Magazine of Civil Engineering. 2019. 86(2). Pp. 30–45
Инженерно-строительный журнал. 2019. № 2(86). С. 30–45

Magazine of Civil Engineering

ISSN 2071-0305

journal homepage: http://engstroy.spbstu.ru/

DOI: 10.18720/MCE.86.4

Nonlinear vibrations of fluid transporting pipelines on a viscoelastic foundation

B.A. Khudayarov*, F.Z. Turaev,
Tashkent institute of irrigation and agricultural mechanization engineers, Tashkent, Uzbekistan
** E-mail: bakht-flpo@yandex.ru*

Keywords: vibration process; foundation; pipeline; mathematical model; numerical algorithm; cylindrical shell

Abstract. The article presents the results of a study of vibration process in pipelines conveying fluid or gas. A mathematical model pipeline was used in the form of cylindrical shell and a viscoelastic foundation in the form of two-parameter model of the Pasternak. The hereditary Boltzmann-Volterra theory of viscoelasticity is used to describe viscoelastic properties. The effects of the parameters of the Pasternak foundations, the singularity in the heredity kernels and geometric parameters of the pipeline on vibrations of structures with viscoelastic properties are numerically investigated. It is found that an account of viscoelastic properties of the pipeline material leads to a decrease in the amplitude and frequency of vibrations by 20–40 %. It is shown that an account of viscoelastic properties of soil foundations leads to a damping of vibration process in pipeline.

1. Introduction

Pipeline systems provide a safe and uninterrupted operation of the objects in fuel and energy industry. The pipelines provide population with basic resources: fresh water, natural gas, oil, etc. Wide networks of pipelines, both domestically and abroad, support the vital functions of states, and are one of the main factors of economic development. The failure of even small sections of pipelines, often accompanied by explosions and fires, can cause serious consequences associated with the loss of the product, the high cost of repairs, and can lead to a significant pollution of the environment.

Currently, the objects of agriculture, oil and gas industry, housing and communal services and others face the problems of repair and restoration of metal pipelines due to the impact of various external factors. One of the ways to solve this problem is to use composite polymer material that has a number of advantages. Due to their characteristics, pipes made of composite materials have found wide application in such areas as housing and communal services, agriculture, oil production and energy industry. They are used in cold and hot water supply systems for pressure and pressure-free systems of domestic and industrial sewerage, in pipeline systems construction in irrigation and melioration, in engineering systems for hydroelectric power plants, etc.

Trunk pipelines for transportation of gas and oil products represent complex engineering structures. When designing underground and underwater pipelines the engineers should correctly evaluate the properties of pipe material and soil foundation.

At present, the problem of vibration processes of pipelines resting on elastic and viscoelastic foundation with a fluid flowing through it is of great theoretical and practical interest. To date, many approaches have been developed to solve these problems, but none of them is able to adequately reflect the real picture of a pipeline – underlying soil interaction. Basically, these approaches describe the individual stages of the processes occurring in the pipelines. There are a significant number of publications devoted to solving the

Khudayarov, B.A., Turaev, F.Z. Nonlinear vibrations of fluid transporting pipelines on a viscoelastic foundation. Magazine of Civil Engineering. 2019. 86(2). Pp. 30–45. DOI: 10.18720/MCE.86.4.

Худаяров Б.А., Тураев Ф.Ж. Нелинейные колебания трубопроводов на вязкоупругом основании, транспортирующего жидкость // Инженерно-строительный журнал. 2019. № 2(86). С. 30–45. DOI: 10.18720/MCE.86.4

problems of calculating the characteristics of elastic and viscoelastic thin-walled structures [1–19]. Results of the theory are compared in some cases with experimental data [6].

Pengyu Jia et al. [8] have studied the effects of crack geometries, pipe geometry, and material properties on the reference strain. An effective empirical formula is proposed to estimate the reference strain. Taolong Xu et al. [9] have conducted combined computational and analytical study to investigate the lateral impact behavior of pressurized pipelines. David Carrier III et al. [10] have analyzed the vibration of a pipeline using the nouniform Winkler soil model with randomized spring constants. W.Q. Chen et al. [11] have studied vibrations of thick beams resting on a Pasternak elastic base. The effects of Poisson's ratio and Pasternak foundation parameters on natural frequencies are analyzed. Deep beam-columns on two-parameter elastic foundation with account of the effect of shear strain, depth change and rotation inertia are analyzed in [12]. Results obtained on the basis of approximate theory are compared with the results obtained by the Timoshenko theory and the classical beam theory. Nonlinear responses of planar motions of fluid-conveying pipe are investigated with allowance for nonlinear elastic foundations [13]. Kameswara Rao Chellapilla [14] has derived an analytical expression for computation of critical velocity of a fluid flowing through a pipeline. The Pasternak two-parameter foundation is used to take into account the effect of foundation properties. The conclusions on the influence of foundation on the critical velocity of a fluid are presented. Haryadi Gunawan Tj et al. [15] have studied vibrations of cylindrical shells partially buried in elastic foundations. The effects of rigidity ratio of foundation and shell are analyzed as well as vibrations of shells on elastic foundations. I. Lottati and A. Kornecki [16] have studied the effect of an elastic foundation and dissipative forces on the stability of fluid-conveying pipes. Results of numerical calculations are compared to the results in previously published papers. The problem of stability of fluid-conveying carbon nanotubes embedded in an elastic medium is considered in [17]. For the critical flow velocity, taking into account the rigidity parameters of the Winkler and Pasternak foundation, analytical expressions are obtained. In [18] a synchronization phenomenon of two equivalent fluid-conveying pipes coupled by a nonlinear spring is studied. On the basis of the Bubnov-Galerkin method the discrete systems of equations are obtained.

At present, there are a number of approaches for improving mechanical model of soil foundation, but, apparently, the simplest mathematical statement of the problem (except for the Winkler model) is the development of the model of two-parametric viscoelastic Pasternak foundation. The model of two-parameter Pasternak foundation, on the one hand, makes it possible to take into account the distribution capacity of soil, and on the other hand it does not complicate the mathematical statement of the problem in comparison with the Winkler model.

From the above review, it can be concluded that the development of adequate models describing viscoelastic properties of structure material and accounting the work of the viscoelastic soil foundation is a rather complex and relevant research task that is directly solved in this paper, along with the construction of appropriate mathematical models.

The aim of this study is to create a mathematical model, a numerical algorithm and a computer program for solving the problem of nonlinear oscillations of viscoelastic thin-walled pipelines of large diameter on the basis of shell theory into account the two-parameter viscoelastic Pasternak foundation.

2. Methods

2.1. Governing equation

Consider the behavior of a thin circular viscoelastic cylindrical shell, with an ideal fluid flowing inside it at a constant velocity. The fluid velocity is U and its direction coincides with the direction of the Ox axis (Figure 1). The impact of external medium is described by the Pasternak model of two-parametric foundation (Figure 2). The Kirchhoff-Love conventional hypotheses are used under the assumption that the deflections are small in comparison with thickness.

Under the assumption in [20] and assuming that, $y = R\theta$, Marguerre equations with respect to displacements u, v, w can be written in the following form:

$$
\begin{gathered}
(1-R^*)\left\{\frac{\partial^2 u}{\partial x^2}+\frac{1-\mu}{2R^2}\frac{\partial^2 u}{\partial\theta^2}+\frac{1+\mu}{2R}\frac{\partial^2 v}{\partial x\partial\theta}+L_1(w)\right\}-\rho\frac{1-\mu^2}{E}\frac{\partial^2 u}{\partial t^2}=0,\\
\left(1-R^*\right)\left\{\frac{1}{R^2}\frac{\partial^2 v}{\partial\theta^2}+\frac{1-\mu}{2}\frac{\partial^2 v}{\partial x^2}+\frac{1+\mu}{2R}\frac{\partial^2 u}{\partial x\partial\theta}+L_2\left(w\right)\right\}-\rho\frac{1-\mu^2}{E}\frac{\partial^2 v}{\partial t^2}=0,\\
D(1-R^*)\nabla^4 w+L_3^*(u,v,w)+(1-R_1^*)\left\{k_1 w-k_2\frac{\partial^2 w}{\partial x^2}\right\}+\rho h\frac{\partial^2 w}{\partial t^2}=q,
\end{gathered}
\tag{1}
$$

where *D* is the cylindrical rigidity of the pipe,

μ is the Poisson's ratio of the pipe material,

E is the modulus of elasticity of the pipe material,

ρ is its density;

k_1, k_2 are the coefficients of the Pasternak's foundation, characterizing the properties of external environment; R is the radius of curvature of the middle surface;

h is the thickness of the pipe wall;

R^* and R_1^* are the integral operators with the Koltunov-Rzhanitsyn relaxation kernels, $R(t)$ and $R_1(t)$, respectively:

$$R(t) = A\cdot\exp(-\beta\cdot t)\cdot t^{\alpha-1};\quad R_1(t) = A_1\cdot\exp(-\beta_1\cdot t)\cdot t^{\alpha_1-1};$$

$A>0,\quad \beta>0,\quad 0<\alpha<1,\ A_1>0,\quad \beta_1>0,\quad 0<\alpha_1<1,\ A, A_1$ – the viscosity parameters;

β, β_1 are the attenuation parameters;

α, α_1 are the singularity parameter determined by experiment

$$R^*\varphi(t) = \int_0^t R(t-\tau)\varphi(\tau)d\tau;\quad R_1^*\varphi(t) = \int_0^t R_1(t-\tau)\varphi(\tau)d\tau,$$

where $R(t-\tau)$, $R_1(t-\tau)$ are relaxation kernel;

t is the time of observation;

τ is the time before observation;

$\varphi(\tau)$ is the functions to be determined; the operators $L_1(w),\ L_2(w),\ L_3^*(u,v,w)$ are:

$$L_1(w) = -\frac{\mu}{R}\frac{\partial w}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2} + \frac{1+\mu}{2R^2}\frac{\partial w}{\partial\theta}\frac{\partial^2 w}{\partial x\partial\theta} + \frac{1-\mu}{2R^2}\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial\theta^2},$$

$$L_2(w) = -\frac{1}{R^2}\frac{\partial w}{\partial x} + \frac{\partial w}{\partial\theta}\frac{\partial^2 w}{\partial\theta^2} + \frac{1+\mu}{2R}\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x\partial\theta} + \frac{1-\mu}{2R}\frac{\partial w}{\partial\theta}\frac{\partial^2 w}{\partial x^2},$$

$$\begin{aligned} L_3^*(u,v,w) &= \left(1-R^*\right)\frac{Eh}{1-\mu^2}\left\{-\frac{\mu}{R}\frac{\partial u}{\partial x} - \frac{1}{R^2}\frac{\partial v}{\partial\theta} + \frac{w}{R^2} - \frac{\mu}{2R}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{1}{R^3}\left(\frac{\partial w}{\partial\theta}\right)^2\right\} - \\ &- \frac{Eh}{1-\mu^2}\frac{\partial}{\partial x}\left\{\frac{\partial w}{\partial x}\left(1-R^*\right)\left[\frac{\partial u}{\partial x} + \frac{\mu}{R}\frac{\partial v}{\partial\theta} - \frac{\mu w}{R}\right] + \frac{1-\mu}{2R}\frac{\partial w}{\partial\theta}\left(1-R^*\right)\left(\frac{1}{R}\frac{\partial u}{\partial\theta} + \frac{\partial v}{\partial x}\right)\right\} - \\ &- \frac{Eh}{1-\mu^2}\frac{1}{R}\frac{\partial}{\partial\theta}\left\{\frac{1}{R}\frac{\partial w}{\partial\theta}\left(1-R^*\right)\left[\mu\frac{\partial u}{\partial x} + \frac{1}{R}\frac{\partial v}{\partial\theta} - \frac{w}{R}\right] + \frac{(1-\mu)}{2}\frac{\partial w}{\partial x}\left(1-R^*\right)\left(\frac{1}{R}\frac{\partial u}{\partial\theta} + \frac{\partial v}{\partial x}\right)\right\}, \end{aligned} \tag{2}$$

q is a pressure of fluid on the pipeline wall [21]:

$$q = -\varphi_{\alpha m}^*\rho\left(\frac{\partial^2 w}{\partial t^2} + U^2\frac{\partial^2 w}{\partial x^2}\right). \tag{3}$$

where $-\varphi_{\alpha m}^*$ is an associated mass of fluid;

m is the number of waves formed along the circumference,

α is the wave number or the constant of phase propagation.

The boundary conditions have the form

$$x=0,\quad x=L:\quad w=0;\quad v=0;\quad N_x=0;\quad M_x=0. \tag{4}$$

Under bending in the middle surface, there arise normal and tangential forces:

$$N_x = \int_{-h/2}^{h/2} \sigma_x dz \quad (x \Leftrightarrow y), \quad N_{xy} = \int_{-h/2}^{h/2} \sigma_{xy} dz. \tag{5}$$

Physical dependence between stresses $\sigma_x, \sigma_y, \tau_{xy}$ and strains $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ is taken in the form [20]:

$$\sigma_x = \frac{E}{1-\mu^2}\left(1-R^*\right)\left(\varepsilon_x + \mu\varepsilon_y\right), \ \sigma_y = \frac{E}{1-\mu^2}\left(1-R^*\right)\left(\varepsilon_y + \mu\varepsilon_y\right), \ \sigma_{xy} = \frac{E}{2\left(1+\mu\right)}\left(1-R^*\right)\varepsilon_{xy}. \tag{6}$$

Here ε_x, ε_y, ε_{xy} are the components of finite strain determined by:

$$\varepsilon_x = \frac{\partial u}{\partial x} - k_x w + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad \varepsilon_y = \frac{\partial v}{\partial y} - k_y w + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2, \quad \varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}, \tag{7}$$

where κ_x, κ_y are the curvature parameters.

Moments M_x, M_y and M_{xy} are determined through the deflection function w:

$$M_x = -D\left(1-R^*\right)\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right),$$

$$M_y = -D\left(1-R^*\right)\left(\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right), \quad M_{xy} = D(1-\mu)\left(1-R^*\right)\frac{\partial^2 w}{\partial x \partial y}. \tag{8}$$

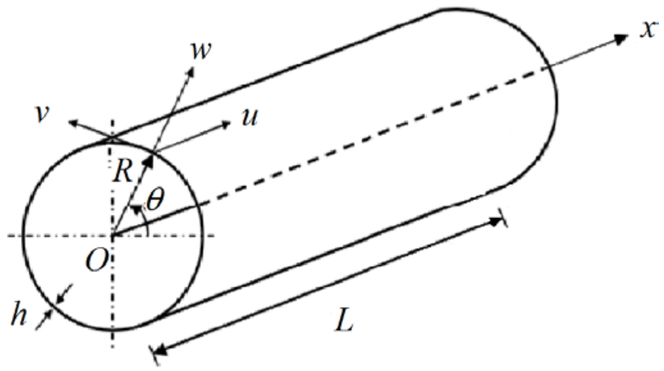


Figure 1. Geometry of the cylindrical shell.

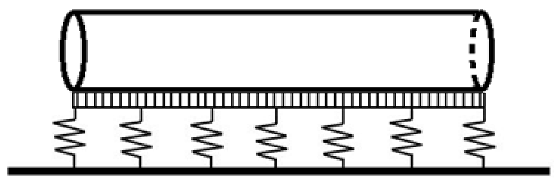


Figure 2. Pasternak's foundation.

2.2. Discret model

The solution of IDE systems in partial derivatives (1) under various boundary conditions and in the presence of singular heredity kernels represents a significant mathematical difficulty. Therefore, the natural way to solve these systems is to discretize them with respect to spatial variables and obtain a system of resolving nonlinear IDE with respect to time functions.

An approximate solution of system (1) is sought for in the form:

$$u\left(x,\theta,t\right) = \sum_{n=1}^{N}\sum_{m=1}^{M} u_{nm}\left(t\right) cos\frac{n\pi x}{L} sinm\theta,$$

$$v\left(x,\theta,t\right) = \sum_{n=1}^{N}\sum_{m=1}^{M} v_{nm}\left(t\right) sin\frac{n\pi x}{L} cosm\theta, \tag{9}$$

$$w\left(x,\theta,t\right) = \sum_{n=1}^{N}\sum_{m=1}^{M} w_{nm}\left(t\right) sin\frac{n\pi x}{L} sinm\theta,$$

where $u_{nm}\left(t\right), v_{nm}\left(t\right), w_{nm}\left(t\right)$ are the unknown time functions.

Substituting (9) in system **(1)** and applying the Bubnov-Galerkin method, the following system of integro-differential equations is obtained:

$$\ddot{u}_{kl}+\left(1-R^*\right)\left\{\left(k^2\pi^2\delta^2\gamma^2+\frac{1-\mu}{2}l^2\delta^2\right)u_{kl}-\frac{1-\mu}{2}kl\pi\gamma\delta^2 v_{kl}+\right.$$

$$+\mu\delta^2\gamma^2 k\pi w_{kl}+\sum_{n,i=1}^{N}\sum_{m,\ r=1}^{M}\left(\frac{ni^2\pi^2}{2}\gamma^3\delta+\frac{1-\mu}{2}\frac{nr^2}{2}\gamma\delta\right)\overline{\Delta}_{1klnmir}w_{nm}w_{ir}-$$

$$\left.-\frac{1+\mu}{2}\sum_{n,i=1}^{N}\sum_{m,r=1}^{M}\frac{imr}{2}\gamma\delta\overline{\Delta}_{2klnmir}w_{nm}w_{ir}\right\}=0\ ,$$

$$\ddot{v}_{kl}+\left(1-R^*\right)\left\{\left[\frac{1-\mu}{2}k^2\pi^2\delta^2\gamma^2+l^2\delta^2\right]v_{kl}-\frac{1+\mu}{2}kl\pi\gamma\delta^2 u_{kl}-l\delta^2 w_{kl}-\right.$$

$$-\sum_{n,i=1}^{N}\sum_{m,r=1}^{M}\frac{mr^2}{2\pi}\delta\overline{\Delta}_{3klnmir}w_{nm}w_{ir}+\frac{1+\mu}{2}\sum_{n,i=1}^{N}\sum_{m,r=1}^{M}\frac{inr\pi}{2}\gamma^2\delta\overline{\Delta}_{4klnmir}w_{nm}w_{ir}-$$

$$\left.-\frac{1-\mu}{2}\sum_{n,i=1}^{N}\sum_{m,r=1}^{M}\frac{i^2m\pi}{2}\gamma^2\delta\overline{\Delta}_{3klnmir}w_{nm}w_{ir}\right\}=0\ ,$$

$$\left(1+\varphi_{\alpha l}^*\right)\ddot{w}_{kl}+\left(1-R^*\right)\left\{\left(\frac{1}{12}\left[k^2\pi^2\gamma^2+l^2\right]^2+\delta^2\right)w_{kl}+\pi\mu\gamma\delta^2 ku_{kl}-l\delta^2 v_{kl}-\right.$$

$$\left.-\frac{\delta}{4\pi}\sum_{n,i=1}^{N}\sum_{m,r=1}^{M}mr\overline{\Delta}_{5klnmir}w_{nm}w_{ir}-\frac{\pi\mu\gamma^2\delta}{4}\sum_{n,i=1}^{N}\sum_{m,r=1}^{M}ni\overline{\Delta}_{6klnmir}w_{nm}w_{ir}\right\}+$$

$$+\frac{1-\mu}{4}\gamma\delta\sum_{n,i=1}^{N}\sum_{m,r=1}^{M}w_{nm}n\left(1-R^*\right)\left[\gamma\pi irv_{ir}-r^2u_{ir}\right]\overline{\Delta}_{6klnmir}+$$

$$+\frac{\delta}{2}\sum_{n,i=1}^{N}\sum_{m,r=1}^{M}mw_{nm}\left(1-R^*\right)\left[ir\mu\gamma u_{ir}-\frac{r^2}{\pi}v_{ir}+\frac{r}{\pi}w_{ir}\right]\overline{\Delta}_{5klnmir}+$$

$$+\frac{1-\mu}{4}\delta\sum_{n,i=1}^{N}\sum_{m,r=1}^{M}mw_{nm}\left(1-R^*\right)\left[ir\gamma u_{ir}-\gamma^2 i^2\pi v_{ir}\right]\overline{\Delta}_{5klnmir}-$$

$$-\frac{\delta}{2}\sum_{n,i=1}^{N}\sum_{m,r=1}^{M}nw_{nm}\left(1-R^*\right)\left[ir\mu\gamma^2\pi v_{ir}-i^2\gamma^3\pi^2u_{ir}-\mu\pi i\gamma^2 w_{ir}\right]\overline{\Delta}_{6klnmir}-$$

$$-\frac{1-\mu}{4}\delta\sum_{n,i=1}^{N}\sum_{m,r=1}^{M}nmw_{nm}\left(1-R^*\right)\left[r\gamma u_{ir}-i\gamma^2\pi v_{ir}\right]\overline{\Delta}_{7klnmir}-$$

$$-\sum_{n,i=1}^{N}\sum_{m,r=1}^{M}m^2w_{nm}\left(1-R^*\right)\left[i\mu\gamma u_{ir}-\frac{r}{\pi}v_{ir}+\frac{1}{\pi}w_{ir}\right]\frac{\delta}{2}\overline{\Delta}_{8klnmir}-$$

$$-\frac{1-\mu}{4}\delta\sum_{n,i=1}^{N}\sum_{m,r=1}^{M}nmw_{nm}\left(1-R^*\right)\left[r\gamma u_{ir}-i\gamma^2\pi v_{ir}\right]\overline{\Delta}_{7klnmir}-$$

$$-\frac{\delta}{2}\sum_{n,i=1}^{N}\sum_{m,r=1}^{M}n^2w_{nm}\left(1-R^*\right)\left[i\gamma^3\pi^2u_{ir}-\mu r\gamma^2\pi v_{ir}+\mu\gamma^2\pi w_{ir}\right]\vec{\Delta}_{8klnmir}-$$

$$-\delta^2M_1^2\gamma^2M_E^2k^2\pi^2w_{kl}+\delta^2\left(1-R_1^*\right)\left(\delta^2k_1+\pi^2k^2\gamma^2k_2\right)w_{kl}=0. \tag{10}$$

$$u_{nm}\left(0\right)=u_{0nm},\quad \dot{u}_{\mathrm{nm}}(0)=\dot{u}_{0nm},\quad v_{nm}\left(0\right)=v_{0nm},\quad \dot{v}_{nm}(0)=\dot{v}_{0nm},$$

$$w_{nm}\left(0\right)=w_{0nm},\quad \dot{w}_{nm}(0)=\dot{w}_{0nm}.$$

Here $\delta = \frac{R}{h}$, $\gamma = \frac{R}{L}$, $M_1 = \frac{U}{V_\infty}$, $M_E = \sqrt{\frac{E}{\rho V_\infty^2}}$, V_∞ is the sound speed $\overline{\Delta}_{1k\ln mir}$, $\overline{\Delta}_{2k\ln mir}$, $\overline{\Delta}_{3k\ln mir}$, $\overline{\Delta}_{4k\ln mir}$, $\overline{\Delta}_{5k\ln mir}$, $\overline{\Delta}_{6k\ln mir}$, $\overline{\Delta}_{7k\ln mir}$, $\overline{\Delta}_{8k\ln mir}$ are the dimensionless coefficients related to coordinate functions and their derivatives; dots over a variable denote the time derivatives of the corresponding order.

2.3. Computational algorithm

Solution of IDE (10) is sought for by a numerical method based on the use of quadrature formulas [22–27]. This method is based on various analytic transformations that make it possible to reduce the initial systems to the systems of integral equations with regular kernels and stable numerical integration ensuring the solution of problems with a high degree of accuracy. Since the integral entering system (10) has a weak Abel-type singularity, it is impossible to use a quadrature formula. Therefore, by changing the variables

$$t - \tau = z^{\frac{1}{\alpha}}, \quad 0 \le z \le t^{\alpha} \quad (0 < \alpha < 1) \tag{11}$$

the integral at the Koltunov-Rzhanitsyn kernel with singularity of the following form

$$A\int_0^t (t - \tau)^{\alpha-1} \exp(-\beta(t-\tau)) w(\tau) d\tau \tag{12}$$

has the form

$$\frac{A}{\alpha}\int_0^{t^\alpha} \exp(-\beta\, z^{\frac{1}{\alpha}}) w(t - z^{\frac{1}{\alpha}}) dz. \tag{13}$$

Note that after the change of variables, the integrand with respect to z becomes regular. Assuming that $t = t_i$, $t_i = i\Delta t$, $I = 1, 2, \ldots$ ($\Delta t = const$ – the integration step) and replacing the integrals by some quadrature formulas (in particular, the trapezoid one), we get

$$\frac{A}{\alpha}\sum_{k=0}^{i} B_k \exp(-\beta\, t_k) w_{i-k}, \tag{14}$$

where the coefficients are $B_0 = \frac{\Delta t^\alpha}{2}$; $\quad B_i = \frac{\Delta t^\alpha (i^\alpha - (i-1)^\alpha)}{2}$;

$$B_k = \frac{\Delta t^\alpha ((k+1)^\alpha - (k-1)^\alpha)}{2}, \quad k = \overline{1, i-1}. \tag{15}$$

Based on this method, an algorithm for the numerical solution of system (10) is described. Integrating the system (10) twice with respect to t, it can be written in integral form; by rational transformation the singularities of the integral operators R^* and R_1^* are eliminated. Then, assuming that $t = t_i$, $t_i = i\cdot\Delta t$, $i = 1, 2, \ldots$ (Δt is the integration step) and replacing the integrals with quadrature trapezoid formulas for the computation of $u_{ikl} = u_{kl}(t_i)$, $v_{ikl} = v_{kl}(t_i)$ and $w_{ikl} = w_{kl}(t_i)$, we obtain the following recurrence formulas for the Koltunov-Rzhanitsyn kernel $\left(R(t) = A\cdot\exp(-\beta t)\cdot t^{\alpha-1},\ 0 < \alpha < 1\right)$:

$$u_{pkl} = u_{okl} + \dot{u}_{okl}\, t_p - \sum_{j=0}^{p-1} A_j \left(t_p - t_j\right)\Bigg\{\varphi_{kl}\left(u_{jkl} - \frac{A}{\alpha}\sum_{s=0}^{j} B_s e^{-\beta t_s} u_{j-s,kl}\right) - \psi_{kl}\left(v_{kl} - \frac{A}{\alpha}\sum_{s=0}^{j} B_s e^{-\beta t_s} v_{j-s,kl}\right) +$$

$$+\,\omega_k\left(w_{jkl} - \frac{A}{\alpha}\sum_{s=0}^{j} B_s e^{-\beta t_s} w_{j-s,kl}\right) + \frac{\delta\gamma}{4}\sum_{n,i=1}\sum_{m,r=1} D_{klnmir}\left(w_{jnm}w_{jir} - \frac{A}{\alpha}\sum_{s=0}^{j} B_s e^{-\beta t_s} w_{j-s,nm} w_{j-s,ir}\right)\Bigg\}$$

$$v_{pkl} = v_{okl} + \dot{v}_{okl}\, t_p - \sum_{j=0}^{p-1} A_j \left(t_p - t_j\right) \Bigg\{ \aleph_{kl} \left(v_{jkl} - \frac{A}{\alpha} \sum_{s=0}^{j} B_s e^{-\beta t_s} v_{j-s,kl} \right) - \psi_{kl} \left(u_{jkl} - \frac{A}{\alpha} \sum_{s=0}^{j} B_s e^{-\beta t_s} v_{j-s,kl} \right) -$$

$$- d_e \left(w_{jkl} - \frac{A}{\alpha} \sum_{s=0}^{j} B_s e^{-\beta t_s} w_{j-s,kl} \right) + \frac{\delta}{4} \sum_{n,i=1}^{N} \sum_{m,r=1}^{M} F_{klnmir} \left(w_{jnm} w_{jir} - \frac{A}{\alpha} \sum_{s=0}^{j} B_s e^{-\beta t_s} w_{j-s,nm} w_{j-s,ir} \right) \Bigg\}$$

$$w_{pkl} = w_{okl} + \dot{w}_{okl}\, t_p - \frac{1}{1+\phi_{\alpha l}^{*}} \sum_{j=0}^{p-1} A_j \left(t_p - t_j\right) \Bigg\{ \Theta_{kl} \left(w_{jke} - \frac{A}{\alpha} \sum_{s=0}^{j} B_s e^{-\beta t_s} w_{j-s,kl} \right) +$$

$$+ \frac{\omega_k}{\gamma} \left(u_{jkl} - \frac{A}{\alpha} \sum_{s=0}^{j} B_s e^{-\beta t_s} u_{j-s,kl} \right) - d_e \left(v_{jkl} - \frac{A}{\alpha} \sum_{s=0}^{j} B_s e^{-\beta t_s} v_{j-s,kl} \right) -$$

$$- \frac{\delta}{4} \sum_{n,i=1}^{N} \sum_{m,r=1}^{M} G_{klnmir} \left(w_{jnm} w_{jir} - \frac{A}{\alpha} \sum_{s=0}^{j} B_s e^{-\beta t_s} w_{j-s,nm} w_{j-s,ir} \right) + \qquad .(16)$$

$$+ \frac{\delta}{4} \sum_{n,i=1}^{N} \sum_{m,r=1}^{M} w_{jnm} \Bigg\langle H_{klnmir} \left(u_{jir} - \frac{A}{\alpha} \sum_{s=0}^{j} B_s e^{-\beta t_s} u_{j-s,ir} \right) + Z_{klnmir} \left(v_{jir} - \frac{A}{\alpha} \sum_{s=0}^{j} B_s e^{-\beta t_s} v_{j-s,ir} \right) +$$

$$+ C_{klnmir} \left(w_{jir} - \frac{A}{\alpha} \sum_{s=0}^{j} B_s e^{-\beta t_s} w_{j-s,ir} \right) \Bigg\rangle - \eta_{kl}{}^2 w_{jkl} + \delta^2 (\delta^2 k_1 + \pi^2 k^2 \gamma^2 k_2) \left(w_{jkl} - \frac{A_1}{\alpha_1} \sum_{s=0}^{j} B_s e^{-\beta t_s} w_{j-s,kl} \right) \Bigg\}$$

$$p = 1, 2, 3, ...; \quad k = 1, 2, ..., N; \quad l = 1, 2, ..., M.$$

Here A_j, B_s are the numerical coefficients that do not depend on the choice of integrands and acquire different values depending on the use of quadrature formulas; φ_{kl}, ψ_{kl}, $\aleph_{kl}$, Θ_{kl}, η_{kl}, ω_k, d_e, D_{klnmir}, G_{klnmir}, C_{klnmir}, F_{klnmir}, H_{klnmir}, Z_{klnmir} are the dimensionless coefficients related to the coordinate functions and their derivatives.

2.4. Example of Test Solutions

Verification of efficiency of the proposed numerical method and programs, based on the solution of test cases, is a necessary stage to confirm the reliability of research results obtained in solving specific problems. The problems for which an exact solution is known [22] have been considered as test cases. Table 1 show a satisfactory agreement of approximate solutions with exact ones; this shows the reliability and high accuracy of calculation results.

Consider a non-linear integro-differential equation of the form

$$\ddot{w} + \lambda_0 \dot{w} + \omega^2 w = q - \lambda_1 \int_0^t R(t-\tau) w(\tau) d\tau - \lambda_2 w \int_0^t R(t-\tau) w(\tau) d\tau - \lambda_3 \int_0^t R(t-\tau) w^2(\tau) d\tau;$$

$$w(0) = 1, \quad \dot{w}(0) = -\beta, \qquad (17)$$

where

$$R(t) = A \exp(-\beta t) t^{\alpha-1}, \quad 0 < \alpha < 1;$$

$$q = \left[\beta^2 + \omega^2 - \lambda_0 \beta - \frac{A t^{\alpha}}{\alpha} \left(\lambda_1 + \left[\lambda_2 + \lambda_3 \right] \exp(-\beta t) \right) \right] \exp(-\beta t)$$

Equation (17) has an exact solution $w = \exp(-\beta t)$, which satisfies the initial conditions.

According to (16), the approximate values $w_n = w(t_n)$ ($t = t_n = n\Delta t$, $n = 0, 1, 2, \ldots$) are found from the relationships

$$w_n = \frac{1}{1+\lambda_0 A_n}\left\{ 1-(\beta-\lambda_0)t_n - \sum_{i=0}^{n-1} A_i \left\langle \lambda_0 w_i - (t_n - t_i)\left[q(t_i) - \omega^2 w_i + \frac{A\lambda_2}{\alpha} w_i \times \right.\right.\right.$$
$$\left.\left.\left. \times \sum_{s=0}^{i} B_s \exp(-\beta t_s) w_{i-s} + \frac{A}{\alpha}\sum_{s=0}^{i} B_s \exp(-\beta t_s) w_{i-s}\left(\lambda_1 + \lambda_3 \exp(-\beta\ t_s) w_{i-s}\right)\right] \right\rangle \right\} \tag{18}$$

$n = 1, 2, \ldots,$; where A_i, B_s are the coefficients of the quadrature formula of trapezoids.

Table 1 gives approximate results of calculations by formulas (18) within the interval from 0 to 1 with Δt = 0.01 step, and exact solutions. The following initial data have been used: λ_0 = 1.1; λ_1 = 1.2; λ_2 = 1.3; λ_3 = 1.4; A = 0.01; β = 0.03; α = 0.01. It follows from the table that the maximum error Δ of calculations performed by described method represents the value $const \cdot \Delta t^2$. The efficiency of this numerical method and programs is shown in other test cases as well.

From the table it follows that the error Δh of calculations performed by described method coincides with the error of the quadrature formulas used and has the same order of smallness relative to the interpolation step (for the trapezoid formula the error of the method with respect to the interpolation step is of second-order, for the Simpson formula – of third order, etc.).

Table 1. Comparison of exact and approximate solutions of IDE.

t	Solution		Δh
	Exact	Approximate	
0	1.000000	1.000000	–
1	0.970445	0.970373	$0.7 \cdot 10^{-4}$
2	0.941764	0.941622	$1.4 \cdot 10^{-4}$
3	0.913931	0.913644	$2.8 \cdot 10^{-4}$
4	0.886920	0.886569	$3.5 \cdot 10^{-4}$
5	0.860707	0.860271	$4.3 \cdot 10^{-4}$
6	0.835270	0.834855	$4.1 \cdot 10^{-4}$
7	0.810584	0.810278	$3 \cdot 10^{-4}$
8	0.786627	0.786113	$5.1 \cdot 10^{-4}$
9	0.763379	0.763126	$2.5 \cdot 10^{-4}$
10	0.740818	0.740509	$3 \cdot 10^{-4}$

3. Results and Discussion

Based on the developed algorithm, a package of applied computer programs in Delphi language has been created. Results of calculations are reflected by the graphs shown in Figures 3–10.

The influence of the viscoelastic properties of material on the vibration process of the pipeline on a two-parameter foundation was investigated (Figure 3, a, b, c). On the ordinate, displacements w (Figure 3, a), u (Figure 3, b), v (Figure 3, c) are plotted. On the abscissa, the parameter of dimensionless time is plotted. The first of these curves is constructed for elastic pipelines $A = 0.0(1)$, the second and the third curves reflect the effect of the viscosity parameter at the following values: $A = 0.05(2); A = 0.1(3)$. The following parameter values were used for calculations: μ = 0.3; V_∞ = 330 m/s; M_1 = 0.1; γ = 0.02; δ = 4; ρ = 7800 kg/cm^3; k_1 = 1; k_2 = 1; N = 5; M = 2.

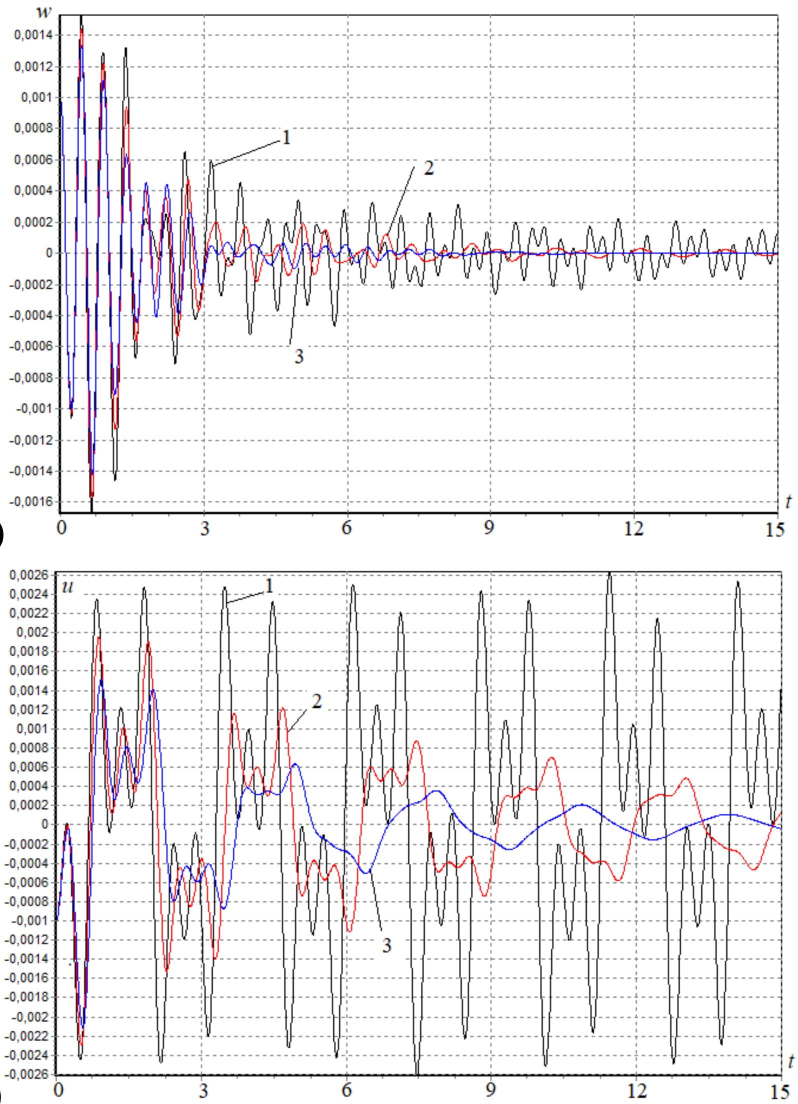
As seen from the figure, the viscoelastic properties of material lead to a decrease in the amplitude and frequency of the pipeline vibration.

Figure 4 shows the effect of rheological parameter α on the vibration process. Calculations have been carried out at α = 0.05; 0.1 and 0.5. The pipeline data and flow parameters were as follows: A = 0.03;

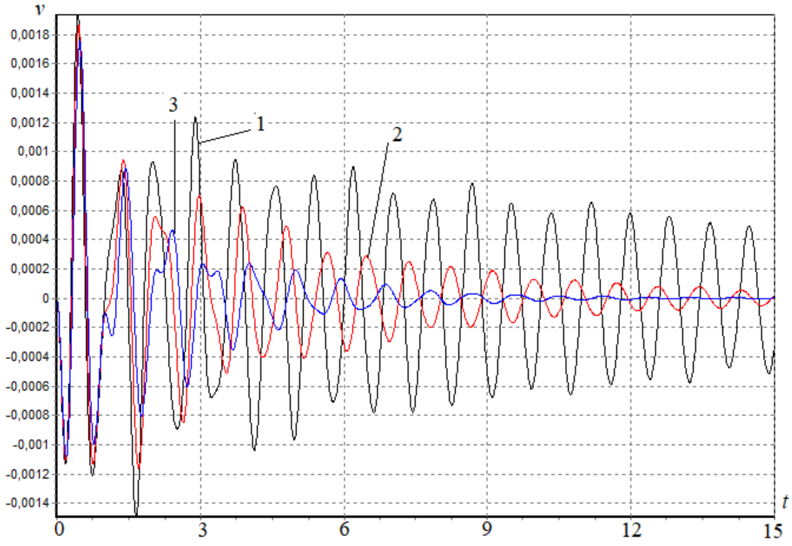
β = 0.005; A_1 = 0.1; α_1 = 0.25; β_1 = 0.005; k_1 = 1; k_2 = 1; $\gamma = 0.02$; $\delta = 3$; $V_\infty = 330$ m/s; $\mu = 0.3$; $M_1 = 0.1$; $\rho = 7800$ kg/cm^3; $k_1 = 1$; $k_2 = 1$; $N = 5$; $M = 2$.

The figure shows that an increase in parameter α leads to an increase in the amplitude and frequency of vibrations. At t = 0,75, 1.5, 2.3 and 3.2 the amplitude of oscillations reaches a maximum value. At t = 1.2 the amplitude of vibrations becomes minimal. Further calculations show that the change in the third rheological viscosity parameter β $(0 < \beta < 1)$ does not have a significant effect on the pipeline vibration process; this confirms the unacceptability of application of exponential relaxation kernels in calculating the dynamic problems of viscoelastic systems. These conclusions and results fully agree with the conclusions and results obtained in [22, 28].

Figure 5 shows the curves corresponding to various values of viscosity parameter of the foundation A_1. On the ordinate the parameter of the pipeline deflection is plotted, on the abscissa – the time parameter. The curves are plotted for the pipeline at the following values of the viscosity parameter: A_1 = 0 (curve 1), A_1 = 0.1 (curve 2). The value of geometric and physical constants is assumed to be: A = 0.05; α = 0.25; β = 0.005; α_1 = 0.25; β_1 = 0.005; k_1 = 1; k_2 = 1; γ = 0.02; δ = 5; μ = 0.3; $V_\infty = 330$ m/s; $M_1 = 0.1$; $\rho = 7800$ kg/cm^3; E = 2·10^5 MPa; $N = 5$; $M = 2$.



a)
b)



c)

Figure 3 (a, b, c). Displacements versus time at A = 0(1); 0.05(2) 0.1(3).

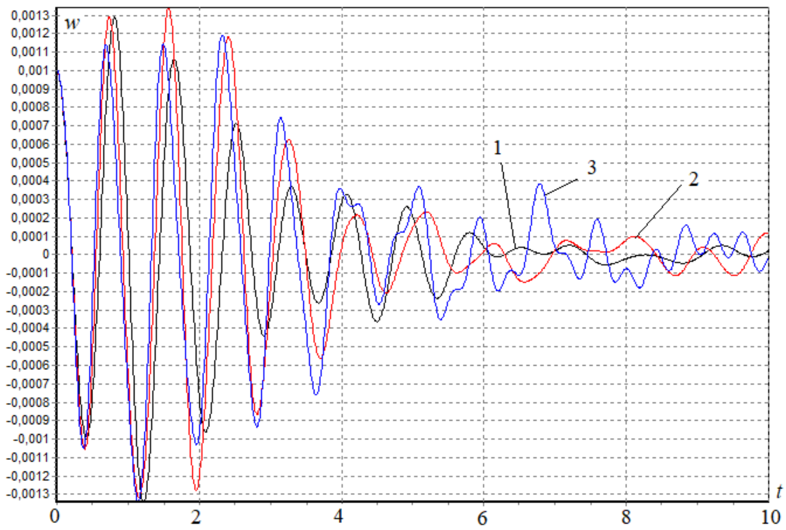


Figure 4. Deflection versus time at α = 0.05(1); 0.1(2); 0.5(3).

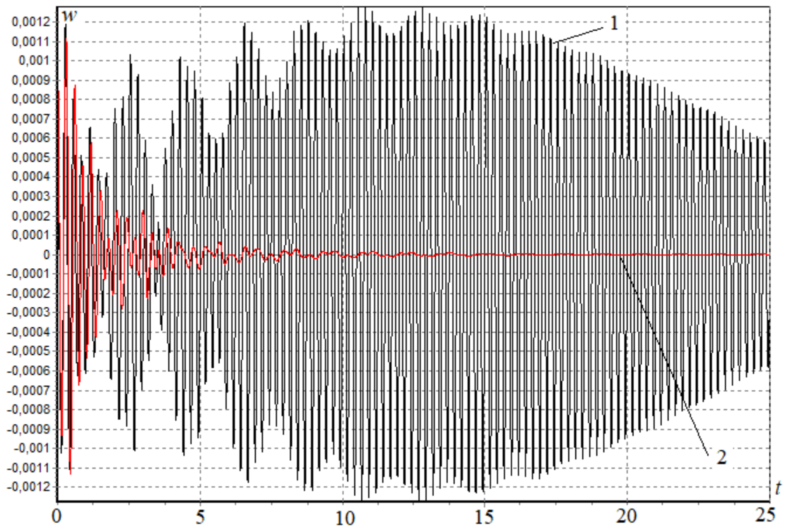


Figure 5. Deflection versus time at A_1 = 0(1); A_1 = 0.1(2).

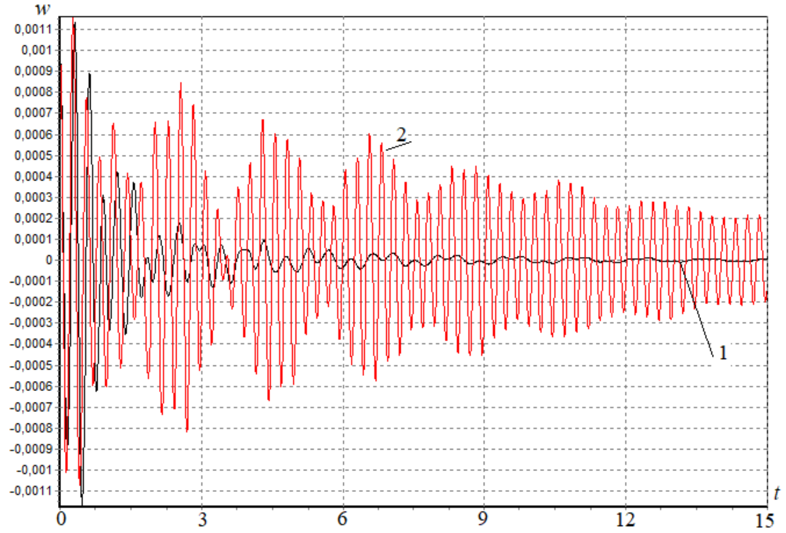


Figure 6. Deflection versus time at α_1 = 0.2 (1); α_2 = 0.75 (2).

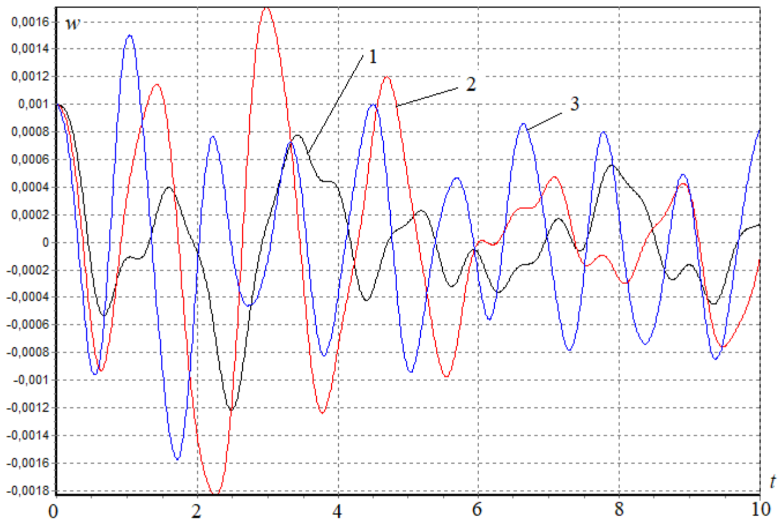


Figure 7. Deflection versus time at γ = 0.01(1); 0.06(2); 0.1(3).

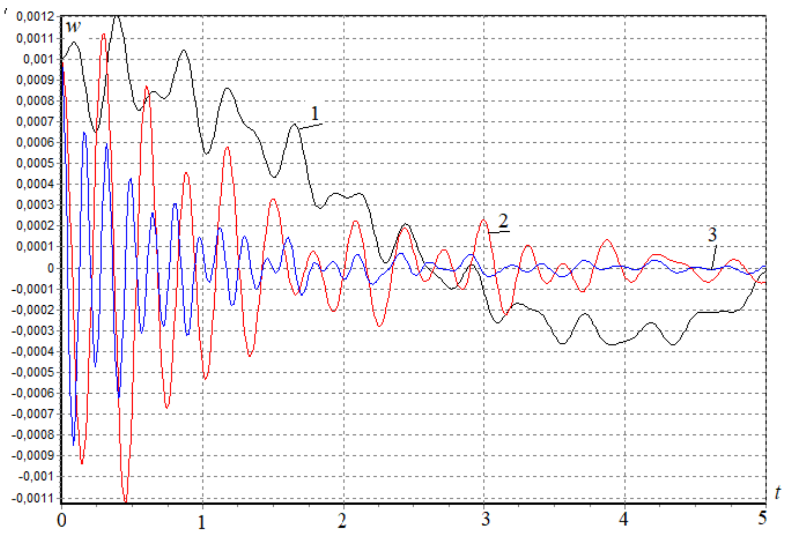


Figure 8. Deflection versus time at the values of k_1 and k_2:
k_1 = 0; k_2 = 0 (curve1); k_1 = 1; k_2 = 1 (curve 2); and k_1 = 3; k_2 = 3 (curve 3)

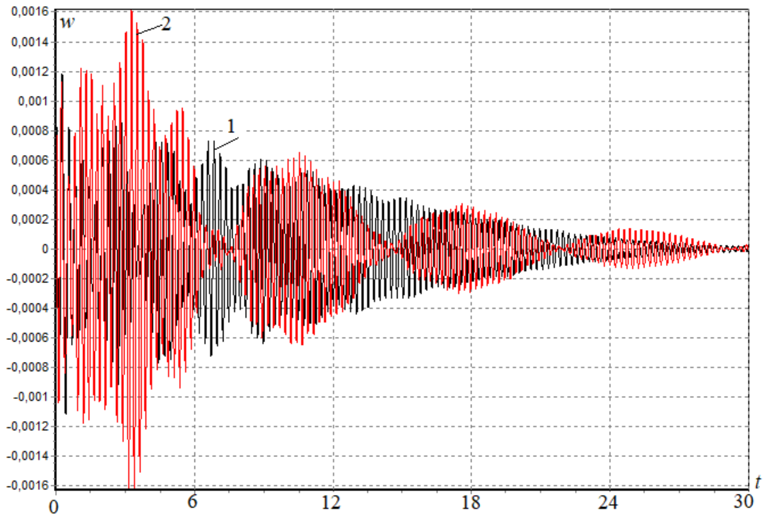


Figure 9. Deflection versus time at various values of M_1: 0.1(1); 1.8(2).

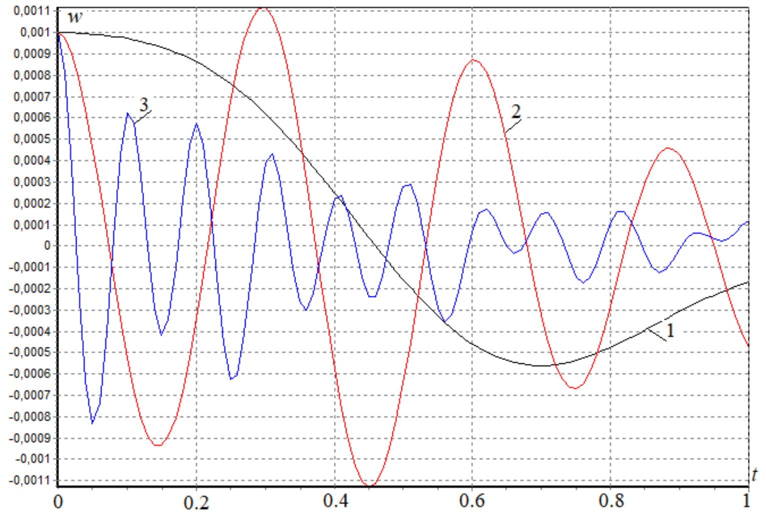


Figure 10. Deflection versus time at various values of δ parameter: 2 (curve 1); 5 (curve 2); 8 (curve 3).

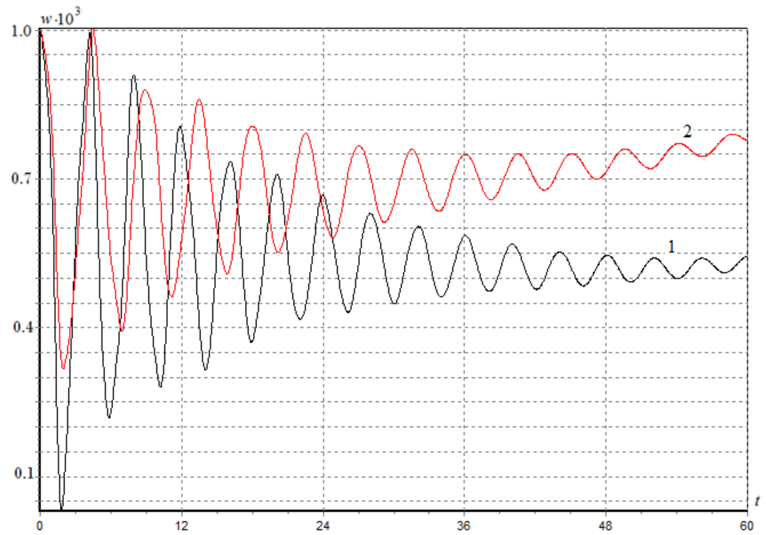


Figure 11. Linear theory (1); nonlinear theory (2).

As seen from Figure 5, an account of viscoelastic properties of soil foundation leads to the damping of vibration process. Though the solution of elastic and viscoelastic problems in the initial period of time differ little from each other, viscoelastic properties exert a significant influence over time. The amplitude of vibrations attenuates, and the vibration phase shifts to the right.

Figure 6 shows the nature of the pipeline motion under various rheological parameters of the foundation α_1. At $\alpha_1 = 0.2$; $\alpha_1 =$ 0.75 the amplitude of pipeline vibration attenuates over time. An increase in rheological parameter $\alpha_1 =$ 0.75 leads to an increase in the frequency and amplitude of pipeline vibrations. The following values of geometric and physical constants are used in calculation: $A = 0.05$; $\alpha = 0.25$; $\beta = 0.005$; $A_1 = 0.1$; $\beta_1 = 0.005$; $k_1 = 1$; $k_2 = 1$; $\gamma = 0.02$; $\delta = 3$; $\mu = 0.3$; $V_\infty = 330$ m/s; $M_1 = 0.1$; $\rho = 7800$ kg/cm^3; $E = 2 \cdot 10^5$ MPa; $N = 5$; $M = 2$.

The influence of parameter γ, equal to the ratio of the radius and length of the pipeline is shown in Figure 7. The numbers indicate the results obtained at the following values of parameter γ: 1 – 0.01; 2 – 0.06; 3 – 0.1. An increase in parameter γ (which corresponds to an increase in the radius or a decrease in the length of the pipeline) causes an increase in the amplitude and frequency of vibrations of the pipeline.

Figure 8 shows the graphs of the function w(t) in time at different values of k_1 and k_2. Curves 1-3 correspond to the values $k_1 = 0$; $k_2 = 0$ (curve 1); $k_1 = 1$; $k_2 = 1$ (curve 2); and $k_1 = 3$; $k_2 = 3$ (curve 3). Analyzing the results obtained, it can be concluded that the presence of a viscoelastic foundation leads to a decrease in the amplitude of vibrations, and the frequency of vibrations increases. At $k_1 = 3$; $k_2 = 3$ (curve 3), the amplitude of vibrations rapidly decays.

The influence of the flow velocity M_1 on the vibration process of the pipelines is studied. Figure 9 shows the graphs of the function $w(t)$ in time at different values of M_1 not exceeding the critical value. The solution is obtained at the following values of physical and geometric coefficients: $A = 0.05$; $\alpha = 0.25$; $\beta = 0.005$; $A_1 = 0.01$; $\alpha_1 =$ 0.25; $\beta_1 = 0.005$; $k_1 = 1$; $k_2 = 1$; $\gamma = 0.02$; $\delta = 5$; $\mu = 0.3$; $V_\infty = 330$ m/s; $\rho = 7800$ kg/cm^3; $E = 2 \cdot 10^5$ MPa. Curves 1 and 2 correspond to the values $M_1 = 0.1$ (curve 1) and $M_1 = 1.8$ (curve 2). Note that with an increase in M_1 at the initial time, the amplitude and frequency of vibrations remain constant. At greater values of M_1 the vibration period increases with time.

Figure 10 shows the time variation of the deflection of the pipeline w at various values of the parameter δ: 2 (curve 1); 5 (curve 5); 8 (curve 3). As seen from the graph, the growth of the parameter δ contributes to a significant decrease in the amplitude of vibrations. An increase in the parameter δ makes it possible to significantly improve the stability of the pipeline.

Figure 11 shows the time variation of the displacement w of the midpoint of viscoelastic cylindrical shell, obtained from various theories: the linear theory (curve 1) and the nonlinear theory (curve 2). According to Figure 11, the results of linear and nonlinear theories differ significantly from each other. Although the solutions of the problems of linear and nonlinear theories differ little in the initial period of time, in the course of time the geometric nonlinearity exerts a significant influence on the solution.

4. Conclusions

It should be noted that the algorithm of the proposed method makes it possible to investigate in detail the influence of viscoelastic properties of structure material, geometric nonlinearities, and Pasternak two-parameter viscoelastic foundation on vibration processes of pipelines with fluid flowing inside.

When studying pipelines vibrations with a flowing fluid, a number of dynamic effects are obtained:

1. It has been established that an account of viscoelastic properties of the pipeline material leads to a decrease in the amplitude and frequency of vibrations by 20–40 %;

2. It is shown that an increase in the geometric parameter γ (which corresponds to an increase in the radius or a decrease in the length of the pipeline) and dimensionless flow velocity M_1 leads to an increase in the amplitude and frequency of vibration;

3. It has been established that an account of viscoelastic foundation leads to a decrease in the amplitude of vibrations, and the frequency of vibration increases.

The obtained results of numerical simulation may be implemented at the enterprises of oil and gas industry, agriculture, housing and communal services and design organizations.

5. Acknowledgement

The study was supported by the National Research Foundation of Uzbekistan (Grant QXA-13-001-2015).

References

1. Stojanović, V., Petković, M.D. Nonlinear dynamic analysis of damaged Reddy–Bickford beams supported on an elastic Pasternak foundation. Journal of Sound and Vibration. 2016. Vol. 385. No. 22. Pp. 239–266.
2. Lee, S.Y., Kes, H.Y. Free vibrations of non-uniform beams resting on non-uniform elastic foundation with general elastic end restraints. Computers and Structures. 1990. Vol. 34. No. 3. Pp. 421–429.
3. De Rosa, M.A. Free vibrations of Timoshenko beams on two-parameter elastic foundation. Computers and Structures. 1995. Vol. 57. Pp. 151–156.
4. Naidu, N.R., Rao, G.V. Vibrations of initially stressed uniform beams on two-parameter elastic foundation. Comput. Struct. 1995. Vol. 57. Pp. 941–943.
5. Ayvaz, Y., Özgan, K. Application of modified Vlasov model to free vibration analysis of beams resting on elastic foundations. J. Sound and Vib. 2002. Vol. 255. Pp. 111–127.
6. Bergant, A., Hou, Q., Keramat, A., Tijsseling, A.S. Water hammer tests in a long PVC pipeline with short steel end sections. Scientific professional Quarterly Spring. 2013. Vol. 1. No. 1. Pp. 23–34.
7. Wu, D., Jing, H., Xu, L., Zhao, L., Han, Y. Theoretical and numerical analysis of the creep crack initiation time considering the constraint effects for pressurized pipelines with axial surface cracks. International Journal of Mechanical Sciences. 2018. Vol. 141. Pp. 262–275.
8. Jia, P., Jing, H., Xu, L., Han, Y., Zhao, L. A modified reference strain method for engineering critical assessment of reeled pipelines. International Journal of Mechanical Sciences. 2016. Vol. 105. Pp. 23–31.
9. Xu, T., Yao, A., Jiang, H., Li, Y., Zeng, X. Dynamic response of buried gas pipeline under excavator loading: Experimental/numerical study. Engineering Failure Analysis. 2018. Vol. 89. Pp. 57–73.
10. Carrier III, W.D. Pipeline Supported on a Nonuniform Winkler Soil Model. Journal of Geotechnical and Geoenvironmental Engineering. 2005. Vol. 131. No. 10.
11. Chen, W.Q., Lü, C.F., Bian, Z.G. A mixed method for bending and free vibration of beams resting on a Pasternak elastic foundation. Applied Mathematical Modelling. 2004. Vol. 28. No. 10. Pp. 877–890.
12. Matsunaga, H. Vibration and buckling of deep beam-columns on two-parameter elastic foundations. Journal of Sound and Vibration. 1999. Vol. 228. No. 2. Pp. 359–376.
13. Qian, Q., Wang, L., Ni, Q. Nonlinear responses of a fluid-conveying pipe embedded in nonlinear elastic foundations. Acta Mechanica Solida Sinica. 2008. Vol. 21. No. 2. Pp. 170–176.
14. Chellapilla, K.R., Simha, H.S. Critical velocity of fluid-conveying pipes resting on two-parameter foundation. Journal of Sound and Vibration. 2007. Vol. 302. No. 1–2. Pp. 387–397.
15. Tj, H.G., Mikami, T., Kanie, S., Sato, M. Free vibration characteristics of cylindrical shells partially buried in elastic foundations. Journal of Sound and Vibration. 2006. Vol. 290. No. 3–5. Pp. 785–793.
16. Lottati, I., Kornecki, A. The effect of an elastic foundation and of dissipative forces on the stability of fluid-conveying pipes. Journal of Sound and Vibration. 1986. Vol. 109. No. 2. Pp. 327–338.
17. Rao, Ch.K., Rao, L.B. Critical velocities in fluid-conveying single-walled carbon nanotubes embedded in an elastic foundation. J. Appl. Mech. Tech. Phy. 2017. Vol. 58. Pp. 743–752.
18. Lü, L., Hu, Y., Wang, X. Dynamical bifurcation and synchronization of two nonlinearly coupled fluid-conveying pipes [Online]. Nonlinear Dynamics. 2015. Vol. 79. Pp. 2715–2734. URL: https://doi.org/10.1007/s11071-014-1842-y
19. Kozhaeva, K.V. Calculation of optimized methods of the river underwater pipeline backfill with the use of APMWinMachine 9.7. Magazine of Civil Engineering. 2016. No. 5. Pp. 42–66. doi: 10.5862/MCE.65.4
20. Grigolyuk, E.I., Mamay, V.I. Nelineynoye deformirovaniye tonkostennykh konstruktsiy [Nonlinear Deformation of Thin-walled Structures]. Moscow: Nauka. Fizmatlit, 1997. 272 p. (rus)
21. Vol'mir, A.S. Obolochki v potoke zhidkosti i gaza [Shell in the flow of liquid and gas]. Moscow: Nauka,1979. 320 p. (rus)
22. Badalov, F.B. Metody resheniya integral'nykh i integro-differentsial'nykh uravneniy nasledstvennoy teorii vyazkouprugosti [Methods for Solving Integral and Integro-differential Equations of the Hereditary Theory of Viscoelasticity]. Tashkent: Mekhnat, 1986. 269 p. (rus)
23. Khudayarov, B.A. Numerical Study of the Dependence of the Critical Flutter Velocity and Time of a Plate on Rheological Parameters. International Applied Mechanics. 2008. Vol. 44. No. 6. Pp. 676–682.
24. Khudayarov, B.A. Numerical Analysis of Nonlinear Flutter of Viscoelastic Plates. International Applied Mechanics. 2005. Vol. 41. No. 5. Pp. 538–542.
25. Khudayarov, B.A. Flutter of Viscoelastic Plate in a Supersonic Gas Flow. International Applied Mechanics. 2010. Vol. 46. No. 4. Pp. 455–460.
26. Badalov, F.B., Khudayarov, B.A., Abdukarimov, A. Effect of the Hereditary Kernel on the Solution of Linear and Nonlinear Dynamic Problems of Hereditary Deformable Systems. Journal of Machinery Manufacture and Reliability. 2007. Vol. 36. No. 4. Pp. 328–335.
27. Khudayarov, B.A., Bandurin, N.G. Nelineynyy flatter vyazkouprugikh ortotropnykh tsilindricheskikh paneley [Nonlinear Flutter of Viscoelastic Orthotropic Cylindrical Panels]. Matematicheskoye modelirovaniye. 2010. Vol. 17. No. 10. Pp. 79–86. (rus)
28. Il'in, V.P., Sokolov, V.G. O svobodnykh kolebaniyakh tsilindricheskikh obolochek s protekayushchey zhidkost'yu [On free oscillations of cylindrical shells with flowing fluid]. Proceedings of the universities: Series Construction and Architecture. 1979. No. 12. Pp. 26–31. (rus)

Contacts:

Bakhtiyar Khudayarov, +998712370986; bakht-flpo@yandex.ru
Fozil Turaev, +998977117666; t.fozil86@mail.ru

Инженерно-строительный журнал

ISSN 2071-0305

сайт журнала: http://engstroy.spbstu.ru/

DOI: 10.18720/MCE.86.4

Нелинейные колебания трубопроводов на вязкоупругом основании, транспортирующего жидкость

Б.А. Худаяров*, Ф.Ж. Тураев,
Ташкентский институт инженеров ирригации и механизации сельского хозяйства, г. Ташкент, Узбекистан
** E-mail: bakht-flpo@yandex.ru*

Ключевые слова: колебательный процесс; основания; трубопровод; математическая модель; численный алгоритм; цилиндрическая оболочка

Аннотация. В статье представлены результаты исследования процесса колебания трубопроводов, транспортирующих жидкость или газ. При исследовании колебаний трубопроводов с протекающей внутри газо-жидкостью используется моделью в виде цилиндрических оболочек и двухпараметрической модели вязкоупругого основания Пастернака. Для описания вязкоупругих свойств использована наследственная теория вязкоупругости Больцмана-Вольтерра. Численно исследованы влияния параметров оснований Пастернака, влияние сингулярности в ядрах наследственности и геометрических параметров трубопровода на колебания конструкций, обладающих вязкоупругими свойствами. Установлено, что учет вязкоупругих свойств материала трубопровода приводит к уменьшению амплитуды и частоты колебаний на 20–40 %. Показано, что учет вязкоупругих свойств оснований грунта приводит к затуханию колебательного процесса трубопровода.

Список литературы

1. Stojanović V., Petković M.D. Nonlinear dynamic analysis of damaged Reddy–Bickford beams supported on an elastic Pasternak foundation // Journal of Sound and Vibration. 2016. Vol. 385. № 22. Pp. 239–266.
2. Lee S.Y., Kes H.Y. Free vibrations of non-uniform beams resting on non-uniform elastic foundation with general elastic end restraints // Computers and Structures. 1990. Vol. 34. №. 3. Pp. 421–429.
3. De Rosa M.A. Free vibrations of Timoshenko beams on two-parameter elastic foundation // Computers and Structures. 1995. Vol. 57. Pp. 151–156.
4. Naidu N.R., Rao G.V. Vibrations of initially stressed uniform beams on two-parameter elastic foundation // Computers and Structures. 1995. Vol. 57. Pp. 941–943.
5. Ayvaz Y., Özgan K. Application of modified Vlasov model to free vibration analysis of beams resting on elastic foundations // J. Sound and Vib. 2002. Vol. 255. Pp. 111–127.
6. Bergant A., Hou Q., Keramat A., Tijsseling A.S. Water hammer tests in a long PVC pipeline with short steel end sections // Scientific professional Quarterly Spring. 2013. Vol. 1. № 1. Pp. 23–34.
7. Wu D., Jing H., Xu L., Zhao L., Han Y. Theoretical and numerical analysis of the creep crack initiation time considering the constraint effects for pressurized pipelines with axial surface cracks // International Journal of Mechanical Sciences. 2018. Vol. 141. Pp. 262–275.
8. Jia P., Jing H., Xu L., Han Y., Zhao L. A modified reference strain method for engineering critical assessment of reeled pipelines // International Journal of Mechanical Sciences. 2016. Vol. 105. Pp. 23–31.
9. Xu T., Yao A., Jiang H., Li Y., Zeng X. Dynamic response of buried gas pipeline under excavator loading: Experimental/numerical study // Engineering Failure Analysis. 2018. Vol. 89. Pp. 57–73.
10. Carrier III W.D. Pipeline Supported on a Nonuniform Winkler Soil Model // Journal of Geotechnical and Geoenvironmental Engineering. 2005. Vol. 131. No 10.
11. Chen W.Q., Lü C.F., Bian Z.G. A mixed method for bending and free vibration of beams resting on a Pasternak elastic foundation. // Applied Mathematical Modelling. 2004. Vol. 28. № 10. Pp. 877–890.
12. Matsunaga H. Vibration and buckling of deep beam-columns on two-parameter elastic foundations // Journal of Sound and Vibration. 1999. Vol. 228. № 2. Pp. 359–376.
13. Qian Q., Wang L., Ni Q. Nonlinear responses of a fluid-conveying pipe embedded in nonlinear elastic foundations // Acta Mechanica Solida Sinica. 2008. Vol. 21. № 2. Pp. 170-176.
14. Chellapilla K.R., Simha H.S. Critical velocity of fluid-conveying pipes resting on two-parameter foundation // Journal of Sound and Vibration. 2007. Vol. 302. № 1–2. Pp. 387–397.

15. Tj H.G., Mikami T., Kanie S., Sato M. Free vibration characteristics of cylindrical shells partially buried in elastic foundations // Journal of Sound and Vibration. 2006. Vol. 290. № 3–5. Pp. 785–793.
16. Lottati I., Kornecki A. The effect of an elastic foundation and of dissipative forces on the stability of fluid-conveying pipes // Journal of Sound and Vibration. 1986. Vol. 109. № 2. Pp. 327–338.
17. Rao Ch. K., Rao L. B. Critical velocities in fluid-conveying single-walled carbon nanotubes embedded in an elastic foundation // J. Appl. Mech. Tech. Phy. 2017. Vol. 58. Pp. 743–752.
18. Lü L., Hu Y., Wang X. Dynamical bifurcation and synchronization of two nonlinearly coupled fluid-conveying pipes [Online] // Nonlinear Dynamics. 2015. Vol. 79. Pp. 2715–2734. URL: https://doi.org/10.1007/s11071-014-1842-y
19. Kozhaeva K.V. Calculation of optimized methods of the river underwater pipeline backfill with the use of APMWinMachine 9.7 // Magazine of Civil Engineering. 2016. № 5. Pp. 42–66. doi: 10.5862/MCE.65.4
20. Григолюк Э.И., Мамай В.И. Нелинейное деформирование тонкостенных конструкций. М.: Наука. Физматлит, 1997. 272 с.
21. Вольмир А.С. Оболочки в потоке жидкости и газа. М.: Наука, 1979. 320 с.
22. Бадалов Ф.Б. Методы решения интегральных и интегро-дифференциальных уравнений наследственной теории вязкоупругости. Ташкент: Мехнат, 1986. 269 с.
23. Khudayarov B.A. Numerical Study of the Dependence of the Critical Flutter Velocity and Time of a Plate on Rheological Parameters // International Applied Mechanics. 2008. Vol. 44. № 6. Pp. 676–682.
24. Khudayarov B.A. Numerical Analysis of Nonlinear Flutter of Viscoelastic Plates. International Applied Mechanics. 2005. Vol. 41. № 5. Pp. 538–542.
25. Khudayarov B.A. Flutter of Viscoelastic Plate in a Supersonic Gas Flow // International Applied Mechanics. 2010. Vol. 46. № 4. Pp. 455–460.
26. Badalov F.B., Khudayarov B.A., Abdukarimov A. Effect of the Hereditary Kernel on the Solution of Linear and Nonlinear Dynamic Problems of Hereditary Deformable Systems // Journal of Machinery Manufacture and Reliability. 2007. Vol. 36. № 4. Pp. 328–335.
27. Худаяров Б.А., Бандурин Н.Г. Нелинейный флаттер вязкоупругих ортотропных цилиндрических панелей. Математическое моделирование. 2010. Т. 17. № 10. С. 79–86.
28. Ильин В.П., Соколов В.Г. О свободных колебаниях цилиндрических оболочек с протекающей жидкостью // Известия вузов: Серия строительство и архитектура. 1979. № 12. С. 26–31.

Контактные данные:

Бахтияр Алимович Худаяров, +998712370986; эл. почта: bakht-flpo@yandex.ru
Фозил Журакулович Тураев, +998977117666; эл. почта: t.fozil86@mail.ru