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Stress state of protective shells in the area of holes due to prestressed reinforcement curvature

Напряженное состояние защитных оболочек в зоне отверстий вследствие кривизны преднапряженных элементов

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Abstract. The stress state around the holes in cylindrical part of prestressed concrete protective shells is examined in this paper. This stress state is caused by general shell prestressing having the aim to compensate internal emergency pressure and the curvature of the reinforcement elements near technological holes in cylindrical part of shell. Presence of technological holes predetermines occurrence of so called “disturbed” stress state of a local nature (stress concentration). The exact solution of the stress concentration problem at any load does not exist even for a plate. So the approximate solution using complex functions and Fourier series for a plate with a hole is proposed. It can be concluded on the basis of the calculation results that the stress concentration due to curvature of reinforcement elements near hole has to be taken into account, since in this case the maximum compressive stress is considerable.

Аннотация. В данной работе рассматривается напряженное состояние конструкции вокруг отверстий в цилиндрической части предварительно напряженных бетонных защитных оболочек. Это напряженное состояние вызвано общим предварительным напряжением оболочки, необходимым для компенсации внутреннего аварийного давления, и кривизной арматурных элементов вблизи технологических отверстий в цилиндрической части оболочки. Наличие технологических отверстий предопределяет возникновение так называемого «возмущенного» напряженного состояния локального характера (концентрация напряжений). Точного решения проблемы концентрации напряжений при любой нагрузке не существует даже для пластины. Поэтому предлагается приближенное решение с использованием комплексных функций и рядов Фурье для пластины с отверстием. На основании результатов расчета можно сделать вывод о том, что необходимо учитывать концентрацию напряжений, вызванную искривлением арматурных элементов вблизи отверстия, поскольку в этом случае максимальное сжимающее напряжение является значительным.

Introduction

Cylindrical part of prestressed concrete protective shells for nuclear power plants with a helical scheme of reinforcement is compressed by two groups of reinforcement elements, oriented over the counter spirals and directed at an angle of 55 degrees to meridian.

The general stress state of the cylindrical part of the shell caused by the preliminary compression forces in areas, sufficiently distant from the bottom and the support ring, is momentless and can be determined in accordance with the membrane theory of thin shells [1–8].

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In areas of technological holes, the largest of which reaches a diameter of 4 m (at elev. 38.1 m), it is necessary to bend the reinforcement elements around the hole. To reduce the prestressing losses from friction trajectory of the reinforcement is made fairly smooth; nevertheless there is considerable pressure of the reinforcement elements (Fig. 1) and the additional stress state in concrete. Of course, in Figure 1 for illustration a simplified scheme of the fragment of rather complicated reinforcement system around the circular hole is presented.

Presence of technological holes predetermines occurrence of so called “disturbed” stress state of a local nature (stress concentration). Thus the stress concentration around the holes is caused by the general compression and by the influence of the curvature of the reinforcement around these holes.

The concentration of the stress caused by a general shells compression is sufficiently investigated [9–23], so the aim of this article is the research of effect of the reinforcement elements curvature for the stress state near the holes , because only a few studies is devoted to this problem [24–25].

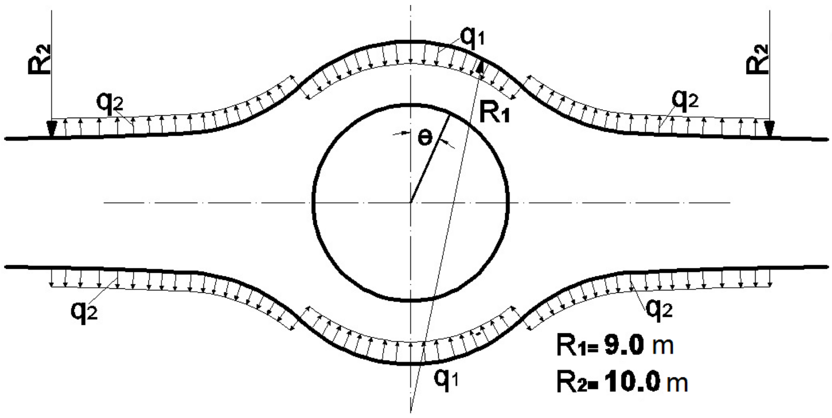

Figure 1. Loads due to curvature of the reinforcement elements

Methods

In the framework of the theory of elasticity a number of special problems of shell theory is currently still not resolved taking into account the complicating factors connected with the mathematical nature of the difficulties. One of such problem is the determination of the stress-strain condition of shells in the area of technological holes. So one has to substitute shell fragment around the hole by the plane fragment in order to solve many engineering problems and consider the plane task. The latter does not lead to significant errors if the fragment is characterized by small curvature, and the hole can be considered as "small". Methods for solving some problems for thin isotropic cylindrical shells (membranes) are described for example in [8, 9, 11]. The holes in these membranes is called small if the following condition take place:

$$\beta a < 1, \tag{1}$$

where $\beta = \sqrt[4]{3(1-\upsilon^2)} / 2\sqrt{Rh}$; *R*, *h* – radius and thickness of the shell; *a* – radius of the hole.

For this problem solution the following sizes are accepted: R = 23.1 m; *h* = 1.2 m; *a* = 2.0 m, which corresponds to βa = 0.25, that is, the hole is certainly small. Note that, if in the above formulas for the stresses around the holes [9, 11] to accept βa = 0, one gets the corresponding formula for a plate of unlimited size.

Comparative calculations show that for the shell with accepted parameters in case of uniaxial tension (or compression) along the cylinder the maximum stress for the shell near hole exceeds corresponding stress for the plate by only 3 % and in case of internal pressure – by 18 %. It can be assumed, that in case of the reinforcement curvature influence the error is less than above.

However, even for plates stress concentration around the holes is one of the most difficult sections of the theory of elasticity, so here analytical solutions in its final form have been obtained for comparatively simple cases of loading [12].

Solution of the problem of stress concentration around the hole can be realized as follows. Due to the task linearity the overall stress-strain state of the plate or membrane can be taken as the sum of "background" stress-strain state and "disturbed" stress-strain state. The state of the plate (or membrane) without holes loaded with an arbitrary system of forces and moments is considered as "background" stress-strain state.

In determining the disturbed state of stress a plate with a hole is considered, and this plate is under influence of the load applied to the hole contour. This load has to be taken equal to stress of the background state, but has to have the opposite direction in order to get zero stress on the hole contour, as well as it should be within the meaning of the problem.

When the superposition of background and disturbed states is realized, the result stress state of the plate with the hole will take place.

In this paper the pressure of the reinforcement elements, caused by its curvature (Fig. 1), for the plate without hole is taken as a background stress state. The intensity of this pressure can be calculated as:

$$q_i = N / R_i, \tag{2}$$

where N – tension force in the reinforcement elements, R_i – radius of curvature of these elements in the plane of the plate. To calculate stress state the actual distribution of the load was replaced by an equivalent in the form of a large number of concentrated forces oriented at different angles to the vertical axis (Fig. 1).

Calculation of the background state of stress from the effects of each of the concentrated forces was carried out according to formulas derived by Melan [1] for the force acting within an infinite plate (without holes) in its plane:

$$\sigma_x = \frac{P}{4\pi}\frac{\cos\theta}{r}\left[-\left(3+\upsilon\right)+2\left(1+\upsilon\right)\sin^2\theta\right]; \tag{3}$$

$$\sigma_y = \frac{P}{4\pi}\frac{\cos\theta}{r}\left[1-\upsilon-2\left(1+\upsilon\right)\sin^2\theta\right]; \tag{4}$$

$$\tau_{xy} = \frac{-P}{4\pi}\frac{\sin\theta}{r}\left[1-\upsilon+2\left(1+\upsilon\right)\cos^2\theta\right]. \tag{5}$$

In the above formulas: r – distance from the force to the point at which the stress is determined; θ – angle is measured from the vertical axis.

The state of stress from all the concentrated forces approximating a distributed load is determined by a superposition of solutions for each of the concentrated forces. Then the transition from Cartesian coordinates to polar has to be fulfilled.

To solve the problem of the second stage (the calculation of the disturbed state) it is advisable to apply the solution of the plane task of elasticity theory with complex variables, which in some cases leads to significant simplifications.

The components of stresses for plane stress state in polar coordinates can be expressed as follows [6]:

$$\sigma_r+\sigma_\theta = 4\,\mathrm{Re}\,F'\left(Z\right) = 2\left[F'\left(Z\right)+\overline{F}'\left(\overline{Z}\right)\right]; \tag{6}$$

$$\sigma_\theta-\sigma_r+2i\tau_{r\theta} = 2\left[\overline{Z}F''\left(Z\right)+\chi''(Z)\right]e^{2i\theta}, \tag{7}$$

where $Z = r\left(\cos\theta+i\sin\theta\right) = re^{i\theta}$; $\overline{Z} = r\left(\cos\theta-i\sin\theta\right) = re^{-i\theta}$; $\overline{Z}$ – conjugate function Z, $F'\left(Z\right)\,u\,\chi'(Z)$ – some analytic functions. Subtracting (7) from (6) we obtain:

$$\sigma_r-i\tau_{r\theta} = F'\left(Z\right)+\overline{F}'\left(\overline{Z}\right)-\left[\overline{Z}F''\left(Z\right)+\chi''(Z)\right]e^{2i\theta}. \tag{8}$$

Let us consider the general solution for unlimited size plate with a circular hole with the origin in the center of the hole. If to run the boundary conditions on the contour of the hole, then σ_r and $\tau_{r\theta}$ will be known when $z = ae^{i\theta}$, where *a* – the radius of the hole, r – distance from the center of the hole (on the contour r = *a*).

Analytic functions $F'(Z), \chi''(Z)$ – can be expanded in power series, so that the functions remain finite at $r \to \infty$

$$F'(Z) = \sum_{n=0}^{\infty} A_n Z^{-n} = A_0 + A_1 \frac{1}{Z} + A_2 \frac{1}{Z^2} + \dots; \tag{9}$$

$$\chi''(Z) = \sum_{n=0}^{\infty} B_n Z^{-n} = B_0 + B_1 \frac{1}{Z} + B_2 \frac{1}{Z^2} + \dots, \tag{10}$$

where A_n and B_n − complex constants (independing from *Z*).

$$\overline{F'(Z)} = \sum_{n=0}^{\infty} \overline{A_n} \overline{Z}^{-n} = \overline{A_0} + \overline{A_1} \frac{1}{\overline{Z}} + \overline{A_2} \frac{1}{\overline{Z}^2} + \dots; \tag{11}$$

$$F''(Z) = -A_1 Z^{-2} - 2A_2 Z^{-3} - \dots - nA_n Z^{-n-1}. \tag{12}$$

Since the stresses σ_r and $\tau_{r\theta}$ must be known on the hole contour (*r* = *a*), the expression $(\sigma_r - i\tau_{r\theta})_{r=a}$ can be expanded in a complex Fourier series [6]:

$$(\sigma_r - i\tau_{r\theta})_{r=a} = \sum_{n=-\infty}^{\infty} C_n e^{in\theta}; \tag{13}$$

the coefficients of which are determined by the formula:

$$C_n = \frac{1}{2\pi} \int_0^{2\pi} \left[\sigma_r(\theta) - i\tau_{r\theta}\right]_{r=a} e^{-in\theta} d\theta; \tag{14}$$

where *n* = 0; 1; −1; 2; − 2; …

Equating the right-hand side of the equation (8), expressed in terms of (9)–(12) and the expression (13), we obtain:

$$\sum_{n=-\infty}^{\infty} C_n e^{in\theta} = \sum_{n=0}^{\infty} \frac{A_n}{a^n} e^{-in\theta} + \sum_{n=0}^{\infty} \frac{\overline{A_n}}{a^n} e^{in\theta} + \sum_{n=0}^{\infty} \frac{nA_n}{a^n} e^{-in\theta} - \sum_{n=0}^{\infty} \frac{B_n}{a^n} e^{-i(n-2)\theta}, \tag{15}$$

since at the hole contour $z = ae^{i\theta}$; $Z^{-n} = \frac{1}{a^n} e^{-in\theta}$.

The last expression can be written as follows:

$$\begin{aligned} &C_0 + C_1 e^{i\theta} + C_{-1} e^{-i\theta} + C_2 e^{2i\theta} + C_{-2} e^{-2i\theta} + C_3 e^{3i\theta} + \dots = \\ &A_0 + \frac{A_1}{a} e^{-i\theta} + \frac{A_2}{a^2} e^{-2i\theta} + \frac{A_3}{a^3} e^{-3i\theta} + \dots \\ &+ \overline{A_0} + \frac{\overline{A_1}}{a} e^{i\theta} + \frac{\overline{A_2}}{a^2} e^{2i\theta} + \frac{\overline{A_3}}{a^3} e^{3i\theta} + \dots \\ &+ \frac{A_1}{a} e^{-i\theta} + \frac{2A_2}{a^2} e^{-2i\theta} + \frac{3A_3}{a^3} e^{-3i\theta} + \dots \\ &-B_0 e^{2i\theta} - \frac{B_1}{a} e^{i\theta} - \frac{B_2}{a^2} e^0 - \frac{B_3}{a^3} e^{-i\theta} - \frac{B_4}{a^4} e^{-2i\theta} + \dots \end{aligned} \tag{16}$$

Equating the coefficients before the same powers of *e* in both parts of the equality, we get:

$$A_0 + \overline{A_0} - \frac{B_2}{a^2} = C_0 \quad \text{(when n = 0)}; \tag{17}$$

$$\frac{\overline{A_1}}{a} - \frac{B_1}{a} = C_1 \quad \text{(when n = 1)}; \tag{18}$$

$$\frac{\overline{A_2}}{a^2} - B_0 = C_2 \quad \text{(when n = 2)}; \tag{19}$$

$$\frac{\overline{A_n}}{a^n} = C_n \quad \text{(when } n \geq 3\text{)}; \tag{20}$$

$$\frac{2A_1}{a} - \frac{B_3}{a^3} = C_{-1} \quad \text{(when n = −1)}; \tag{21}$$

$$\frac{1+n}{a^n} A_n - \frac{B_{n+2}}{a^{n+2}} = C_{-n}. \tag{22}$$

Constants A_0, $\overline{A_0}$ equal to each other, since the imaginary part determines the displacement of a rigid body, and in the analysis of strain and stress can be assumed to be equal zero [6]:

$$r\left(\cos\theta + i\sin\theta\right) = r\left(\cos\theta - i\sin\theta\right) = r\left(\cos\theta\right), \tag{23}$$

that is $A_0 + \overline{A_0} = 2A_0$.

From the condition of the uniqueness of the value of the displacement $\vartheta_r + i\vartheta_\theta$, should be

$$A_1 = -\frac{1+\upsilon}{3-\upsilon}\overline{B_1}; \tag{24}$$

whence

$$\overline{A_1} = -\frac{1+\upsilon}{3-\upsilon}B_1; \tag{25}$$

Substituting (25) into (18) we obtain:

$$B_1 = -\frac{(3-\upsilon)c_1 a}{4}; \tag{26}$$

Substituting (26) into (25) we obtain:

$$\overline{A_1} = -\frac{1+\upsilon}{3-\upsilon}B_1 = -\frac{1+\upsilon}{3-\upsilon}\left(-\frac{(3-\upsilon)c_1 a}{4}\right) = \frac{\left(1+\upsilon\right)c_1 a}{4}; \tag{27}$$

or

$$\overline{A_1} = \frac{\left(1+\upsilon\right)c_1 a}{4}. \tag{28}$$

From (19) we get:

$$\overline{A_2} = B_0 a^2 + C_2 a^2; \tag{29}$$

$$A_2 = \overline{B_0}a^2 + \overline{C_2}a^2; \qquad (30)$$

Taking into account, that $A_0 + \overline{A_0} = 2A_0$, from (17):

$$2A_0 - \frac{B_2}{a^2} = C_0 \qquad (31)$$

$$B_2 = 2A_0a^2 - C_0a^2; \qquad (32)$$

From equations (20) and (22):

$$A_n = C_n a^n \text{ when } n \geq 3; \qquad (33)$$

$$B_n = (n-1)a^2 A_{n-2} - a^n C_{-n+2}, \text{ when } n \geq 3. \qquad (34)$$

Thus, all members of the functions $F'(Z), \chi''(Z), \overline{F}'(Z), F''(Z)$, are known (expressed in terms C_n), and the problem reduces to the determination of the coefficients C_n by the formula (14), which can be written as follows:

$$C_n = \frac{1}{2\pi}\int_0^{2\pi}\left[\sigma_r(\theta) - i\tau_{r\theta}\right]_{r=a} e^{-in\theta}d\theta = \\ = \frac{1}{2\pi}\int_0^{2\pi}\left[\sigma_r(\theta) - i\tau_{r\theta}\right]_{r=a}(\cos n\theta - i\sin n\theta)\,d\theta. \qquad (35)$$

Results and Discussion

When the plate is loading by the pressure of curved near the hole reinforcing elements (Fig. 1), coefficients in the complex Fourier series were determined by numerical integration in accordance with expression (35). Convergence was studied by doubling the number of intervals between concentrated forces. The calculation results show that for the scheme of loading, which has two axes of symmetry (Fig. 1.), it is sufficient to restrict 8–10 members of series (9)–(12).

The calculations were performed for the reinforcement elements with the prestressing force of 8000 *kN.* Accordingly accepted radius of curvature distributed load intensity are $q_1 = 888,9\,kN/m,\ q_2 = 800\,kN/m.$ According to the calculations, the maximum value of the tangential forces near the hole, induced by curvature of two reinforcement elements, when $\theta = \frac{\pi}{2}$ is about 1900 *kN*/*m*.

Since the influence of the curvature of individual prestressed elements near holes was hardly studied by other authors, the force from curvature of two elements is compared with the vertical force of general compression which is necessary to compensate internal emergency pressure *p* = 0,4 *MPa*. This force in the shell with a hole, but excluding influence of the curvature of reinforcement elements near hole, is approximately determined as 3*pR*/2 [26–37], where *R* is the radius of the cilindrical part of the shell. This force at *R* = 23.1 m is equal to 13860 *kN/m*.

It can be concluded on the basis of the calculation results that the stress concentration due to curvature of reinforcement elements near holes has to be take into consideration, since in this case the maximum compressive stress may achieve about 15% of the maximum stresses caused by general compression.

$$3\frac{pR}{2} = 3\frac{400 \cdot 23,11}{2} = 13860\,kN/m. \qquad (36)$$

Conclusions

1. Analysis of existing analytical solutions related to the study of stresses around the holes in the cylindrical shells and plates, showed that, when examined relations between the radius of the shell and the radius of the hole take place, solutions for the plates is allowed to use.

2. It can be concluded on the basis of the calculation results that the stress concentration due to curvature of reinforcement elements near hole has to be take into account, since in this case the maximum compressive stress may achieve about 15 % of the maximum stresses caused by general compression. At the same time, a significant value of tensile stresses, caused by curvature of reinforcement, may result in inadequate general compression in some areas of the shell around the holes.

3. The method developed for calculation of the stress state near the holes of plates using complex functions with some improvements can be used in any case of the load distribution in plane of the plate.

References

1. Timoshenko S.P., Gudyer Dzh. *Teoriya uprugosti* [Elasticity theory]. Moscow: Nauka, 1979. 576 p. (rus)
2. Timoshenko S.P., Voynovskiy-Kriger S. *Plastinki i obolochki* [Plates and Shells]. Moscow: Nauka, 1966. 636 p. (rus)
3. Sokolov V.A. *Zashchitnyye obolochki reaktornykh otdeleniy AES* [Protective shells of NPP reactor compartment]. St. Petersburg: Izdatelstvo Politekhnicheskogo universiteta Petra Velikogo, 2007. 106 p. (rus)
4. Musabayev T.T. *Nelineynaya teoriya rascheta zhelezobetonnykh obolochek i plastin* [The nonlinear theory of concrete shells and plates calculation]. PhD dissertation. Saint-Petersburg State University of Architecture and Civil Engineering. St. Petersburg. 1999. (rus)
5. Medvedev V.N., Ulyanov A.N., Kiselev A.S., Kiselev A.S., Lopanchuk A.A., Nefedov S.S. *Analiz predelnoy prochnosti zashchitnoy obolochki energobloka VVER-1000* [Analysis of protective shell ultimate strength of power block VVER-1000]. Proceedings of IBRAE RAS. 2008. No. 6. Pp. 122–130. (rus)
6. Van Tszi-De. *Prikladnaya teoriya uprugosti* [Applied elasticity]. Moscow: Fizmatgiz, 1959. 400 p. (rus)
7. Korobov L.A., Zharkov A.F., Shernik A.O. *Issledovanie zhelezobetonnykh zashchitnykh obolochek AES. Preduprezhdenie o vozmozhnykh avariyakh na AES Rossii* [Research of concrete protective shell of NPP. Warning of possible accidents at Russian NPP]. Moscow: Sputnik plyus, 2011. 256 p. (rus)
8. Lurye A.I. *Statika tonkostennykh uprugikh obolochek* [Statics of thin elastic shells]. Moscow : Gostekhizdat, 1947. 252 p. (rus)
9. Pirogov I.M. Vliyanie krivizny naraspredelenie napriazhenij okolo otverstiya v tsilindricheskoj obolochke [Effect of curvature on the stress distribution near the hole inthe cylindrical shell]. *Applied Mechanics.* 1965. Vol. 1. No. 12. Pp. 116–119. (rus)
10. Solovey N.A., Krivenko O.P., Malygina O.A. Konechnoelementnye modeli issledovaniya nelineynogo deformirovaniya obolochek stupenchato-peremennoy tolshchiny s otverstiyami, kanalami i vyemkami [Finite element models for the analysis of nonlinear deformation of shells stepwise-variable thickness with holes, channels and cavities]. *Magazine of Civil Engineering.* 2015. No. 1. Pp. 56–69. (rus)
11. Guz A.N., Chernyshenko I.S., Chekhov Val.N., Chekhov Vik.N., Shnerenko K.I. *Tsilindricheskie obolochki, oslablennye otverstiyami* [Impaired by holes cylindrical shell]. Kiev: Naukova Dumka, 1974. 272 p. (rus)
12. Savin G.N. *Raspredelenie napryazheniy okolo otverstiy* [Stress distribution around holes]. Kiev: Naukova Dumka, 1968. 891 p. (rus)
13. Sigova E.M. *Chislennoe reshenie geometricheski*

Литература

1. Тимошенко С.П., Гудьер Дж. Теория упругости. М.: Наука, 1979. 576 с.
2. Тимошенко С.П., Войновский-Кригер С. Пластинки и оболочки. М.: Наука, 1966. 636 с.
3. Соколов В.А. Защитные оболочки реакторных отделений АЭС. СПб: Издательство Политехнического университета Петра Великого, 2007. 106 с.
4. Мусабаев Т.Т. Нелинейная теория расчета железобетонных оболочек и пластин. Дисс. на соиск. учен. степ. к.т.н.: Спец. 05.23.17. СПб, 1999. 421 с.
5. Медведев В.Н., Ульянов А.Н., Киселев А.С., Киселев А.С., Лопанчук А.А., Нефедов С.С. Анализ предельной прочности защитной оболочки энергоблока ВВЭР-1000 // Труды ИБРАЭ РАН. 2008. № 6. С. 122–130.
6. Ван Ц.-Д. Прикладная теория упругости. М. Государственное издательство физико-математической литературы, 1959. 400 с.
7. Коробов Л.А., Жарков А.Ф., Шерник А.О. Исследование железобетонных защитных оболочек АЭС. Предупреждения о возможных аварях на АЭС России. М.: Спутник плюс, 2011. 256 с.
8. Лурье А.И. Статика тонкостенных упругих оболочек. М.: Гостехиздат, 1947. 252 с.
9. Пирогов И.М. Влияние кривизны на распределение напряжений около отверстия в цилиндрической оболочке // Applied Mechanics. 1965. Vol. 1. № 12. С. 116–119.
10. Соловей Н.А., Кривенко О.П., Малыгина О.А. Конечноэлементные модели исследования нелинейного деформирования оболочек ступечато-переменной толщины с отверстиями, каналами и выемками // Инженерно-строительный журнал. 2015. № 1. С. 56–69.
11. Гуз А.Н., Чернышенко И.С., Чехов Вал.Н., Чехов Вик.Н. Шнеренко К.И. Цилиндрические оболочки, ослабленные отверстиями. Киев: Наукова Думка, 1974. 272 с.
12. Савин Г.Н. Распределение напряжений около отверстий. Киев: Наукова Думка, 1968. 891 с.
13. Сигова Е.М. Численное решение геометрически нелинейной задачи о напряженно-деформируемом состоянии цилиндрической облочки с отверстием. Томск: Издательство Томского политехнического университета, 2013. 468 с.
14. Довбня Е.Н., Крупко Н.А. Влияние кругового отверстия на напряженное состояние оболочки произвольной гауссовской кривизны // Вестник ПНИПУ. № 1. С. 108–125.
15. Васильев В.В., Федоров Л.В. Задача геометрической теории упругости о концентрации напряжений в пластине с круговым отверстием // Механика твердого тела. 2008. № 4. С. 6–18.

nelineynoy zadachi o napryazhenno-deformirovannom sostoyanii tsilindricheskoy obolochki s otverstiem [numerical solution geometrically nonlinear problem of stress strain behavior of cylindrical shell with a hole]. Tomsk : Izdatelstvo Tomskogo politekhnicheskogo universiteta, 2013. 468 p. (rus)

14. Dovbnya Ye.N., Krupko N.A. Vliyaniye krugovogo otverstiya na napryazhennoye sostoyaniye obolochki proizvolnoy gaussovoy krivizny [Influence of circular hole on the shell stress state for arbitrary Gaussian curvature]. *PNRPU Mechanics Bulletin.* 2014. No. 1. Pp. 108–125. (rus)
15. Vasilyev V.V., Fedorov L.V. Zadacha geometricheskoy teorii uprugosti o kontsentratsii napryazheniy v plastine s krugovym otverstiyem [The geometric theory problem of elasticity of the stress concentration at the plate with a circular hole]. *Mechanics of Solids.* 2008. No. 4. Pp. 6–18. (rus)
16. Trofimov V.N., Abdulova A.S., Isayeva Yu.A., Podkina N.S. Issledovaniye kontsentratsii napryazheniy sostoyaniya v plastinakh s otverstiyami [Research of stress concentration of state in plates with holes]. *Proceedings of the XI International Scientific and Practical Conference.* Kursk: University Book, 2014. Vol. 4. Pp. 205–208. (rus)
17. Nizomov D.N. Khodzhiboyev A.A. Khodzhiboyev O.A. Napryazhenno-deformirovannoye sostoyaniye anizotropnoy plastiny, oslablennoy otverstiyem [Stress-strain state of anisotropic plate weakened by a hole]. *Structural Mechanics of Engineering Constructions and Buildings.* 2013. No. 4. Pp. 22–28. (rus)
18. Timerbayev R.M., Khayrullin F.S., Khakimov R.G. Zadacha o deformirovanii krugloy plastiny s otverstiyem [The problem of the deformation of the circular plate with hole]. *Vestnik Kazanskogo Tekhnologicheskogo Universiteta.* 2013. Vol. 16. No. 6. Pp. 179–182. (rus)
19. Semykina T.D., Vulman S.A. Vliyaniye na napryazhennoye sostoyaniye beskonechnoy plastiny kasatelnykh napryazheniy, raspredelennykh po krugovomu otverstiyu [Influence of shear stresses distributed over a circular hole to the stress state of an infinite plate]. *Vestnik ChGPU im. I.Ya. Yakovleva.* 2012. Vol. 12. No 2. Pp. 83–87.
20. Nizomov D.N., Khodzhiboyev A.A., Khodzhiboyev O.A. Kontsentratsiya napryazheniy vokrug otverstiya v anizotropnoy plastine [Simulation of the stress-strain state of excavation boundaries in fractured massifs]. *Scientific and Technical Journal on Construction and Architecture.* 2011. No. 6. Pp. 307–311.
21. Makarov E.V., Monahov I.A., Nefedova I.V. Dvuosnoye rastyazheniye plastiny s krugovym otverstiyem [Biaxial stretching of the plate a circular hole]. *Bulletin of Russian Peoples' Friendship University. Series Engineering Researches.* 2015. No 3. Pp. 17–22.
22. Poluektov V.A., Mirenkov V.Ye., Shutov V.A. Napryazhennoye sostoyaniye plastin s otverstiyem [Stress state of plates with a hole]. *News of higher educational institutions. Construction.* 2014. Vol. 671. No .11. Pp. 5–9.
23. Bok Kh.K.A. *Konstruirovaniye i issledovaniye napryazhenno-deformirovannogo sostoyaniya plastin i obolochek s otverstiyami variatsionno-raznostnym metodom* [Design and research of stress-strain state of plates and shells with holes by variational-difference method]. PhD dissertation. RUDN University. Moscow. 2005. (rus)
24. Ulyanov A.N., Medvedev V.N Printsipy konstruirovaniya zon tekhnologicheskikh prokhodok [Design principles of technological holes areas]. *Proceedings of IBRAE RAS.* 2008. No. 6. Pp. 8–10. (rus)
25. Ulyanov A.N., Medvedev V.N., Kiselev A.S. Vliyaniye otgibov armaturnykh elementov na napryazhennoye sostoyaniye zashchitnykh obolochek AES v zone tekhnologicheskikh prokhodok [Influence of offset bend to the NPP protective shell stress state in technological holes areas]. *Proceedings of IBRAE RAS.* 2008. No. 6. Pp. 11–

16. Трофимов В.Н., Абдулова А.С., Исаева Ю.А. Исследование концентраций напряженний состояния в пластинах с отверстиями // Сборник научных трудов XI Международной научно-практической конференции. Курск: Университетская книга, 2014. Т. 4. С. 205–208.
17. Низомов Д.Н., Ходжибоев А.А., Ходжибоев О.А. Напряженно-деформированное состояние анизотропной пластины, ослабленной отверстием // Строительная механика иженерных конструкций и сооружений. 2013. № 4. С. 22–28.
18. Тимербаев Р.М. Хайруллин Ф.С., Хакимов Р.Г. Задача о деформировании круглой пластины с отверстием // Вестник Казанского технологического унвиерситета. 2013. Т. 16. № 6. С. 179–182.
19. Семыкина Т.Д., Вульман С.А. Влияние на напряженное состояние бесконечной пластины касательных напряжений, распределенных по круговому отверстию // Вестник Чувашского государственного педагогического университета им. И.Я. Яковлева. 2012. Т. 12. № 2. С. 83–87.
20. Низомов Д.Н., Ходжибоев А.А., Ходжибоев О.А. Концентрация напряжений вокруг отверстия в анизотропной пластине // Вестник МГСУ. 2011. № 6. С. 307–311.
21. Макаров Е.В., Монахов И.А., Нефедова И.В. Двуосное растяжение пластины с круговым отверстием // Вестник РУДН. Серия: Инженерные исследования. 2015. № 3. С. 17–22.
22. Полуэктов В.А., Миренков В.Е., Шутов В.А. Напряженное состояние пластин с отверстием // Известия ВУЗов. Строительство. 2014. № 11. С. 5–9.
23. Бок Х.К.А. Конструирование и исследование напряженно-деформируемого состояния пластин и оболочек с отверстиями вариационно-разностным методом. Дисс... на соиск. учен. степ. к.т.н.: Спец. 05.23.17.Москва, 2005. 231 с.
24. Ульянов А.Н., Медведев В.Н. Принципы конструирования зон технологичеких проходок // Труды ИБРАЭ РАН. 2008. № 6. С. 8–10.
25. Ульянов А.Н., Медведев В.Н., Киселев А.С. Влияние отгибов арматурных элементов на напряженное состояние защитных оболочек АЭС в зоне технологических проходок // Труды ИБРАЭ РАН. 2008. № 6. С. 11–14.
26. Kulka F. Prestressing systems for secondary and primary containment structures // Nuclear Engineering and Design. 1968. Vol. 8. № 4. Pp. 435–439.
27. Walser A. Capability of a prestressed concrete containment for internal pressure load // Nuclear Engineering and Design. 1984. Vol. 82. № 1. Pp. 25–35.
28. Ashar H., Naus D.J. Overview of the use of prestressed concrete in U.S. nuclear power plants. Nuclear Engineering and Design. 1983. Vol. 75. № 3. Pp. 425–437
29. Prinja N.K., Ogunbadejo A., Jonathan Sadeghi, Edoardo Patelli. Structural reliability of pre-stressed concrete containments // Nuclear Engineering and Design. 2016. Vol. 82. № 1. Pp. 36–46.
30. Noh S.-H., Kwak H.-G., Jung R. Effects of No Stiffness Inside Unbonded Tendon Ducts on the Behavior of Prestressed Concrete Containment Vessels // Nuclear Engineering and Technology. Vol. 48. № 3. Pp. 805–819.
31. Takeda H., Kusabuka M., Imoto K., Takumi K., Soejima M.. Numerical algorithm for local failure mechanism of pre-stressed concrete containment vessel wall with penetration // Nuclear Engineering and Design. 1996. Vol. 166. № 3. Pp. 389–401.
32. Sun Z., Liu S., Lin S., Xie Yo. Strength monitoring of a prestressed concrete containment with grouted tendons // Nuclear Engineering and Design. 2002. Vol. 216. № 1–3.

Sokolov V.A., Strachov D.A., Sinyakov L.N., Vasiutina S.V. Stress state of protective shells in the area of holes due to prestressed reinforcement curvature. *Magazine of Civil Engineering*. 2017. No. 2. Pp. 33–41. doi: 10.18720/MCE.70.4

14. (rus)

26. Kulka F. Prestressing systems for secondary and primary containment structures. *Nuclear Engineering and Design.* 1968. Vol. 8. No. 4. Pp. 435–439.
27. Walser A. Capability of a prestressed concrete containment for internal pressure load. *Nuclear Engineering and Design*. 1984. Vol. 82. No. 1. Pp. 25–35.
28. Ashar H., Naus D.J. Overview of the use of prestressed concrete in U.S. nuclear power plants. *Nuclear Engineering and Design.* 1983. Vol. 75. No. 3. Pp. 425–437
29. Prinja N.K., Ogunbadejo A., Jonathan Sadeghi, Edoardo Patelli. Structural reliability of pre-stressed concrete containments. *Nuclear Engineering and Design.* 2016. Vol. 82. No. 1. Pp. 36–46.
30. Noh S.-H., Kwak H.-G., Jung R. Effects of No Stiffness Inside Unbonded Tendon Ducts on the Behavior of Prestressed Concrete Containment Vessels. *Nuclear Engineering and Technology.* Vol. 48. No. 3. Pp. 805–819.
31. Takeda H., Kusabuka M., Imoto K., Takumi K., Soejima M.. Numerical algorithm for local failure mechanism of pre-stressed concrete containment vessel wall with penetration. *Nuclear Engineering and Design.* 1996. Vol. 166. No. 3. Pp. 389–401.
32. Sun Z., Liu S., Lin S., Xie Yo. Strength monitoring of a prestressed concrete containment with grouted tendons. *Nuclear Engineering and Design*. 2002. Vol. 216. No. 1-3. Pp. 213–220.
33. Kwak H.-G., Kwon Ya. Nonlinear analysis of containment structure based on modified tendon model. *Annals of Nuclear Energy.* 2016. Vol. 92. Pp. 113–126.
34. Bílý P., Kohoutková A. Sensitivity analysis of numerical model of prestressed concrete containment. *Nuclear Engineering and Design.* 2015. Vol. 295. Pp. 204–214.
35. Hu Hs.-T., Lin Yu-H. Ultimate analysis of PWR prestressed concrete containment subjected to internal pressure. *International Journal of Pressure Vessels and Piping.* 2006. Vol. 83. No. 3. Pp. 161–167.
36. Hofstetter G., Mang H.A. Collapse load analysis of prestressed concrete surface structures with unbonded tendons by the finite element method. *Finite Elements in Analysis and Design.* 1989. Vol. 5. No. 2. Pp. 141–165.
37. Pandey M.D. Reliability-based assessment of integrity of bonded prestressed concrete containment structures. *Nuclear Engineering and Design.* 1997. Vol. 176. No. 3. Pp. 247–260.

Pp. 213–220.

33. Kwak H.-G., Kwon Ya. Nonlinear analysis of containment structure based on modified tendon model // Annals of Nuclear Energy. 2016. Vol. 92. Pp. 113–126.
34. Bílý P., Kohoutková A. Sensitivity analysis of numerical model of prestressed concrete containment // Nuclear Engineering and Design. 2015. Vol. 295. Pp. 204–214.
35. Hu Hs.-T., Lin Yu-H. Ultimate analysis of PWR prestressed concrete containment subjected to internal pressure // International Journal of Pressure Vessels and Piping. 2006. Vol. 83. № 3. Pp. 161–167.
36. Hofstetter G., Mang H.A. Collapse load analysis of prestressed concrete surface structures with unbonded tendons by the finite element method // Finite Elements in Analysis and Design. 1989. Vol. 5. № 2. Pp. 141–165.
37. Pandey M.D. Reliability-based assessment of integrity of bonded prestressed concrete containment structures // Nuclear Engineering and Design. 1997. Vol. 176. № 3. Pp. 247–260.

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