A SMALL-AMPLITUDE CRYSTALLINE UNDULATOR BASED ON 20 GEV ELECTRONS AND POSITRONS: SIMULATIONS

This paper presents the results of numerical simulations of a crystalline undulator based on the channeling of 20 GeV electrons and positrons. The device considered is characterized by a small amplitude and a short period of periodic bending. Calculations have been performed accounting for all-atom interactions using the MBN EXPLORER software package. The effect of low crystal thickness (less than a channeling oscillations period) on radiation spectrum was studied. A new scheme to produce a high-energy radiation was proposed. It is based on the short-period small-amplitude crystalline undulator and allows decreasing the intensity of the non-undulator part of the spectrum.

SMALL-AMPLITUDE CRYSTALLINE UNDULATOR, ELECTRON, POSITRON, CHANNELING, SIMULATION, CRYSTALLINE TARGET.

1. Introduction

A crystalline undulator (CU) [1] is a device producing high-energy radiation by propagating charged relativistic particles through a periodically bent oriented crystalline medium. The feasibility of creating such a device has been predicted theoretically [2 – 4] and now is being studied experimentally [5]. This concept is based on the channeling effect that involves the propagation of projectiles in oriented crystals along crystalline planes or axes. Projectiles in channels interact with crystalline medium and oscillate around the center of the channel. The shape of the channel is responsible for the amplitude and the period of channeling oscillations and the corresponding radiation. Channeling in bent crystals leads to a special type of radiation which depends on the parameters of crystal bending. In recent years two main types of crystalline undulators have been discussed: small-amplitude short-period (SASP) undulators [5 – 7], and large-amplitude large-period (LALP) undulators [5, 8 – 10]. The bending amplitude is considered high if its value is more than the interplanar distance in the crystal. The bending period is considered large if its value is larger than that of the period of channeling oscillations in the channel. Short-period undulators can be used to generate the radiation with photon energies higher than the characteristic energy of the channeling radiation.

In recent series of experiments at Mainzer Microtron [5] with 600 and 855 MeV electrons the effect of a small-amplitude short-period undulator was observed. Another set of experiments with diamond crystalline undulators is planned within the E-212 collaboration at the SLAC facility (Stanford Linear Accelerator Center, USA) with 10 – 20 GeV electron beam. The current experiments with small-amplitude short-period undulators are based on thin silicon (or diamond) crystals doped with small amount of germanium atoms. These crystals are produced using the Molecular Beam Epitaxy (MBE) technology.

The cases of LALP and SASP undulators can be distinguished by the value of the parameter \( C = \varepsilon(RU_{\text{max}}') \). In the case of an LALP undulator, the value of this parameter is considered small, \( C << 1 \), because the projectile has to follow bends of crystalline planes or axes to produce appropriate radiation. In this case an increase in the bending amplitude leads to an increase in the radiation intensity, but also causes an increase in \( C \) being a limiting factor. In SASP undulators the \( C \) parameter can also be formally calculated and its value is more than 1. In this
case projectiles are unable to follow bends of the crystalline medium. What produces the undulator radiation here is the periodic force acting on the projectile due to the rapid change of the interplanar potential. Then the growth of the bending amplitude also boosts the intensity of radiation but reduces the number of channeling projectiles. The amplitude in this instance is limited to a half of the interplanar distance.

In previous papers [11 – 13] the methodology of simulation of channeling with MBN Explorer was presented and applied for straight and bent crystals for sub-GeV energies. The simulations for multi-GeV energies were also described for bent [14] and periodically bent [6] crystals.

MBN Explorer implements a full-atom model of the three-dimensional motion of projectiles in the crystalline medium. In this work, the crystal parameters were taken from experiment [5] as a starting point for the simulations. With these parameters the simulation of propagation of projectiles through the crystals was performed. Using a quasi-classical approach to the calculation of a radiation spectrum the radiation of projectiles was calculated and compared for different crystals. It is shown that for crystals with a thickness less than channeling oscillations period the channeling radiation effect can be suppressed, while undulator radiation can still be generated. The parameters of this radiation are studied numerically.

2. Physical model

The simulation of propagation of a relativistic projectile in a crystalline medium was performed by solving the classical relativistic equations of motion. The force acting on a projectile was calculated as a sum of its interactions with neighboring atoms of the medium. Each atom of the medium was considered as fixed in space as its velocity is much lower than the speed of projectile. Interaction of the projectile with atoms was modeled using the classical Molière [15] interaction potential for screened charges. The atom positions are conditioned by the crystal grid and random displacement due to thermal vibrations. We took the crystalline grid parameters and thermal vibrations from literature [16].

In order to simulate the propagation of a whole beam, the parameters of individual projectiles are randomly sampled. These parameters include an entry point of the projectile into the crystal and an angle between a particle initial velocity and a beam direction. The width of the angular distribution is determined by beam emittance. A random shift of the atoms of the medium due to thermal vibration also causes the sampling of the crystal parameters.

An accelerated motion of charged projectiles in crystal channels produces radiation which can be characterized by a radiation spectrum differential with respect to the photon energy \( h\omega \) and integrated over the given angular aperture \( \theta_c \).

The simulated trajectories were used to compute the spectral distribution of the emitted radiation. For each set of simulated trajectories of the total number \( N_0 \) the spectral distribution emitted within the cone \( \theta < \theta_c \) with respect to the incident beam was calculated as follows:

\[
\frac{dE(\theta < \theta_c)}{h\omega} = \frac{1}{N_0} \sum_{j=1}^{N_0} \int_0^{\theta_c} d\phi \int_0^{\theta_c} d\theta \frac{d^3E_j}{h\omega d\Omega},
\]

here \( d^3E_j / h\omega d\Omega \) is the spectral-angular distribution emitted by a particle moving along the \( j \)-th trajectory.

The sum is carried out over all simulated trajectories, i.e. it takes into account the contribution of the channeling segments of the trajectories as well as of those corresponding to the non-channeling regime.

In order to calculate \( d^3E_j / h\omega d\Omega \) a general quasi-classical method developed by Baier and Katkov [17] was used. The quasi-classical approach explicitly takes into account the quantum corrections due to the radiative recoil. The method is applicable in the whole range of the emitted photon energies, except for the extreme high-energy tail of the spectrum where \( (1 - h\omega / \omega) < 1 \).

Within the framework of the quasi-classical approach, the spectral distribution in energy radiated in the given direction \( n \) by an ultra-relativistic particle is given by the following expression (see Ref. [18] for the details):

\[
\frac{d^3E_j}{h\omega d\Omega} = \frac{\alpha q^2 \omega^2}{8\pi^2} \int_{t_1}^{t_2} dt_1 \int_{t_1}^{t_2} dt_2 e^{i(\omega(t_2) - \omega(t_1))} \times
\]

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\[ \times \left( 1 + (1 + u)^2 \right) \left( \frac{v(t_1) \cdot v(t_2)}{c^2} - 1 \right) + \frac{u^2}{\gamma^2} \right), \tag{2} \]

where

\[ u = \frac{h_0}{e - h_0}, \quad \psi(t) = t - \frac{n \cdot r(t)}{c}, \quad \omega' = \omega(1 + u). \]

In the classical limit, \( u \to 0, \ \omega' = \omega \). These equations allow us to compute the emission spectrum for each simulated trajectory. Averaging the spectrum over all trajectories allows calculating the spectrum of the beam, and determines the statistical error.

3. Numerical results

Let us consider planar channeling in straight and periodically bent crystals of silicon and diamond. The thickness of the crystal is 4 μm, the bending period \( \lambda_u \) is 0.4 μm and the bending amplitude \( a \) is 0.4 Å which is lower than a half of the interplanar distance in both cases. The motion of charged projectiles in the straight crystal corresponds to a classical channeling regime. The second case corresponds to a SASP crystalline undulator regime. The beam is oriented in a (110) plane, avoiding axial channeling directions.

The results of the simulation of radiation of 20 GeV electrons and positrons are compared for the straight and periodically bent diamond and silicon crystals (Fig. 1). At this energy, the natural emission angle is \( \theta_c = 1/\gamma = 25.6 \mu \text{rad} \). The taken value of beam emittance was \( \psi = 5 \mu \text{rad} \). The taken angular aperture of the detector \( \theta_{\text{max}} = 150 \mu \text{rad} \) was 5.8 times higher than the natural emission angle, and covering most of the radiation of projectiles.

In both pictures (see Fig. 1) the rise of each curve (below 1 GeV) corresponds to the channeling radiation, while the peak at \( \sim 6 \text{ GeV} \) as well as other ones corresponds to the undulator radiation. Bending of a crystal leads to a significant suppression of the channeling peak. This effect can be described in terms of an average potential model of channeling. With increase in the bending amplitude the potential well depth decreases but its width grows. This leads to a decrease in the number of channeling projectiles.

Fig. 1. Radiation spectra for 20 GeV electrons and positrons in the straight and the periodically bent diamond (a) and silicon (b) crystals; \( a = 0.4 \, \text{Å}, \lambda_u = 0.4 \, \text{μm} \)
Another factor of suppression of the channeling radiation arises from the high parameter values being unfit for thin crystals; for example, the characteristic periods of channeling oscillations in simulated trajectories for positrons are equal to 9 μm in C(110) channels and 11.5 μm in Si(110) channels at the energy of projectiles of 20 GeV; these periods are longer than some crystal thickness values. This difference causes the suppression of the channeling effect in thin crystals. The comparison between the radiation spectra for a thick (24 μm) crystal and a thin (4 μm) one is shown in Fig. 2. The values for the thinner crystal are multiplied by a factor of 6 for comparison. As can be seen from the plots, the undulator peaks of both curves nearly coincide, while the curve shapes in the region of lower photon energies are significantly different. The channeling produces a sharp peak of radiation around the energy of 150 MeV for a thick crystal, while the corresponding peak is absent for a less thickness, and the radiation is produced by a synchrotron effect.

The radiation emission by projectiles in this simulation is directed strongly along the beam. For the small-aperture undulator the radiation is emitted within a narrow energy range. The dependence of the emitted radiation spectrum on the detector aperture $\theta_{\text{max}}$ is shown in Fig. 3. In the case of the narrow aperture $\theta_{\text{max}} = 10$ μrad the peak in the spectrum is rather sharp. With an increase in the aperture the absolute value of the number of high-energy photons grows, but the peak width grows as well. At $\theta_{\text{max}} = 150$ μrad the radiation spectrum saturates and does not change at higher aperture values.
### 3.1. Crystalline undulator stack

The effect of suppression of channeling radiation in short-periodically bent crystals can be used in order to produce undulator radiation with a higher efficiency. To increase the energy of undulator radiation without increasing channeling radiation, it is possible to use a stack of short crystalline undulators instead of one long undulator. The scheme of such a crystalline system is shown in Fig. 4. In this system, the projectile passes several layers of periodically bent crystalline medium, the radiation produced at all layers adds to the total radiation produced by projectile. For an SASP undulator the thickness of the layers could be taken in the interval between the bending period $\lambda_u$ and the characteristic channeling period of projectile. Such choice of the parameters leads to the absence of full channeling oscillation periods in each channeling segment of trajectory of projectile which results in the suppression of channeling radiation. The effect of undulator radiation in the system remains and grows with an increase in the number of layers.

To simulate the effect of channeling in CU stack the following system was modeled. A set of layers ($l = 4 \mu m$) of periodically bent crystals was put into the simulation box with gaps ($l' = 4 \mu m$) between layers, the period of crystal bending was set to $\lambda = 0.4 \mu m$. Such a system corresponds to a case of the crystalline undulator stack. In this instance the projectiles are captured in the first crystal, escape from the crystal at some point, and have to be captured again in the next crystal layer. The process of the recapturing of the projectiles in the channeling mode leads to the increase in angular dispersion in velocity of projectiles and the decrease in the number of channeling particles as the number of layers increases.

The thickness of layers in this simulation is set below the channeling oscillations period (11.5 $\mu m$) but above the period of crystal channeling oscillations.

Fig. 4. The scheme of the CU stack made of $1, 2, ..., n$ periodically bent crystal layers, each of the thickness $l$. The quantity $l'$ stands for the gap between the layers, $\lambda_u$ is the bending period. Arrowed lines illustrate the trajectories of projectiles.

Fig. 5. Radiation spectra for the small (a) and large (b) apertures calculated for different number of layers ($l = 4 \mu m$) in a stack: 1 (curve 1), 4 (2), 8 (4), 12 (5); $\theta_{\text{max}} = 250 \mu \text{rad}$ (a) and 15.6 $\mu \text{rad}$ (b); $a = 0.4 \AA$, $\lambda_u = 0.4 \mu m$. 

\[ a = 0.4 \AA, \lambda_u = 0.4 \mu m \]
bending (0.4 μm). Such a choice of the crystal thickness leads to the suppression of channeling radiation due to the lack of full oscillation periods, but the undulator radiation remains.

The comparison between radiation spectra for the different number of layers in a stack is given in Fig. 5. The energy of both synchrotron and undulator radiation grows linearly with an increase in the number of stack layers up to 4 (the case for a small aperture). For the higher number of stack layers the angular distribution of projectile velocities becomes wider and the radiation in a narrow cone saturates. For a higher aperture the growth of the peak intensity continues.

The radiation from a single crystal undulator (the crystal is 24 μm thick) is compared to a stack of them (6 × 4 μm) in Fig. 6. It can be seen, that in the stack case the channeling peak in the radiation spectrum is suppressed, while the undulator peak is nearly the same as the one for the case of a single crystal.

4. Summary

This paper deals with the simulation of photon emission by 20 GeV electrons and positrons in a small-amplitude crystalline undulator. The obtained results have been presented and thoroughly discussed. The simulations were performed using the MBN EXPLORER software package with accounting for all-atom interactions of projectiles. It is shown that for high-energy projectiles in thin periodically bent silicon and diamond crystals the effect of channeling is highly suppressed, but the motion in a periodically bent medium generates intensive radiation. This regime is favorable for the construction of a crystalline undulator based on light sources as the energy of a projectile is transferred mostly to the undulator radiation.

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Сушко Г.Б., Король А.В., Соловьев А.В. МОДЕЛИРОВАНИЕ МАЛОАМПЛИТУДНОГО КРИСТАЛЛИЧЕСКОГО ОНДУЛЯТОРА НА ОСНОВЕ ЭЛЕКТРОНОВ И ПОЗИТРОНОВ С ЭНЕРГИЕЙ 20 ГэВ.

В статье приводятся результаты численного моделирования малоамплитудного кристаллического ондулятора на основе электронов и позитронов с энергией 20 ГэВ. Расчеты были проведены с использованием пакета программ MBN Explorer, учитывающего взаимодействие налетающих частиц с кристаллом на атомарном уровне. Рассмотрен случай тонких кристаллических мишень, толщина которых не превышает характерного периода осцилляции частиц при каналировании. Предложена схема получения интенсивного излучения высоких энергий, на основе малоамплитудного ондулятора, которая позволяет снизить уровень фонового излучения.

МАЛОАМПЛИТУДНЫЙ КРИСТАЛЛИЧЕСКИЙ ОНДУЛЯТОР, ЭЛЕКТРОН, ПОЗИТРОН, КАНАЛИРОВАНИЕ, ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ, КРИСТАЛЛИЧЕСКАЯ МИШЕНЬ.


