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Semi-rigid steel beam-to-column connections

Податливые соединения стальных балок с колоннами

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Key words: semi-rigid joint; steel frame; beam; stiffness; rotational angle; support moment

Ключевые слова: податливый узел; стальной каркас; ригель; жесткость; угол поворота; опорный момент

Abstract. Steel frameworks are widely used construction of multistory buildings for different purposes. In the practical framework calculations, the girder-column joint connections are taken as either absolutely rigid or hinged ones. The analysis of the actual behaviour of the frame joint connections shows that they normally occupy an intermediate position in the joints classification into "rigid" and "hinged" ones, i.e. they have certain pliability. Such pliability is characterized by different grades of stiffness that depends on a specific design solution of a joint. Therefore, to avoid possible material errors, the statistical calculations of frames should consider layouts with the joints that are able to support the corresponding amount of bending moments. This article contains the results of experimental and theoretical research of the actual behaviour of the girder-column connection semi-rigid joints using ABAQUS 6.13 computing complex, which enables us to solve problems by the finite elements method with due regard to the geometrical and physical nonlinearity. We consider the design of a beam-to-column connection with connecting elements in the form of paired vertical angles bolted to the beam wall and to the column flange. Based on the comparative analysis of the results of the numerical analysis and on the experimental data, the actual behaviour of the structure has been found and the stiffness of the joint type to be considered has been determined.

Аннотация. В строительстве многоэтажных зданий различного назначения широко применяются стальные каркасы. Узловые сопряжения ригелей с колоннами при практических расчетах рам принимаются либо абсолютно жесткими, либо шарнирными. Анализ действительной работы узловых соединений рам показывает, что они, как правило, занимают промежуточное положение в классификации узлов на «жесткие» и «шарнирные», то есть обладают некоторой податливостью. Такая податливость характеризуется различной степенью жесткости, зависящей от конкретного конструктивного решения узла. Поэтому во избежание возможно существенных ошибок в статических расчетах рам должны рассматриваться схемы с узлами, способными воспринимать соответствующую долю изгибающих моментов. В настоящей статье приводятся результаты экспериментально-теоретических исследований действительной работы податливых узлов сопряжения ригелей с колоннами с использованием вычислительного комплекса ABAQUS 6.13, позволяющего решать задачи методом конечных элементов с учетом геометрической и физической нелинейности. Рассматривается конструкция узла «ригель-колонна» с соединительными элементами в виде парных вертикальных уголков, прикрепляемых к стенке балки и полке колонны с помощью болтов. На основе сравнительного анализа результатов численного расчета и экспериментальных данных выявлена действительная работа конструкции и определена жесткость рассматриваемого типа узла.

Introduction

Metal frameworks of multistory buildings and facilities are represented a complex of structural elements connected in joints. The reliability of a facility in general is equally determined by the reliability of its separate load-carrying structural elements and the faultless work of their joint connections.

Local forces that are characterized by significant bearing reactions in the form of concentrated forces and bending moments occur in the places of connection between girders and columns in a framework building system under operational loads. Flowing from one element to another within small contact areas, those forces lead to uneven distribution of stress within the joint area, which causes

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development of excessive deformations, as well as occurrence and development of cracks, etc. That is an evidence of the fact that the girder-column connections are the most critical joints in the framework buildings [4, 5–7, 9, 12–14].

As of today, for practical calculation of frames in the framework building systems, the beam-to-column joints are normally taken as divided into the following two types: absolutely rigid and hinged. The results of experimental and theoretical research on the behaviour of beam-to-column connection joints in steel frameworks show that they have certain pliability that is characterized by different grades of stiffness affecting both the actual behaviour of the frame as a whole and the distribution of the metal in its main elements, columns and girders.

The results of both experimental and theoretical research of the actual behaviour of semi-rigid beam-column joints by both foreign and domestic scientists clearly demonstrate the influence of the stiffness of such connections on the load-bearing capacity of the columns. Thus, the academic papers state that the load-bearing capacity of the columns in the frames with semi-rigid joints are underestimated at the average of 40 % in comparison to the frames that have hinged beam-to-column connections. On the other hand, the load-bearing capacity of the columns in the frames that have semi-rigid joints might be unreasonably increased, if they are considered as rigid [10, 11, 14–16, 23, 24, 27, 35].

Semi-rigid beam-to-column connection joints in braced frameworks are sufficiently diverse in terms of design solution; however, they are all characterized by the presence of steel plate flexible elements that contribute to relatively free rotation of the beam within a joint when working under load. In addition, the connection elements show the development of plastic deformations, and the connected elements show a change in the stress and strain-state in comparison to rigid joints. It is particularly important to take into account the grade of stiffness of the beam-to-column joint connections in the structures working beyond the elastic limit.

It is obviously possible to receive a reliable picture of the stress-strain state of joints of frames in an elastic-plastic stage of work on the basis of the numerical methods of calculation which are widely used at design of buildings today. So the analysis of numerical researches of semi-rigid joints taking into account geometrical and physical nonlinearity is provided in works [1, 6, 17–20, 22, 25–34].

Taking into consideration the fact that the joint pliability depends on a considerable variety of factors conditioned by the peculiarities of the design solution of a beam-to-column connection, it is quite difficult to assess. However, many scientists have attempted to resolve this problem to a greater or lesser degree of proximity; based on the results of their experimental and theoretical research, it can be concluded that the main factor that determines the joint pliability is the deformation of the connection elements, which can be up to 80% of the total joint deformation and directly depends on its design solution. The rest of the deformations occurring in a semi-rigid beam-to-column connection, such as a bend of a column flange, section shear and column bend deformations within the joint area, etc., have practically no influence on its flexibility [7, 10, 11, 15, 23, 27, 35].

The foreign and domestic experience of design and construction of multistory buildings shows that the design solutions of beam-to-column connection joints with double angles are the most appropriate for braced frameworks due to their constructability and low metal consumption.

The purpose of this work was studying stress-strain state and destruction of bolted joint with double angles in an elastic-plastic stage of work on the basis of the comparative analysis of numerical calculation with use of the ABAQUS 6.13 [2] computer system and experimental data [35].

The objectives of the research included the following:

- assessment of the limit state of the structure;
- study of the distribution of internal forces within the joint areas of a frame fragment;
- identification of the locations of the highest stress concentration within a joint;
- analysis of the strain state of connection angles, beam and column;
- angle stiffness assessment.

Methods

The study of the stress-strain state considered type of joint was carried out on the example of the constructive decision with connecting elements in the form of the double angles bolted to the beam wall and to the column flange (Fig. 1) as an example.

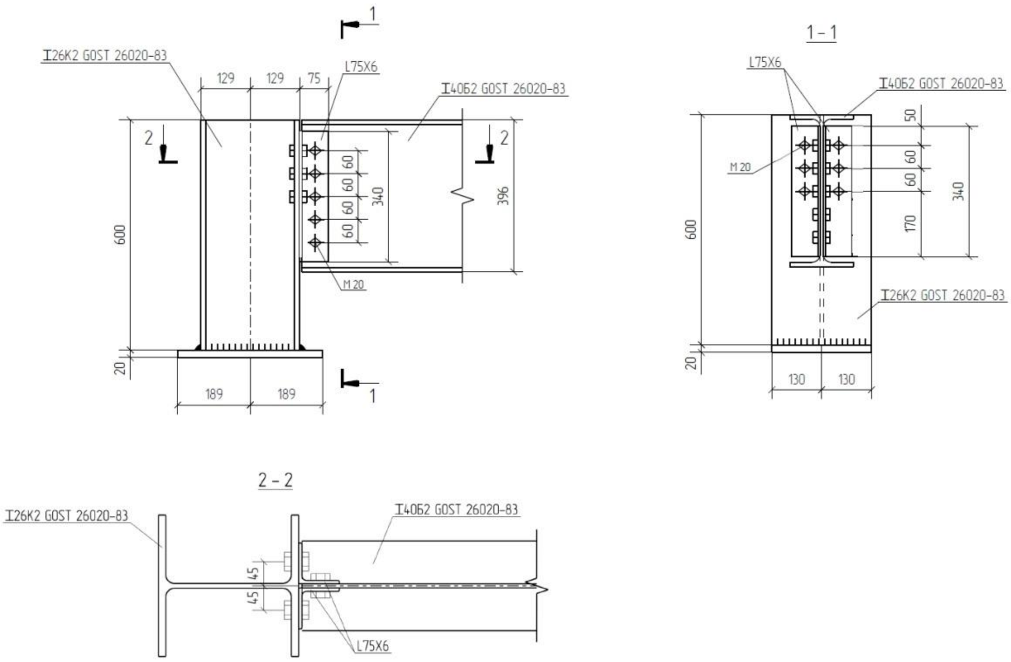


Figure 1. A design solution of a bolted joint with double vertical angles

To make the experimental model as close to the natural conditions as possible, it was made in the form of a U-shaped framework fragment with a beam made of a normal (Б type per Russian classification) wide-flanged I-beam and posts made of a column (К type per Russian classification) wide-flanged beams. Taking into consideration the fact that the influence of the column stiffness on the joint behaviour is insignificant, the fragment height was determined as possibly small. We took sufficient height of the beam section to allow significant angles of rotation and deflection. To increase the load and extend the stage of beam elastic work, the load was applied on span quarters (Fig. 2).

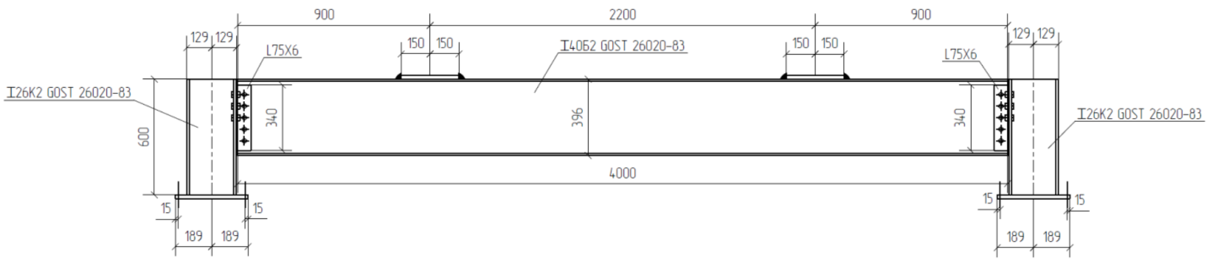
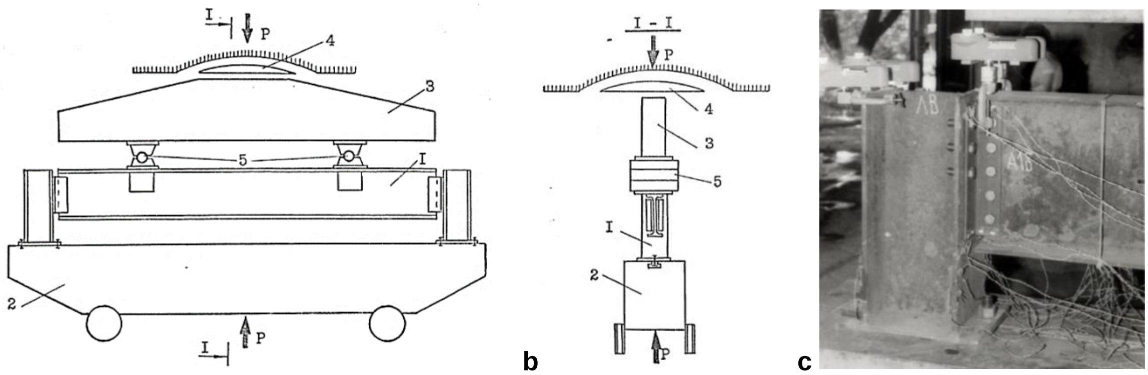


Figure 2. Scheme of the structure to be considered

Figure 3 shows a scheme of sample testing in a natural experiment, as well as a frame joint with measuring instruments.



a

Figure 3. Static testing of a frame fragment
a – scheme of static testing of frame fragment; b – frame joint with measurement instruments;
1 – frame; 2 – large cart; 3 – cross-arm; 4 – press ball hinge; 5 – hinges

The experiment was carried out using M20 normal precision bolts of strength grade 5.8 without pretension. Double vertical angels 75x6 (length 340 mm) were made of S345 low-alloy steel, and the beam (40Б2 profile) was made of S245 steel.

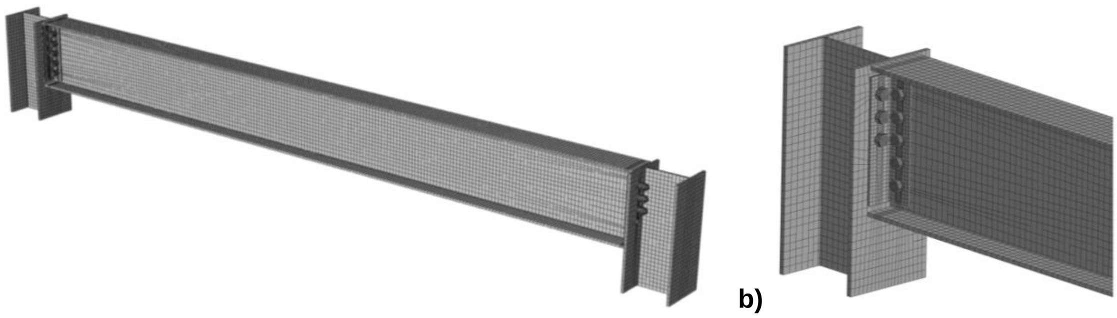
The numerical research was carried out using ABAQUS 6.13 computing complex, which allows to resolve the problems of the structural mechanics and the materials resistance applying the finite elements method. An explicit solver (Abaqus/Explicit) [2] was used to solve the problem considering the geometrical and physical nonlinearity and to compare the results to the experiment data [35].

According to the testing, the load was applied on two nodes located in the gravity centre of absolutely rigid bodies in a quasi-static manner during 10 seconds with a limit total value of 1000 kN.

The structural elements and the joint parts were made using a standard model (Plasticity), that allows to consider not only the physical nonlinearity and the descending unloading branch during the material's work beyond the elastic limit, but also the structure damage (Ductile Damage) when achieving the limit stress values typical for steel [3] in the joints elements.

The finite-element computing model was formed by volume octagonal finite elements (C3D8). The mesh was condensed using tetrahedrons (C3D10) in order to obtain a more precise picture of the stress state of the joints for the connection parts and the beam area adjacent to a joint. The mesh was performed under the condition of including no less than 2 elements based on the thickness of the parts.

Figure 4 shows the finite-element model.

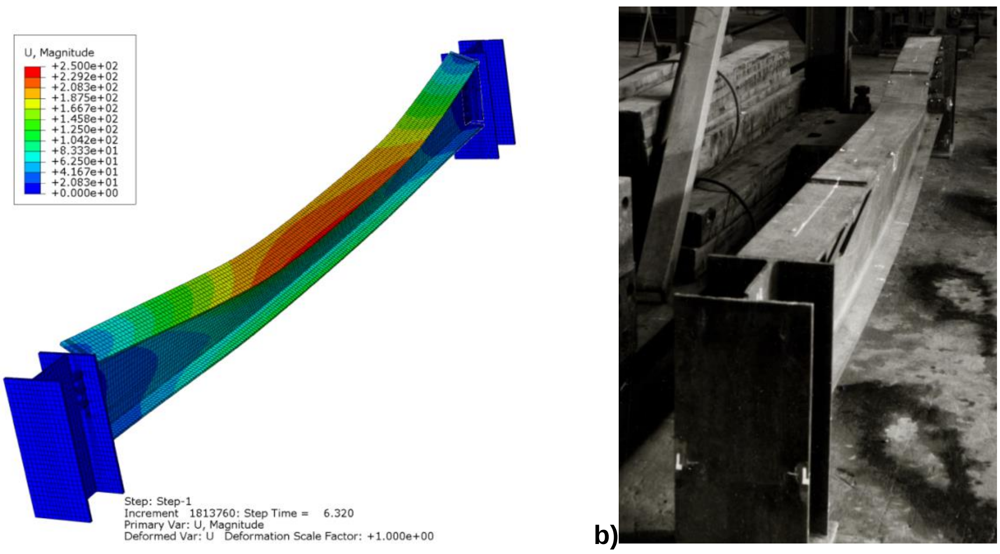


a) b)

Figure 4. Finite-element model:
a – general view of the frame; b – beam-to-column connection joint

Results and Discussion

It was established experimentally [35] that the exhaustion of the load-bearing capacity of the structure consisted in the loss of beam stability at the plastic stage of work of the flange in the simple bending area. In addition, local loss of flange stability was noted as well (Fig. 5b).



a) b)

Figure 5. Structure limit state:
a – numerical analysis; b – experiment

The numerical analysis confirmed the nature of damage of the tested structure established by the testing (Fig. 5a). The results of the numerical calculation showed that the beam transition into the elastic-plastic stage of work occurs at P = 500 kN; however, no structural damage occurs in this case, and the structure keeps working in case of further load increasing. This can be explained by the restraining effect of the neighbouring "rigid" areas on the development of deformations in the plastic areas. Joint damage in the form of angle rupture close to the back edge occurred at the load of P = 632 kN (Fig. 6).

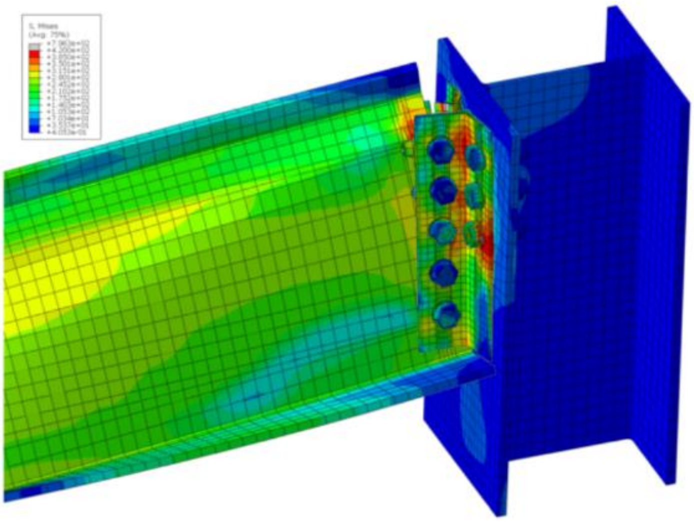


Figure 6. Joint damage at P=632 kN (Abaqus)

The general nature of the structure's work under load reflects the "load-deflection" dependence. The load deflection dependence graphs (fig. 7) obtained on the basis of the experiment [35] and numerical calculation data show that the elastic-plastic stage of work of the beam within the wide area of a simple bend occurs at the load of 500 kN, which is characterized by a sharp increase in the inclination angle at that moment.

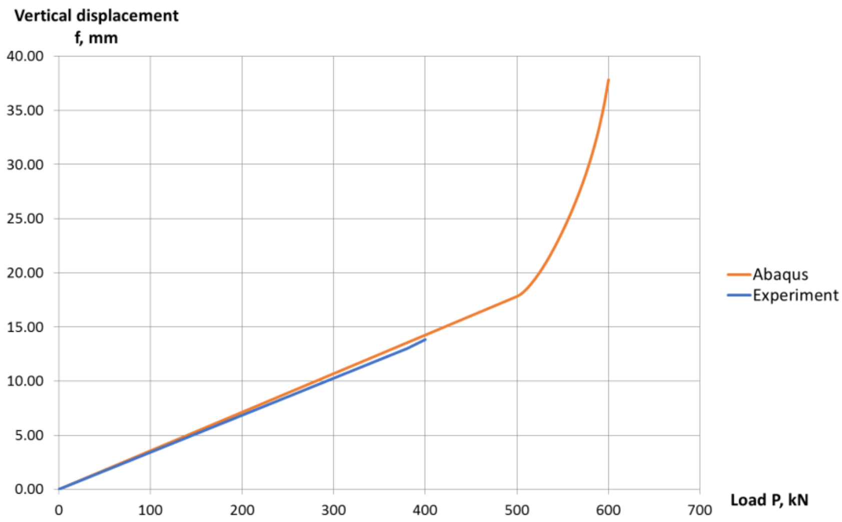


Figure 7. The "load-deflection" dependence graph

There is no conflict between the numerical calculation and the experiment data [35], and the difference between the corresponding values does not exceed 4% (Table 1).

Table 1. The comparison of the vertical displacements of the beam middle point

Load *P*, kN	Vertical displacement of the beam middle point *f*, mm		Difference, %
	Abaqus	*Experiment*	
0	0	0	0
48	1.71	1.64	-3.99 %
100	3.56	3.42	-3.99 %
148	5.27	5.06	-3.99 %
200	7.13	6.84	-3.99 %
248	8.84	8.48	-3.99 %
300	10.69	10.26	-3.99 %
348	12.40	11.91	-3.99 %
400	14.25	13.83	-2.94 %
448	15.96	-	-
500	17.82	-	-

Figure 8 contains the graphs of dependence of the rotational angle of the beam support section on the load that were made based on the experimental data [35] and the numerical calculation results. Sufficient similarity is noted at the stage of elastic work of the joint. The figure shows that the numerical curve (Abaqus) has two typical break points corresponding to the load (P_{lim}) at which a plastic mechanism is formed within the connection angles, and to the load (P_T) at which the primary yield occurs in the area of a simple bending of the beam. At this moment, a plastic hinge is formed within the joint, and the nature of the "load-rotational angle" dependence changes, since the load-bearing joint loses the capacity of elastic restraint and the girder works in the frame system as a simply supported beam.

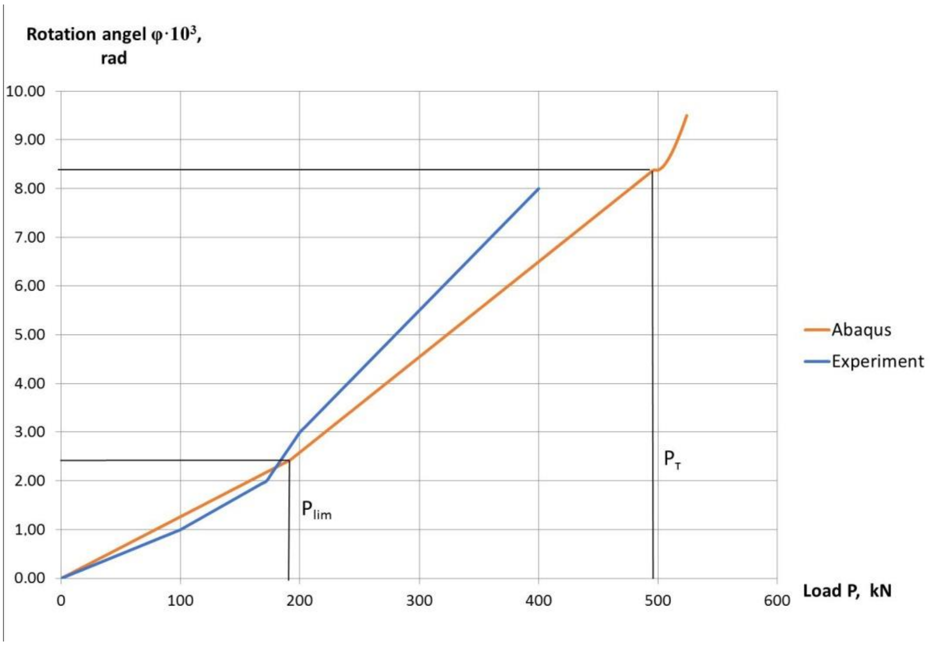
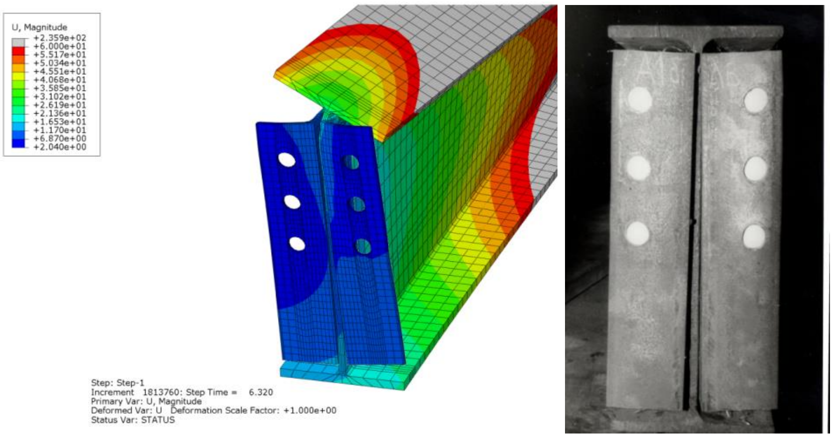


Figure 8. The graph of "load-rotation angle" dependence

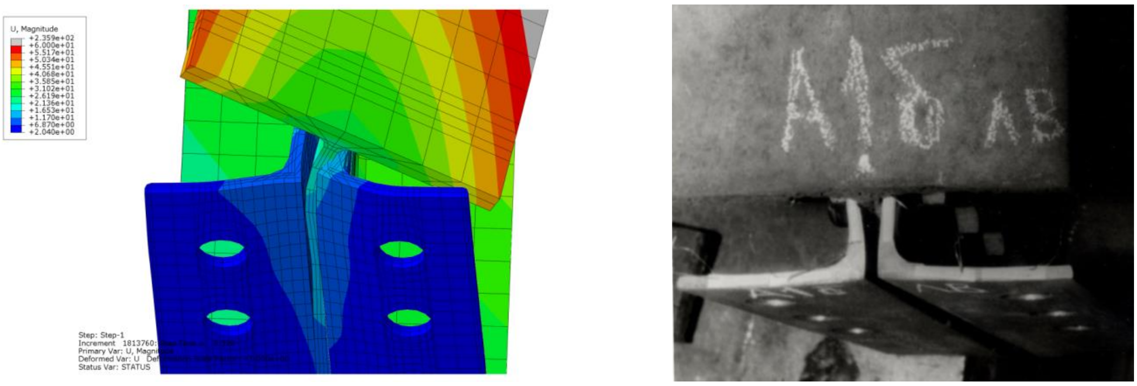
Table 2. The comparison of the values of rotational angel of support cross-section

Load *P*, kN	Rotational angel of support cross-section $\varphi \cdot 10^3$, rad		Difference, %
	Abaqus	*Experiment*	
0	0	0	0
48	0.61	0.48	-21.09 %
100	1.27	1.00	-21.09 %
148	1.88	1.67	-11.14 %
200	2.59	3.00	13.68 %
248	3.53	4.20	15.99 %
300	4.55	5.50	17.36 %
348	5.48	6.70	18.15 %
400	6.50	8.00	18.74 %
448	7.44	-	-
500	8.38	-	-

Beside a significant bevelling (horizontal displacement of the angle back side in respect of the column flange), the deformations of the connection angles bolted to the beam and the column are characterized by a strong approximation of the back sides in the upper and their separation from each other in the lower part of the joint. In this case, the angles rotate within the plane of the column flange. This is explained by the fact that, beside the bending moment out of the plane of the "column" angle flange, there is a moment acting in the plane of the flange, rotating the connection angle. Therefore, a couple of forces acting in this area brings the angle back sides together in the upper part and separates them in the lower part. The deformations of the holes of the connected elements, which are inherent to bolted connections, contribute to that to a large extent. According to the testing results, the value of the limit bevelling of the angles amounted to 11 mm [35], and according to the results of the numerical calculation, it amounted to 7 mm.



a)



b)

Figure 9. Strain state of joint elements:
a – strain state of angles (view of column flanges); b – strain state of angles (top view)

The stress state of the tested structure is clearly demonstrated by the graphs of "load-span moment" dependence (Fig. 10). The linear dependence of the "load-span moment" curves can be seen up to the load of P = 500 kN. At this moment, the initial yield occurs in the simple bending area. In addition, the bending moment grows disproportionally to the load applied. Then, the beam's work reaches the next stage corresponding to the formation of an elastic hinge in the beam span section, when the span moment reaches its limit value and remains constant for some period until the exhaustion of the load-bearing capacity of the beam.

The pictures of the stress state of the beam at P = 500 kN in the form of isofields of normal and equivalent stresses obtained through a numerical calculation, as well as of the vertical displacement, are given on figures 11 and 12 respectively.

Table 3 contains the values of the span moment M_{sp} obtained both through a numerical analysis and experimentally [35], the difference between which does not exceed 6 %.

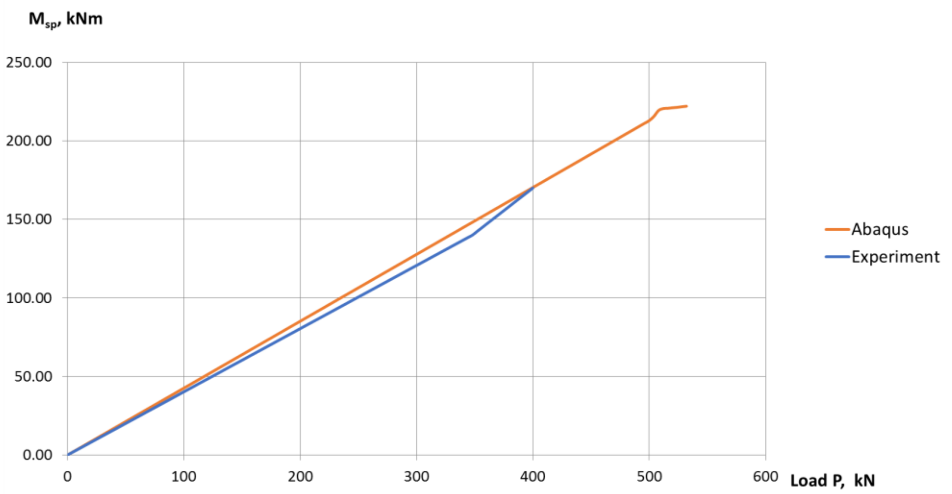
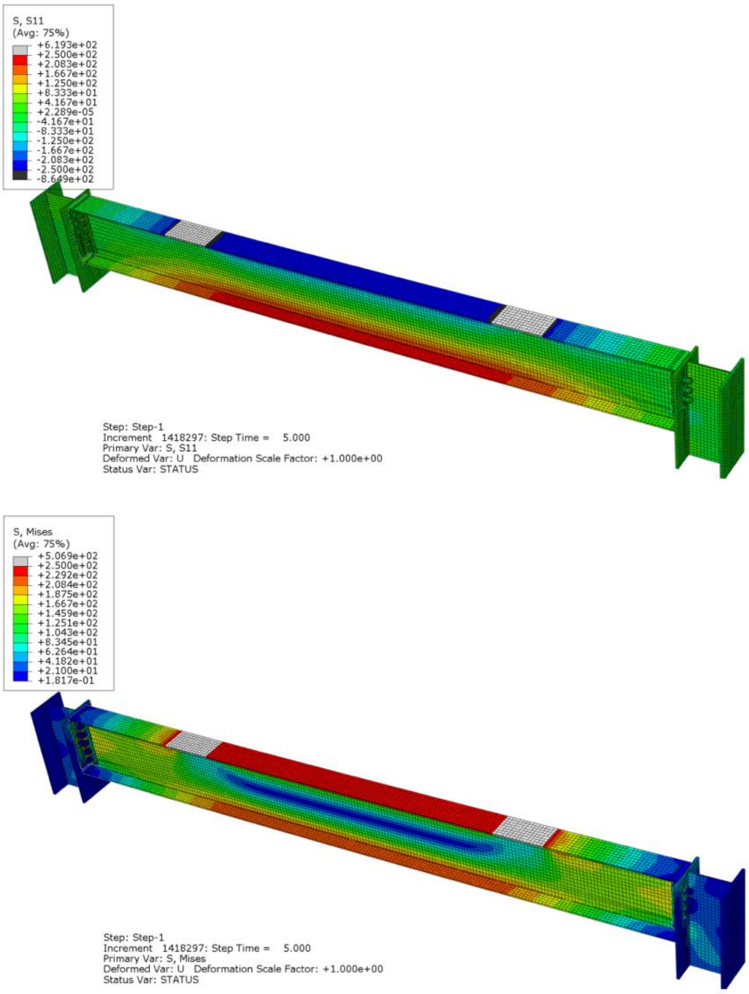


Figure 10. The graph of "span moment-load" dependence

Table 3. The comparison of the values of span moment

Load *P*, kN	**Span moment M_{sp}, kNm**		***Difference, %***
	Abaqus	*Experiment*	
0	0	0	0
48	20.44	19.31	-5.52%
100	42.58	40.23	-5.52%
148	63.02	59.54	-5.52%
200	85.16	80.46	-5.52%
248	105.60	99.77	-5.52%
300	127.74	120.69	-5.52%
348	148.18	140.00	-5.52%
400	170.32	170.00	-0.19%
448	190.76	-	-
500	212.91	-	-



a)

b)

Figure 11. Stress state of the beam at P = 500 kN (numerical analysis): contour plots of normal (a) and equivalent (b) stresses within the beam (MPa)

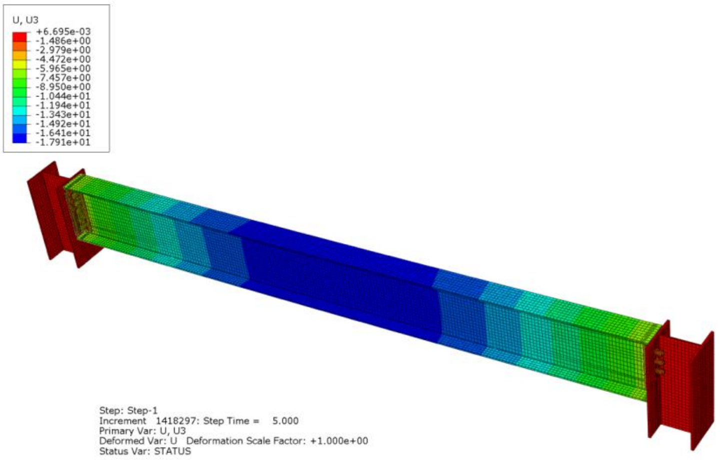


Figure 12. Vertical displacement (mm) of the beam at P = 500 kN (numerical analysis)

Numerical studies have revealed the general picture of the stress-state of connection angles and the adjacent column areas and beam (Figs.13, 14). The figures show that a strong bending nature of the stress-state is inherent to the "beam" flanges of angles. The horizontal stresses acting in the points along the diagonals of the "column" flanges of angles characterize the bending nature of deformation of these flanges from the plane with a restraint along the bolts axis and next to the angles curve.

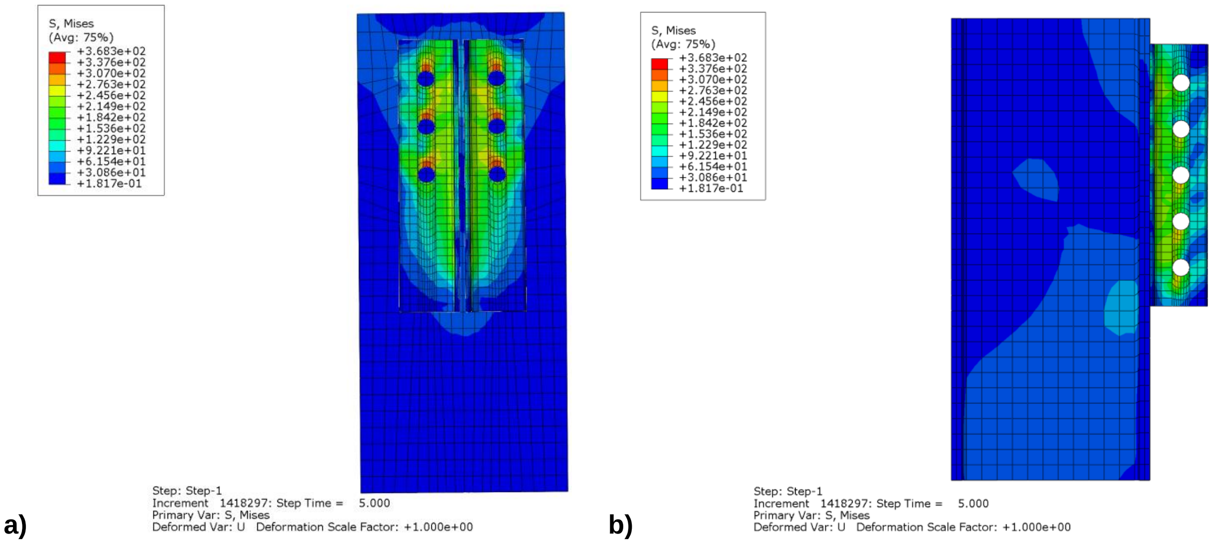


Figure 13. Stress state of the connection angles (numerical analysis): at P = 500 kN: a – "column" flanges; b – "beam" flanges

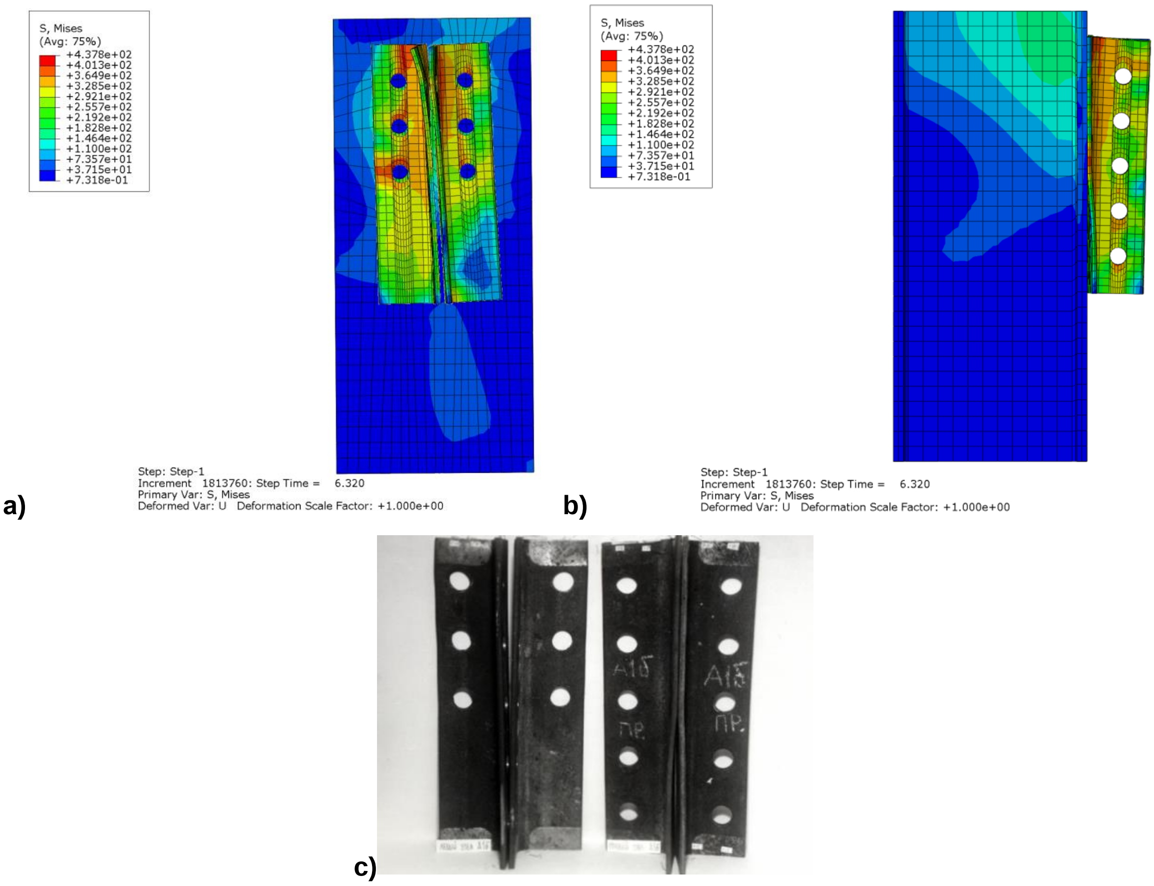


Figure 14. Stress state of the connection angles with damage of the following: a – "column" flanges (numerical analysis); b – "beam" flanges; c – "column" and "beam" flanges (experiment)

The structure of the beam-to-column bolted connection being considered has certain stiffness allowing the support moments to develop and be transferred to the column. The graphs of dependence of the support moment M_{sup} on the load made on the basis of the results of the numerical calculation and testing, are given on figure 15. The comparison of the results of the numerical analysis and the experiment [35] are given in Table 4.

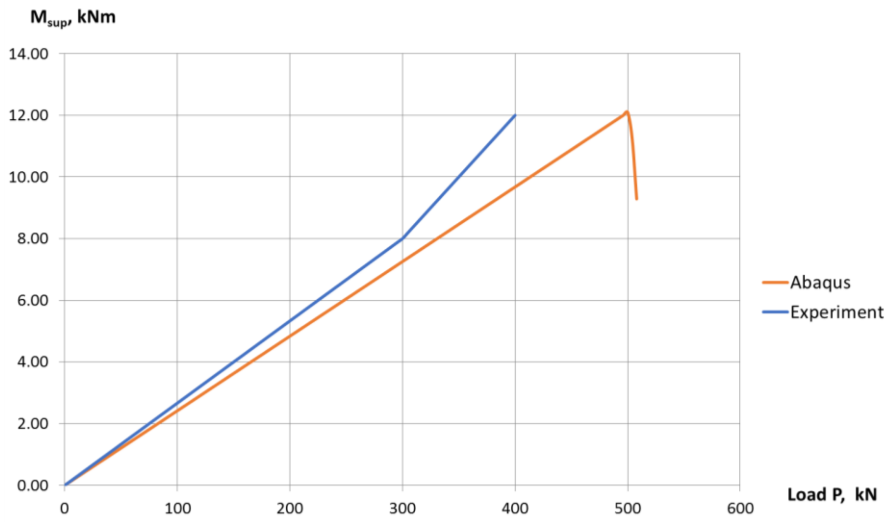


Figure 15. The graph of "support moment-load" dependence

Table 4. The comparison of the values of support moment

Load P, kN	**Support moment M_{sup}, kNm**		***Difference, %***	$k = \dfrac{M_{sup}}{M_{sup,theor}}$	
	Abaqus	*Experiment*		*Abaqus*	*Experiment*
0	0	0	0	-	-
48	1.16	1.28	9.29%	0.069	0.076
100	2.42	2.67	9.29%	0.069	0.076
148	3.58	3.95	9.29%	0.069	0.076
200	4.84	5.33	9.29%	0.069	0.076
248	6.00	6.61	9.29%	0.069	0.076
300	7.26	8.00	9.29%	0.069	0.076
348	8.42	9.92	15.14%	0.069	0.082
400	9.68	12.00	19.37%	0.069	0.086
448	10.84	14.40	24.74%	0.069	0.092
500	12.09	17.00	28.86%	0.069	0.097

The C factor of the beam restrain on support, which is equal to the relation of the support moment to the corresponding rotational angle of the beam end within the joint, is taken as the stiffness property of the semi-rigid frame joints:

$$C = \frac{M}{\varphi}, \tag{1}$$

where M is the bending moment acting on the support;

φ is the rotation angle of support cross-section of beam.

The qualitative analysis of the results of the numerical and experimental studies of the behaviour of a beam-to-column bolted joint connection with double vertical angles allowed us to determine the stiffness of the joint type being considered.

The graphs of "stiffness-load" dependence and the comparison of the numerical calculation and the testing results are given on figure 15 and in table 5 respectively.

It has been noted that the angle stiffness determined based on the experiment data exceeds the numerical one significantly at the initial stage of structure loading. However, the difference between their values is levelled and reaches the average of 15 % as the load increases.

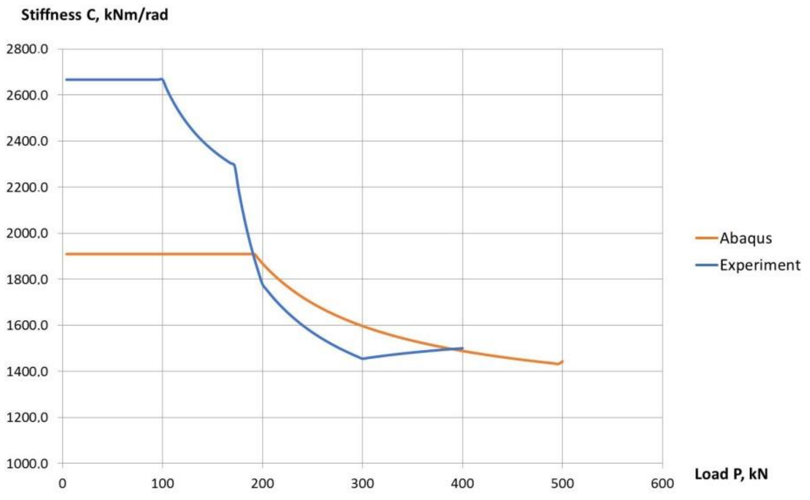


Figure 16. The graph of "stiffness-load" dependence

Table 5 The comparison of the joint stiffness

Load P, kN	Stiffness of the joint C, kNm/rad		***Difference, %***
	Abaqus	*Experiment*	
0	0	0	0
48	1,908.7	2,666.6	28.42%
100	1,908.7	2,666.6	28.42%
148	1,908.7	2,367.9	19.40%
200	1,868.1	1,777.7	-4.84%
248	1,700.2	1,574.6	-7.39%
300	1,596.6	1,454.5	-8.90%
348	1,535.0	1,480.6	-3.55%
400	1,488.4	1,500.0	0.77%
448	1,456.7	-	-
500	1,443.3	-	-

To verify the assessment of the pliability of the joint type being considered, the stiffness was determined using the methodology proposed by the authors of the paper [27], where the factor characterizing the beam restrain on support is calculated by the following formula:

$$k = \frac{M_{\sup}}{M_{\sup,theor}} , \tag{2}$$

where M_{sup} is the bending moment acting on the support;

$M_{sup,\ theor}$ is the bending moment acting on the absolutely rigid support.

The values of the k factor obtained based on the testing and numerical calculation results are given in Table 4.

For approval of joint stiffness estimating, the coefficient of beam restraint on the support was calculated by the formula suggested in [35]:

$$C = \frac{3}{2E}(\frac{S}{th})^3, \tag{3}$$

where E – elastic modulus of steel;

S – distance between butt of “column” flange of connecting angel and axes of bolts;

t – thickness of connecting angel;

g – height of connecting angel.

Stiffness of considered joint obtained by the kinematic theory of limit equilibrium method represented in [27] is 1789 kNm/rad. Stiffness was calculated in the moment of formation of “plastic” hinge in the “column” part of connecting angel when limit moment is acting in joint. This situation occurred in load about 200 kN, which is grafically demonstrated by rhe graphs of the “stiffness-load” dependences in Figure 16.

Conclusions

1. The picture of the stress-strain state and damage of semi-rigid joints of beam-to-column connection with double vertical angles is found.

2. It is established that the main factor determining the deformability of a joint is the deformation of the angles, which accounts for up to 87 % of the total deformation of the connection.

3. In the "column" flanges of connection angles, an early transition (at $P = 0.3\ P_{max}$) of the metal into the elastic-plastic stage with subsequent formation of linear plastic hinges in this area is noted.

4. The strain state of the connection angles is mainly characterized by a bend out of the plane of the flange attached to the column.

5. The design solution of the joints with vertical angles allows large angles of rotation of the beam support section, the value of which at the elastic stage of work achieves 75 % of the beam rotation angle under the condition of its simple support.

6. The assessment of the joint stiffness carried out allows to note that the structure with double vertical angles is able to support up to 8 % of the bending span moment within the beam.

In conclusion, it should be noted that the design solutions of the beam-to-column connection joints with double vertical angles, that are traditionally taken as hinged ones in the calculations of connection frameworks of multistory buildings, can be considered as semi-rigid joints supporting the corresponding support moment, the value of which depends on the peculiarities of their design solution, which is confirmed by the results of this research.

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