i. e. $\subset \varepsilon_i^{(0)} \neq \varepsilon_i$. Otherwise, assuming they are equal, it will be necessary to prove the properties of the *additive* components for each case of a set of new variables $z_k^{(i)}$ [5] ratios $\varepsilon_i = \varepsilon_i^{(0)} + R_s(\alpha, \beta, \gamma)$.

Correction coefficients α, β, γ in (11) are common (ordinary) *OLS* estimates which solve the system of normal equations constructed by the so-called «*mechanical*» method based on the source of the linear multiple regression equation

$$\begin{cases} \sum_{i=1}^{n} \varepsilon_i^{(0)} z_1^{(i)} = \alpha \sum_{i=1}^{n} (z_1^{(i)})^2 + \beta \sum_{i=1}^{n} z_1^{(i)} z_2^{(i)} + \gamma \sum_{i=1}^{n} z_1^{(i)} z_3^{(i)}; \\ \sum_{i=1}^{n} \varepsilon_i^{(0)} z_2^{(i)} = \alpha \sum_{i=1}^{n} z_1^{(i)} z_2^{(i)} + \beta \sum_{i=1}^{n} (z_2^{(i)})^2 + \gamma \sum_{i=1}^{n} z_2^{(i)} z_3^{(i)}; \\ \sum_{i=1}^{n} \varepsilon_i^{(0)} z_3^{(i)} = \alpha \sum_{i=1}^{n} z_1^{(i)} z_3^{(i)} + \beta \sum_{i=1}^{n} z_2^{(i)} z_3^{(i)} + \gamma \sum_{i=1}^{n} (z_3^{(i)})^2. \end{cases} \quad (12)$$

The algorithm for solving systems of normal equations by the Gauss–Jordan method consists of the series of similar steps, each of which are made in the mode of action of turn-based iteration of the procedure using the following steps:

1. Check whether the system of equations is inconsistent; if the system contains some contradictory equation, it is inconsistent;

2. Check the possibility of reducing the number of equations; if the system contains the trivial equation, it is eliminated;

3. If the system of equations is permitted, then write the general solution of the system, and if necessary, the particular solutions;

4. If the system is not permitted, the equation does not contain the allowed unknown, choose the respective resolution element and convert this item by the Jordan method. Further re-transfer to the n. 1 incremental iteration.

Methods for solving systems of linear algebraic equations of the type shown in (12), are mainly divided into two groups of methods:

a) *exact methods*, which are finite algorithms for computing the roots (matrix method, Cramer's rule, Gauss et al.);

b) *iterative methods* to get the roots with a given accuracy by converging processes.

It should be noted that the reduced form of the system of normal equations of linear type on the right contains the terms containing unknown quantities of corrections, while the order of the arguments in each equation must be strictly identical.

Parameters $\hat{a}_0, \hat{a}_1, \hat{a}_2$ defined by successive approximation have statistical properties of unbiased parameters, i. e. $E(\hat{a}_{kn}) = \hat{a}_k$; $E\{|\hat{a}_{kn} - a_k|\} = 0$, and the expectation coincides with the true estimate of the parameter. In other words, mathematical conditional expectation (MCE) sampling rate is zero for $\subset n\uparrow$ and/or $n \to N$, where N is the number of specific general population, such as the set of all families of St. Petersburg in the specified range of income levels. Everything happens for a linear regression model with respect to the residual vector.

However, it should be recognized that in the most general case, these parameters are not optimal estimates as they do not exhibit an asymptotic increase in the volume efficiency in the sample $\{n\}\uparrow$, i. e. are not fully effective estimates in the traditional sense of the properties of the *OLS* estimates. Therefore, an additional check should be carried out at the level of significance of 5.0 % ($p < 0.05$) as the parameters of the regression, and actually the 2nd Tornquist equation.

The level of *p*-values confirms how such an event would be unexpected (the fact) that the data correspond to the null hypothesis H_0. Small *p*-values indicate greater surprise for the discovery of this fact and the reasons for the refusal by analyst H_0. The hypothesis H_0 should be rejected when *p*-values less than the value of 0.05.

In order to assess how reliably the parameters $\hat{a}_0, \hat{a}_1, \hat{a}_2$ of the study reflect the formation of demand for an expensive tourist product and whether these values are the result of the impact of random variables, we calculated the average error of sample parameters – the calculated error in the general case $\Delta\hat{a}_k$ with the corresponding dispersion characteristics $\sigma_{\hat{a}_k}$, σ_x, σ_y, and the pair correlation coefficient $r_{y|x}$, and Student's *t*-tests

$$\Delta\hat{a}_k = \frac{|\hat{a}_k|}{\sigma_{\hat{a}_k}} = \frac{|\hat{a}_k|\,\sigma_x\sqrt{n-2}}{\sigma_y\sqrt{1-r_{y|x}^2}}. \qquad (13)$$

The conducted calculations showed a relatively high quality of analytical smoothing – the coefficient of determination was equal to $d_y = 0.8376$, i. e. in general variability of the resultant variable 83.76 % it can *explain* the variation of the income factor x_i.

The calculated characteristics of the parameters $\hat{a}_0, \hat{a}_1, \hat{a}_2$, are shown in Tab. 2.

Due to the fact that a circle of people with incomes well above 200 thousand rub. per month per family member forms quite a significant group of the population of Russia, of particular interest is the capacity assessment of the market of expensive tourist products (individually developed routes to destinations in expensive, prestigious and world-renowned resorts and so on.) for solvent, successful Russians vacationing and traveling several times a year. The study of the capacity of the exclusive Russian market can be made by considering the above-built economic-statistical model.

The consumer market has impressive volumes, since Russia holds one of the first places in the world for the inequality of distribution of property and income. This is stated in the annual 2013 study of the *Credit Suisse Research Institute* (*CSRI*) which reported that in Russia, 110 billionaires control up to 35 % of the wealth of all households, while the average in the world is 1–2 %. The number of US dollar millionaires is estimated in Russia at 84,000 persons [8].

Table 2

№ p/p	Name countable characteristics	Parameters of the 2nd Tornquist equation, thous. Rub.		
		$\hat{a}_0$	$\hat{a}_1$	$\hat{a}_2$
1	The level of the true parameter	633.3940	133.3840	35.7895
2	The estimated error parameter from equation (13)	2.026808	0.5736	0.1365
3	The lower limit of the confidence interval	627.3134	131.6059	35.3815
4	The upper limit of the confidence interval	639.4746	135.1621	36.1975
5	*p*-values at 5 % significance level	0.000347	0.000211	0.000076

Together with a world leader in market research, the research firm *AC Nielsen*, *CSRI* published the results of the second annual survey *Emerging Consumer Survey*, a detailed study of consumer sentiment in the *BRIC* (*Brazil, Russia, India and China*) countries. The study sought to identify the specificity of the cost structure and consumer preferences of the population in these countries, which are at the heart of structural changes in world demand. The main structural feature that characterizes the consumers of these countries and their optimism is in the transition from essential to more discretionary spending (*discretionary spending* is the spending on big ticket items), which is more typical for developed economies. It reflects the global changes in the balance of consumer spending.

Despite the fact that the prices of exported raw materials in recent years were still favorable for the Russian economy, a noticeable effect on the average consumer spending is not observed. The level of optimism is still one of the lowest in the *BRIC* countries. Taking into account the income inequality, the growth opportunities in Russia were provided for the general population with the highest income in the segment of discretionary purchases.

The *CSRI* analysts recorded an increase in Russian discretionary spending in different categories, particularly in the category of expensive goods, including technology products, luxury goods and real estate abroad. It has two simple explanations. Firstly, due to a marked significant income inequality and prominent inflation, high-income consumers still actively make expensive purchases. Second, the domestic market of discretionary items such as watches, smartphones, antiques, fashion clothing, cosmetics, perfumes, tourist products remains clearly *underutilized*, with a further structural growth. Our ordinary millionaire will continue to increase their discretionary spending.

Conclusion. The considerations expressed in this article lead to three local conclusions on the applicability of the procedure in question in dealing with similar ones on the content of economic and statistical research problems of supply and demand for a high level of tourist products [7]:

1. In the epistemological and methodological aspects the formulated problem of estimating the parameters by a typical nonlinear technique (sequential iterative procedure) does not give a final decision, i. e. the results are incomplete and not fully statistically accurate.

2. Because of the inherent rounding even reliable and accurate measures of economic parameters are approximate. When using iterative procedures, an approximation error is added to the actual implementation of the method, and its effectiveness depends on the more or less successful choice of initial conditions approximation and the convergence rate of the iteration process.

3. Finally, a common approach to stochastic approximation of a nonlinear function is understood by the authors through the use of Taylor series and polynomials search for suitable forms. In particular, the linearization of the equations of simple regression with typical nonlinearity must be carried out by the power series expansion with all members of the first order of smallness eliminated from further calculations.

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