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Stress-strain state of three-layered shallow shells under conditions of nonlinear creep

Напряженно-деформированное состояние трехслойных пологих оболочек в условиях нелинейной ползучести

A.S. Chepurnenko,
Don State Technical University, Rostov-on-Don, Russia

Канд. техн. наук, ст. преподаватель А.С. Чепурненко,
Донской государственный технический университет, г.Ростов-на-Дону, Россия

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Abstract. The resolving equations were obtained and a calculation technique was developed with allowance for the nonlinear creep of three-layer plates and shallow shells with a lightweight filler. The problem was reduced to a system of three differential equations with respect to the stress function, displacement and deflection function. An example is given of calculating a rectangular planar shell in the form of an elliptical paraboloid. The solution was performed numerically by the finite difference method in combination with the Euler method for determining creep strains. The linear Maxwell-Thompson equation and the Maxwell-Gurevich nonlinear equation were used as the creep law. There were no significant discrepancies between the results obtained on the basis of the linear and nonlinear theory. It was established that, as the curvature of the shell increases, the creep of the aggregate has a lesser effect on the deflection value. It was revealed that for shells of greater curvature with constant displacements a redistribution of stresses and internal forces occurs. The bending and twisting moments decrease, and the longitudinal and shearing forces increase. In the aggregate, the tangential stresses relax, while in the sheaths the normal and tangential stresses increase.

Аннотация. Получены разрешающие уравнения и разработана методика расчета с учетом нелинейной ползучести трехслойных пластин и пологих оболочек с легким заполнителем. Задача свелась к системе из трех дифференциальных уравнений относительно функции напряжений, функции перемещений и прогиба. Приведен пример расчета прямоугольной в плане пологой оболочки в форме эллиптического параболоида. Решение выполнялось численно методом конечных разностей в сочетании с методом Эйлера для определения деформаций ползучести. В качестве закона ползучести использовано линейное уравнение Максвелла-Томпсона и нелинейное уравнение Максвелла-Гуревича. Существенных расхождений между результатами, полученными на основе линейной и нелинейной теории, не выявлено. Установлено, что с увеличением кривизны оболочки ползучесть заполнителя оказывает меньшее влияние на величину прогиба. Выявлено, что для оболочек большей кривизны при постоянных перемещениях происходит перераспределение напряжений и внутренних усилий. Изгибающие и крутящие моменты убывают, а продольные и сдвигающие силы возрастают. В заполнителе происходит релаксация касательных напряжений, а в обшивках нормальные и касательные напряжения возрастают.

Introduction

Three-layer structures with lightweight filler are widely used in various industries, including civil and industrial construction, aircraft construction, shipbuilding, etc. With the same flexural rigidity, such structures are much lighter then single-layer panels. As a filler of three-layered structures, polymeric materials are widely used, for which, in addition to elastic properties, viscoelasticity is characteristic. Therefore, to adequately describe the stress-strain state of three-layer structures, it is necessary to involve the apparatus of the theory of creep. There is a large number of papers devoted to the calculation taking into account the creep of three-layer beams, including [1–7]. As for the plates and shells, in most papers the calculation is considered only in the elastic stage [8–10]. In [11–12] the solution of the problem of axisymmetric bending of circular plates with a nonlinear elastic filler is given. In this case, only

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instantaneous nonlinearity is taken into account. In [13], a three-layered shell model with a lightweight filler is used to describe the linear creep of a reinforced concrete structure. The linear creep of three-layer plates and shells is also investigated in the author's papers [14–16].

In this paper, we will consider the technique for calculating plates and shallow shells, suitable for arbitrary creep laws, including nonlinear ones.

Methods

The element of the three-layered shallow shell is shown in Figure 1. In the calculation, we will use the technical theory of three-layer structures, according to which the bending and twisting moments, as well as the shear and longitudinal forces are completely perceived by the carrier layers. The filler only works on shear, taking transverse forces. The thickness of the carrier layers δ is the same and small compared to the total shell thickness h.

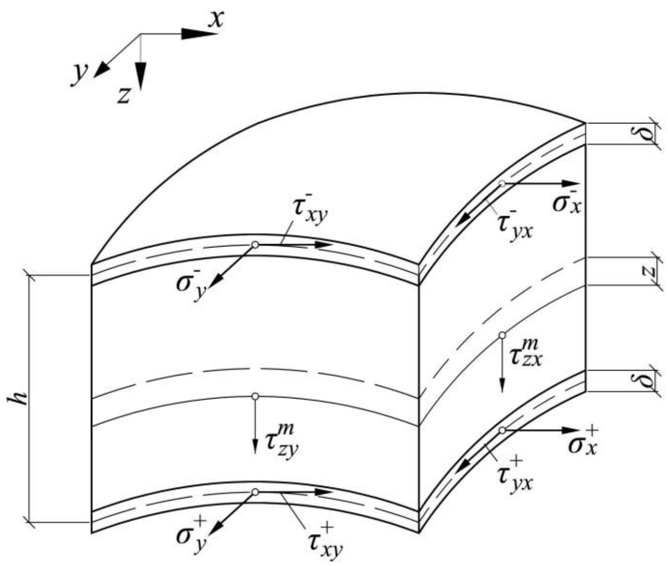

Figure 1. Element of a three-layered shallow shell

Equilibrium equations for the element of a three-layered shallow shell are written in the form:

$$\begin{gathered}\frac{\partial N_x}{\partial x}+\frac{\partial S}{\partial y}=0;\ \frac{\partial S}{\partial x}+\frac{\partial N_y}{\partial y}=0;\\ \frac{\partial M_x}{\partial x}+\frac{\partial H}{\partial y}-Q_x=0;\ \frac{\partial M_y}{\partial y}+\frac{\partial H}{\partial x}-Q_y=0;\\ \frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}-k_xN_x-k_yN_y+q=0,\end{gathered} \tag{1}$$

where N_x, N_y – longitudinal forces; S – shear force; M_x, M_y – bending moments; H – torque; Q_x, Q_y – transverse forces; q – surface load, $k_x \approx -\frac{\partial^2 z}{\partial x^2}, k_y \approx -\frac{\partial^2 z}{\partial y^2}$ – principal curvatures.

To satisfy the first two equilibrium equations in (1), we introduce the stress function according to the formulas:

$$N_x=\frac{\partial^2\Phi}{\partial y^2},\qquad N_y=\frac{\partial^2\Phi}{\partial x^2},\qquad S=-\frac{\partial^2\Phi}{\partial x\partial y}. \tag{2}$$

Bending moments and torque are related to the stresses in the carrier layers as follows:

$$M_x=(\sigma_x^+-\sigma_x^-)\delta\frac{h}{2};\ M_y=(\sigma_y^+-\sigma_y^-)\delta\frac{h}{2};\ H=(\tau_{xy}^+-\tau_{xy}^-)\delta\frac{h}{2}. \tag{3}$$

Longitudinal and shear forces are written as:

$$N_x=(\sigma_x^++\sigma_x^-)\delta;\ N_y=(\sigma_y^++\sigma_y^-)\delta;\ S=(\tau_{xy}^++\tau_{xy}^-)\delta. \tag{4}$$

For tangential stresses in the aggregate, a uniform thickness distribution is adopted:

$$Q_x=\tau_{zx}^m h;\ Q_y=\tau_{zy}^m h. \tag{5}$$

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Deformations of the carrier layers can be written as:

$$\varepsilon_x^{+(-)} = \frac{\partial u^{+(-)}}{\partial x} + k_x w;$$
$$\varepsilon_y^{+(-)} = \frac{\partial v^{+(-)}}{\partial y} + k_y w; \quad (6)$$
$$\gamma_{xy}^{+(-)} = \frac{\partial u^{+(-)}}{\partial y} + \frac{\partial v^{+(-)}}{\partial x},$$

where $u^{+(-)}$, $v^{+(-)}$ – displacements of the lower (upper) skin along the axes x and y, w – deflection.

For the displacements of the filler, a linear thickness distribution is adopted:

$$u^m = \frac{u^- + u^+}{2} + \frac{u^+ - u^-}{h} z = u + \alpha z;$$
$$v^m = \frac{v^- + v^+}{2} + \frac{v^+ - v^-}{h} z = v + \beta z. \quad (7)$$

Shear strains of the filler are defined as follows:

$$\gamma_{zx}^m = \frac{\partial u^m}{\partial z} + \frac{\partial w}{\partial x} = \alpha + \frac{\partial w}{\partial x};$$
$$\gamma_{zy}^m = \frac{\partial v^m}{\partial z} + \frac{\partial w}{\partial y} = \beta + \frac{\partial w}{\partial y}. \quad (8)$$

When calculating, we assume that the carrier layers work elastically, and the middle layer is viscoelastic. The stresses in the carrier layers of the shell are determined as follows:

$$\sigma_x^{+(-)} = \frac{E}{1-\nu^2}\left(\varepsilon_x^{+(-)} + \nu\varepsilon_y^{+(-)}\right);$$
$$\sigma_y^{+(-)} = \frac{E}{1-\nu^2}\left(\varepsilon_y^{+(-)} + \nu\varepsilon_x^{+(-)}\right); \quad (9)$$
$$\tau_{xy}^{+(-)} = \frac{E}{2(1+\nu)} \gamma_{xy}^{+(-)}.$$

Deformations of the filler represent the sum of elastic deformations and creep strains:

$$\gamma_{zx}^m = \frac{\tau_{zx}^m}{G_m} + \gamma_{zx}^*; \; \gamma_{zy}^m = \frac{\tau_{zy}^m}{G_m} + \gamma_{zy}^*, \quad (10)$$

where G_m – shear modulus of the middle layer, γ_{zx}^* and γ_{zy}^* – creep strains of the middle layer.

We express from (9) the stresses through deformations:

$$\tau_{zx}^m = G_m(\gamma_{zx}^m - \gamma_{zx}^*) = G_m\left(\alpha + \frac{\partial w}{\partial x} - \gamma_{zx}^*\right);$$
$$\tau_{zy}^m = G_m(\gamma_{zy}^m - \gamma_{zy}^*) = G_m\left(\beta + \frac{\partial w}{\partial y} - \gamma_{zy}^*\right). \quad (11)$$

Then the transverse forces will take the form:

$$Q_x = G_m h\left(\alpha + \frac{\partial w}{\partial x} - \gamma_{zx}^*\right);$$
$$Q_y = G_m h\left(\beta + \frac{\partial w}{\partial y} - \gamma_{zy}^*\right). \quad (12)$$

Substituting (6) into (9) and then (9) into (3), we obtain:

$$M_x = D\left(\frac{\partial \alpha}{\partial x} + \nu\frac{\partial \beta}{\partial y}\right);$$
$$M_y = D\left(\nu\frac{\partial \alpha}{\partial x} + \frac{\partial \beta}{\partial y}\right); \quad (13)$$
$$H = \frac{D(1-\nu)}{2}\left(\frac{\partial \alpha}{\partial y} + \frac{\partial \beta}{\partial x}\right),$$

where $D = \frac{E\delta h^2}{2(1-\nu^2)}$ – cylindrical rigidity of a three-layer shell.

For longitudinal forces, taking into account (9) and (4), we can write:

$$N_x = \frac{E\delta}{1-\nu^2}\left(\varepsilon_x^+ + \varepsilon_x^- + \nu(\varepsilon_x^+ + \varepsilon_x^-)\right) = \frac{E\delta}{1-\nu^2}\left(\frac{\partial u^+}{\partial x} + \frac{\partial u^-}{\partial x} + \right.$$
$$\left. +2\left(k_x + \nu k_y\right)w + \nu\left(\frac{\partial v^+}{\partial y} + \frac{\partial v^-}{\partial y}\right)\right) = \frac{2E\delta}{1-\nu^2}\left(\frac{\partial u}{\partial x} + \nu\frac{\partial v}{\partial y} + \left(k_x + \nu k_y\right)w\right);$$
$$N_y = \frac{2E\delta}{1-\nu^2}\left(\frac{\partial v}{\partial y} + \nu\frac{\partial u}{\partial x} + \left(k_y + \nu k_x\right)w\right); \tag{14}$$
$$S = \frac{E\delta}{1+\nu}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right).$$

By analogy with the average displacements u and v, we introduce the values of the average deformations of the carrier layers by the formulas:

$$\varepsilon_x^0 = \frac{\partial u}{\partial x}; \quad \varepsilon_y^0 = \frac{\partial w}{\partial y}; \quad \gamma_{xy}^0 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}. \tag{15}$$

For the values introduced by formulas (15), the deformation compatibility equation is valid:

$$\frac{\partial^2 \varepsilon_x^0}{\partial y^2} + \frac{\partial^2 \varepsilon_y^0}{\partial x^2} = \frac{\partial^2 \gamma_{xy}^0}{\partial x \partial y}. \tag{16}$$

We express from the relations (14) the values ε_x^0, ε_y^0 и γ_{xy}^0:

$$\varepsilon_x^0 = \frac{1}{2E\delta}\left(N_x - \nu N_y\right) - k_x w = \frac{1}{2E\delta}\left(\frac{\partial^2 \Phi}{\partial y^2} - \nu\frac{\partial^2 \Phi}{\partial x^2}\right) - k_x w;$$
$$\varepsilon_y^0 = \frac{1}{2E\delta}\left(N_y - \nu N_x\right) - k_y w = \frac{1}{2E\delta}\left(\frac{\partial^2 \Phi}{\partial x^2} - \nu\frac{\partial^2 \Phi}{\partial y^2}\right) - k_y w; \tag{17}$$
$$\gamma_{xy}^0 = \frac{1+\nu}{E\delta}S = -\frac{1+\nu}{E\delta}\frac{\partial^2 \Phi}{\partial x \partial y}.$$

Substituting (17) into the deformation compatibility equation (16), we obtain the first resolving equation:

$$\frac{1}{2E\delta}\nabla^4\Phi - k_x\frac{\partial^2 w}{\partial y^2} - k_y\frac{\partial^2 w}{\partial x^2} = 0. \tag{18}$$

To obtain the second resolving equation, we substitute the formulas of the transverse forces (12) into the last equation of equilibrium in (1):

$$G_m h\left(\frac{\partial \alpha}{\partial x} + \frac{\partial \beta}{\partial y} + \nabla^2 w - \frac{\partial \gamma_{zx}^*}{\partial x} - \frac{\partial \gamma_{zy}^*}{\partial y}\right) = -q + k_x\frac{\partial^2 \Phi}{\partial y^2} + k_y\frac{\partial^2 \Phi}{\partial x^2}. \tag{19}$$

We introduce the displacement function F by the formula:

$$F = \frac{\partial \alpha}{\partial x} + \frac{\partial \beta}{\partial y}. \tag{20}$$

Then equation (19) takes the form:

$$\nabla^2 w - \frac{1}{G_m h}\left(k_x\frac{\partial^2 \Phi}{\partial y^2} + k_y\frac{\partial^2 \Phi}{\partial x^2}\right) = -\frac{q}{G_m h} - F + \frac{\partial \gamma_{zx}^*}{\partial x} + \frac{\partial \gamma_{zy}^*}{\partial y}. \tag{21}$$

We exclude the transverse forces from the last three equilibrium equations in (1):

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 H}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} - k_x N_x - k_y N_y + q = 0. \tag{22}$$

The third resolving equation is obtained by substituting (13) into (22):

$$D\nabla^2 F = -q + k_x\frac{\partial^2 \Phi}{\partial y^2} + k_y\frac{\partial^2 \Phi}{\partial x^2}. \tag{23}$$

Thus, the problem of calculating a shallow three-layer shell reduces to a system of three differential equations (18), (21), and (23). Instead of two second-order equations (21) and (23) with respect to the functions w, F and Φ, we can obtain a fourth-order equation with respect to the deflection and stress function. For this we express from (21) the value F:

$$F=-\nabla^2 w+\frac{1}{G_m h}\left(k_x\frac{\partial^2\Phi}{\partial y^2}+k_y\frac{\partial^2\Phi}{\partial x^2}-q\right)+\frac{\partial\gamma_{zx}^*}{\partial x}+\frac{\partial\gamma_{zy}^*}{\partial y}. \tag{24}$$

Further we substitute (24) into (23):

$$\begin{gathered}\nabla^4 w-\frac{1}{G_m h}\nabla^2\left(k_x\frac{\partial^2\Phi}{\partial y^2}+k_y\frac{\partial^2\Phi}{\partial x^2}\right)+\frac{1}{D}\left(k_x\frac{\partial^2\Phi}{\partial y^2}+k_y\frac{\partial^2\Phi}{\partial x^2}\right)=\\ =\frac{q}{D}-\frac{1}{G_m h}\nabla^2 q+\nabla^2\left(\frac{\partial\gamma_{zx}^*}{\partial x}+\frac{\partial\gamma_{zy}^*}{\partial y}\right).\end{gathered} \tag{25}$$

The solution of the system of equations (18) and (25) makes it possible to determine the deflection, as well as longitudinal and shearing forces. However, to calculate the bending moments, torque and shear forces, it is required to find the functions α and β. To obtain the resolving equations for α and β, we substitute (13) in the third and fourth equation (1):

$$\begin{gathered}Q_x=\frac{\partial M_x}{\partial x}+\frac{\partial H}{\partial y}=D\left(\frac{\partial^2\alpha}{\partial x^2}+\frac{1+\nu}{2}\frac{\partial^2\beta}{\partial x\partial y}+\frac{1-\nu}{2}\frac{\partial^2\alpha}{\partial y^2}\right);\\ Q_y=\frac{\partial M_y}{\partial y}+\frac{\partial H}{\partial x}=D\left(\frac{\partial^2\beta}{\partial y^2}+\frac{1+\nu}{2}\frac{\partial^2\alpha}{\partial x\partial y}+\frac{1-\nu}{2}\frac{\partial^2\beta}{\partial x^2}\right).\end{gathered} \tag{26}$$

Using the displacement function F, we exclude the function β from the first equation in (26), and the function α from the second:

$$\begin{gathered}Q_x=\frac{D}{2}\left((1-\nu)\nabla^2\alpha+(1+\nu)\frac{\partial F}{\partial x}\right);\\ Q_y=\frac{D}{2}\left((1-\nu)\nabla^2\beta+(1+\nu)\frac{\partial F}{\partial y}\right).\end{gathered} \tag{27}$$

Equating (27) to (12), we obtain:

$$\begin{gathered}\nabla^2\alpha-\frac{2G_m h}{D(1-\nu)}\alpha=\frac{2G_{ь}h}{D(1-\nu)}\left(\frac{\partial w}{\partial x}-\gamma_{zx}^*\right)-\frac{1+\nu}{1-\nu}\frac{\partial F}{\partial x}.\\ \nabla^2\beta-\frac{2G_m h}{D(1-\nu)}\beta=\frac{2G_m h}{D(1-\nu)}\left(\frac{\partial w}{\partial y}-\gamma_{zy}^*\right)-\frac{1+\nu}{1-\nu}\frac{\partial F}{\partial y}.\end{gathered} \tag{28}$$

Thus, to determine the functions α and β, it is necessary to first find the displacement function F, therefore, the use of Eq. (25) instead of (21) and (23) is inexpedient.

After calculating the longitudinal forces, bending and twisting moments stresses in the carrier layers can be found by the formulas:

$$\begin{gathered}\sigma_x^+=\frac{N_x}{2\delta}+\frac{M_x}{h\delta};\quad \sigma_x^-=\frac{N_x}{2\delta}-\frac{M_x}{h\delta};\\ \sigma_y^+=\frac{N_y}{2\delta}+\frac{M_y}{h\delta};\quad \sigma_y^-=\frac{N_y}{2\delta}-\frac{M_y}{h\delta};\\ \tau_{xy}^+=\frac{S}{2\delta}+\frac{H}{h\delta};\quad \tau_{xy}^-=\frac{S}{2\delta}-\frac{H}{h\delta}.\end{gathered} \tag{29}$$

We consider the calculation technique using the example of a three-layered shallow shell rectangular in plan, the surface of which is an elliptical paraboloid (Fig. 2). The equation of the shell surface is:

$$z=f\left[\frac{f_1}{f}\left(2\frac{x}{a}-1\right)^2+\frac{f_2}{f}\left(2\frac{y}{b}-1\right)^2-1\right], \tag{30}$$

where $f=f_1+f_2$ – shell elevation.

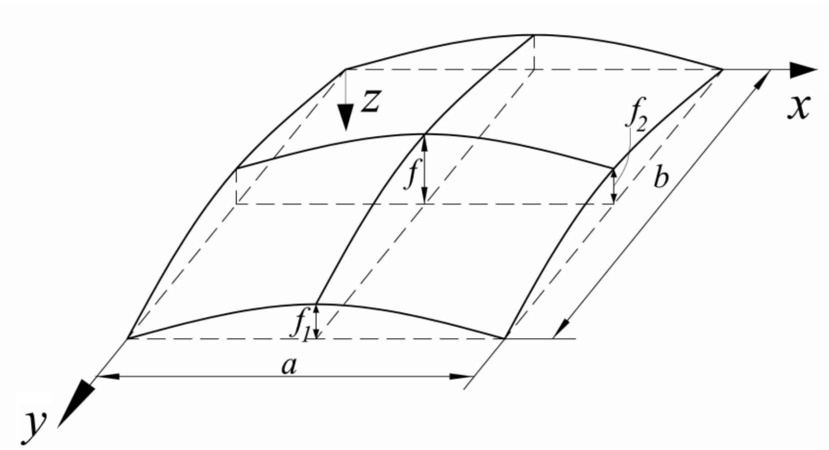

Figure 2. Rectangular in the plan shallow shell in the form of an elliptical paraboloid

The principal curvatures of the considered shell are determined as follows:

$$k_x = -\frac{\partial^2 z}{\partial x^2} = -\frac{8f_1}{a^2};$$
$$k_y = -\frac{\partial^2 z}{\partial y^2} = -\frac{8f_2}{b^2}. \quad (31)$$

In the calculations, we assume that along the contour the shell is connected to diaphragms absolutely rigid in their plane and flexible from it. The boundary conditions on the edges have the form:

at $x = 0$, $x = a$:

$$w = 0;\ M_x = 0;\ N_x = \frac{\partial^2 \Phi}{\partial y^2} = 0;\ v^+ = v^- = 0. \quad (32)$$

at $y = 0$, $y = b$:

$$w = 0;\ M_y = 0;\ N_y = \frac{\partial^2 \Phi}{\partial x^2} = 0;\ u^+ = u^- = 0. \quad (33)$$

From the last equality in (32) it follows that at the edges $x = 0$ and $x = a$:

$$\beta = \frac{v^+ - v^-}{h} = 0. \quad (34)$$

Then on these edges the derivative $\frac{\partial \beta}{\partial y}$ automatically vanishes. In order for the bending moment M_x to be zero it is necessary that the derivative $\frac{\partial \alpha}{\partial x}$ is equal to zero. Then for the edges $x = 0$, $x = a$ we can write:

$$w = 0;\ \frac{\partial^2 \Phi}{\partial y^2} = 0;\ F = \frac{\partial \alpha}{\partial x} + \frac{\partial \beta}{\partial y} = 0. \quad (35)$$

Similarly for the edges $y = 0$ and $y = b$:

$$w = 0;\ \frac{\partial^2 \Phi}{\partial x^2} = 0;\ F = 0. \quad (36)$$

To solve the system of differential equations (18), (21) and (23), one more boundary condition is necessary with respect to the stress function Φ. As this condition we use the equality of Φ function to zero at the edges.

The boundary conditions for the α function have the form:

at $x = 0, x = a$: $\frac{\partial \alpha}{\partial x} = 0$;

at $y = 0, y = b$: $\alpha = 0$.

For the function β, the boundary conditions are written as:

at $x = 0, x = a$: $\beta = 0$;

at $y = 0, y = b$: $\frac{\partial \beta}{\partial y} = 0$.

The system of equations (18), (21) and (23) was solved numerically by the finite difference method in combination with Euler's method for determining creep strains. Calculations were performed in the Matlab package. The time interval at which the creep process was considered we divided into n steps Δt. The first step was the solution of the elastic problem with $t = 0$, $\gamma_{zx}^* = 0$, $\gamma_{zy}^* = 0$. After defining the functions Φ, w and F the numerical solution of equations (28) was performed. Next, the stresses in the shells and filler were determined. If the creep law is given in differential form, then the stresses can be used to calculate the growth rates of creep strains, as well as creep strains at time $t + \Delta t$ by linear approximation:

$$\gamma_{t+\Delta t}^* = \gamma_t^* + \frac{\partial \gamma^*}{\partial t}\Delta t. \tag{37}$$

Note that for three-layered plates in comparison with shells, the calculation is much simpler, since instead of a system of three differential equations, it is sufficient to solve successively the following two equations:

$$D\nabla^2 F = -q;$$
$$\nabla^2 w = -\frac{q}{G_m h} - F + \frac{\partial \gamma_{zx}^*}{\partial x} + \frac{\partial \gamma_{zy}^*}{\partial y}. \tag{38}$$

The first equation in (38) does not include creep strains, which implies that the displacement function F for a three-layer plate does not depend on time, and it is not necessary to solve this equation at every time step, but only once. After determining the functions F and w, the functions α and β are determined from Eqs. (28).

Results and Discussion

The calculation was made for a rectangular shell with dimensions $a = b = 3\ \mathrm{m}$, overall thickness $h = 8\ \mathrm{cm}$, loaded by uniformly distributed over the area the load q. Carrier layers of the shell were steel with thickness of 1 mm ($E = 2 \cdot 10^5\ \mathrm{MPa}, \nu = 0.3$). The middle layer is a rigid polyurethane foam ($G_m = 4.85$ MPa).

As the creep law, the Maxwell-Thompson linear equation was used, as well as the nonlinear Maxwell-Gurevich equation. For uniaxial tension (compression), the Maxwell-Thompson equation has the form [17]:

$$nE\frac{\partial \varepsilon}{\partial t} + H\varepsilon = n\frac{\partial \sigma}{\partial t} + \sigma, \tag{39}$$

where E and H – respectively, instantaneous and long-term elastic moduli, n – relaxation time, σ – normal stress, ε – full strain.

For the case of pure shear, equation (39) can be written in the form:

$$nG_m\frac{\partial \gamma}{\partial t} + H\gamma = n\frac{\partial \tau}{\partial t} + \tau. \tag{40}$$

Here G_m and H are respectively the instantaneous and long-term shear moduli, τ is the tangential stress, and γ is the total shear deformation.

Representing the total deformation as the sum of elastic deformation and creep deformation, we can express from (40) the growth rate of creep strains in the following form:

$$\frac{\partial \gamma_i^*}{\partial t} = \frac{1}{\kappa}\left[\left(1 - \frac{H}{G_m}\right)\tau_i - H\gamma_i^*\right], \tag{41}$$

where $\kappa = nG_m$ – coefficient of viscosity of the filler.

To determine the deformations γ_{zx}^* and γ_{zy}^* it suffices to substitute the corresponding indices in (41) instead of i.

The Maxwell-Gurevich equation in the case of a triaxial stress state is written in the form [18]:

$$\frac{\partial \varepsilon_{ij}^*}{\partial t} = \frac{f_{ij}^*}{\eta^*}, \qquad i = x, y, z, \qquad j = x, y, z, \tag{42}$$

where f_{ij} – stress function, η^* – relaxation viscosity.

$$f_{ij}^{*} = \frac{3}{2}\left(\sigma_{ij} - p\delta_{ij}\right) - E_{\infty}\varepsilon_{ij}^{*};$$
$$\eta^{*} = \eta_0^{*}\ \exp\left(-\frac{|f_{max}^{*}|}{m^{*}}\right), \tag{43}$$

where δ_{ij} – Kronecker symbol, $p = (\sigma_x + \sigma_y + \sigma_z)/3$ – mean stress, m^* – the relaxation constant, called the velocity modulus, η_0^* – initial relaxation viscosity, E_∞ – high elasticity modulus.

When using equation (43), it is necessary to bear in mind that $\varepsilon_{zx}^* = \frac{1}{2}\gamma_{zx}^*$ and $\varepsilon_{zy}^* = \frac{1}{2}\gamma_{zy}^*$.

The results of tests of rigid polyurethane foam in shear creep are presented in [1–2]. The creep curves in the above studies are approximated by the Findley power law, which in the case of uniaxial tension is:

$$\varepsilon = \sigma\left(\frac{1}{E_e} + \frac{1}{E_t}t^n\right), \tag{44}$$

where E_e and E_t – respectively, elastic and viscoelastic modulus of deformation of the material.

The disadvantage of this law is that the time in it is contained in an explicit form, which can lead to contradictory results. A method for determining the relaxation constants of a material on the basis of the Maxwell-Gurevich equation is given in [19–20]. Using this technique, as well as the results presented in [1–2], the author obtained the following values of the relaxation constants: $E_\infty = 27.38$ MPa, $\eta_0^* = 1.43 \cdot 10^4$ MPa · h, $m^* = 0.0218$ MPa.

When processing creep curves on the basis of the Maxwell-Thompson equation, the relaxation parameters were chosen so that at the end of the creep process the solution using this equation coincided with the solution based on the Maxwell-Gurevich equation. As a result we obtained the values $\kappa = 1118$ MPa · h, $H = 3.17$ MPa.

The magnitude of the load was assumed constant: $q = 2$ kPa. The curvature of the shell was varied by varying the amount of elevation f. The growth curves of the largest deflection relative to the deflection at $t = 0$ with different values of the ratio $f\ /\ a$ are shown in Fig. 3. The dashed lines correspond to the result based on the Maxwell-Gurevich equation, continuous - to the Maxwell-Thompson equation.

In his Ph.D. thesis, the author studied the influence of the curvature of the shell on the growth of deflection under linear creep by the example of a spherical three-layer shell. For the analysis, finite element modeling was used. As a result, it was found that with increasing curvature, the creep effect decreases, and for shells of large curvature, the creep of the aggregate has no effect on the amount of deflection. Similar results are observed in Figure 3. Creep of the aggregate does not have a noticeable effect on the deflection already at $f/a = 1/15$. We recall that it is customary to refer to shallow shells such that have $f/a \le 1/5$. The results obtained on the basis of the linear and nonlinear theory differ insignificantly, especially for shells with greater curvature.

If the displacements in time practically do not change, then in the presence of viscoelastic properties of the material, the stresses can not be constant. Figure 4 is a graph of the change in time of the greatest value of tangential stresses τ_{zx}^m at $f/a = 1/15$. As before, the dashed line corresponds to the result based on the Maxwell-Gurevich equation, continuous - to the Maxwell-Thompson equation. It is seen from the presented graph that in the aggregate with constant shear deformations the stress relaxation occurs. Bending and twisting moments also decrease in time. The graphs of their changes are shown in Figure 5. The greatest bending moment in the creep process decreased by 32.7 %, and the torque – by 27 %.

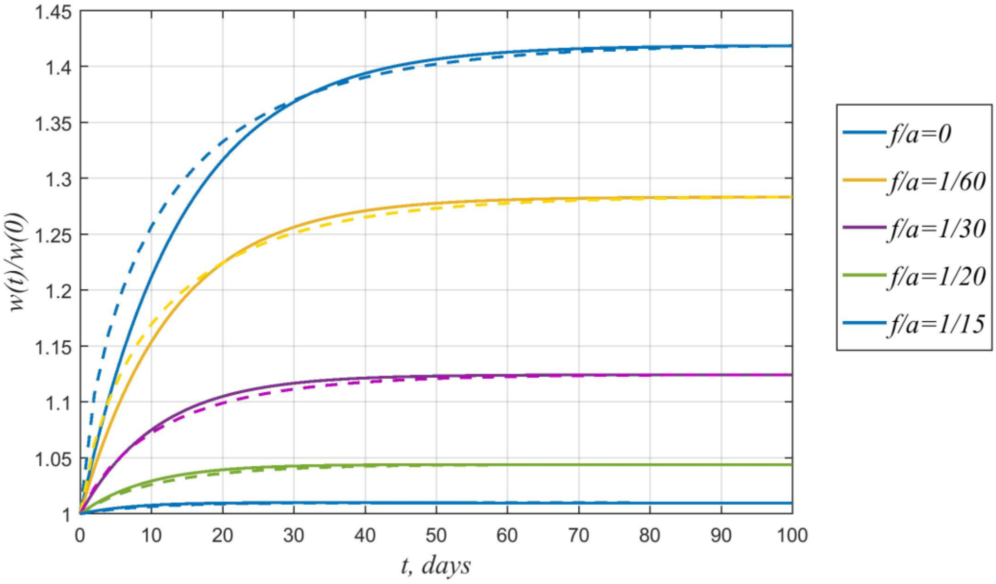

Figure 3. Graphs of the growth of the deflection for different values of the shell elevation: solid lines – Maxwell-Thompson equation, dashed lines – Maxwell-Gurevich equation

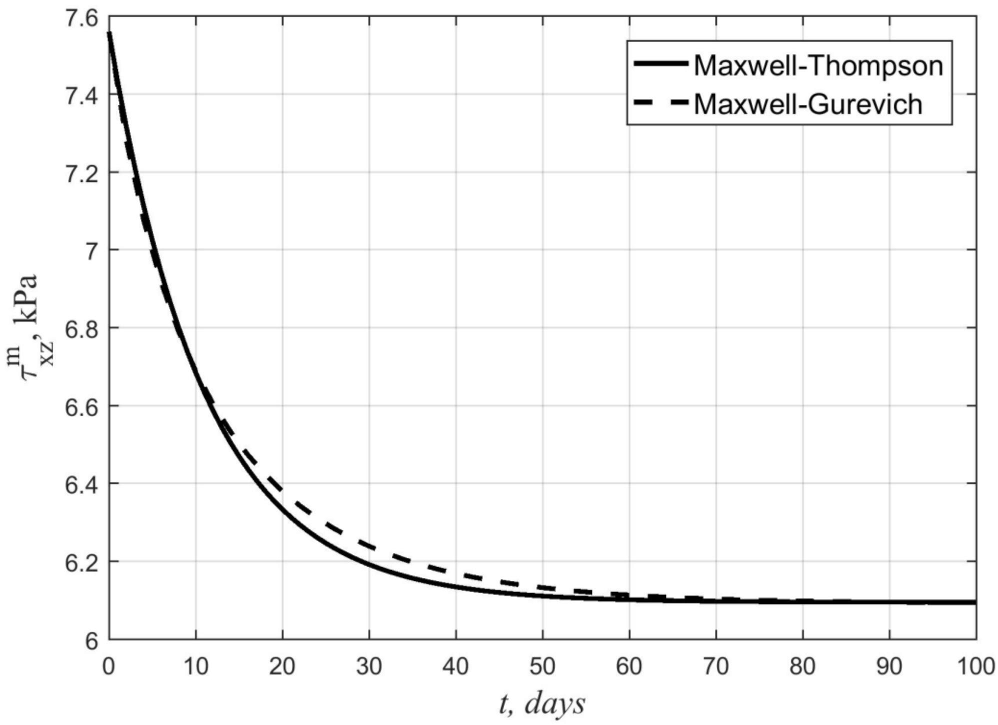

Figure 4. Change in time of the greatest tangential stresses in the aggregate

Longitudinal and shear forces increase during the creep process. The graphs of their growth are shown in Figure 6. The longitudinal force N_x increased by 8.33 %, and the shear force S – by 12.4 %. Figure 7 is a graph of the change in time of the largest values of the normal stresses σ_x in the upper and lower skin. In the upper skin, the stresses are practically constant, and in the lower skin they increase by 17.4 %. The tangential stresses increase both in the upper and the lower skin, as it can be seen from Figure 8. In the upper skin, the stresses τ_{xy} increased by 18.8 %, and in the lower skin by 7.71 %.

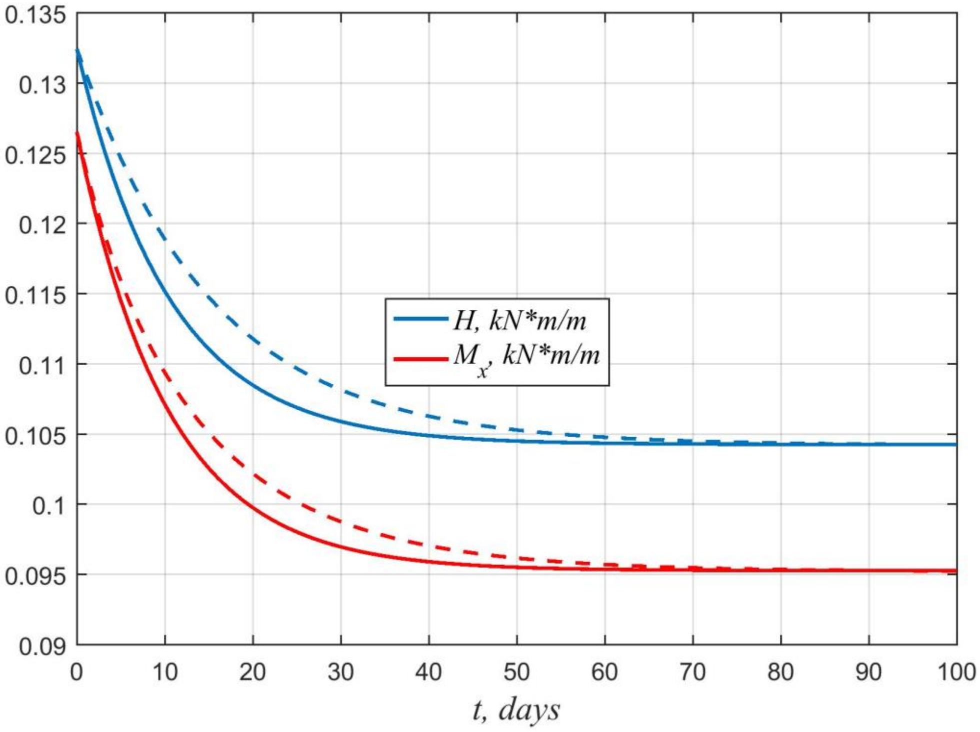

Figure 5. Change in time of the greatest bending and twisting moments: solid lines – Maxwell-Thompson equation, dashed lines – Maxwell-Gurevich equation

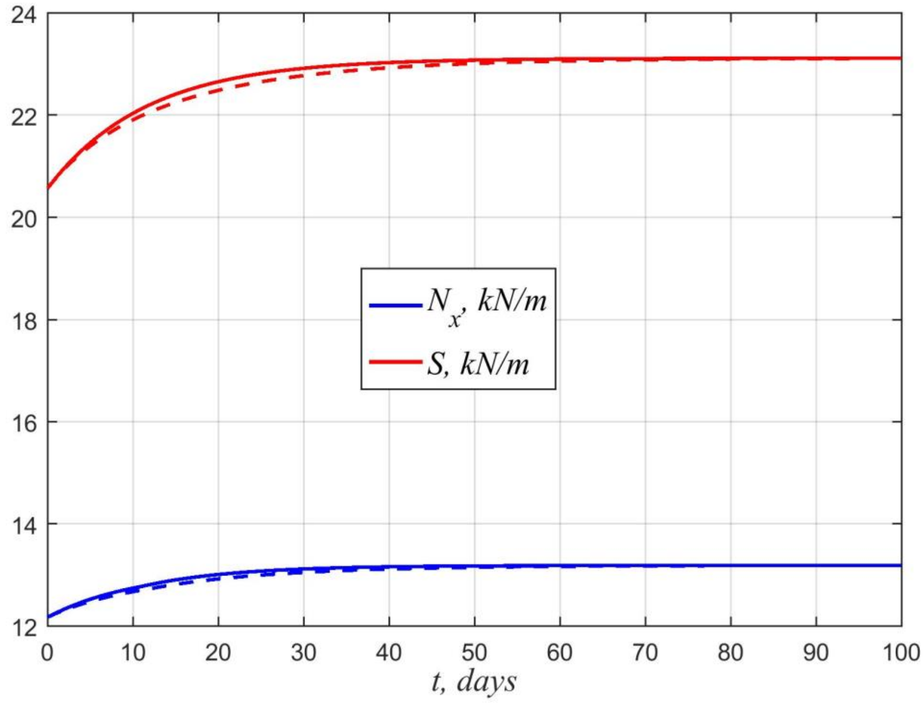

Figure 6. Change in time of the greatest longitudinal and shear forces

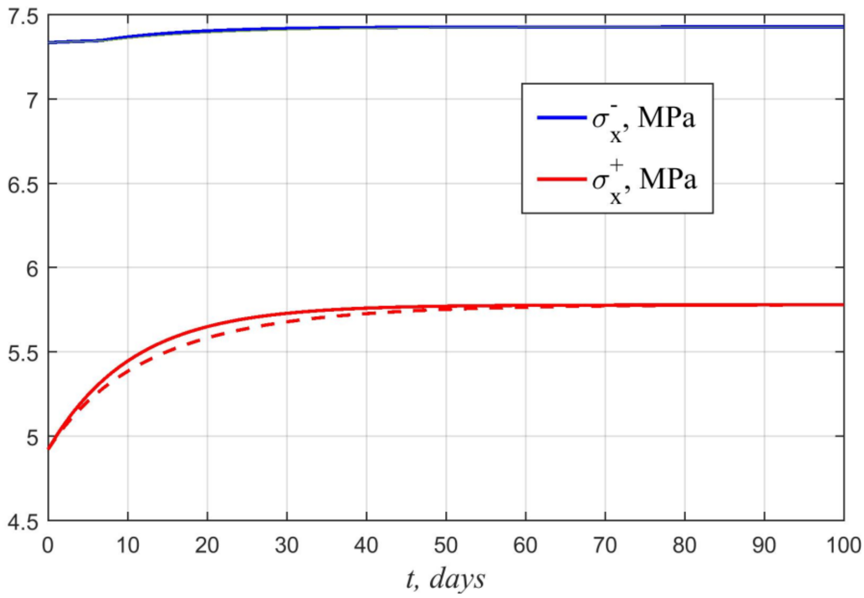

Figure 7. Change in time of the greatest normal stresses in the carrier layers

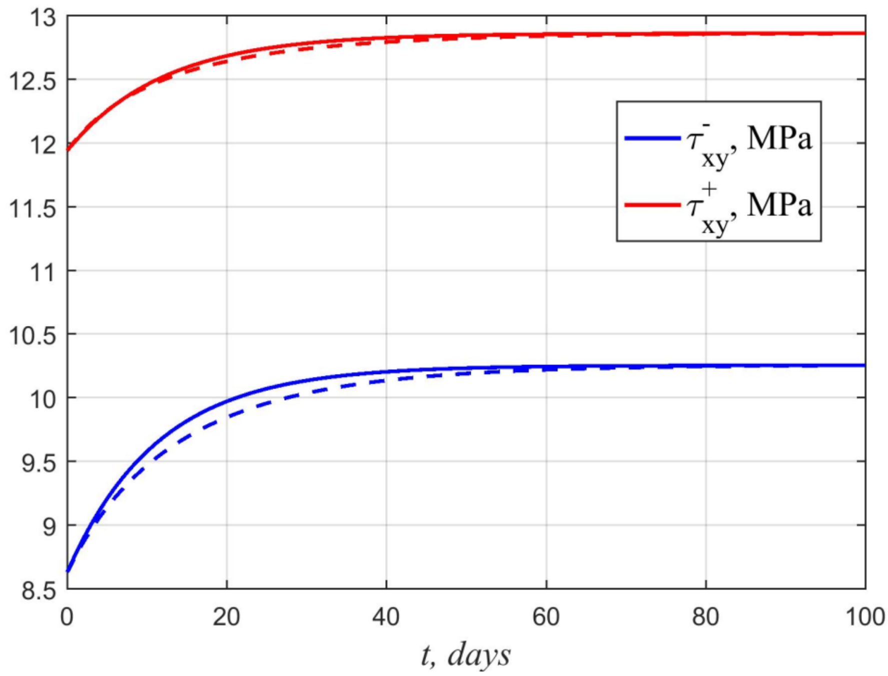

Figure 8. Change in time of the greatest tangential stresses in the shells

Conclusions

The obtained resolving equations are universal and allow to calculate three-layer plates and shallow shells under an arbitrary creep law. Using the example of a three-layer shell in the form of an elliptical paraboloid, it was shown that, with increasing of curvature, the effect of creep of the aggregate on the deflection amount decreases and is practically absent even at $f\ /\ a\ =\ 1/15$. At the same time, with constant displacements, redistribution of stresses and internal forces occurs. The bending and twisting moments decrease, and the longitudinal and shear forces increase. As for stresses, in the middle layer relaxation occurs, and in carrier layers stresses increase.

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Anton Chepurnenko,
+7(918)571-87-38; anton_chepurnenk@mail.ru

Антон Сергеевич Чепурненко,
+7(918)571-87-38;
эл. почта: anton_chepurnenk@mail.ru

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