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## Dynamic buckling of stiffened orthotropic shell structures

### Динамическая устойчивость подкрепленных ортотропных оболочечных конструкций

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**Ключевые слова:** подкрепленные оболочки; устойчивость; ортотропия; динамическое нагружение; метод размазывания жесткости; метод Канторовича

**Abstract.** Explored orthotropic shallow shells of double curvature, as well as cylindrical panels that are reinforced from the concave side by an orthogonal grid of stiffeners. The external transverse load acting on the structure is uniformly distributed and has a linear dependency on time. A geometrically nonlinear variant of the model which also takes into account orthotropy of the material and transverse shears are considered. The model is presented as a functional of total deformation energy of the shell. The algorithm for studying the mathematical model is based on the L.V. Kantorovich method and the Rosenbrock method. The proposed algorithm was implemented in the analytical computing environment Maple 2016. The calculations showed a significant increase in critical load values for the loss of stability when the shell is reinforced with stiffeners.

**Аннотация.** Рассматриваются ортотропные пологие оболочки двоякой кривизны, а также цилиндрические панели, которые подкреплены со стороны вогнутости ортогональной сеткой ребер жесткости. Внешняя поперечная нагрузка, действующая на оболочечную конструкцию, равномерно распределена и имеет линейную зависимость от времени. Рассмотрен геометрически нелинейный вариант модели, учитывающий также ортотропию материала и поперечные сдвиги. Модель представлена в виде функционала полной энергии деформации оболочки. Алгоритм исследования математической модели основан на методе Л. В. Канторовича и методе Розенброка. Предлагаемый алгоритм реализован в среде аналитических вычислений Maple 2016. Расчеты показали значительное увеличение значений критической нагрузки потери устойчивости, когда оболочка усилена ребрами жесткости.

## 1. Introduction

For thin-walled shell structures used in construction and other industrial fields, dynamic loading [1–7] can lead to a loss of stability. Thereby such structures are reinforced by stiffeners [1–4], which makes it possible to increase stability and reduce stress concentration in danger zones, redistributing them throughout the area of the structure.

The main objectives of the study of shell structures under dynamic loading are to investigate their stability, strength and vibrations as evidenced by review articles and monographs [8–16]. One should mention the extensive review article of E.A. Kogan and A.A. Yurchenko [8] which is devoted to free and forced nonlinear oscillations of multilayered thin elastic plates and shells under periodic loading.

Buckling of shells under dynamic loading is considered in [1–4, 17–28], moreover in [1, 2, 18–21] equations in mixed form were used. Most of the buckling studies of shells and panels were carried out for cylindrical shells [3, 4, 7, 20–26], less often for shallow shells of double curvature [22, 24].

The paper of D.H. Bich, D.V. Dung and V.H. Nam [22] presents a semi-analytical approach to investigate the nonlinear dynamic of imperfect eccentrically stiffened functionally graded shallow shells taking into account the damping subjected to mechanical loads. The functionally graded shallow shells are simply supported at edges and are reinforced by transversal and longitudinal stiffeners on internal or external surface. The formulation is based on the classical thin shell theory with the geometrical nonlinearity

in von Karman–Donnell sense and the smeared stiffeners technique. The nonlinear critical dynamic buckling loads are found according to the Budiansky–Roth criterion.

According to the type of external action, structures under axial compression are more frequently investigated [1, 4, 17–19, 21–23], whereas structures under uniformly distributed transverse loading are studied less often [1, 22].

The process of deformation of reinforced shell structures made of orthotropic materials under dynamic loading has not yet been studied sufficiently.

The purpose of this work is to analyze the stability of some variants of stiffened shell structures made of modern orthotropic materials under dynamic loading.

In accordance with this purpose, the following tasks are set:

1. To modify the mathematical model of deformation of orthotropic shell structures under dynamic loading, using the method of constructive anisotropy to smear the stiffness of the ribs.
2. To develop a computer program, based on the L.V. Kantorovich method and the Rosenbrock method for solving the buckling problem of the shells.
3. To analyze the buckling process of an orthotropic double-curved shallow shell and an orthotropic cylindrical shell panel, reinforced by stiffening ribs, under dynamic loading.

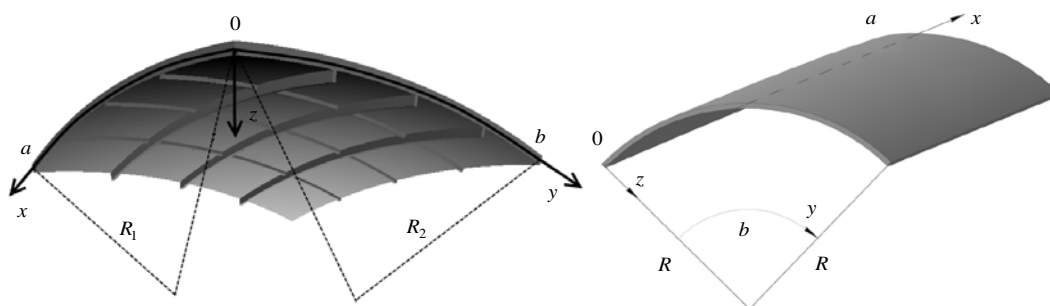
## 2. Methods

### 2.1. Mathematical model

We will examine orthotropic shallow shells of double curvature, as well as cylindrical panels that are reinforced from the concave side by an orthogonal grid of stiffeners. The external transverse load acting on the structure is uniformly distributed and has a linear dependency on time.

We will use a geometrically nonlinear variant of the model, which also takes into account orthotropy of the material and transverse shears (based on Mindlin–Reissner model). In this case, the unknown functions are three displacement functions  $U = U(x, y, t)$ ,  $V = V(x, y, t)$ ,  $W = W(x, y, t)$  and two functions of the angles of rotation of the normal  $\Psi_x = \Psi_x(x, y, t)$ ,  $\Psi_y = \Psi_y(x, y, t)$ .

We shall consider orthotropic shell constructions of an arbitrary form of thickness  $h$ , under the influence of an external uniformly distributed lateral load  $q = q(x, y, t)$  (Figure 1).



**Figure 1. Schematic representation of a double-curved shallow shell [29] and cylindrical panel**

The model is presented as a functional of total deformation energy of the shell:

$$I = \int_{t_0}^{t_1} (K - E_p) dt, \quad (1)$$

where  $K$  is the kinetic deformation energy of the system, and  $E_p$  is the functional of the static problem [30], equal to the difference in the potential deformation energy of the system and the work of external forces. For reinforced shells, they can be represented in the form:

$$K = \frac{\rho}{2} \int_0^a \int_0^b \left\{ (h + F_p) \left[ \left( \frac{\partial U}{\partial t} \right)^2 + \left( \frac{\partial V}{\partial t} \right)^2 + \left( \frac{\partial W}{\partial t} \right)^2 \right] + 2S_p \left[ \frac{\partial U}{\partial t} \frac{\partial \Psi_x}{\partial t} + \frac{\partial V}{\partial t} \frac{\partial \Psi_y}{\partial t} \right] + \right. \\ \left. + \left( \frac{h^3}{12} + J_p \right) \left[ \left( \frac{\partial \Psi_x}{\partial t} \right)^2 + \left( \frac{\partial \Psi_y}{\partial t} \right)^2 \right] \right\} AB dx dy, \quad (2)$$

$$E_p = \frac{E_1}{2(1 - \mu_{12}\mu_{21})} \int_0^a \int_0^b \left\{ (h + F_x) \varepsilon_x^2 + \bar{G}_2 (h + F_y) \varepsilon_y^2 + \mu_{21} (2h + F_x + F_y) \varepsilon_x \varepsilon_y + \bar{G}_{12} \left( h + \frac{F_x}{2} + \frac{F_y}{2} \right) \gamma_{xy}^2 + \right. \\ + \bar{G}_{13} k (h + F_x) (\Psi_x - \theta_1)^2 + \bar{G}_{23} k (h + F_y) (\Psi_y - \theta_2)^2 + 2S_x \varepsilon_x \chi_1 + \mu_{21} (S_x + S_y) \varepsilon_x \chi_2 + \mu_{21} (S_x + S_y) \varepsilon_y \chi_1 + \\ + 2\bar{G}_2 S_y \varepsilon_y \chi_2 + 2\bar{G}_{12} (S_x + S_y) \gamma_{xy} \chi_{12} + \left( \frac{h^3}{12} + J_x \right) \chi_1^2 + \bar{G}_2 \left( \frac{h^3}{12} + J_y \right) \chi_2^2 + \mu_{21} \left( \frac{h^3}{6} + J_x + J_y \right) \chi_1 \chi_2 + \\ \left. + 2\bar{G}_{12} \left( \frac{h^3}{6} + J_x + J_y \right) \chi_{12}^2 - 2 \frac{q(1 - \mu_{12}\mu_{21})}{E_1} W \right\} AB dx dy, \quad (3)$$

where  $\varepsilon_x, \varepsilon_y$  – are axial strains along the  $x$  and  $y$  coordinates of the median surface;  $\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$  – are the shear strains in the  $xOy, xOz, yOz$  planes respectively;  $\chi_1, \chi_2, \chi_{12}$  – are functions of change of curvatures and torsion;  $\rho$  – is density;  $A, B$  – Lamé parameters;

$$\theta_1 = - \left( \frac{1}{A} \frac{\partial W}{\partial x} + k_x U \right), \quad \theta_2 = - \left( \frac{1}{B} \frac{\partial W}{\partial y} + k_y V \right), \quad k = \frac{5}{6}, \\ \bar{G}_2 = \frac{E_2}{E_1}, \quad \bar{G}_{12} = \frac{G_{12}(1 - \mu_{12}\mu_{21})}{E_1}, \quad \bar{G}_{13} = \frac{G_{13}(1 - \mu_{12}\mu_{21})}{E_1}, \quad \bar{G}_{23} = \frac{G_{23}(1 - \mu_{12}\mu_{21})}{E_1}.$$

Here  $k_x = 1/R_1, k_y = 1/R_2$  – are primary curvatures of the shell along the  $x$  and  $y$  axes;  $R_1, R_2$  – are the principal radii of curvature, which characterizing the geometry of the shell;  $E_1, E_2, \mu_{12}, \mu_{21}$  – elastic moduli and Poisson's coefficients;  $G_{12}, G_{13}, G_{23}$  – shear moduli in planes  $xOy, xOz, yOz$  respectively.

There are different approaches to “smearing” the rigidity of the ribs [1, 2, 31]. Next, we will use the approach, based on the works of V.V. Karpov [32, 33]. Then the variables  $F_x, F_y, F_p, S_x, S_y, S_p, J_x, J_y, J_p$  – are the area of the cross-sectional or longitudinal section of the stiffener per unit length of the cross-section; the static moment of the area; and the moment of inertia of this cross-section, which have the form

$$F_x = \sum_{i=1}^n \frac{h^i r_i}{bB} + \sum_{j=1}^m \left( \frac{h^j r_j}{aA} - \sum_{i=1}^n \frac{h^{ij} r_i r_j}{aAbB} \right) \frac{r_j}{aA}, \quad F_y = \sum_{j=1}^m \frac{h^j r_j}{aA} + \sum_{i=1}^n \left( \frac{h^i r_i}{bB} - \sum_{j=1}^m \frac{h^{ij} r_i r_j}{aAbB} \right) \frac{r_i}{bB}, \\ S_x = \sum_{i=1}^n \frac{S^i r_i}{bB} + \sum_{j=1}^m \left( \frac{S^j r_j}{aA} - \sum_{i=1}^n \frac{S^{ij} r_i r_j}{aAbB} \right) \frac{r_j}{aA}, \quad S_y = \sum_{j=1}^m \frac{S^j r_j}{aA} + \sum_{i=1}^n \left( \frac{S^i r_i}{bB} - \sum_{j=1}^m \frac{S^{ij} r_i r_j}{aAbB} \right) \frac{r_i}{bB}, \quad (4) \\ J_x = \sum_{i=1}^n \frac{J^i r_i}{bB} + \sum_{j=1}^m \left( \frac{J^j r_j}{aA} - \sum_{i=1}^n \frac{J^{ij} r_i r_j}{aAbB} \right) \frac{r_j}{aA}, \quad J_y = \sum_{j=1}^m \frac{J^j r_j}{aA} + \sum_{i=1}^n \left( \frac{J^i r_i}{bB} - \sum_{j=1}^m \frac{J^{ij} r_i r_j}{aAbB} \right) \frac{r_i}{bB},$$

$$F_p = \sum_{i=1}^n \frac{h^i r_i}{bB} + \sum_{j=1}^m \frac{h^j r_j}{aA} - \sum_{j=1}^m \sum_{i=1}^n \frac{h^{ij} r_i r_j}{aAbB}, \quad S_p = \sum_{i=1}^n \frac{S^i r_i}{bB} + \sum_{j=1}^m \frac{S^j r_j}{aA} - \sum_{j=1}^m \sum_{i=1}^n \frac{S^{ij} r_i r_j}{aAbB},$$

$$J_p = \sum_{i=1}^n \frac{J^i r_i}{bB} + \sum_{j=1}^m \frac{J^j r_j}{aA} - \sum_{j=1}^m \sum_{i=1}^n \frac{J^{ij} r_i r_j}{aAbB},$$

where

$$S^i = 0.5h^i(h+h^i), \quad S^j = 0.5h^j(h+h^j), \quad S^{ij} = 0.5h^{ij}(h+h^{ij}),$$

$$J^i = 0.25h^2h^i + 0.5h(h^i)^2 + \frac{1}{3}(h^i)^3, \quad J^j = 0.25h^2h^j + 0.5h(h^j)^2 + \frac{1}{3}(h^j)^3,$$

$$J^{ij} = 0.25h^2h^{ij} + 0.5h(h^{ij})^2 + \frac{1}{3}(h^{ij})^3.$$

Here  $h^i, h^j$  are the height of the stiffener; indices  $i$  and  $j$  indicate the order number of the stiffener located parallel to the  $x$  and  $y$  axes, respectively;  $n, m$  are the number of stiffeners;  $h^{ij} = \min\{h^i, h^j\}$ .

The method of constructive anisotropy can be used if the ratio of distance between ribs to shell length is not more than 0.1.

## 2.2. Algorithm

The algorithm for studying the mathematical model is based on the L.V. Kantorovich method (for reducing the system of differential equations to a system of ordinary differential equations (ODE) with respect to functions of only one variable  $t$ ).

Likewise, the system thus derived is called a multidimensional variant of the Euler–Lagrange equation:

$$\frac{d}{dt} \frac{\partial(K - E_p)}{\partial \dot{X}_k(t)} - \frac{\partial(K - E_p)}{\partial X_k(t)} = 0, \quad k = 1, 2, \dots, 5N, \quad (5)$$

where  $X(t) = (U_i(t), V_i(t), W_i(t), \Psi_{x_i}(t), \Psi_{y_i}(t))^T$ ,  $i = 1, \dots, N$ , and the dot denotes the time derivative.

Moreover,  $U_i - \Psi_{y_i}$  are unknown functions of the variable  $t$ .

Since the derivatives of the required function with respect to variable  $t$  are contained only in the expression for the kinetic energy, and the functions themselves only in the expression for  $E_p$ , then the following is true

$$\frac{d}{dt} \frac{\partial K}{\partial \dot{X}_k(t)} + \frac{\partial E_p}{\partial X_k(t)} = 0, \quad k = 1, 2, \dots, 5N. \quad (6)$$

The resulting system of ODE is a rigid system of equations, and therefore it is necessary to select the most effective numerical method for solving it. In many works, the Runge-Kutta method is used for this [2, 20–22]. Research that has been conducted has shown that for the class of problems being studied, the optimal in terms of stability, accuracy and speed is the Rosenbrock method [34].

The proposed algorithm was implemented in the analytical computing environment Maple 2016.

## 3. Results and Discussion

We consider isotropic and orthotropic cylindrical panels that have fixed-pin joints along the contour. The transverse load acting on the structure is uniformly distributed and linearly dependent on time:  $q = q(x, y, t) = A_1 t$ , where  $A_1$  is loading speed.

Inflection of the “load-deflection” curve is the criterion for loss of stability of the shell under dynamic loading.

We shall consider shell structures with two sets of geometric parameters and reinforced by different number of stiffeners:

- double-curved shallow shell ( $A = 1, B = 1, k_x = 1/R_1, k_y = 1/R_2$ ) with  $a = b = 120h$ , turning angle  $b = 0.4$  rad., radii of curvature  $R_1 = R_2 = 445h$  and thickness  $h = 0.09$  m, with loading speed  $A_1 = 0.35$  MPa/s. Material – E-Glass/Epoxy [35].
- cylindrical shell panel ( $A = 1, B = R, k_x = 0, k_y = 1/R$ ) with length  $a = 150h$ , turning angle  $b = 0.4$  rad., radius  $R = 250h$  and thickness  $h = 0.01$  m, with loading speed  $A_1 = 0.19$  MPa/s. The direction of the orthotropy axis 2 coincides with the direction of the generatrix (the  $x$  axis). Material – AS/3501 Graphite/Epoxy [35].

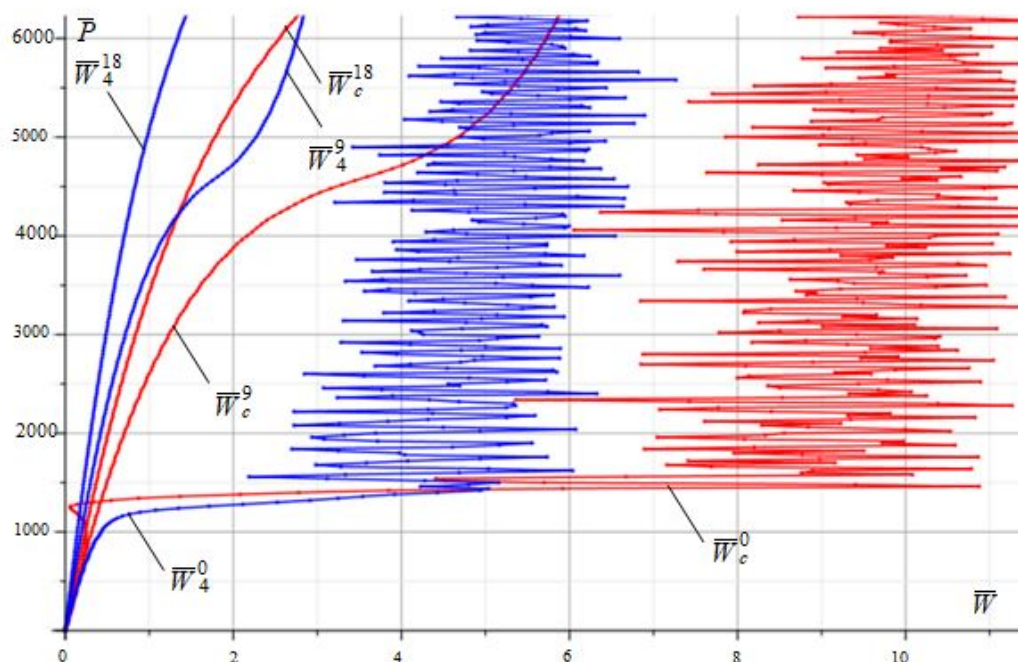
Material parameters are given in Table 1. The shells are hinged-fixed along the outline and reinforced with stiffeners with height  $3h$  and width  $2h$ . All calculations were performed for  $N = 9$ .

**Table 1. Material parameters [35]**

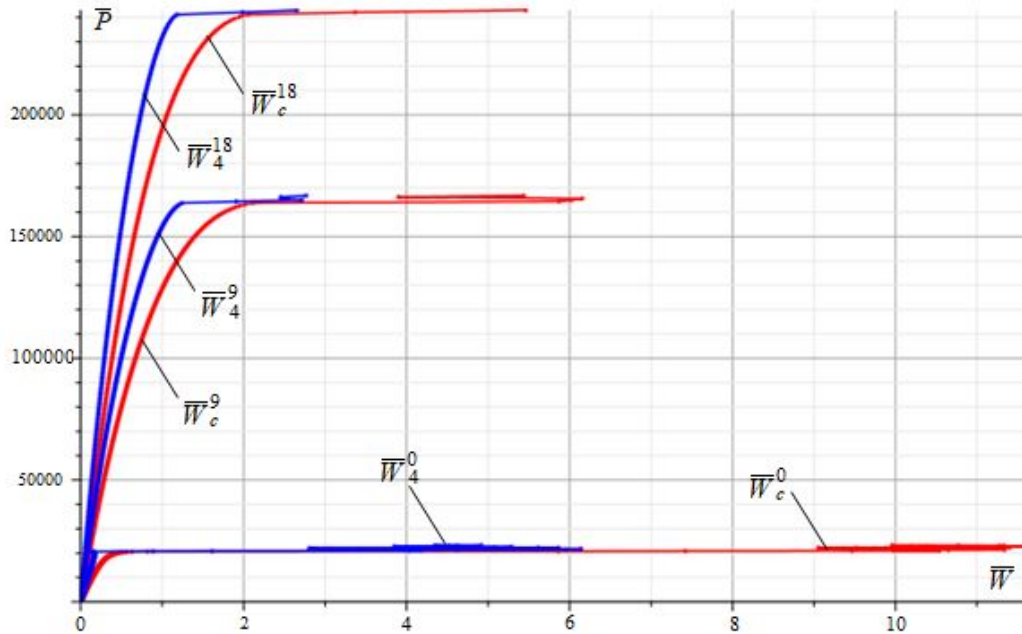
Material	$E_1, \text{MPa}$	$E_2, \text{MPa}$	$G_{12}, \text{MPa}$	$\mu_{12}$	$\rho, \text{kg/m}^3$
E-Glass/Epoxy	$0.607 \cdot 10^5$	$0.248 \cdot 10^5$	$0.12 \cdot 10^5$	0.23	1800
AS/3501 Graphite/Epoxy	$1.38 \cdot 10^5$	$0.0896 \cdot 10^5$	$0.071 \cdot 10^5$	0.3	1540

“Load-deflection” curves for the double-curved shallow shell are given in Figure 2, and for the cylindrical shell panel are given in Figure 3 (red color – deflection in the point  $x = a/2, y = b/2$ , blue color – deflection in the point  $x = a/4, y = b/4$ ). Further on the graphs, all results will be presented in dimensionless parameters:

$$\bar{t} = \frac{h}{a^2 A^2} \sqrt{\frac{E_1}{(1 - \mu_{12}\mu_{21})\rho}} \cdot t, \quad \bar{W} = \frac{W}{h}, \quad \bar{P} = \frac{a^4 A^4 q}{h^4 E_1}. \quad (7)$$



**Figure 2. The “load-deflection” relations for the doubly-curved shallow shell, reinforced by stiffeners (0x0, 9x9, 18x18)**



**Figure 3.** The “load-deflection” relations for the panel of cylindrical shell, reinforced by stiffeners (0×0, 9×9, 18×18)

Load values for loss of stability  $q_{cr}$  for all considered cases are given in Table 2.

**Table 2.** Load values for loss of stability  $q_{cr}$

Shell	Number of stiffeners	$\bar{P}$	$q_{cr}, \text{MPa}$
1. Double-curved shallow shell	0×0	1360	0.163
	9×9	–	–
	18×18	–	–
2. Cylindrical panel	0×0	20700	0.366
	9×9	163800	2.899
	18×18	242100	4.285

To compare the obtained solution with the results from other sources, the author has analyzed various studies on this topic [1–4, 17–28, 36–37], but it was not possible to find a solution to this problem in this formulation.

#### 4. Conclusions

As a result of the present work, the following conclusions can be outlined:

1. The mathematical model of deformation of orthotropic shell structures under dynamic loading and the method of constructive anisotropy to smear the stiffness of the ribs are modified and used.
2. A computer program, based on the L.V. Kantorovich method and the Rosenbrock method for solving the buckling problem of the shells, are developed.
3. The buckling of some variants of doubly-curved shallow shells and cylindrical panels, made of modern orthotropic materials, under dynamic loading was analyzed.
4. The calculations showed a significant increase in critical load values for the loss of stability when the shell is reinforced with stiffeners, as well as nonuniformity of the distribution of deflections and stresses with regard to orthotropy of the material.
5. The proposed technique for studying buckling under dynamic loading can be used to analyze orthotropic shell structures, supported by a large number of stiffeners.



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