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ON THE INITIAL STATE OF THE UNIVERSE IN THE THEORY OF INFLATION

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A toy quantum model of the inflationary universe is considered in which the role of cosmic time is played by the inflaton scalar field (its logarithm). Based on a variant of the positive energy theorem in General Relativity for the case of a closed universe, a strictly positive energy of space is introduced. The principle of minimum of the energy of space is proposed which determines a ground state as well as the excited states of the universe in quantum cosmology. According to this principle, the Beginning of the universe does exist as a state of minimal excitation of the energy of space. The initial proto-inflation quantum state of the universe is defined as a state of minimal excitation of the energy of space provided that the potential energy of the inflaton scalar field is large at the Beginning. Simultaneously, quanta of space energy excitation are introduced and the expansion of the universe can be considered as the birth of these quanta. Quantum birth of the ordinary matter becomes significant when the potential energy of the inflaton scalar field comes down to zero value.

Key words: universe; theory of the inflation; energy of space; time; quantum; matter

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О НАЧАЛЬНОМ СОСТОЯНИИ ВСЕЛЕННОЙ В ТЕОРИИ ИНФЛЯЦИИ

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Рассмотрена квантовая модель инфляционной Вселенной, в которой роль космического времени играет инфлатонное скалярное поле (его логарифм). На основании варианта теоремы положительности энергии в общей теории относительности для случая замкнутой Вселенной вводится строго положительная энергия пространства. Предложен принцип минимума этой энергии для определения основного и возбужденных состояний Вселенной в квантовой космологии. Согласно этому принципу, существует начало Вселенной как состояние минимального возбуждения энергии пространства. Начальное квантовое состояние Вселенной перед инфляцией определяется как состояние минимального возбуждения пространства при условии, что инфлатонное скалярное поле имеет большую потенциальную энергию в своем начале. Вместе с основным состоянием вводятся кванты возбуждения энергии пространства, так что расширение Вселенной может рассматриваться как рождение квантов возбуждения. Квантовое рождение обычной материи становится существенным, когда потенциальная энергия инфлатонного скалярного поля падает до нуля.

Ключевые слова: Вселенная; теория инфляции; энергия пространства; время; квант; материя

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Introduction

The modern cosmological paradigm includes (as an inevitable part) the existence of an inflation stage with the exponential expansion of the universe [1 – 4]. A quantum epoch takes its own place substantially at the beginning of the universe in just the same way. Inflation theories do not explicitly specify the initial state of the universe and its size before the inflation. For instance, the inflation may be supposed to begin exactly following the quantum epoch with the Planck initial size [5]. On the other hand, in different quantum theories of the Beginning [6, 7] the classical inflation stage is considered as a natural continuation of the history of the universe. According to the Vilenkin tunneling theory [6], the de Sitter stage of the exponential expansion of the universe is sewn together with the de Sitter instantion at a definite radius determined by a vacuum energy density ρ_v in the framework of the Grand Unified Theory. In order to accommodate the tunneling theory with inflaton theories where an effective scalar field (inflaton) is present, the vacuum energy density should be identified with an initial value of the inflaton potential energy. In Ref. [8], the classical stage of the inflation in the quasi-classical approximation of the Hartle-Hawking no-boundary wave function of the universe was obtained as well.

In both approaches the dynamics of the scale factor of the universe at the inflaton stage was considered as a classical one. However, quantum effects, for instance, the ordinary matter creation, are important at the final stages. This also concerns the dynamics of the scale factor of the universe. In order to formulate quantum dynamics of the scale factor, a cosmic time should be defined in the quantum universe. In Ref. [9], a canonical time parameter related to the slow-rolling inflaton scalar field was introduced in the minisuperspace model of the universe. As a result, the Wheeler-DeWitt (WDW) equation for quantum geometry [10] took the form of the Schrödinger equation with that cosmic time. This equation implies the exponential growth of the average volume of the universe, provided the initial state of the scale factor is a Gauss wave packet. The width of the packet is an arbitrary parameter in that approach.

In the present work a principle of minimal energy of space is used in order to determine the ground state of the universe and to fix the width of the initial wave packet.

In fact, in recent years a heated argument based on the chaotic theory of inflation [11] has developed on the debated topic whether there was a beginning of the universe or whether it did not exist at all [12, 13]. Taking into account the positivity of the energy of space in a closed universe [14] we have come to conclusion that a ground state with minimal excitation of the energy of space does exist and it can be taken as a Beginning of the universe. At the same time, the ground state is a state of maximal (Planckian) vacuum energy density. It is similar to Planck energy density in the loop quantum gravity [15], which serves as a “quantum bridge” between large classical universes, one contracting and the other expanding. The minimal energy principle was used first for definition of the ground state of the universe in Ref. [16]. Notice that this state is not stationary and evolves with time. For instance, it admits quantum fluctuations in which a universe with high initial value ϕ_0 of the inflaton scalar field may be nucleated. The space energy in such a proto-inflation quantum state remains minimal admitted by the Hamiltonian constraint in a closed universe.

In this paper we propose a toy model for determination of the proto-inflation quantum state of the universe and its subsequent quantum dynamics on the condition that a cosmic time related to the inflaton scalar field is introduced [9]. Together with the initial ground state of space, excited states are introduced as well, in terms of which the exponential expansion obtained in Ref. [9] is represented as quanta of space birth. It is pertinent to note that these quanta are not the same as those of a spatial volume obtained in the loop quantum gravity [17]. Here, we call quanta the excitations of the space energy.

Minisuperspace quantum inflationary model of the universe

Let us consider a homogeneous Friedman-Robertson-Walker (FRW) model of the universe with the metric

$$ds^2 = N^2(t)dt^2 - a^2(t) \times \times (d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\varphi^2)), \quad (1)$$

and a scalar field ϕ with the four degree potential

$$V(\phi) = \lambda\phi^4 / 4$$

described by the action

$$I = \int dt \left[\frac{1}{2g}\left(\frac{\dot{a}^2 a}{N} - aN \right) - 2\pi^2 a^3 \left(\frac{\dot{\phi}^2}{2N} - \frac{\lambda\phi^4}{4} \right) \right]. \quad (2)$$

Here N is the lapse function [18], a is the scale factor of the universe, $g = 2G / 3\pi$ (G is the Newton gravitational constant).

By varying the action I with respect to the lapse function N, we obtain the Hamiltonian constraint equation

$$H = \frac{1}{2}\left(\frac{gp_a^2}{a} + \frac{a}{g} \right) - \frac{1}{2}\left(\frac{p_\phi^2}{2\pi^2 a^3} + 2\pi^2 a^3 \frac{\lambda\varphi^4}{2} \right) \approx 0, \quad (3)$$

where p_a and p_ϕ are canonical conjugate momenta to a and φ, respectively.

In conventional quantum theory Eq. (3) is replaced by the WDW equation for a quantum state

$$\widehat{H}\psi = 0$$

with

$$p_a \to (\hbar / i)\partial / \partial a, \quad p_\phi \to (\hbar / i)\partial / \partial\phi.$$

A problem of ordering of non-commuting multipliers (in the first round brackets) will arise in the definition of the operator $\widehat{H}$. It will be solved below.

We are interested in the slow-roll regime with the slowly varying scalar field ϕ. So, we shall consider the momentum

$$p_\phi = 2\pi^2 a^3 \frac{\dot{\phi}}{N}. \quad (4)$$

to be small, and, following Ref. [9], replace the kinetic energy of the scalar field in the constraint (3) by

$$-\frac{1}{3}\sqrt{\frac{\lambda}{g}}\phi p_\phi. \quad (5)$$

Then, a formula

$$t = -3\sqrt{\frac{g}{\lambda}} \ln\frac{\phi}{\phi_0}. \quad (6)$$

gives an appropriate canonical time parameter [9], where ϕ_0 is an initial value of ϕ.

According to Ref. [9], let us choose the following prescription for the operator ordering:

$$\frac{p_a^2}{a} \to -\hbar^2 \frac{1}{\sqrt{a}}\frac{\partial}{\partial a}\frac{1}{\sqrt{a}}\frac{\partial}{\partial a} \equiv -\hbar^2 \frac{\partial^2}{\partial x^2}, \quad (7)$$

where the new variable

$$x = \pm(2 / 3)a^{3/2},$$

being a square root of the volume $V / 2\pi^2$, will be considered as arbitrary real $x \in (-\infty, \infty)$.

Now, according to our choice of the cosmic time (6) which is canonically conjugate to the kinetic energy of the scalar field (5), the WDW equation in the slow-roll regime can be replaced by the time-dependent Schrödinger equation:

$$-i\hbar\frac{\partial\psi}{\partial t} = \frac{1}{2}\left[-g\hbar^2\frac{\partial^2}{\partial x^2} - 2\pi^2\frac{\lambda\phi^2(t)}{2}\frac{9}{4}x^2 \right]\psi, \quad (8)$$

where

$$\phi(t) = \phi_0 e^{-\mu t}, \ \mu = \frac{1}{3}\sqrt{\frac{\lambda}{g}}. \quad (9)$$

Here, following Ref. [9], we have neglected the spatial curvature term in Eq. (8).

Proto-inflation quantum state of the universe

Of concern to us is the quantum state of the universe at the moment $t = 0$ of the cosmic time, when the inflaton scalar field takes its maximal value ϕ_0. In the classical theory, the initial proto-inflation scale factor of the universe should be taken as the end point of the Vilenkin tunnel path determined as the non-zero solution of the equation

$$U(a) \equiv \frac{a}{g} - 4\pi^2 a^3 V(\phi_0) = 0. \quad (10)$$

The spatial curvature term is important in this definition of the initial radius of the universe before inflation:

$$a_V = \left(\frac{3}{2}\right)^{\frac{2}{3}} x_V^{\frac{2}{3}} \equiv \frac{1}{2\pi\sqrt{gV(\phi_0)}}. \quad (11)$$

According to Eq. (11), the initial pre-inflation radius of the universe is inversely proportional to the square root of a gravitational energy density $gV(\phi_0)$ related to the initial scalar field value ϕ_0. Therefore, the initial value ϕ_0 of the inflaton scalar field is related both to the beginning of the cosmic time and to the energy found for the universe before the inflation.

One can read Eq. (10) as a balance between the gravitational energy of the vacuum condensate of the initial scalar field and an "elastic energy" of the spatial curvature of the universe [18] in the initial state. This simple classical energy balance is a capsule version of the gravitational constraints in a closed universe. In the general case, it can be formulated as a variant of the positive energy theorem for a closed universe [19], which has the form of an equality of two strictly positive quantities. One of them, the square of an eigenvalue of a 3D Dirac operator in a spatial slice, which includes the "elastic energy" of the curvature, we relate to an energy of space. The second one is a positive definite energy of all physical degrees of freedom of a closed universe, including the transversal degrees of freedom of gravitational field. In Refs. [14, 16] this positive energy theorem is used to define the ground state of the universe in quantum cosmology, with a minimal energy of space and, respectively, a minimal energy of its matter content. It is this ground state that we will take in the present work as a proto-inflation initial state of the universe.

In the simple minisuperspace model given by the Hamiltonian constraint (3) the first term represents the energy of space, and the second one does the energy of the scalar field. In the classical theory, the constraint implies that both energies compensate each other. In the definition of a ground state of the universe, given in Refs. [14, 16], the equality of the quantum average values of these energies takes into account an additional condition in the minimal principle.

Returning to the inflation scenario with the only slow-rolling inflaton scalar field, considered here as a classical one, now we formulate the following principle.

Principle of the space energy minimum. *The quantity*

$$\frac{\langle\psi|\widehat{H}_x|\psi\rangle}{\langle\psi|\psi\rangle} + L\langle\psi|\widehat{H}|\psi\rangle \qquad (12)$$

must be extremal with respect to the variations of a quantum state of the universe $\psi(x)$ *and the Lagrangian multiplier L.*

Here

$$\widehat{H} \equiv \frac{1}{2}\left(\widehat{H}_x - 2\pi^2\frac{\lambda\phi_0^2}{2}\frac{9}{4}x^2\right) \qquad (13)$$

and

$$\widehat{H}_x = -g\hbar^2\frac{\partial^2}{\partial x^2} \qquad (14)$$

is the operator of the energy of space.

The kinetic energy of the inflaton scalar field is taken here to be equal to zero.

Being a positive definite, the operator of the space energy (14) has a minimal positive average value. The corresponding state of minimal energy can be approximated by a probe function, which would be taken as a Gauss wave packet (as well as that in Ref. [9]):

$$\psi_0(x) = A\exp\left(-\frac{x^2}{4\chi_0^2}\right), \qquad (15)$$

where χ_0 is a variation parameter.

For this state, $\langle x^2\rangle_0 = \chi_0^2$. This parameter is determined by the quantum constraint equation

$$\langle\psi\,|\,\widehat{H}\,|\psi\rangle = \frac{1}{2}\left(\frac{g\hbar^2}{2\chi_0^2} - 2\pi^2\frac{\lambda\phi_0^2}{2}\frac{9}{4}2\chi_0^2\right) = 0 \quad (16)$$

which gives an estimation of the pre-inflation volume of the universe:

$$\chi_0^2 = \frac{4}{9}\langle a^3\rangle_0 = \frac{2}{9\pi}l_{Pl}^2 a_V \qquad (17)$$

where $l_{Pl} \equiv \sqrt{G\hbar}$ is the Planck length.

As usual, apart from the ground state, there exists a set of extremal solutions, which obey the equation

$$[(\tilde{L}+1)\widehat{H}_x - \tilde{L}9\pi^2V(\varphi_0)x^2]\psi = W\psi \quad (18)$$

where

$$\tilde{L} = L\langle\psi|\psi\rangle,$$

plus the quantum constraint Eq. (16) for the Lagrangian multiplier L.

Solutions of Eq. (18) are eigenstates of a quantum harmonic oscillator, on the condition that $-1 < \tilde{L} < 1$:

$$\frac{1}{2}\left[-g\hbar^2\frac{\partial}{\partial x^2} + 9\pi^2\eta V(\varphi_0)x^2\right]\psi_n = \frac{W_n}{1-\tilde{L}}\psi_n \text{ ,(19)}$$

$$\eta \equiv \frac{\tilde{L}}{1-\tilde{L}}. \quad (19)$$

From Eq. (19), we obtain the energy spectrum of space

$$W_n = \frac{3}{2}\pi\hbar\sqrt{gV(\phi_0)}\left(n+\frac{1}{2}\right), \quad (20)$$

and the corresponding values of the universe's volume:

$$\left\langle a^2\right\rangle_n = \frac{1}{\pi} l_{Pl}^2 a_V \left(n+\frac{1}{2}\right) \quad (21)$$

with $\tilde{L} = 1/2$, which implies the constraint Eq. (16). Corresponding eigenfunctions are the following [20]:

$$\psi_n(x) = A_n H_n\left(\frac{x}{\sqrt{2}\chi_0}\right)\exp\left(-\frac{x^2}{4\chi_0^2}\right). \quad (22)$$

Inflation as the birth of the space quanta

So then, we suppose that the state of minimal excitation of space (15) is the initial proto-inflation state of the universe. It is not stationary for the Hamiltonian (13) and, therefore, will evolve with time. Let us consider the evolution at small times when the inflaton scalar field can be considered to be constant ($\phi = \phi_0$). The kernel of the evolution operator in this case would be calculated in the same way as that for the ordinary harmonic oscillator [21]. It becomes as follows:

$$K(x_1, T; x_0, 0) = F(T)\exp\left(\frac{i}{\hbar} I_{Cl}\right),$$
$$F(T) = \sqrt{\frac{\gamma}{2\pi g\hbar \sinh(\gamma T)}} \quad (23)$$

where

$$I_{Cl} = \frac{1}{2g}\int_0^T (\dot{x}^2 + \gamma^2 x^2)dt,\ \gamma^2 \equiv 9\pi^2 gV(\phi_0). \quad (24)$$

Here I_{Cl} is the action of the geometrical variable $x(t)$ calculated on a classical trajectory with the end points $(x_0, 0)$ and (x_1, T); it equals

$$I_{Cl} = \frac{\gamma}{2g\sinh(\gamma T)} \times$$
$$\times \left[(x_0^2 + x_1^2)\cosh(\gamma T) - 2x_0 x_1\right]. \quad (25)$$

Evolution of the initial ground state with time is described in the form

$$\psi(T, x_1) = \int K(x_1, T; x_0, 0)\psi_0(x_0)dx_0. \quad (26)$$

Some simple, but rather great calculations in volume give the following:

$$\psi(T, x_1) = AF(T)D(T) \times$$
$$\times \exp\left\{-\frac{x_1^2}{4\chi^2(T)} + i\frac{x_1^2}{2}\Gamma\right\}, \quad (27)$$
$$\chi^2(T) \equiv \chi_0^2\left[\cosh^2(\gamma T) + \left(\frac{g\hbar\sinh(\gamma T)}{4\gamma\chi_0}\right)^2\right],$$

$$\Gamma = \frac{\gamma}{g\hbar}\coth(\gamma T) \times$$
$$\times\left[1 - \frac{1}{\cosh^2(\gamma T) + \left(\frac{g\hbar\sinh(\gamma T)}{4\gamma\chi_0}\right)^2}\right]. \quad (28)$$

According to Eqs. (27), the packet width grows at large times exponentially with high accuracy:

$$D(T) = 2\chi\sqrt{\frac{\pi}{1 - 2i\gamma\chi_0^2\coth(\gamma T)/g\hbar}}. \quad (29)$$

Such behavior also results in the average volume of the universe:

$$\left\langle a^3\right\rangle \sim \exp(2\gamma T),$$

which confirms the inflation scenario, but it is of interest to describe this behavior in the context of the birth of space quanta introduced earlier.

Here we shall restrict ourselves to calculating the probability of the universe to remain at the initial ground state with time T. The amplitude of the outcome equals

$$S_{00} = A^2 F(T)D(T)Q(T) \quad (30)$$

where

$$Q(T) = \sqrt{\frac{2\pi}{R(T)}},$$
$$R(T) = \frac{1}{2\chi_0^2}\left[1 + \frac{\chi_0^2}{\chi^2(T)}\right] + i\Gamma. \quad (31)$$

The last multiplier $Q(T)$ in Eq. (30) remains limited with time, as well as the third one, $D(T)$. It is the second multiplier $F(T)$ that provides the exponential damping of the amplitude:

$$F(T) \sim e^{-\gamma T/2}. \qquad (32)$$

Since the space evolution described by the Schrödinger Eq. (8) is unitary, this result indicates the fast growth of the space quanta number. According to Eq. (30), this number increases exponentially:

$$\langle n \rangle \sim \exp(2\gamma T).$$

Having started with an initial state of the universe of minimal excitation at a given value of the inflaton scalar field ϕ_0, we find out its expansion as the birth of the space energy quanta.

Now, our interest is in the birth of matter from the original ground state. The presence of matter, different from the inflaton scalar field, can be recorded in our model as an additional part of the Hamiltonian constraint (3); for example,

$$H_\phi = \frac{1}{2}\left(\frac{p_\phi^2}{2\pi^2 a^3} + 2\pi^2 a^3 \frac{m\phi^2}{2}\right) \qquad (33)$$

for a scalar field ϕ.

Starting with its ground state, we do not obtain any sensible birth of its quanta at the inflation stage, when the potential energy of the inflaton scalar field ϕ dominates. However, according to the following quantum constraint equation

$$\langle \psi | \hat{H}_x | \psi \rangle = 0,$$

the energy of matter $\langle \psi | \hat{H}_\varphi | \psi \rangle$ is bound to be sufficiently high at the end of the inflation stage, when $\phi \approx 0$, in order to compensate the energy of space $\langle \psi | \hat{H}_x | \psi \rangle$.

The process of the birth of quanta of matter will be the subject of another work.

Summary

In conclusion, a toy model for description of the early universe in the framework of the energy approach to the quantum state and its dynamics has been developed in the present work. The Hamiltonian representation of General Relativity as a difference of the two strictly positive quantities underpinned the proposed approach.

One of these quantities, the square of the Dirac operator, relates to the scale factor of the universe, and we call it the space energy. The approach provided definite answers to the problems concerning the Beginning.

First, based on the space energy bounded below we took its ground state as the Beginning of the universe. Therefore, according to the energy approach, the Beginning does exist. In fact, this state is not stationary and evolves with time. For instance, the nucleation of the universe with the observed large-scale structure can be considered as the result of a quantum fluctuation of the inflaton scalar field up to a high value but with minimal excitation of the space energy. Thereby, the initial proto-inflation quantum state of geometry becomes definite for the subsequent inflation stage with the exponential expansion of the universe. In the case of a homogeneous model of the universe, in the present work this expansion is described by means of a Schrödinger equation with a cosmic time determined by the inflaton scalar field (its logarithm). Apart from the ground state the principle of minimal space energy determines a set of excited states with the definite quantum number of space as well. These states form a full orthonormal basis in a space of physical states of geometry. Starting from the initial ground state, the quantum number of space grows exponentially with cosmic time. At the final stage of inflation, when the potential energy of the inflaton tends to zero, the energy of the ordinary matter compensates the energy of space.

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