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Dynamic interaction of high-speed trains with span structures and flexible support

Динамическое взаимодействие высокоскоростных поездов с пролетными строениями и гибкими опорами

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Abstract. To ensure reliable and safe operation of the bridge structure throughout the life cycle, it is necessary to analyze and take into account many important factors, including the interaction of the main load-bearing structures. The presented work has new theoretical information, which gives the main provisions in the field of design of artificial structures for high-speed railroads. To analyze the system of a multi-span split bridge design using flexible intermediate supports of a flyover type, the Newton method (algorithm) is used. The basic data on the interaction of span structures and the design of flexible supports that are not taken into account in the design of facilities are not specified or regulated in the basic normative space, either by domestic JV standards or by foreign European EN standards, including national normative bases of the CIS countries. Harmonic analysis of the recording of the interaction of high-speed rolling stock and the joint operation of the main bridge structures of man-made structures is necessary for the design of high-speed railroads of transport infrastructure, especially in conditions of high-speed rolling stock. The article proposes a methodology for taking into account the interaction of the elements of the "bridge-train" system and determines the directions for further research to take into account the joint work and optimization of the basic designs of modern bridges and rolling stock in the region of high train speeds.

Аннотация. Для обеспечения надежной и безопасной работы мостового сооружения на протяжении всего жизненного цикла необходим анализ и учет многих важных факторов, в том числе и учет взаимодействия основных несущих конструкций. Представленная работа обладает новыми сведениями теоретического характера, дающими основные положения в области проектирования искусственных сооружений для скоростных железнодорожных магистралей. Для анализа системы многопролетной разрезной конструкции моста с применением гибких промежуточных опор эстакадного типа используется метод (алгоритм) Ньютона. Представлены основные данные о взаимодействии пролетных строений и конструкции гибких опор, которые не учитываются при проектировании сооружений, не оговариваются и не регламентированы в базовом нормативном пространстве, как отечественными нормами СП, так и зарубежными европейскими нормами EN, включая национальные нормативные базы стран СНГ. Гармонический анализ учета взаимодействия высокоскоростного подвижного состава и совместной работы основных мостовых конструкций искусственных сооружений необходим для проектирования скоростных железнодорожных магистралей транспортной инфраструктуры, особенно в условиях

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высокоскоростного движения подвижного состава. В статье предложена методика учета взаимодействия элементов системы «мост-поезд» и определены направления дальнейших исследований для учета совместной работы и оптимизации основных конструкций современных мостов и подвижного состава в области высоких скоростей движения поездов.

Introduction

According to preliminary forecast, the total length of high-speed mainlines (HSRM) will reach 60 thousand kilometers until 2020 in the world. Nowadays, China possesses the densest and developed network of high-speed railway mainlines in the world, holds first place in the world – 40% of total length of HSRM, will hold this position until 2020 increasing the length till 30 thousand kilometers, and 38 thousand kilometers until 2025. As a rule, more than half of the total length HSRM consists of elevates, 48-80% tunnels and bridges.

The normative documents used in the design of bridges operating in the Russian Federation [1, 2] and the countries of the near and far abroad [2, 3] formulate only general basic requirements for the corresponding projected structure that is part of the railway network. The above normative documents do not take into account and do not reflect the effect of the interaction of high-speed rolling stock and basic structures of bridges (towers and span structures), these norms do not apply at all to the design of bridges intended for high-speed traffic – HSRM. The main drawback is that the regulatory framework is not updated for the design of high-speed railways.

In compliance with the program for creating national standards, leading project Institute Gyprostroymost (The largest organization of total bridge engineering design in Russian Federation), there was prepared first redaction of Project for set of rules “Artificial constructions of high-speed railway lines. The rules of projecting and building” [5, 6].

To develop project-specific design codes (PSDC) for designing and building the infrastructure of the Moscow to Kazan high-speed long-distance railway line in order to create an up-to-date regulatory document for designing bridges on high-speed long-distance railway lines [7, 8].

In the works dedicated to analysis of bridge construction works of HSRM, as a rule, the dynamical reaction of the bridge is not evaluated on longitudinal train action that is necessary for constructions with (flexible) supports, characterized for high-speed mainlines.

Dynamic interaction of train loading on bridges usually either brings to analysis of span fluctuations or is considered as a interaction of spans and trains, as in 60-70s years [9, 10]; 80-90s years [11,12]; and 2000s [13–15].

Method

Influence of solid characteristics support of various types of railway bridges while constructing “bridge-train” models is not practically taken into account.

With the development of high-speed railways, the dynamic behaviour of trains and bridges has been studied more thoroughly. However, it is difficult to find papers in the scientific, normative and technical literatures about the lateral response of high-speed trains travel over long viaducts with pier.

The dynamic interaction between high-speed train and bridge is studied by theoretical analysis and field experiment. The main purpose of this paper is to develop a simple-model moving wheel/rail contact element, so that the sticking, sliding, and separation modes of the wheel/rail contact can be appropriately simulated. The three-dimensional (3D) contact finite element analysis for a realistic wheel and rail was used to accurately model the wheel/rail contact stiffness [16].

A computational model of train-bridge system with 24 m-span PC box girders are simulated. The dynamic responses of the bridge such as dynamic deflections, lateral amplitudes, lateral and vertical accelerations, lateral pier amplitudes, and the vehicle responses such as derail factors, offload factors, wheel/rail forces and car-body accelerations are calculated [17, 18]. Experimental and theoretical studies have been performed to determine the dynamic behavior of bridges crossed by the Korean high-speed train (KHST) [19, 20].

In the article of Russian authors deals with peculiar features of dynamic interaction of high-speed train loading and beam superstructures of bridges on the basis of numerical experiment. It also presents dependence of the quan tity of dynamic influence of trains on bridges from the speed of their movement, dynamic characteristics of the superstructures (more than 10 basic types and their lengths from 2.55 m to

90 m), as well as the results of pilot studies of the operation of steel superstructures at 250 km/h speed of movement of the high-speed train "Sapsan" [21].

In the article of Spain authors influence of pier height on the response of train and bridge is also studied. Continuous bridges, straight and constant section deck viaducts with variable height and tapered piers are the structures which have been considered. The height of the tested viaduct ranges between 60 and 120 m [22–25].

To solve the problem of oscillations of a railway beam bridge in its plane, taking into account the work of the bridge, the following assumptions were made.

1. A multi-span bridge is viewed as a system with a finite number of degrees of freedom.

2. The bases of the supports are perfectly elastic (the movements of the foundation of the support and the reaction of the base are connected by a linear dependence).

3. The bridge cloth is modeled by an elastic bar fixed to the elastic at the ends of the structure and lying on the base, elastic in the longitudinal direction. The last assumption is based, first, on the fact that, with terminal fasteners and elastic gaskets, it is possible for the track to work elastically at relatively large longitudinal displacements of the rails, and secondly because the values of the experimental relative displacements of the rails on the bridges at longitudinal effects of trains are very small.

4. When simulating a temporary mobile longitudinal load, the latter is assumed to be a one-dimensional system with one degree of freedom, which simplifies the solution of the problem, making it possible to obtain reliable results.

5. Dynamic impact on the bridge along the track axis is realized with a change in the time of traction forces or braking of the train f_π. For example, the traction force of a locomotive can be expressed as

$$f_\Pi(t) = F_0(1 - e^{-t\gamma}); \tag{1.1}$$

and the intensity of the brake load

$$\tau(t) = \tau(1 - e^{-\delta t}). \tag{1.2}$$

Here F_0, τ – the maximum values of the relevant factors (for example, for the freight train τ = 0.1q, where q – the uniformly distributed vertical train load), γ, δ – coefficients, t – time.

It is assumed that at the time of the longitudinal load on the structure the train is on the bridge, and the transmission of the horizontal longitudinal force in the case of split beam bridges is carried out through the fixed support parts of the beams and the path. Thus, between the rolling stock and the rail way of the bridge, a frictional coupling that varies in time is assumed. Between the beams of span structures and the supports in the places where the beams are supported on the movable bearing parts, friction bonds are also provided.

Under the assumed assumptions, the calculation scheme of, for example, the four-span bridge will have the form shown in Figure 1. In accordance with the design scheme, the bridge is an elastic system with elastic connections between track rails and span structures and with frictional connections in the moving support parts.

Equations of free vibrations of a bridge as a conservative system can be obtained by making Lagrange equations of the second kind, which have the form

$$\frac{d}{dt}\left(\frac{d\tau_w}{dq_j}\right) + \frac{d\pi_w}{dq_j} = 0; \tag{1.3}$$

where q_j – displacement in the direction of the j-th generalized coordinate;

π_w – potential energy of deformation of the bridge;

τ_w – kinetic energy of the structure oscillations.

To determine π_w, τ_w it is necessary to establish the dependence of the crossings of the sections of the track $u(x)$ from generalized coordinate's q_j. Vertical displacements of the centers and longitudinal displacements of the ends of span beams, vertical and longitudinal displacements of the top of supports, as well as displacement along the axis of the path of points located in the middle part of each intermediate support are accepted as generalized coordinates.

We will determine the desired displacements of the points of the track as a function of the displacement of the top of the beams of the span structures. To compose the differential equation of motion of the elastic rod (on the elastic in the direction along the bridge base), which simulates the rail track on the bridge, consider the element of the rod with the stiffness EA_P of length dx. It can be seen from the figure that

$$dN = r(u - v)dx;$$

where

$$\frac{dN}{dx} = r\left[u(x) - v(x)\right]. \tag{1.4}$$

Here *r[u(x) - v(x)]* is the linear resistance of the track to the shear; *r* – coefficient of proportionality, characterizing the elastic properties of the path in the direction along the axis of the bridge (longitudinal modulus of elasticity of the under-rail base). Taking into account that, according to Hooke's law, the elementary displacement of a bar from a pair of rails with rigidity EA_P is determined by expression

$$du = \frac{Ndx}{EA_p};$$

$$\frac{du}{dx} = \frac{N}{EA_p}; \tag{1.5}$$

$$\frac{d^2u}{dx^2} = \frac{dN}{dx} \cdot \frac{1}{EA_p}. \tag{1.6}$$

Equating the expressions for *dN/dx*, obtained by (1.4) and (1.6), we obtain the required differential equation:

$$\frac{d^2u}{dx^2} = \frac{r}{EA_p}(u - v). \tag{1.7}$$

We represent the resulting equation in the following form

$$u''(x) - \gamma^2 \cdot u(x) = -\gamma^2 \cdot v(x); \tag{1.8}$$

where, $\gamma^2 = \dfrac{r}{EA_p}$.

The boundary conditions for equation (1.7) are obtained by writing down the relations for the normal stresses at the ends of the rail track:

$$\sigma(0) = \frac{R \cdot u(0)}{A_p}; \qquad \sigma(L) = \frac{-R \cdot u(L)}{A_p}. \tag{1.9}$$

Representing the equation (1.9) in the form

$$\sigma(0) = E\frac{du}{dx_{x=0}}, \qquad \sigma(L) = E\frac{du}{dx_{x=L}}, \tag{1.10}$$

we find the following parameters

$$\left.\frac{du}{dx}\right|_{x=0} = \frac{R \cdot u(0)}{EA_p}; \quad \left.\frac{du}{dx}\right|_{x=L} = \frac{-R \cdot u(L)}{EA_p}. \tag{1.11}$$

The solution of equation (1.7) taking into account condition (1.11) has the form

$$u(\xi) = -\gamma^2 \cdot \int_0^L G(x,\xi) \cdot v(x) dx; \tag{1.12}$$

where $G(x,\xi)$ – the Green's function, taking into account the boundary conditions (1.11), has the form

$$G(x,\xi) = \begin{cases} A(\xi) \cdot U(x) \text{ if } 0 \le x \le \xi \\ B(\xi) \cdot V(x) \text{ if } \xi \le x \le L \end{cases}. \tag{1.13}$$

Here

$$A(\xi) = a_1 \cdot sh\,\gamma\xi + a_2 \cdot ch\,\gamma\xi; \tag{1.14}$$

$$B(\xi) = b_1 \cdot sh\,\gamma\xi + b_2 \cdot ch\,\gamma\xi; \tag{1.14}$$

Where

$$a_1 = \frac{\gamma \cdot sh\gamma L + h \cdot ch\gamma L}{D}; \; a_2 = -\frac{h \cdot sh\gamma L + \gamma \cdot ch\gamma L}{D}; \tag{1.15}$$

$$h = R/(EA_p);$$

$$b_1 = h/D; \; b_2 = \gamma/D, \quad D = \gamma(\gamma^2 + h^2)sh\gamma L + 2\gamma^2 h.ch\gamma L;$$

$$U(x) = hsh\,\gamma x + \gamma\, ch\,\gamma x;$$

$$V(x) = (h.ch\,\gamma L + \gamma .sh\,\gamma L)\, sh\,\gamma x - (h.sh\,\gamma L + \gamma\, ch\,\gamma L)ch\,\gamma x.$$

As regards the second factor of the integrand in equation (1.12), which represents the displacement of the points of the top of the i-th span structure $v_i(x)$, it is determined on the basis of the following considerations. We will assume that the span structure is bent along the half-wave of the sinusoid, that is, the vertical displacement of the point of the *i*-th span structure with abscissa *x* is expressed by the dependence

$$y(x) = y_{max} \sin\frac{\pi x}{l_i}.$$

It is clear that the longitudinal displacement of the lower belt of the movable end of the beam from its deflection y will amount to is $2\varphi e$. Moving the movable end of the beam along the upper fiber will be

$$2\varphi e - \varphi H_B = \left(\frac{2\pi}{l} e - \frac{\pi H_B}{l}\right) y = \frac{2\pi}{l}\left(e - \frac{H_B}{2}\right) y;$$

$$\varphi = \frac{\pi}{l} y.$$

where *e*, H_b – the eccentricity and height of the span of the span structure, respectively.

Longitudinal displacements of the points of the top of the beam of the *i*-th span during its deflection are given by

$$v_i^B(x) = \left(A + B\cos\frac{\pi}{l_i} x\right);$$

$$x \,E\, \{0, 1_i\}. \tag{1.16}$$

where:

$$A + B = \varphi_i H_{Bi};$$ (1.17)

$$A - B = 2\varphi_i e_i - \varphi_i H_{Bs}.$$

By solving the system of equations (1.17), we find

$$A = \varphi_i e_i, \qquad B = -\varphi_i e_i + \varphi_i H_{Bi}.$$ (1.18)

Substituting equation (1.18) into (1.16), we obtain the current displacement of the top of the i-th beam from its sagging along the sinusoid:

$$v_i^{*B}(x) = \frac{\pi}{l_i} y_i (e_i - (e_i - H_{Bi}) \cos\frac{\pi}{l_i} x) = v_i^{B}(x) y_i.$$ (1.19)

Accordingly, the longitudinal displacement of the beam bottom is determined by the expression

$$v_i^{*H} = v_i^{*B}(x) - \varphi_i\ H_{Bi} \cos\frac{\pi}{l_i} x.$$ (1.20)

The expression for the total longitudinal displacement of the top of the i-th beam (taking into account its deformation both under the action of longitudinal forces and the compliance of the supports in the vertical direction) will have the form

$$v_i(x) = v_i^{B}(x)(y_i - (\frac{y_i^{right}}{2} + \frac{y_i^{left}}{2}) + y_i^{H} + \frac{y_i^{\Pi} - y_i^{H}}{l_i} x;$$ (1.21)

where y^{right}, y^{left} – vertical displacement of the right and left ends of the beam of the i-th span, respectively; y_i^{Π}, y_i^{H} – longitudinal displacement, respectively, of the movable and fixed end of the beam of the i-th span in the level of the centers of gravity of the section; y_i – vertical deflection of the middle of the beam of the i-th span.

Now, in the case of a multi-span bridge or viaduct (see, for example, Fig. 2), the solution of equation (1.7) can be written in the form

$$-\frac{1}{\gamma^2} u(\xi) = \int_0^{L_1} G(x,\xi) v_1(x) dx + \int_{L_1}^{L_2} G(x,\xi) v_2(x) dx + \int_{L_2}^{L_3} G(x,\xi) v_3(x) dx + \dots \int_{L_{k-1}}^{L_k} G(x,\xi) v_k(x) dx$$ (1.22)

$(0 \le x \le k_1)$, *k* – number of spans.

Figure 1 shows

$$v_1(x) = v_1^B(x)\left(q_1 - \frac{q_3}{2}\right) + \frac{q_2}{l_1} x \qquad (0 \le x \le L_1);$$

$$v_2(x) = v_2^B(x)\left(q_6 - \frac{q_3 - q_8}{2}\right) + q_4 + \frac{q_7 - q_4}{l_2} x \qquad (L_1 \le x \le L_2);$$

$$v_3(x) = v_3^B(x)\left(q_{11} - \frac{q_{13} - q_{18}}{2}\right) + q_{9+} \frac{q_{12} - q_9}{l_3} x \qquad (L_2 \le x \le L_3).$$

$$v_i^B(x) = \frac{\pi}{l_i}\left[e_i - (e_i - H_{Hi}) \cos\frac{\pi}{l_i}(x - L_{i-1})\right].$$ (1.23)

Considering, as an example, a four-span viaduct as a system with seventeen degrees of freedom (see Figure 1), we can, based on the above, write an expression for the displacement of the S point of the railroad track of the bridge in the form

$$-\frac{1}{\gamma^2}u(\xi s)=q_1\int_0^{L_1}G(x,\xi_S)v_1^B(x)dx+q_2\int_0^{L_1}G(x,\xi_S)\frac{x}{l_1}dx-$$
$$-\frac{1}{2}q_3\int_0^{L_1}G(x,\xi_S)v_1^B(x)dx-\frac{1}{2}q_3\int_{L_1}^{L_2}G(x,\xi_S)v_2^B(x)dx+$$
$$+q_4\int_{L_2}^{L_3}G(x,\xi_S)\left(1-\frac{x-L_1}{L_2}\right)dx+q_6\int_{L_1}^{L_2}G(x,\xi_S)v(x)dx+q_7\int_{L_1}^{L_2}G(x,\xi_S)\frac{x-L_1}{L_2}dx-$$
$$-\frac{1}{2}q_8\int_{L_1}^{L_2}G(x,\xi_S)v_2^B(x)dx-\frac{1}{2}q_8\int_{L_2}^{L3}G(x,\xi_S)v_3^B(x)dx+q_9\int_{L_2}^{L3}G(x,\xi_S)\left(1-\frac{x-L_2}{l_3}\right)dx+ \quad (1.24)$$
$$+q_{11}\int_{L_2}^{L3}G(x,\xi_S)v_3^B(x)dx+q_{12}\int_{L_2}^{L3}G(x,\xi_S)\frac{x-L_2}{l_3}dx-\frac{1}{2}q_{13}\int_{L_2}^{L3}G(x,\xi_S)v_3^B(x)dx-$$
$$-\frac{1}{2}q_{13}\int_{L3}^{L4}G(x,\xi_S)v_4^B(x)dx+q_{14}\int_{L_3}^{L4}G(x,\xi_S)\left(1-\frac{x-L_3}{l_4}\right)dx+$$
$$+q_{16}\int_{L_3}^{L4}G(x,\xi_S)v_4^B(x)dx+q_{17}\int_{L_3}^{L4}G(x,\xi_S)\frac{x-L_3}{l_3}dx.$$

where L_{1-1}, L_1 – the abscissa of the initial and final reference sections of the i-th span structure, respectively (Figure 1).

We divide the rail whip with a length equal to the length of the viaduct into m sections and we will consider its displacements at m + 1 boundary points of these sections (Figure 1). We introduce the vector {u} of displacements of the rail track at these points

$$\{u(\xi)\}=\{u(\xi_1=0)\ \ u(\xi_2)\ \ldots\ u(\xi_{m+1})\}=\{u_1\ u_2\ \ldots\ u_{m+1}\}; \quad (1.25)$$

then the equation (1.24) for a system with n degrees of freedom in the matrix form is written in the form

$$u=-\gamma^2\sum G(x,\xi)\,a_i(x)d(x)q \quad (1.26)$$

where $q = \{q_1\ q_2\ \ldots\ q_n\}$ – column of generalized coordinates of the system;

u – a column of offsets of track points within the structure;

$a_i(x)$ – i-th matrix-string, given for the calculation scheme in Figure 2.

Finally, the equation 1.24 for a system with n degrees of freedom in the matrix form is written in the form

$$u=\Phi\cdot q; \quad (1.27)$$

where Φ – the matrix of coefficients j determined from expression

$$\varphi_j^{(S)}=-\gamma^2\cdot\sum_{1=1}^{k}G(x,\xi_S)\cdot f_{ij}(x)dx \quad (1.28)$$

(S=1, 2 ..., m+1; i=1, 2 …, k; j=1, 2 …, n).

where $G(x,\xi_s)$ – function of the effect of the movements of the base on the displacements of the rails of the bridge web, resiliently fixed at the ends of the bridge (Green's function);

ξ_S – abscissa of the s-th point of the track;

f_{ij} – a function that determines the longitudinal displacement of the top of the i-th span when the qj of the system is moved;

m – number of sections of railroad lashing on the bridge;

k – number of spans;

n – number of degrees of freedom of the system.

The matrix Φ can be represented in the form

$$\Phi = \begin{bmatrix} \varphi_1^{(1)} & \varphi_2^{(1)} & .....\varphi_j^{(1)} & ....\varphi_n^{(1)} \\ \varphi_1^{(2)} & \varphi_2^{(2)} & .....\varphi_j^{(2)} & ....\varphi_n^{(2)} \\ \ldots\ldots\ldots & & & \\ \varphi_1^{(m+1)} & \varphi_2^{(m+1)} & .....\varphi_j^{(m+1)} & ......\varphi_{n1}^{(m+1)} \end{bmatrix} \qquad (1.29)$$

The coefficients $\varphi_j^{(S)}$ for the example of the four-span bridge (Fig. 2) have the form

$$\varphi_1^{(1)} = -\gamma^2 \int_0^{L_1} G(x,\xi_1) v_1^B(x) dx;$$
$$\varphi_2^{(1)} = -\gamma^2 \int_0^{L_1} G(x,\xi_1) v_1^B(x) \frac{1}{l_1} x dx; \qquad (1.30)$$

and so on in accordance with equation (1.24).

Returning to the Lagrange equations (1.36), we note that the potential strain energy of the railway bridge can be expressed by the formula

$$\pi_w = \pi_r + \pi_{ec} + \pi_b + \pi_{pier}; \qquad (1.31)$$

where, π_r, π_{ec}, π_b, π_{pier} – the potential energy of deformation, respectively, of the rails of the track, elastic connections between rails and beams of span structures, beams of span structures, supports.

The kinetic energy of the oscillations of the bridge can be expressed by the dependence

$$\tau_w = \tau_b + \tau_{pier}; \qquad (1.32)$$

where τ_b, τ_{pier} – kinetic energy of oscillations, respectively, of span structures and supports.

In the matrix form, expression (1.31) can be represented in the form

$$\pi_w=(1/2)q^T \Pi\, q; \qquad (1.33)$$

where Π – matrix of coefficients under generalized coordinates q of the system.

The expression (1.33) can be written in the matrix form

$$\tau_w=(1/2)q^{*T} T\, q^*; \qquad (1.34)$$

where T – matrix of coefficients for the vector q*.

Substituting the expressions (1.33) and (1.34) into the Lagrange equations (1.36), we obtain a system of ordinary differential equations of the form

$$Tq^{**}+\Pi q=0. \qquad (1.35)$$

For $q = z \cdot sinwt$, instead of (1.35), we have

$$Tz - (1/w^2)\Pi z =0. \qquad (1.36)$$

It is known that the shapes and the corresponding frequencies of free oscillations described by the system of equations (1.36) are obtained by analyzing the eigenvectors z and the corresponding frequencies of their own problem

$$(\mathrm{T} - \lambda\, \Pi)\mathrm{z} = 0; \qquad (1.37)$$

where in $\lambda = 1/w^2$, w – frequency of free oscillations.

On the basis of the above methodology, it is possible to obtain a solution to the problem of free oscillations of a beam rail bridge structure in its plane, taking into account the influence of the track.

Analysis of forced oscillations of bridges with longitudinal effects of a train

In non-stationary (transient) modes of train traffic movement, characteristic for real operating conditions, there are forced oscillations of the structure in its plane.

The oscillations of the train-bridge system from the longitudinal effects of the train under the assumptions made earlier will be described by a system of nonlinear differential equations of the form

$$Mp^{**} + \frac{M_B g}{m} f_П I^T\{ I\, sign\, [\, (v+p^*) - Фq^*]\} + (M-M_B)\cdot g\cdot f_П\cdot sign(v+p^*)=0;$$
$$T^{-1}\, T\, q^{**} + T^{-1} B\, q^* + T^{-1} Пq + T^{-1} Q = T^{-1} w\, \frac{M_B g}{m} f_П I^T\{ I\ \ sign\, [(v+p^*) - Ф\, q^*]\}. \qquad (2.1)$$

where q, q^*, q^{**} – the column of the generalized coordinates, velocities and accelerations of the bridge, respectively, as a system with n degrees of freedom (see Figure 1);

T, B, and P – the matrix of the coefficients of inertia, resistance and rigidity, respectively;

Q – column of generalized frictional forces in the movable supporting parts of span beams;

I – column of the form $\{0,7 \;\; 1 \;\; 1 \ldots 0,7\}$ $m + 1$, reflecting the discreteness of the mobile load;

v – the speed of the moving load relative to the fixed path;

p – the longitudinal displacement of the moving load band caused by the bridge oscillations;

g – the acceleration due to gravity;

M, M_B – the mass of the temporary mobile load is respectively common and on the bridge;

$f_П$ – function describing the impact on the bridge on the side of the train longitudinal load (for example, braking);

$Ф$ – the matrix characterizing the elasticity of the under-rail base along the axis of the path, formed from expression (1.29);

w – column of the form $\{1\ 1\ 1 \ldots 1\}_n$.

As a result of the calculation, the movements of the points of the structure along the directions of the generalized coordinates q are determined, the values of the forces acting on the structure and applied along the directions of the generalized coordinates are determined for any instant of time by the formula

$$s = П\cdot q; \qquad (2.2)$$

and the longitudinal forces at the $m + 1$ point of the track on the bridge according to formula

$$N_{m+1} = R_o\cdot Ф\cdot q; \qquad (2.3)$$

where

$$R_o = \frac{EA_p}{L/m}\, K. \qquad (2.4)$$

where EA_P – the linear rigidity of rail track sections on a bridge of length L;

$$K = \begin{bmatrix} -1 & 1 & & & & \\ 1 & -2 & 1 & & & \\ & \ldots & 1 & -2 & 1 & \\ & & & \ldots & & \\ & & & & 1 & -1 \end{bmatrix}_{m+1} \qquad (2.5)$$

Thus, the proposed method of dynamic calculation of a beam rail bridge on the longitudinal effects of a train includes the following steps.

Смирнов В.Н., Шестакова Е.Б., Чижов С.В., Антонюк А.А., Ледяев Л.А., Индейкин И.А., Евтюков Е.С. Динамическое взаимодействие высокоскоростных поездов с пролетными строениями и гибкими опорами // Инженерно-строительный журнал. 2017. № 8(76). С. 115–129.

1. We construct the matrices *T, П*, *Ф* (according to 1.29),

2. A matrix of coefficients of resistance B is formed.

3. The columns of external loads are determined, as well as the generalized friction forces Q in the moving support parts.

4. The system of non-linear differential equations (2.1), which describes the oscillations of the interacting "train-bridge" system, is solved numerically, as a result of which the displacements along directions of the generalized coordinates qj are determined.

5. According to the expression (2.2), the forces acting on the structure along the directions of the generalized coordinates are determined for the longitudinal action of the rolling stock (braking or starting from the place).

6. According to the formula (2.3), axial dynamic forces are sought in the rails of the track on the bridge, which arise when braking or starting from the place of temporary loading.

7. The values of the dynamic coefficients to the longitudinal loads on the supports are determined by the formula $1 + \mu = \dfrac{q_j^{dyn}}{q_j^{st}}$.

Where q_j^{dyn}, q_j^{st} – the dynamic and static displacement of the support, respectively, in the direction of the corresponding generalized coordinate (static displacements q_j^{st} are obtained, for example, by assuming a coefficient $\gamma = 0$ for the function f_P determined by (2.1), which ensures a "quasistatic" effect of the train load).

According to the methodology outlined above, a dynamic calculation of a four-span railway bridge with reinforced concrete beams of span structures of 27.6 m in length, intermediate supports of 17.85 m in height and ballast rides (see Figure 1) is performed when the train brakes with a vertical load equal to 80 kN / m of the way. The module of longitudinal elasticity of the under-rail base was adopted *U* = 0 (the work of the rail track for longitudinal actions is not taken into account), as well as *U* = 4.5 MPa and *U* = 26 MPa (in the case of bridges with riding on wooden cross bars with crutches and riding on reinforced concrete sleepers with crushed stone ballast and separate fasteners).

The resistance of the path along the ends of the bridge was not taken into account (in Figure 1 *R* = 0). Integration of the system equations (2.1) was carried out numerically by the Runge-Kutta method using a special program. Some results of the calculations are given below. The role of the connections between the rail and the sub-rail base on the dynamic reaction of supports is evaluated. In Figure 2 is a graph of the dynamic movements of the top of the support along the axis of the path under the longitudinal action of the rolling stock. The vibrational nature of the motion of the support is seen, and in the absence of elastic bonds between the rails and beams of the span structures (*U* = 0), the support oscillations are realized at a frequency of about 3.5 Hertz (circular frequency *w* = 22.5 1 / s), and in the presence of such bonds (*U* = 26 MPa) the frequency increases to 9.7 Hertz. The values of the dynamic displacements of the support, calculated with the account of the work of the bridge cloth for longitudinal forces, are several times smaller than the corresponding displacements found by the traditional method - without taking into account the work of the path on the bridge (at *U* = 0).

In Fig. 2, it is shown that the largest values of the amplitude of the oscillations of the top of the support are observed in the step-like form of the longitudinal action function, when the resistivity of the train increases almost instantaneously (from $f_П$ = 0.01 to $f_П$ = 0.1), which is theoretically possible and sometimes adopted in dynamic calculations. However, the practical implementation of the $f_П$ transition from the minimum to the regulated by the norms is stretched in time, which is confirmed experimentally.

In order to reflect this circumstance in the computational model, the coefficient of friction $f_П$ in exponential calculations varied exponentially (in practice, changing discrete values of fn were determined at different instants of time).

Due to the gradual increase in the value of $f_П$ in time, the dynamic displacements of the top of the supports with longitudinal actions of the rolling stock are significantly reduced in comparison with the case of instantaneous changes in the values of $f_П$.

As can be seen from the figure, with static application of the braking force, the amount of displacement of the top of the support (without taking into account the path) was 1.2 mm. The dynamic nature of the process of interaction between the train and the bridge caused an increase in movement by 10-12% (Figure 1). This circumstance gives grounds to assume in the future (with the accumulation of a pachet and experimental material) the possibility of introducing corresponding dynamic coefficients to the longitudinal loads on the supports, the values of which depend on the dynamic properties of the system and the nature of the interaction of the temporary load and the structure. The recommendations of the norms for assigning the values of the dynamic coefficient for the supports appear to be poorly grounded.

Result and Discussion

While passing of train by bridges sprung crew experience the dynamic interaction specified with rebound deflection of spans and other irregularities on roads. Especially, adverse conditions are created while passing of rolling-stock by multi-span bridge with equal spans. Spans can experience in this case essential vertical fluctuations, especially in case of one-type crew. On these occasions dynamic processes, leaked into system “train-bridge”, in substantial measure may depend on solid, inertial and dissipative characteristic intermediate support which are projected for moderns elevates in the form of eased constructions of reduced rigidity. To define dynamic forces in supports as elements of the system is becoming more controversial task.

Calculating scheme of the system “bridge-train” is shown on the Figure 1. Elevate is presented in the view of system of discrete lumps, each of them has 2 levels of freedom. Every crew in case of this dynamic task also owns 2 levels of freedom, having an opportunity of vertical moving (bumping) and turning (galloping). Foundations are accepted absolutely hard.

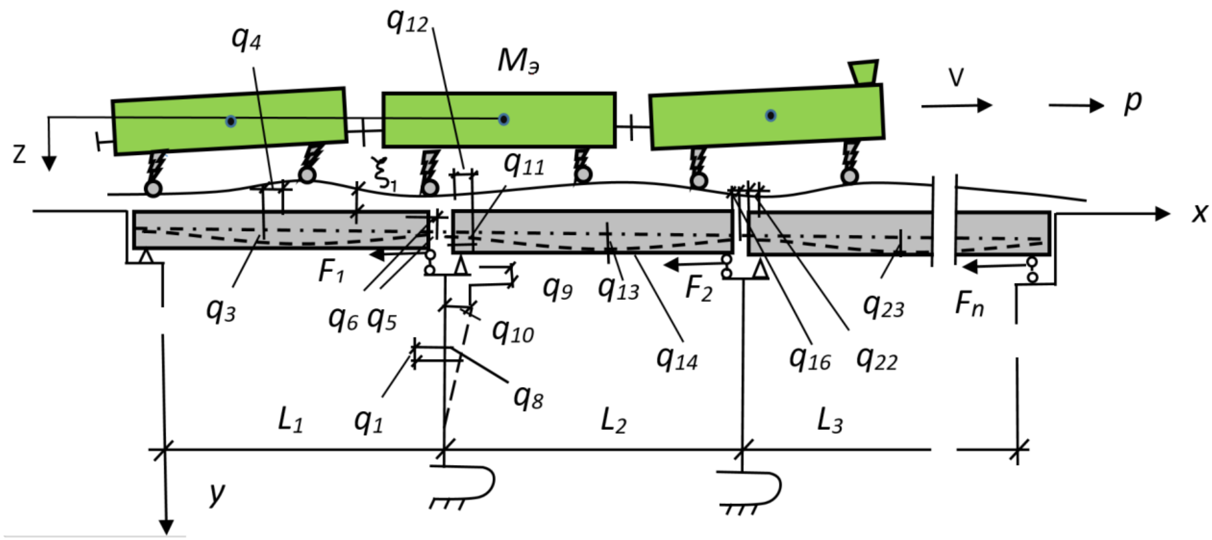

Figure 1. Calculating scheme of the system “bridge-train”

By accepted assumptions differential equations of system’s fluctuations “bridge-train” may be considered as:

$$\left\{ \begin{array}{l} q_3 = -\left(M_3 \ddot{q}_3 + \beta_3 \dot{q}_3\right)\left(\delta_{33} + \frac{\delta_{55}}{2}\right) - M_5 \ddot{q}_5 \delta_{35} - M_7 \ddot{q}_7 \delta_{37} - M_9 \ddot{q}_9 \delta_{39} - \\ -\sum_1^m \frac{M\text{э}}{2}(\ddot{z}_1 - g)\delta_{31(i)} - \sum_1^n \frac{M\text{э}}{2}(\ddot{z}_2 - g)\delta_{32(i)} + S_1 F_1 sign\left(S_1 \dot{q}_3 + \dot{q}_6 - \dot{q}_{12}\right) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \end{array} \right\}$$

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$$q_{10} = -\left(M_{10} \ddot{q}_{10} + \beta_{10} \dot{q}_{10} \right) \delta_{10,10} - M_8 \ddot{q}_8 \delta_{10,8} - M_{12} \ddot{q}_{12} \delta_{10,12} - M_{14} \ddot{q}_{14} \delta_{10,14} -$$

$$M_{16} \ddot{q}_{16} \delta_{10,16} + F_1 sign\left(S_1 \dot{q}_3 + \dot{q}_6 - \dot{q}_{12} \right) + F_2 sign\left(S_2 \dot{q}_{13} + \dot{q}_{16} - \dot{q}_{22} \right)$$

Here Z_1 – absolute movement of body i-th crew on point 1 (first by passing in a run), defined by rigidity spring and irregularities of passage.

q_j – movement of j-th point of the bridge by directions of summing coordinations.

δ_{kl} – solitary movement of the point "k" of the bridge from force, given on point *l*;

$M_Э$ – mass i-th crew;

M_i –th discrete lump of construction;

F_1- friction force of movable bearing parts, put on i-th support, from permanent and temporary loading;

$$\delta_{31,i} = \delta_{33i} \sin \pi u_1 + \frac{\delta_{55}}{2} u_1 \qquad \left(u_1 = \frac{x_1}{l_1} = \frac{vt}{l_1} \right);$$

$$\delta_{32,i} = \delta_{33} \sin \pi u_2 + \frac{\delta_{55}}{2} u_2 \qquad \left(u2 = \frac{x2}{l2} = \frac{vt}{l2} \right);$$

v – speed of train's running;

S – coefficient of passing from vertical movement in the middle of spans to longitudinal movement of construction on joint hinge's level of support part.

The solution of nonlinear differential equation system provides to get the image of dynamic interaction elements of the system "bridge-train" on conditions of high-speed running and to reveal the influence of support parameters on construction's dynamical work.

Conclusions

1. To ensure reliable and safe operation of a bridge structure under conditions of high-speed movement within the limits of long-term operation throughout the life cycle, it is necessary to take into account the joint operation of the rolling stock and the main load-bearing bridge structures: span structures and supports.
2. Harmonic analysis of the recording of the interaction of high-speed rolling stock and the joint operation of the main bridge structures of man-made structures is necessary from the point of view of optimal design of high-speed railroads of transport infrastructure, especially in conditions of high-speed rolling stock movement.
3. As a result of using the above methodology for calculating the interaction of the elements of the "bridge-train" system, the dependence of the dynamic characteristics of bridge structures on the inertial and rigid parameters of the supports is revealed, which makes it possible to design the supports of the flyover with the provision of their minimum possible massiveness (and, consequently, material capacity). This significantly reduces the material consumption of the support and the laboriousness of its erection and, as a consequence, increases the economy of the structure, ensuring operational reliability and durability.
4. The proposed method for the bridge design makes it possible to determine the stress-strain state of the elements of the structure (including the rail unshackled path) with longitudinal influences to avoid the appearance of unacceptable efforts in the rails and at the same time to ensure the design of non-material-intensive economical supports, avoiding excessive reserves of massive railroad bridge supports, and with optimal geometric parameters and ensuring the necessary operational parameters of the structure for reliability and durability awns. Thus, by varying the geometric parameters of the supports, the designer can evaluate the dynamic response under the influence of the railway train load under different initial data and choose the optimal design solution for the bridge structure taking into account economic indicators.

5. The results of the calculation procedure presented in this article can be used in the future for practical applications and for a deeper study:

– It is recommended to update the current standards taking into account the interaction of the elements of the "bridge-train" system and the criterion of the economics of the support of bridge structures;
– Development of recommendations and an album of technical solutions for new railroad bridge supports operated at high speeds of railway transport;
– It is recommended to apply this method in software tools Revit and SOFiSTiK or others to expand the basic kit for modeling the dynamic effects on a bridge structure from high-speed trains.

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