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Design of steel beams cross three directions with sprengel

Проектирование стальных перекрестных балок трех направлений со шпренгелем

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Key words: dimensional steel construction; cross beams with spatial tightening; terms beams strength; terms strength puffs; deflection of the structure

Ключевые слова: пространственная стальная конструкция; перекрестные балки с пространственной затяжкой; условия прочности балок; условия прочности затяжек; прогиб конструкции

Abstract. The article presents one of the possible formulations and solutions of the optimization problem of designing the cross-beams of the combined system, reinforced bongs. The basis for efforts as functions of variable parameters was a displacement method. Approximate analytical depending forces and displacements in beams, and the puffs. Analytical expressions possible to formulate the problem of how to design optimization. The article points out that the problem was posed and solved for a fixed geometry puffs and beams. This narrows the area of optimal solutions, but greatly simplifies the solution itself. The objective function – linear. Restrictions – nonlinear functions, therefore considered the problem relates to the problems of nonlinear programming. Optimum obtained in one of the points of intersection of two active constraints – for tightening strength and maximum permissible deflection of the structure. Because of this discrete mathematical optimum mix does not coincide with the physical, so the solution obtained approximately.

Аннотация. В статье приводится одна из возможных постановок и решений оптимизационной задачи проектирования комбинированной системы перекрестных балок, усиленных затяжками. Основой для получения усилий как функций варьируемых параметров явился метод перемещений. Получены приближенные аналитические зависимости усилий и перемещений, как в балках, так и в затяжках. Аналитические выражения позволили сформулировать задачу проектирования как оптимизационную. В статье обращается внимание на то, что задача ставилась и решалась при фиксированной геометрии затяжек и балок. Это суживает область оптимальных решений, однако существенно упрощает само решение. Целевая функция – линейна. Ограничения – нелинейные функции, поэтому рассмотренная задача относится к задачам нелинейного программирования. Оптимум получен в одной из точек пересечения двух активных ограничений – по прочности затяжки и по предельно допустимому прогибу конструкции. Вследствие дискретности сортамента математический оптимум не совпадает с физическим, поэтому полученное решение приближенно.

Introduction

Cross beams three areas have long established themselves as expressive architectural coatings are widely used in practice [1–5]. The task of structural design can be supplied as an optimization. This formulation combines the strength and rigidity of conditions in a single task.

Advantages of this generalization are that a minimum while achieving highest objective function and, in addition, it is clear – conditions which are decisive for the design. An important advantage in that the process of designing a number of unrelated directly manual calculation methods is naturally transferred to the computer base.

To date, the time period studied in detail the flat load-bearing structures, including both conventional ties and ties with prestressed [6–11].

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The spatial beam structures with ties studied enough and need to be studied [12].

Methods

Considers cross continuous steel beams three directions hinged at its ends to the hexagonal plan. The system of cross-beams of rolled I-beams supported by a permanent stiffening spatial Sprengel (Fig. 1).

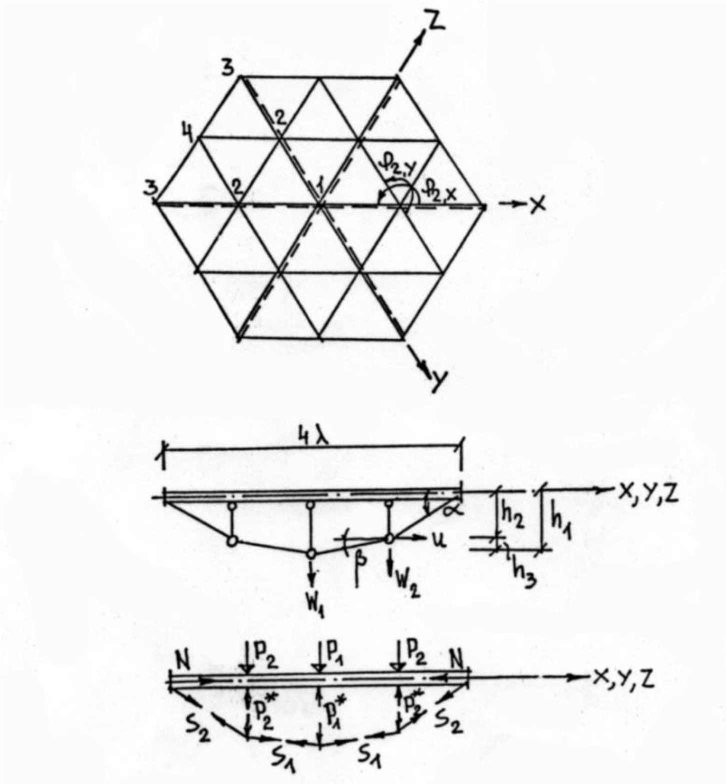

Figure 1. The investigated construction

A uniformly distributed load applied to the symmetric beams crossing sites in the form of concentrated forces. Torsion beams and shear deformations are neglected. Design Conditions coupling beams correspond to the case only a vertical interaction between them. The adopted scheme reflects well the real more complex contact conditions and significantly reduces the number of unknowns [13]. The material works elastically in accordance with Hooke's law. Stability of beams is considered wealthy. A deformation resistant is neglected.

Variable parameters are positive values:

$x_1 = \mathrm{A}$ – the cross sectional area of beams; $x_2 = \mathrm{A}_{\circ}$ – area of ties; $x_3 = \rho$ – core distance; $x_4 = h_6$ – the height of the beams; $x_5 = J$ – moment of inertia of beams.

The objective function, without the expense of influence on steel racks and therefore roughly expressing the volume of steel, can be written:

$$Z = 30\lambda x_1 + 6\lambda\left(\frac{1}{cos\alpha} + \frac{1}{cos\beta}\right) x_2 => min \quad (1)$$

A feature of optimization problems bearing structures as opposed to the economic problems, a small number of restrictions, but there is cumbersome analytical expressions of these restrictions. Sometimes itself getting analytical expressions in closed form is not simple and often independent research task.

Restrictions on the objective function (1) form three groups of equalities and inequalities:

I. Equality satisfying the conditions of equilibrium, physical and geometrical equations:

$$r \cdot [w] + [p] = 0, \quad (2)$$

where r – the coefficient matrix of the canonical equations of motion method comprising variable parameters;

[w] – the displacement vector components;

[p] – load vector.

The expanded form of equations (2) can be written as (3)

$$\begin{bmatrix} (72\frac{EJ}{\lambda^3}+36\frac{E_0A_0}{\lambda}\sin^2\beta\cos\beta) & -(72\frac{EJ}{\lambda^3}+6\frac{E_0A_0}{\lambda}\sin^2\beta\cos\beta) & -36\frac{EJ}{\lambda^2} & 0 & -6\sin\beta\cos^2\beta\frac{E_0A_0}{\lambda} \\ -(12\frac{EJ}{\lambda^3}+6\frac{E_0A_0}{\lambda}\sin^2\beta\cos\beta) & \left[21\frac{EJ}{\lambda^3}+\frac{E_0A_0}{\lambda}(\sin^2\beta\cos\beta+\sin^2\alpha\cos\alpha)\right] & 3\frac{EJ}{\lambda^2} & -6\frac{EJ}{\lambda^2} & \frac{E_0A_0}{\lambda}(\sin\beta\cos^2\beta\sin\alpha\cos^2\alpha) \\ -6\frac{EJ}{\lambda^2} & 3\frac{EJ}{\lambda^2} & 7\frac{EJ}{\lambda} & 0 & 0 \\ 0 & -3\frac{EJ}{\lambda^2} & 0 & 5\frac{EJ}{\lambda} & 0 \\ -6\frac{E_0A_0}{\lambda}\sin\beta\cos^2\beta & (\sin\beta\cos^2\beta+\sin\alpha\cos^2\alpha)\frac{E_0A_0}{\lambda} & 0 & 0 & (\cos^3\beta+\cos^3\alpha)\frac{E_0A_0}{\lambda} \end{bmatrix} \tag{3}$$

$$x\begin{bmatrix} W_1 \\ W_2 \\ \varphi_{2x} \\ \varphi_{2y} \\ u \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where W_1 – vertical movement of the i-th node

φ_{2i} – rotation angles of the node 2

u – horizontal movement of the coupling assembly of ties

II. Inequalities express terms of strength and stiffness of individual bearing elements and structure as a whole:

In particular:

1) The condition of strength beams according to the criterion of normal stresses

$$g_1 = 1 - \frac{M_1}{W_x R_y} - \frac{N}{A R_y} \geq 0\,, \tag{4}$$

where W_x – section modulus;

N – the longitudinal force in the beam;

i – beam cross section under consideration;

R_y - standardized resistance of steel.

2) Conditions strength of ties

$$g_2 = 1 - \frac{S_2}{A_0 R_y} \geq 0, \tag{5}$$

where S_2 – the greatest force in the tie.

3) Conditions design stiffness

$$g_3 = 1 - \frac{W_1}{[f]} \geq 0, \tag{6}$$

where $[f] = \frac{1}{400} * l$ – allowable amount of deflection.

III. Inequalities, reflecting the peculiarities of the constructive form

$$g_4 = A > 0 \tag{7}$$

$$g_5 = A_0 > 0 \tag{8}$$

$$g_6 = A - \mathrm{A}_0 > 0 \tag{9}$$

Condition (4–9) are natural for the designer, but it is very important to the mathematical formalization of the problem. In order to reduce the number of constraints can be eliminated system (2) of the problem, and then analytically approximated expression of internal forces as continuous functions of variable parameters. For simplicity, we will solve the problem fixed geometry of ties ensemble:

$$h_1 = \frac{9}{24}\lambda;\ \ h_2 = \frac{\lambda}{3}\quad \text{and denote}\ \ K = \frac{E_0}{E} * \frac{A_0}{J} * \lambda^2$$

The dimensionless parameter "K" reflects the impact of the tightening of all varying sizes.

In this case the system of equations (3) can be rewritten in the form (10).

$$\begin{bmatrix} (72+0.0622К) & -(72+0.01036К) & -36 & 0 & -0.249168К \\ -(12+0.01036К) & (21+0.09649К) & 3 & -6 & 0.326076К \\ -6 & 3 & 7 & 0 & 0 \\ 0 & -3 & 0 & 5 & 0 \\ -0.249168К & 0.326076К & 0 & 0 & 1.85102К \end{bmatrix} \text{x} \begin{bmatrix} \widehat{W_1} \\ \widehat{W_2} \\ \widehat{\varphi_{2x}} \\ \widehat{\varphi_{2y}} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (10)$$

The constancy of geometry narrows the area of optimal solutions, but greatly simplifies the approximation expression. The solutions of the system (10) for different values of "K" are presented in Table 1. As can be seen from Table 1 tightening spatial trussed significantly increases rigidity, and greatly reduces the internal forces in the beams lowering steel consumption. Table 1 allows to obtain approximate analytical expressions for beams, and for delay. With an accuracy of less than 5 % in the range 100 ≤ K ≤ 250 true formula (11–13) (for A <100 can quadratic approximation):

$$N = (2.41925 + 0.002927K)P \qquad (11)$$

$$S_2 = (2.54867 + 0.003093K)P \qquad (12)$$

$$M_1 = (0.13 - 0.00032K)P\lambda \qquad (13)$$

Table 1. The solutions of the system (10) for different values of "K"

Deformation and strength factors			System without ties K=0	$K = \frac{E_0 A_0}{EJ_x} * \lambda^2$ System with ties K=100	K=150	K=200	K=250	Factor
Linear displacement	Vertical	W_1	0.561	0.131	0.095	0.075	0.062	$\frac{P\lambda^3}{EJ_x}$
		W_2	0.390	0.090	0.065	0.050	0.041	
	Horizontal	u_x	-	0.002	0.001	0.001	0.001	
The angular displacement		$\varphi_{2,x}$	0.313	0.074	0.054	0.043	0.035	$\frac{P\lambda^2}{EJ_x}$
		$\varphi_{2,y}$	0.234	0.054	0.039	0.030	0.025	
bending moments		M_1	0.397	0.101	0.076	0.062	0.053	$P\lambda$
		$M_{2,x}$	0.230	0.047	0.032	0.023	0.018	
		$M_{2,y}$	0.468	0.108	0.078	0.060	0.049	
Transverse force		Q_{1-2}	0.167	0.054	0.044	0.039	0.036	P
		Q_{2-3}	0.230	0.047	0.032	0.023	0.018	
		Q_{2-4}	0.468	0.108	0.078	0.060	0.049	
Efforts in ties		S_1	-	2.714	2.942	3.073	3.154	
		S_2	-	2.859	3.099	3.237	3.328	
Largest strength in the longitudinal beams		N	-	2.712	2.939	3.070	3.151	
Efforts in spacers		P_1^*	-	0.678	0.734	0.767	0.787	
		P_2^*	-	0.791	0.857	0.896	0.919	

The deflection at the geometric center of the "K" = 100 ÷ 250 is approximately

$$W_1 = (0.1777 - 0.000464K)\frac{P^H \lambda^3}{EJ} \qquad (14)$$

The analytical expressions are substituted into the limit II and III Group.

Having specific values: P=10т ; $\lambda = 6m$; $E = 21 * 10^6\ t/m^2$;

$[f] = \frac{l}{400} = \frac{\lambda}{100}; \frac{P}{pH} = 1.2; R_y = R_0 = 21000\ t/m^2$

and approximate dependence: $J \approx \frac{A\rho h_6}{2} = \frac{x_1 x_3 x_4}{2}$; $h_6 \approx \frac{\rho}{0.32}$; $x_3 \approx 1.818\sqrt{x_1}$ can be reduced to the task with five varying parameters to two independent parameters x_1 и x_2. Thus the objective functions with a fixed geometry and flow of ties neglecting steel rack written more specifically:

$$Z = 180x_1 + 74.16x_2 \ => \ min \quad (15)$$

Restrictions:

The strength of beams, which receive the thrust of ties:

$$g_1 = 1 - \frac{0.00115}{x_1} - \frac{0.0000996x_2}{x_1^3} - \frac{0.000204}{x_1\sqrt{x_1}} + \frac{0.0000035x_2}{x_1^3\sqrt{x_1}} \geq 0 \quad (16)$$

Durability of ties:

$$g_2 = 1 - \frac{0.00121}{x_2} - \frac{0.000102}{x_1^2} \geq 0 \quad (17)$$

The rigidity of the structure:

$$g_3 = 1 - \frac{0.000049}{x_1^2} + \frac{0.000000073x_2}{x_1^4} \geq 0 \quad (18)$$

Restrictions defined physical meaning of the problem:

$$g_4 = x_1 > 0 \quad (19)$$

$$g_5 = x_2 > 0 \quad (20)$$

Restrictions defined structural features form:

$$g_6 = x_1 - x_2 > 0 \quad (21)$$

Characteristic mathematical model is reduced to four positions:

1. The mathematical model described by continuous and differentiable functions.

2. The objective function – linear.

3. Restrictions on the objective function – nonlinear.

4. The problem relates to a class of constrained optimization problems.

The scientific literature on the issues of language and methods of solving nonlinear programming problems is extensive [14–16], and implemented by the mathematical ideas very diverse [17–20].

Results and Discussion

One possible solution to this problem of constrained optimization is to convert it into unconstrained optimization problem. To do this, enter all six additional restrictions are non-negative variables $u_i^2 \geq 0$, thus converting inequality constraints limit equality. Further, writing the Lagrangian:

$$F(x, m, u) = Z(x) + \sum_{i=1}^{8} m_i[g_i(x) + u_i^2 - b_i], \quad (22)$$

determine the necessary conditions for the existence of extremum, the so-called Kuhn-Tucker conditions:

$$\frac{\partial F}{\partial x_j} = 0 \qquad j = 1, 2$$

$$\frac{\partial F}{\partial m_i} = 0 \qquad i=1,\dots,6 \quad (23)$$

$$\frac{\partial F}{\partial u_i} = 2m_i u_i = 0 \qquad i=1,\dots,6$$

In our case, we get a system of 14 nonlinear equations. Among all those roots only examine the system that satisfy the system of constraints (16) – (21),i.e. ,

$$x_1^* = 0.00682\,m^2;\ x_2^* = 0.0016\,m^2;\ \ m_1^* = 0.1124;\ m_2^* = 0.6231$$

The Hessian matrix at the extremum point is positive:

$$H = \begin{bmatrix} \dfrac{\partial^2 F}{\partial x_1^{*2}} & \dfrac{\partial^2 F}{\partial x_1^* \partial x_2^*} \\ \dfrac{\partial^2 F}{\partial x_2^* \partial x_1^*} & \dfrac{\partial^2 F}{\partial x_2^{*2}} \end{bmatrix} = \begin{bmatrix} 24509 & 12669 \\ 12669 & 66408 \end{bmatrix} = 146709{\times}10^4 > 0$$

Therefore, found a fixed point is a point of mathematical minimum.

Because of this discrete mix (I №40 B1) [21], the actual point of optimum

$x_1^{**} = 0.00746;\ x_2^{**} = \ 0.0017$ It is somewhat different from the mathematical.

As seen in Figure 2. Mathematics optimum is at the border of the feasible region at the intersection of two active constraints – on the condition tightening strength and maximum allowable deflection structure.

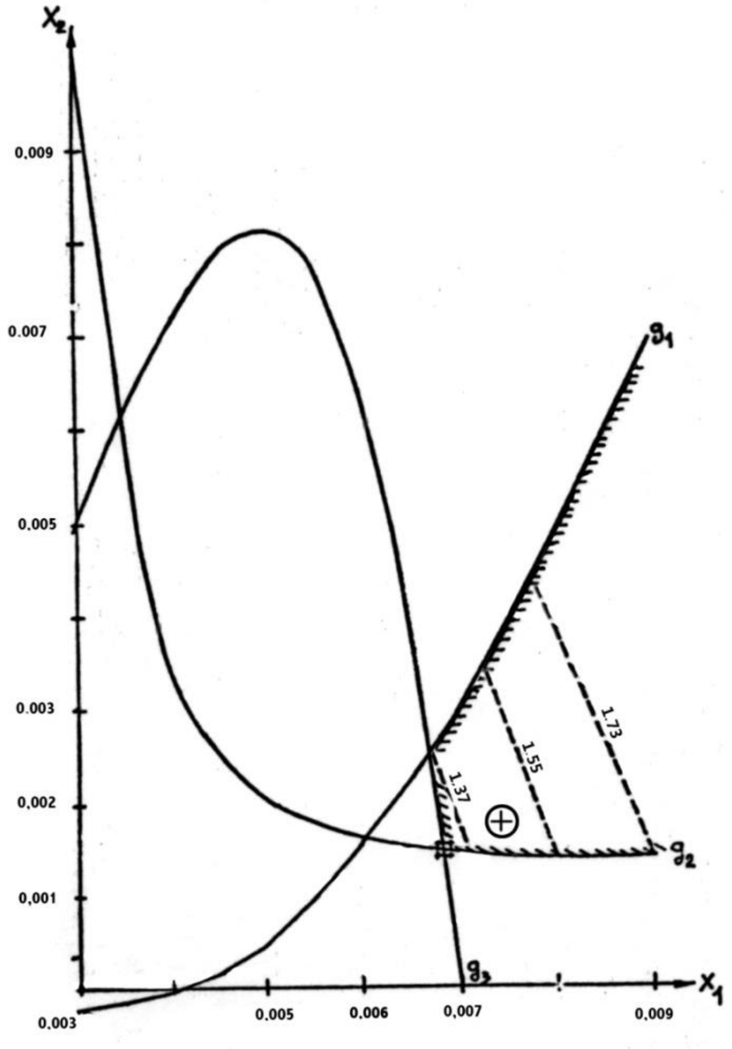

Figure 2. Graphical interpretation of the problem being solved

where – Point of mathematical optimum;

– the optimum point in connection with the readability of the assortment;

– line levels of the objective function;

– restrictions

– the range of permissible values

The results of the decisions shows that the parameter K = 247.9, and is within the accepted linear approximations (11–14).Terms beams strength g_1 are not critical and are beyond the tolerance range varying variables, so a more accurate calculation of the deformed scheme does not lead to the results of other solutions. In particular, for the case settlement of the deformed scheme causes an additional increase in stress in the beams by 3.5 %, which is less than the permissible 5% according to the norms. [22]

The slope of the line levels of the objective function shows that the most effective way to further reduce the consumption of steel is to increase the height of the Sprengel.

Steel flow without weight ratio of spacers and the construction is 25.6 kg/m2, which is a very good indicator.

Conslusion

The solution provided by the problem is not unique. It is worth considering, and other formulations, and use other methods of solution. The accumulation of experience solving similar close to the practice, the tasks, the proposed methodology will improve and clarify many of the questions of the real design.

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