Theory of determining the frequency of natural oscillations of span structures

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Abstract. The article is devoted to the development of mathematical dependence of determination the natural oscillation frequencies of reinforced concrete span structures according to the physical, mechanical, geometric and energy characteristics of their elements and materials. Carrying out and processing the results of the experimental studies as well as the analysis of literature revealed the ways of reducing the error of the existing mathematical dependences in determining the natural oscillation frequencies of the span structures. The use of Griffith's energy approach, the account of the nonlinear law of stiffness changes and the subsequent approximation of the experimental data made it possible to reduce the error of the mathematical dependence in the process of determination the natural oscillation frequencies of span structures by 4.4 times. The effect obtained makes it possible to apply the developed mathematical dependence to monitor the technical condition of the reinforced concrete span structures and the numerical determination of the boundaries of their technical condition categories.

1. Introduction

The object of research is span-bearing reinforced concrete structures, which make up 40% of the total volume of building structures. One of the ways to assess their technical condition is vibration diagnostics.

Today vibrodiagnostics is an extremely popular is an extremely popular method of technical monitoring because it gives the possibility to assess the technical condition of different systems. Also it can be used to assess seismic loads on buildings and structures, both of natural (earthquakes, wind loads) [1, 2] and of man-made origin (aircraft take-off, launch of space rockets, traffic, etc.) [3].

Among the advantages of vibrodiagnostics the reduced (compared to other methods of nondestructive control) errors [4, 5] can be called. In addition, the natural oscillations frequency as a diagnostic feature of vibrodiagnostics takes into account the physico-mechanical, geometric, structural and other factors that characterize the decrease of the bearing capacity of the structural element in the assemblage.

By means of vibrodiagnostics the technical condition of various buildings and structures [6−8] can be assessed. On the one hand, they include unique buildings, such as launch facilities of rocket and space complexes, the source of structural vibrations in them being the launch of the carrier rocket [9]. On the other hand, it can be buildings of historical heritage [10, 11], bridge structures [12, 13] or concrete columns [14], the source of oscillation in them is the load of man-made origin (technoseismic).

In the process of assessing the technical condition category of buildings and structures the relationship of diagnostic signs of vibrodiagnostics with defects and destruction occurring in structures is of special interest [15].

The article [16] gives the analysis of the results of the vibration method used to determine the degree of damage in the structures. The software module developed for the localization of these damages is also given. The article shows that the natural frequencies of structure vibration, as well as the forms of their

oscillations are the most important characteristics that allow to obtain information about the state of the structure.

In [17] a new feature is developed and proposed for the vibration methods – the index of damage. This step has been done in order to detect and locate damage in composite beams. The new method is compared with similar methods, for example, the deformation energy method.

The article [18] discusses the possibility of using the computational and intelligent methods of structural monitoring of the composite materials. It is emphasized that the composite materials, unlike metal, can have an ideal appearance without visible damage, although they may have many microdefects inside. In this case, it is necessary to use the SHM (Structural Health Monitoring) system.

In [19] the problem of damage detection in building constructions at external excitation is considered. The paper proposes a new index called the modal participation coefficient which is obtained on the basis of experimental data on the environmental vibration assessment.

In article [20] it is shown that at the origin of microcracks or at the development of the main crack in a structure the potential energy of deformation which can create an acoustic signal is released. This signal can be recorded on the surface of the product as an acoustic emission.

As the literature review shows, the relationship of vibrodiagnostic features with the characteristics of defects and damages formed in the structure during operation, as well as with the energy of crack formation has not been considered up to date.

In addition, there is no data concerning the relationship of diagnostic features with the structure, phase composition of the construction material, as well as the presence of microdefects in it which are not detected during the visual inspection.

The present international normative documents [21, 22] do not include the quantitative indicators that determine the categories of technical condition of buildings and structures and the safety of their operation.

The existing methods of quality monitoring of the building structure technical condition require the determination of a large number of physical and mechanical characteristics and geometric parameters of elements and materials of structures for control calculations. The access to building structures for determining these characteristics is difficult in most of the buildings and structures in use (ceramic tiles, plaster, drywall, flooring, suspended ceilings, etc.).

Then it will be urgent to develop the quantitative characteristics of the transition of one category of the technical condition of the structure into another, which will increase the accuracy and efficiency of determining its technical condition. The basis of the development is the experimental determination of the natural oscillation frequency.

The goal of the study: to increase the reliability of monitoring the technical condition of load-bearing reinforced concrete structures by using vibration diagnostics.

Research problem: on the basis of the conducted experimental studies, we obtain a mathematical dependence of the natural vibration frequency of span structures on the physical-mechanical, geometric, and energy characteristics of their elements and materials; error of the obtained mathematical dependence should be significantly reduced in comparison with the existing ones.

2. Methods

From the analysis of the literature it follows that the accumulation of defects and damages in the span concrete beam causes a decrease in its rigidity and the frequency of its own oscillations under the load action [23].

Analysis of the possibility of calculating the natural oscillation frequency of the structure shows that there is an analytical formula for determining the dependence of the natural oscillation frequency of span structures on a number of parameters, including the spatial stiffness of the section (1), [24]. However, the error in the determination of the natural oscillation frequencies for this dependence is as high as 35 %.

$$\lambda = \frac{\phi}{L_0^2}\sqrt{\frac{D}{m}}, \tag{1}$$

where $D = EI$ is spatial stiffness of the structure (it changes at the formation and the development of defects and damages), Nm^2; E is reduced modulus of elasticity of the material of construction, Pa; I is moment of inertia of the cross section, m^4; φ is dimensionless frequency factor depending on the type of the construction

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and the method of its fixing; L_0 is construction span, m; m is the structure mass distributed in the span per unit length (takes into account the actual load on the structure), Hsec^2/m^2.

The task of the study was to develop a mathematical formula of determining the frequency of natural oscillations of span concrete structures with reduced error for its further application to control their technical condition.

The main scientific idea of the work is as follows.

1. To date the following relationship, Formula (2), is used to calculate the spatial stiffness D in Formula (1). This dependence makes it possible to determine the value of stiffness only at the final stage of the structure operation, Fig. 1, due to the fact that it takes into account the physical and mechanical characteristics of the structure's material prior to their destruction.

$$D = E_b \left(I + I_s \alpha + I_s' \alpha \right),$$ (2)

where E_b is the deformation module of the compressed concrete, Pa; I is the moment of inertia of the concrete section relative to the center of gravity of the reduced cross-section of the element, m^4; I_s, I_s' is moments of inertia of the cross-sectional areas, respectively, of the stretched and compressed reinforcement relatively to the center of gravity of the reduced cross-section of the element, m^4; α is coefficient of reduction of reinforcement to concrete.
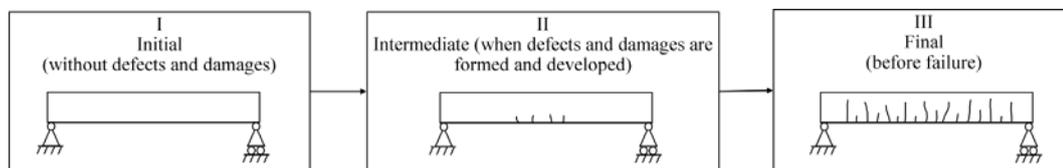

Figure 1. Stages of the span structure operation.

The work suggested that to reduce large errors in the determination of the natural oscillation frequency (Formula 1) it is necessary to consider the stiffness changes over the full life cycle of the construction, which in turn will allow to take into account nonlinear work of concrete during the operation of long-span structures. This can be realized if we take into account the law of rigidity variability in the process of calculation.

In the Formula 1 the paper proposes to use the dependence in order to calculate the spatial stiffness (3), [9]. The choice is dictated by the fact that it includes the law of rigidity variability. This law is based on the force approach and is taken into account by the value of the ratio of the bending moment from the external load to the limiting bending moment perceived by the cross section (M/M_{max}).

$$D = kD_0 \left\langle 1 - \left[1 - \frac{D_{min}}{kD_0} \right] \frac{M}{M_{max}} \right\rangle,$$ (3)

where k is correction factor depending on the duration of the load; D_0 is stiffness of the cross section of the element under the assumption of a linear relationship between stresses and strains and the absence of cracks, Nm^2; D_{min} is the minimum value of the section stiffness in the state preceding its destruction, Nm^2; M is bending moment from the external load, Nm; M_{max} is the ultimate bending moment resisted by the section, Nm.

In addition, this mathematical dependence deals with two stiffness: the stiffness of the cross section of the element under the assumption of a linear relationship between loads and strains and the absence of cracks, D_0, and the minimum value of the stiffness of the section in the state preceding its destruction, D_{min}.

It is assumed that the use of Formula (3) in Formula (1) will lead to a significant reduction in the error in determining the natural oscillation frequencies of span structures. This effect should be especially evident at the initial and final stage of the design, since the dependence (3) includes the rigidity of the structure, D at these stages of its operation. But, since the law of rigidity variability (M/M_{max}) is based on the force approach which does not reflect the nonlinear operation of concrete at the intermediate stage of the structure operation the error is likely to change insignificantly.

2. It is assumed that since the decrease in the frequency of natural oscillations and spatial stiffness is connected with the formation of defects and damage, then in the dependence (3) it is necessary to use the energy law reflecting the mechanics of destruction of the construction material. Further reduction of the error can be realized by using energy rather than force approach, for example Griffith's approach [24] (Formula 4)

in order to reflect the law of rigidity variability. This dependence is derived by Griffith for brittle materials which are known to include concrete.

$$\Delta U = 2l\gamma, \tag{4}$$

where l is crack length, m; γ is surface crack energy, Nm.

The energy approach can take into account not only the geometric characteristics of structures (in terms of l) but also the physical and chemical characteristics of the material (in terms of γ). Their joint contribution will allow to take into account the nonlinear operation of concrete and reduce the error of numerical determination of the natural oscillation frequency of span structures.

3. Results and Discussion

To confirm the assumptions made in the work it is necessary to solve the following steps:

1) while determining (on the basis of experimental data) the frequency of natural oscillations of the span structures by substituting Formula (3) in Formula (1), Formula (5) obtained allows to assess the magnitude of the error

$$\lambda = \phi \sqrt{\frac{kD_0 \left\langle 1 - \left[1 - \dfrac{D_{\min}}{kD_0}\right] \dfrac{M}{M_{\max}} \right\rangle}{mL_0^4}}; \tag{5}$$

2) to carry out the experimental investigations in order to establish the relationship between the natural frequency of the span structures and the energy of destruction according to Griffith, and correspondingly, the characteristics that are reflected in this formula: defects and damages emerging in construction is indicator l and physico-chemical characteristics of the materials of these structures is figure γ.

3) to identify the type of function describing the nonlinearity of changes in the law of stiffness variability based on Griffith's energy approach; to derive a new mathematical dependence and determine the magnitude of its error.

In accordance with the tasks stated, at the first stage the experimental studies were carried out in order to establish the relationship between the natural oscillation frequency of the structure and the index $M/M_{\max}$ (Formula 3).

The experiment was carried out on the reinforced concrete beams of factory production 1 PB-10-1P, their parameters being shown in Table 1 and Fig. 2 and 3.

Table 1. Geometrical and physical-mechanical parameters of beams 1 PB-10-1P and their elements.

Brand	Dimensions, mm									Weight, kg	E_b, MPa	E_s, MPa
	L	L_0	b	b'	h	h_0	a	a'	d			
1 PB-10-1P	1030	930	120	80	65	53	12	18	4	20	3×10^4	2×10^5
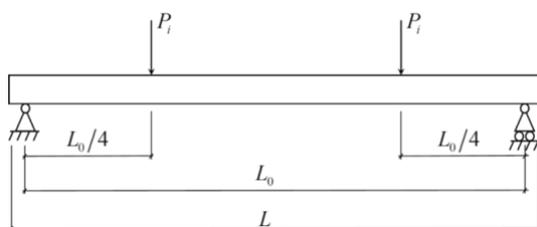
Figure 2. The rated scheme of the beam's 1 PB-10-1P.
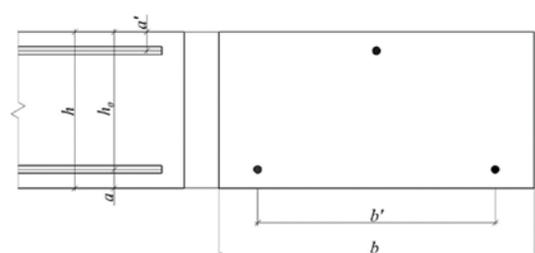
Figure 3. The beam cut 1 PB-10-1P.

To ensure a constant bending moment in the middle of the span the rated scheme shown in Fig. 2 was used.

At the initial stage the sample without defects and damages was fixed on the stand (Fig. 4) and the first forms of natural oscillations of the sample were excited while determining its natural frequencies (Fig. 5) with the device VIC-3-2.

Figure 4. Fixing the test sample on the stand.

**Figure 5. Excitation of the first forms
of the sample oscillation with the determination
of its natural frequencies.**

Next, a step-by-step application a static load to the sample was performed (Fig. 6) on the test press IP-100 with the increments of not more than 10 % of the destructive load, followed by a repetition of the initial stage. At the final stage a destructive load was applied to the sample and the natural oscillation frequency of the beam was estimated.

Figure 6. Load application to the sample according to the diagram (Figure 2).

The readings obtained were recorded into the table. After each step of load application the value M/M_{max} was calculated and a map of defects was drawn up: the total length of the cracks was determined; the spalling of the concrete compressed zone, the pulling of the reinforcement and its rupture in the stretched zone, and so on were fixed.

Then the dependence of the beam natural oscillation frequency on the ratio value of the bending moments was constructed, M/M_{max}, Fig. 7.
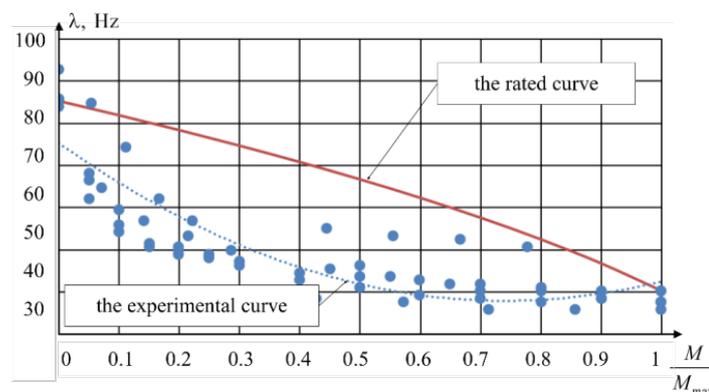
**Figure 7. The rated and experimental graphs of the dependence
of the natural oscillation frequency on the relative bending moment.**

As we can see from the figure, the error of the rated determination of the natural oscillation frequency in accordance with the proposed Formula (5) at the initial (without defects) and final (before destruction) stage of the construction operation is greatly reduced (as shown by the comparison of experimental and calculated data, from 35 to 8 %, i.e. 4.4 times), which confirmed earlier assumptions and differs significantly from the

results of other authors [4]. From the figure it also follows that, unlike the experimental curve, the rated one has a linear character. This special feature may be the reason why the error of the mathematical model remains the same at the intermediate stage of operation of the beam, which also confirms the assumptions made. This implies the need to move to a nonlinear form of dependence describing the law of stiffness changes in Formula 5. Proceeding from the nature of the experimental curve, it is assumed to be described by the exponential dependence properly.

At the second stage, in accordance with the tasks of the study, it was necessary to establish a correlation between the frequency of natural vibrations of the beam and the energy of destruction according to Griffith and the geometric and physical-chemical characteristics of the beam's material (reinforced concrete).

Table 2 shows the relationship obtained on the basis of the experimental data between the natural oscillation frequency of the span beam and the relative length of the cracks formed in it when the load is applied. This relationship is obtained in the work for the first time. The presence of cracks was assessed visually. The table shows that with an increase in the relative length of cracks during the transition from the initial to the final stage of the beam operation the frequency of the natural oscillations is reduced by 2.1 times.

Table 2. The relationship between the natural oscillation frequency of the beam and the relative cracks length.

The natural frequency, λ, Hz	82	72	63	57	50	45	42	41	40	38	38
Relative total cracks length	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1

Then, in order to identify the relationship of the natural oscillation frequencies of the span structures with the presence of microdefects in them and with the physical and chemical properties of the materials of these structures, electron microscopy of the samples (Fig. 8) at the initial and the final stages of the span structure, as well as their derivatographic analysis (Fig. 9, Table 3) were performed.

However it was assumed that the magnitude λ will be interconnected not only with the length of the visually defined crack but with the presence of microcracks, their contribution to the process of reducing the bearing capacity and the subsequent destruction of the concrete beam being very large. In other words, if there are no visible cracks on the surface of the concrete, and the number of invisible microcracks increases and the bearing capacity of the span structure falls, this effect will affect the value of the frequency of its own oscillations.
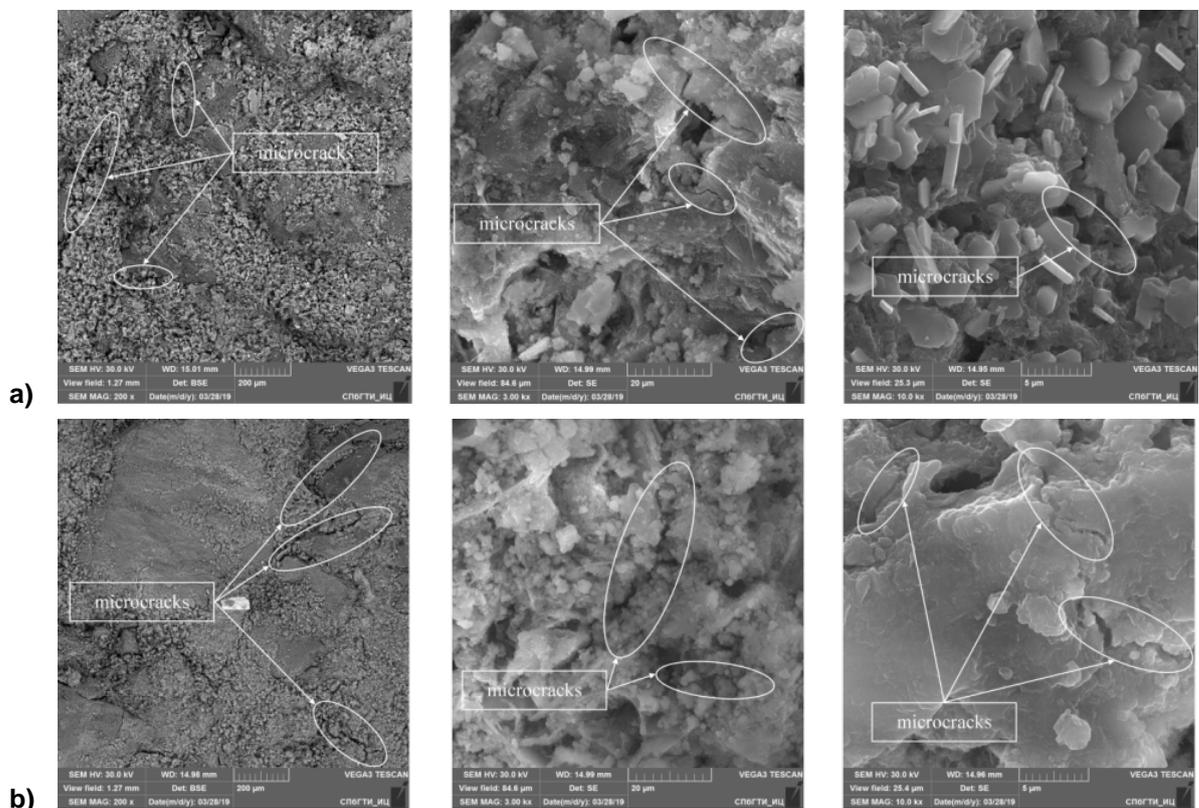
Figure 8. Results of concrete samples data processing by means of the electron microscopy: a) the initial stage of the structure operation, b) the final stage of the structure operation.

a)   b)
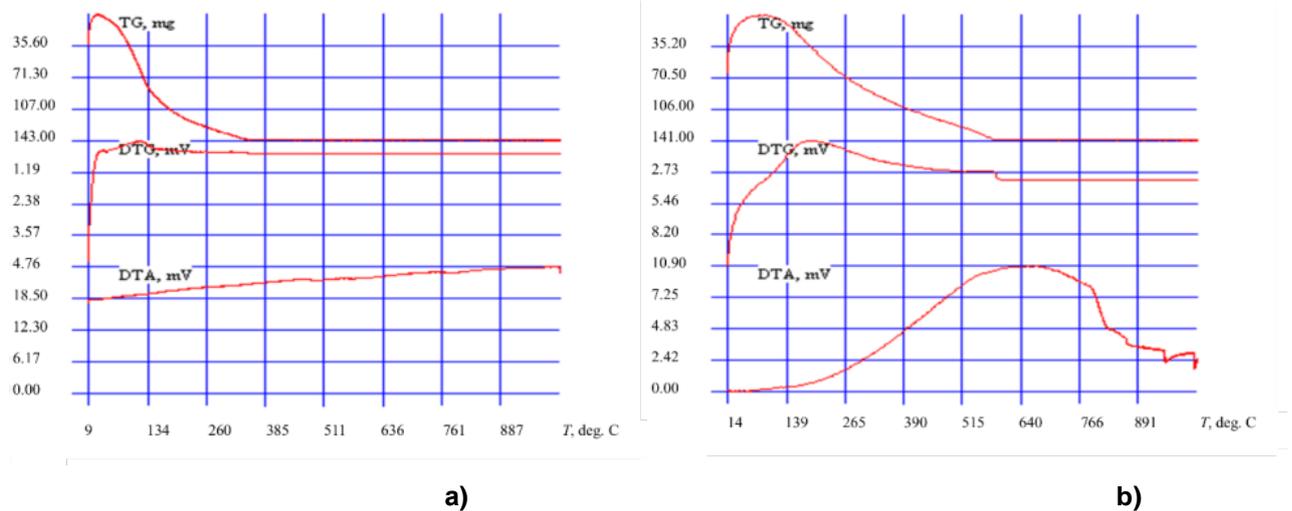
Figure 9. Data of the derivatographical analysis of samples:
a) the initial stage of structure operation, b) the final phase of structure operation.

According to the results of analysis of microscopic tests the relative density of microcracks, ρ_{rel}, m^{-1}, as the ratio of the total length of cracks of the structure to its surface area was obtained (Table 3).

Table 3. Results of physical and chemical analysis of samples.

Stage of the structure operation	Natural oscillation frequency, Hz	Total density of macrocracks, ρ_{rel}, m^{-1}	Total density of microcracks, ρ_{rel}, m^{-1}	Loss of water in the gel phase
Initial	85	0	6800	67
Final	40	22	77109	33

Table 3 shows that the total density of microcracks increases by more than an order of magnitude with a decrease in the natural oscillation frequency of the span structure by 2.1 times. In this case the total density of microcracks increases only by 2 times. This may indicate that the presence of macrocracks plays an important role in reducing the frequency of natural vibrations of the concrete structure and this value can reflect the internal invisible processes of destruction in the material.

The derivatographic analysis of the samples carried out showed that the amount of water in the gel phase at the final stage of the structure's operation as regards the initial one falls by 2 times. This fact can probably be explained by the fact that the gel phase as a component of the cement stone of the span beam is more brittle in relation to other phases and is a source of the formation and development of cracks. In this case the destruction of this phase occurs as shown also by the decrease in the amount of chemically bound water in it [25].

Physical and chemical studies carried out confirm the assumption about the interrelation of both geometric and physico-chemical characteristics of the material with the frequency of natural oscillations of the superstructure made from this material. These results are obtained for the first time.

Further it was necessary to determine the value of the surface energy of cracks included in Griffith's formula. Due to the complexity of determining this value on the edge angle of wetting because of the capillary-porous structure of the concrete, a search was made for the physical value of the material surface by which it can be determined. Analysis of the literature data showed that when increasing the surface hardness of non-metallic inorganic materials their surface energy also increases [26].

It was suggested that if we derive a mathematical relationship between these indicators, then after measuring the hardness of the crack surface, it is possible to determine its surface energy.

Proceeding from the above mentioned conclusions, the results of the approximation of the experimental data (Fig. 10) gave the possibility to obtain for the first time the empirical dependence of the surface energy of nonmetallic bodies of different nature on their hardness according to Mohs scale, Formula (6).

$$\gamma = 0.35e^{0.32H}Mohs.$$ (6)
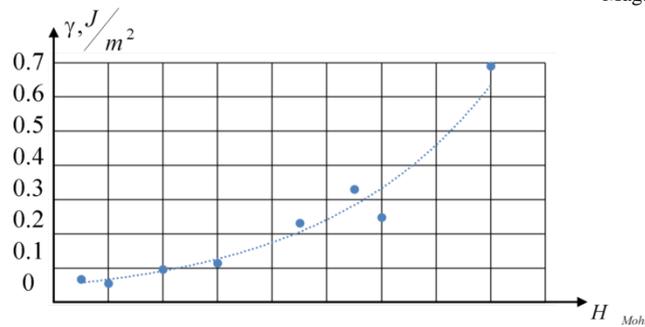
**Figure 10. Dependence of surface energy of non-metallic materials
on their hardness according to Mohs scale.**

The dependence obtained was used to calculate the surface energy of cracks in the experimental beam, which allowed to calculate the relative failure energy according to Griffith, $\Delta U/\Delta U_{max}$.

At the third stage, the mathematical processing of experimental data by using the regression analysis was carried out in order to identify the type of function describing the nonlinearity of the change of the law of stiffness variability on the basis of Griffith's energy.

The output parameter and the variable factors are shown in Table 4.

As an output parameter the stiffness variable function obtained from Formula (5) was used.

Table 4. Initial data for the construction of mathematical equations.

$Y = \dfrac{kD_0 - \dfrac{\lambda^2 L_0^4 m}{\phi^2}}{kD_0 - D_{min}}$	the stiffness variability function
$X_1 = \dfrac{P}{P_{max}}$	the relative load
$X_2 = \dfrac{n}{n_{max}}$	the relative number of cracks
$X_3 = \dfrac{\Delta U}{\Delta U_{max}}$	the relative energy according to Griffith
$X_4 = \dfrac{a_{crc}}{a_{crc_{max}}}$	the relative crack's width

Linear and exponential types of equations were used in the construction of the models. For the convenience of the coefficients comparison in the regression equations relative variable parameters were used.

The obtained regression equations of various types and classes are given below. For each equation the calculated and tabular values of Fisher's criterion are given, by means of their comparison the adequacy of the equations obtained was estimated.

A linear model describing the individual influence of each of the four factors X_1, X_2, X_3, X_4

$$Y = 0.68 - 0.04X_1 + 0.93X_2 - 2.40X_3 + 0.80X_4. \tag{5}$$

Fisher's criterion rated is 2.92.

Fisher's criterion table $F(60.56) = 1.55$.

2.92 > 1.55, hence the model describes the experimental data adequately.

An exponential model describing the individual influence of each of the four factors X_1, X_2, X_3, X_4

$$Y = e^{0.02 - 3.54X_1 + 2.67X_2 - 6.53X_3 + 1.62X_4}.$$ (6)

Fisher's criterion rated is 24.01.

Fisher's criterion table $F(60.56) = 1.55$.

24.01 > 1.55, therefore, the model describes the experimental data adequately and properly.

From the Formulas (7 and 8) it follows that the most significant factor is the relative energy of crack formation according to Griffith, and the nonlinear exponential dependence (8) is more adequate according to Fisher's criterion in regard to the linear one, which confirms the previously stated assumption.

Further, we approximates the experimental data by the least squares method to obtain the dependence of the natural frequency of the structures on the relative Griffith energy, Formula 9.

$$\lambda = \phi \sqrt{\frac{kD_0 \left\langle 1 - \left[1 - \dfrac{D_{\min}}{kD_0} \right] \left(1 - e^{-5\frac{\Delta U}{\Delta U_{\max}}} \right) \right\rangle}{mL_0^4}}.$$ (9)

The experimental and new calculated curves obtained from the approximation results are shown in Fig. 11. The figure shows that the error has significantly decreased (as shown by the comparison of experimental and calculated data, the error has decreased up to 8 % at all stages of the structure operation), which confirms the previously stated assumption. In comparison with the existing mathematical dependence [7], the error is reduced by 4.4 times.
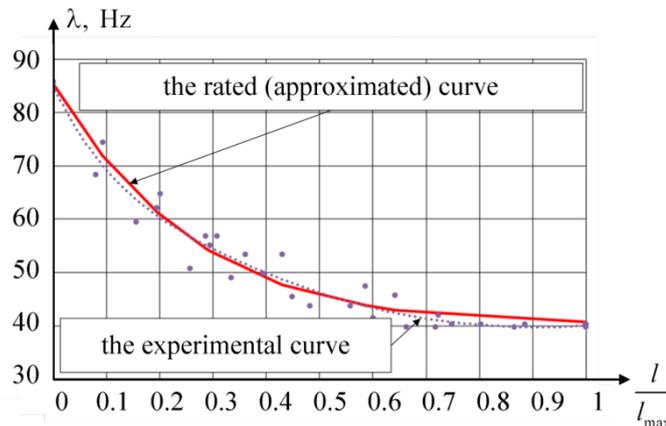
Figure 11. The dependence of the natural oscillation frequency of the beam on the relative energies according to Griffith.

The validity of the obtained model application for the real structures and buildings is justified by the application of the theory of similarity [9]:

1) the physical similarity of the mathematical dependence developed is taken into account by the corresponding coefficients in Formulas 1 and 3 which it includes;

2) the law of the stiffness variability on the basis of Griffith energy used in the mathematical dependence developed is chosen in the form of dimensionless quantity.

4. Conclusions

1. On the basis of Griffith's energy approach the mathematical dependence of the determination of the natural oscillation frequency of the span structures is developed.

2. It is shown that the error of the mathematical dependence developed is reduced by 4.4 times in comparison with the existing formulas.

3. It is shown that the value of the natural oscillation frequency of the span beam is reduced by 2.1 times in the presence of defects and destruction at the final stage of its operation in regard to the initial one.

4. According to the physico-chemical investigations it is shown that at the final stage of the structure operation the relative density of microcracks in the span beam increases by an order of magnitude in regard

to the initial stage, the relative density of macrocracks increases by 2 times, the amount of gel water being reduced by 2 times.

5. For the first time the mathematical formula linking the surface energy of nonmetallic inorganic materials with the hardness of their surface is obtained.

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