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RECONSTRUCTING QUINTESSENCE SCALAR FIELD MODEL FROM NEW HOLOGRAPHIC DARK ENERGY IN BIANCHI TYPE I UNIVERSE

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Abstract. In this paper, we investigate the cosmic evolution of a spatially homogeneous and anisotropic Bianchi type I universe filled with new holographic dark energy (NHDE) and cold dark matter (CDM) within the framework of General Relativity by considering both the components of the universe to be interacting with each other. To obtain the exact solutions of Einstein's field equations, we consider two expansion laws: an exponential expansion and a power-law volumetric expansion. The evolutions of some parameters of cosmological importance are studied for both the models corresponding to the exponential expansion and the power-law volumetric expansion. We observe that in both the models the anisotropy parameter decreases as time evolves and tends to zero at late times. The model corresponding to exponential expansion behaves like CDM model and the model corresponding to power-law volumetric expansion behaves like quintessence holographic dark energy model at late time. We also compare the equation of state (EoS) and energy density of our interacting NHDE model with that of quintessence scalar field and establish a correspondence between them. The quintessence potential is reconstructed which depicts the observed accelerated expansion of the universe.

Keywords: Bianchi type I universe, new holographic dark energy, quintessence, EoS parameter

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РЕКОНСТРУКЦИЯ МОДЕЛИ СКАЛЯРНОГО ПОЛЯ КВИНТЭССЕНЦИИ ДЛЯ НОВОЙ ГОЛОГРАФИЧЕСКОЙ МОДЕЛИ ТЕМНОЙ ЭНЕРГИИ ВО ВСЕЛЕННОЙ БИАНКИ ТИПА I

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Аннотация. В рамках общей теории относительности в работе исследована космическая эволюция однородной по пространству и анизотропной Вселенной Бианки типа I, заполненной новой голографической темной энергией (НГТЭ) и холодной темной материей (ХТМ); при этом сделано предположение, что оба компонента Вселенной взаимодействуют между собой. Чтобы получить точные решения полевых уравнений Эйнштейна, рассмотрены два закона расширения: экспоненциальное и степенное объемное. Для обоих случаев изучено изменение некоторых ключевых космологических параметров. Установлено, что для обеих моделей анизотропный параметр уменьшается со временем и в итоге стремится к нулю. Оказалось, что модель, соответствующая экспоненциальному расширению, ведет себя как модель ХТМ, а модель, соответствующая степенному объемному расширению, – как модель ГТЭ позднего периода. Проведено также сравнение уравнения состояния и плотности энергии взаимодействия в НТГЭ (наша модель) и в модели скалярного поля квинтэссенции и найдено соответствие между ними. Потенциал квинтэссенции реконструирован таким образом, чтобы он описывал наблюдаемое ускоренное расширение Вселенной.

Ключевые слова: Вселенная Бианки типа I, новая голографическая темная энергия, квинтэссенция, уравнение состояния

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1. Introduction

Until the late 20th century, no consensus has been reached about the expansion rate of the universe. However, various cosmological and astrophysical observations such as Supernovae Type Ia (SN Ia) [1 – 3], Cosmic Microwave Background (CMB) [4, 8], Baryon Acoustic Oscillations (BAO) [9], Large Scale Structure (LSS) [10 – 12] and their cross relations make it clear that our universe is currently undergoing a phase of accelerated expansion. A strange kind of physical entity is supposed to be the cause of this late time cosmic acceleration which is termed as dark energy.

A dark energy candidate which can simply explain the late time cosmic acceleration is the so called cosmological constant Λ introduced by Einstein in his field equations. However, due to its non-evolving nature, it suffers from some theoretical challenges such as the fine-tuning and the cosmic coincidence problems. Therefore, quintessence [13 – 15], a minimally coupled homogeneous scalar field which provides a solution to the fine-tuning problem and also to the coincidence problem by means of tracker solutions, is considered as dark energy candidate. Different dynamical dark energy models such as phantom [16], *k*-essence [17], tachyon [18], dilatonic ghost condensate [19] and the interacting exotic fluid models such as Chaplygin gas models [20 – 23] etc. have also been investigated in the literature.

Holographic dark energy is another candidate of considerable interest which emerges from the Holographic Principle, first put forwarded by Gerard ’t Hooft [24] in the context of black hole physics. This principle states that the number of degrees of freedom directly related to the entropy of a system scales with the enclosing surface area of the system and not with its volume. Based on the effective Quantum Field Theory, Cohen et al. [25] proposed a relationship between the ultraviolet (UV) cutoff and the infrared (IR) cutoff of a system due to the limit set by the formation of a black hole which in turn gives an upper bound on the zero-point energy of a system. When the whole universe is taken into account, this zero-point energy density has the same order of magnitude as the dark energy density which is referred to as the holographic dark energy.

In the cosmological context, a new version of the Holographic Principle was first proposed by Fischler and Susskind [26] which states that at any point during cosmological evolution the gravitational entropy within a closed surface should not exceed the particle entropy that passes through the past light-cone of that surface. Granda and Oliveros [27] proposed a new holographic dark energy density of the form

$$\rho_{\mathrm{NHDE}} \approx \alpha H^2 + \beta \dot{H},$$

where H is the Hubble parameter, and the two constants α and β are to be determined so as to satisfy the restrictions entailed by the current observational data.

In Ref. [27], they showed that this new dark energy model can explain the current cosmic acceleration and being consistent with the observational data. In Ref. [28], these authors also established correspondence between quintessence, tachyon, *k*-essence and dilaton dark energy models with this new holographic dark energy in the flat Friedman – Robertson – Walker (FRW) universe. Many other researchers have since investigated several aspects of new holographic dark energy (NHDE) in an isotropic as well as in anisotropic background and studied the correspondence between the scalar fields such as quintessence, *k*-essence and NHDE models in cosmology [28 – 46].

The goal of this paper is to investigate a spatially homogeneous and anisotropic Bianchi type I universe filled with interacting cold dark matter and new holographic dark energy within the framework of General Relativity.

The reason why an anisotropic universe is considered in our investigation is that although our universe is homogeneous and isotropic at large scale, recent experimental tests like Wilkinson Microwave Anisotropy Probe (WMAP) [7, 8], Cosmic Background Explorers (COBE) [47] and Planck Collaboration results [48] support the existence of an anisotropic phase in the evolution of the universe that approaches an isotropic one.

The paper is organized as follows: In Section 2, we derive the field equations for the Bianchi type I line-element and find the expression for the equation-of-state (EoS) parameter for interacting cold dark matter and new holographic dark energy. In Section 3, we solve the Einstein field equations by considering two expansion laws viz. an exponential expansion and a volumetric expansion law. We construct two different models corresponding to these two expansion laws. In this section, we also obtain the expressions for some parameters of cosmological interest and discuss some physical and geometrical properties of both the models. In Section 4, we establish the correspondence between the quintessence and the new holographic dark energy for the model with volumetric expansion law. We conclude the paper with a brief discussion in Section 5.

2. Metric and field equations

A spatially homogeneous and anisotropic Bianchi type I universe is described by the line-element

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2, \tag{1}$$

where A, B, C are functions of the cosmic time t alone.

We assume that the universe is filled with two interacting components: cold dark matter (CDM) and new holographic dark energy (NHDE).

In natural units ($8\pi G = 1$, $c = 1$), Einstein's field equations are

$$R_{ij} - \frac{1}{2} g_{ij} R = -(T_{ij} + \overline{T}_{ij}), \tag{2}$$

where R_{ij} is the Ricci tensor; R is the Ricci scalar; T_{ij} is the energy-momentum tensor for cold dark matter given by

$$T_{ij} = \rho_m u_i u_j, \tag{3}$$

and $\overline{T}_{ij}$ is the energy-momentum tensor for new holographic dark energy given by

$$\overline{T}_{ij} = (\rho_{\mathrm{NHDE}} + p_{\mathrm{NHDE}}) u_i u_j + g_{ij} p_{\mathrm{NHDE}}. \tag{4}$$

Here, ρ_m is the energy density of cold dark matter, ρ_{NHDE} and p_{NHDE} are respectively the energy density and the pressure of the new holographic dark energy.

In a comoving coordinate system, Eq. (2) with Eqs. (3) and (4) for the metric (1) lead to the following system of field equations:

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=-p_{\text{NHDE}}, \tag{5}$$

$$\frac{\ddot{C}}{C}+\frac{\ddot{A}}{A}+\frac{\dot{C}\dot{A}}{CA}=-p_{\text{NHDE}}, \tag{6}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=-p_{\text{NHDE}}, \tag{7}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}=\rho_m+\rho_{\text{NHDE}}, \tag{8}$$

where an over dot denotes differentiation with respect to t.

Following Ref. [27], we consider the new holographic dark energy density as

$$\rho_{\text{NHDE}}=3M_p^2(\alpha H^2+\beta\dot{H}), \tag{9}$$

where $M_p^{-2} = 8\pi G = 1$; α, β are constants.

The pressure of the NHDE is given by

$$p_{\text{NHDE}}=\omega_{\text{NHDE}}\rho_{\text{NHDE}}, \tag{10}$$

where ω_{NHDE} is the EoS parameter of NHDE.

In a universe where dark matter and dark energy are interacting with each other, their energy densities do not conserve separately. So, when CDM and NHDE are interacting the continuity equations can be obtained as

$$\dot{\rho}_m+3H\rho_m=Q, \tag{11}$$

$$\dot{\rho}_{\text{NHDE}}+3H(\rho_{\text{NHDE}}+p_{\text{NHDE}})=-Q, \tag{12}$$

where Q represents the interaction term between CDM and NHDE.

A natural choice for the interaction term available in the literature is $Q \propto H\rho_m$, $Q \propto H\rho_{\text{NHDE}}$ or a combination of these. We take

$$Q=3H(\gamma\rho_{\text{NHDE}}+\delta\rho_m), \tag{13}$$

where γ, δ are coupling constants.

For $\gamma = 0$, we have $Q = 3H\delta\rho_m$, and for $\delta = 0$, we have $Q = 3H\gamma\rho_{\text{NHDE}}$. CDM and NHDE are non-interacting for $\gamma = 0 = \delta$.

From Eqs. (9), (10), (12) and (13), we obtain

$$\omega_{\text{NHDE}}=-1-\frac{2\alpha H\dot{H}+\beta\ddot{H}}{3H(\alpha H^2+\beta\dot{H})}-\left(\gamma+\frac{\delta\rho_m}{\rho_{\text{NHDE}}}\right). \tag{14}$$

3. Solutions of the field equations

From Eqs. (5) – (8), we derive

$$A(t)=a_1V^{\frac{1}{3}}\exp(b_1\int V^{-1}dt), \tag{15}$$

$$B(t)=a_2V^{\frac{1}{3}}\exp(b_2\int V^{-1}dt), \tag{16}$$

$$C(t)=a_3V^{\frac{1}{3}}\exp(b_3\int V^{-1}dt), \tag{17}$$

where $a_1a_2a_3 = 1$, $b_1 + b_1 + b_1 = 0$, and $V = ABC$ is the volume of the universe.

Now, as we have four equations in five unknown parameters A, B, C, ρ_m and p_{NHDE}, therefore, we need one extra condition to obtain an exact solution of the field equations. In view of Eqs. (15) – (17), we consider an exponential expansion law given by

$$V = ce^{3lt}, \tag{18}$$

and also a volumetric expansion law given by

$$V = ct^{3n}, \tag{19}$$

where c, l, n are positive constants.

A model for exponential expansion (Model 1). Using (18) in Eqs. (15) – (17), we get

$$A = a_1 c^{\frac{1}{3}} e^{lt} \exp\left[-\frac{b_1}{3cl} e^{-3lt}\right], \tag{20}$$

$$B = a_2 c^{\frac{1}{3}} e^{lt} \exp\left[-\frac{b_2}{3cl} e^{-3lt}\right], \tag{21}$$

$$C = a_3 c^{\frac{1}{3}} e^{lt} \exp\left[-\frac{b_3}{3cl} e^{-3lt}\right]. \tag{22}$$

For this model, the directional Hubble parameters H_i and the mean Hubble parameter H are obtained as

$$H_1 = \frac{\dot{A}}{A} = l + \frac{b_1}{c} e^{-3lt}, \tag{23}$$

$$H_2 = \frac{\dot{B}}{B} = l + \frac{b_2}{c} e^{-3lt}, \tag{24}$$

$$H_3 = \frac{\dot{C}}{C} = l + \frac{b_3}{c} e^{-3lt}, \tag{25}$$

$$H = l. \tag{26}$$

The scalar of expansion θ, the spatial volume V, the shear scalar σ^2, the deceleration parameter q and the anisotropy parameter A_m for this model are obtained as

$$\theta = 3l, \tag{27}$$

$$\sigma^2 = \frac{M}{2c^2} e^{-6lt}, \tag{28}$$

$$q = -1, \tag{29}$$

$$A_m = \frac{M}{3l^2 c^2} e^{-6lt}, \tag{30}$$

where $M = b_1^2 + b_2^2 + b_3^2$.

From Eq. (9) we obtain

$$\rho_{\text{NHDE}} = 3\alpha l^2, \tag{31}$$

and from Eq. (11), using Eqs. (13) and (26), we get

$$\rho_m = \frac{3\alpha\gamma l^2}{1-\delta} + de^{-3lt(1-\delta)}, \tag{32}$$

where d is an integrating constant and $\delta \neq 1$.

Therefore, for this model, the total energy density and the EoS parameter are given by

$$\Omega = \Omega_{\text{NHDE}} + \Omega_m = \frac{\rho_{\text{NHDE}}}{3H^2} + \frac{\rho_m}{3H^2} = \alpha + \frac{\alpha\gamma}{1-\delta} + \frac{de^{-3lt(1-\delta)}}{3l^2}, \quad (33)$$

$$\omega_{\text{NHDE}} = -1 - \gamma - \delta\left(\frac{\gamma}{1-\delta} + \frac{de^{-3lt(1-\delta)}}{3\alpha l^2}\right), \quad (34)$$

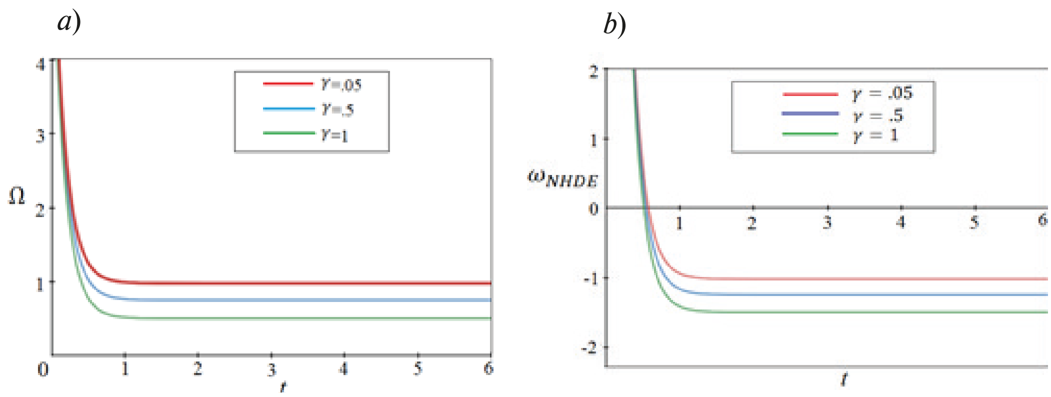

Fig. 1. Graphs of the total energy density Ω (*a*) and the EoS parameter (*b*) vs cosmic time *t* at different γ: 0.05 (red lines), 0.5 (blue ones) and 1.0 (green ones); $\alpha = 1$, $\delta = -1$, $d = 100$, $l = 1$

It is evident from the graphs in Fig. 1, that for small values of γ, our model approaches a flat isotropic universe in the course of time and behaves like a ΛCDM model. However, the model does not represent a flat isotropic universe for $\delta = 0$ and sufficiently large values of γ.

A model for volumetric expansion law (Model 2). Using Eq. (19) in Eqs. (15) – (17), we obtain

$$A(t) = a_1 c^{\frac{1}{3}} t^n \exp\left[\frac{b_1}{c(1-3n)} t^{-3n+1}\right], \quad (35)$$

$$B(t) = a_2 c^{\frac{1}{3}} t^n \exp\left[\frac{b_2}{c(1-3n)} t^{-3n+1}\right], \quad (36)$$

$$C(t) = a_3 c^{\frac{1}{3}} t^n \exp\left[\frac{b_3}{c(1-3n)} t^{-3n+1}\right], \quad (37)$$

where $n \neq 1/3$.

The directional Hubble parameters H_i and the mean Hubble parameter H for this model are obtained as

$$H_1 = nt^{-1} + \frac{b_1}{c} t^{-3n}, \quad (38)$$

$$H_2 = nt^{-1} + \frac{b_2}{c} t^{-3n}, \quad (39)$$

$$H_3 = nt^{-1} + \frac{b_3}{c} t^{-3n}, \quad (40)$$

$$H = \frac{n}{t}. \quad (41)$$

The scalar of expansion θ, the spatial volume V, the shear scalar σ^2, the deceleration parameter q and the anisotropy parameter A_m are obtained as

$$\theta = \frac{3n}{t}, \tag{42}$$

$$\sigma^2 = \frac{1}{2c^2} M t^{-6n}, \tag{43}$$

$$q = -1 + \frac{1}{n}, \tag{44}$$

$$A_m = \frac{1}{3} \frac{M}{c^2 n^2 t^4} \tag{45}$$

where $M = b_1^2 + b_2^2 + b_3^2$.

The cosmological observations indicate that the value of the deceleration parameter lies in the range $-1 < q < 0$, so it is clear from Eq. (44) that for an accelerating expansion of the universe, we must take $n > 1$. Moreover, Eq. (45) reveals that the anisotropy parameter is a decreasing function of cosmic time and tends to zero in the course of time.

The NHDE density for this model is obtained from Eq. (9), as

$$\rho_{\text{NHDE}} = \frac{3(\alpha n^2 - \beta n)}{t^2}. \tag{46}$$

Using Eqs. (13), (41) and (46) in Eq. (11), we obtain

$$\rho_m = \frac{9n\gamma(\alpha n^2 - \beta n)}{3n(1-\delta) - 2} t^{-2} + d t^{-3n(1-\delta)}, \tag{47}$$

where d is an integrating constant.

Therefore, for this model, the total energy density and the EoS parameters are given by

$$\Omega = \Omega_{\text{NHDE}} + \Omega_m = \frac{\rho_{\text{NHDE}}}{3H^2} + \frac{\rho_m}{3H^2} = \alpha - \frac{\beta}{n} + \frac{3\gamma(\alpha n - \beta)}{3n(1-\delta) - 2} + \frac{d t^{-3n(1-\delta)-2}}{3n^2}, \tag{48}$$

$$\omega_{\text{NHDE}} = -1 + \frac{2}{3n} - \left\{ \gamma + \delta \left(\frac{3n\gamma}{3n(1-\delta) - 2} + \frac{d t^{-3n(1-\delta)-2}}{3(\alpha n^2 - \beta n)} \right) \right\}. \tag{49}$$

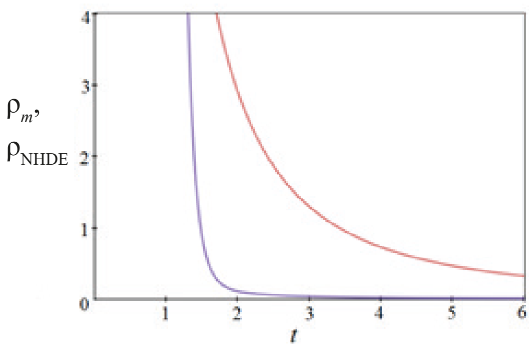

Fig. 2. Graphs of the new holographic dark energy density ρ_{NHDE} (the red line) and the cold dark matter density ρ_m (the blue one) vs cosmic time t; $d = 100$, $\delta = -1$, $n = 2$, $\alpha = 1$, $\beta = 0.05$, $\gamma = 0.05$

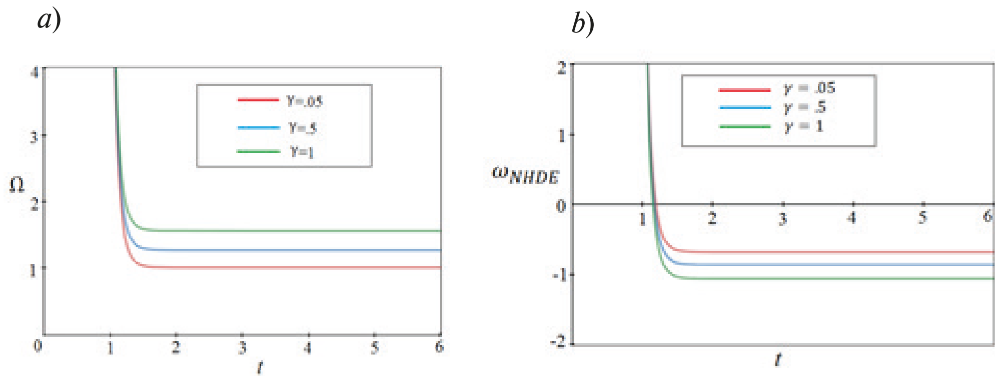

Fig. 3. Graphs of the total energy density Ω (*a*) and the EoS parameter ω_{NHDE} (*b*) vs cosmic time *t* at different γ: 0.05 (red lines), 0.5 (blue ones) and 1.0 (green ones); $n = 2$, $\alpha = 1$, $\beta = 0.05$, $\delta = -1$, $d = 100$

It is clear from the graphs in Fig. 2 that both the CDM density ρ_m and the NHDE density ρ_{NHDE} are decreasing functions of cosmic time. The former tends to zero as time evolves while the latter is near to zero in the course of time. For $\gamma = 0$ and for sufficiently small values of γ this model approaches a flat universe and the EoS parameter enters into quintessence region $-1 < \omega_{\text{NHDE}} < -1/3$ at a later time (Fig. 3). The same as Model 1, this model fails to represent the current universe for sufficiently large values of γ and for $\delta = 0$.

Hence, to consider the interaction between NHDE and CDM, it suffices to take the coupling parameter δ as the interacting term and $\gamma = 0$. Thus, putting $\gamma = 0$ in Eq. (49), we obtain

$$\omega_{\text{NHDE}} = -1 + \frac{2}{3n} - \delta \frac{dt^{-3n(1-\delta)-2}}{3(\alpha n^2 - \beta n)}. \tag{50}$$

4. Correspondence between new holographic dark energy and quintessence scalar field model

Quintessence is described by an ordinary scalar field φ minimally coupled to gravity and self-interaction described by a potential $V(\varphi)$ that leads to late time cosmic acceleration.

The action for the quintessence scalar field φ is given by an expression

$$S = \int d^4x\sqrt{-g}[-\frac{1}{2}g^{ij}\partial_i\varphi\partial_j\varphi - V(\varphi)]. \tag{51}$$

The energy density and pressure of the scalar field are given by expression

$$\rho(\varphi) = \frac{1}{2}\dot{\varphi}^2 + V(\varphi), \tag{52}$$

$$p(\varphi) = \frac{1}{2}\dot{\varphi}^2 - V(\varphi). \tag{53}$$

Using Eqs. (52) and (53), the EoS for the quintessence scalar field is obtained as

$$\omega_\varphi = \frac{p(\varphi)}{\rho(\varphi)} = \frac{\dot{\varphi}^2 - 2V(\varphi)}{\dot{\varphi}^2 + 2V(\varphi)}. \tag{54}$$

In order to establish the correspondence between the new holographic dark energy and the quintessence scalar field model, we compare the EoS and the dark energy density for the interacting new holographic dark energy and quintessence dark energy models.

Thus, comparing Eqs. (50) and (54), we obtain

$$-1+\frac{2}{3n}-\delta\frac{dt^{-3n(1-\delta)-2}}{3(\alpha n^2-\beta n)}=\frac{\dot{\varphi}^2-2V(\varphi)}{\dot{\varphi}^2+2V(\varphi)}. \tag{55}$$

Also, comparing Eqs. (46) and (52), we obtain

$$\frac{3(\alpha n^2-\beta n)}{t^2}=\frac{1}{2}\dot{\varphi}^2+V(\varphi). \tag{56}$$

Again, from Eq. (55), we derive

$$\dot{\varphi}^2=\frac{4(\alpha n-\beta)-2\delta dt^{-3n(1-\delta)-2}}{6(\alpha n^2-\beta n)-2(\alpha n-\beta)+\delta dt^{-3n(1-\delta)-2}}V(\varphi). \tag{57}$$

Using Eq. (56) in Eq. (57), we obtain

$$V(\varphi)=\frac{3(\alpha n^2-\beta n)}{t^2}\frac{6(\alpha n^2-\beta n)-2(\alpha n-\beta)+\delta dt^{-3n(1-\delta)-2}}{6(\alpha n^2-\beta n)+2(\alpha n-\beta)-\delta dt^{-3n(1-\delta)-2}}. \tag{58}$$

This type of potential can produce an accelerated expansion of the universe. Thus a correspondence between the interacting new holographic dark energy and quintessence scalar field model is established.

5. Conclusion

In this work, we study a spatially homogeneous and anisotropic Bianchi type I universe filled with interacting the new holographic dark energy and the cold dark matter by taking the coupling parameter Q as

$$Q = 3H(\gamma\rho_{\text{NHDE}} + \delta\rho_m).$$

To obtain the exact solutions of the Einstein field equations, we consider two expansion laws: an exponential expansion and a power-law volumetric one; they correspond to Models 1 and 2, respectively.

Considering Model 1, we find that the average Hubble parameter and the NHDE density are constant and the deceleration parameter equals −1. The anisotropy parameter decreases as time evolves and tends to zero at late time. The total energy density for this model approaches 1 for $\gamma \to 0$. Thus, for small values of γ, this model approaches a flat, isotropic universe at late time and the EoS parameter approaches −1 showing thereby that the Model 1 behaves like a ΛCDM model.

In Model 2, the Hubble parameter, NHDE density and CDM one are decreasing functions of cosmic time. We also observe that the anisotropy parameter decreases as time evolves and tends to zero. Hence, we conclude that the anisotropy of our universe dies out in the course of the evolution to reach the present isotropic phase. It can be also seen from Fig. 3,*a* that the total energy density approaches 1, and from Fig. 3,*b* we see that the EoS parameter of this model lies in the quintessence region

$$-1 < \omega_{\text{NHDE}} < -1/3$$

for $\gamma \to 0$.

So, this model behaves like a quintessence holographic dark energy model.

In both the models we define the interaction between NHDE and CDM by taking

$$Q = 3H(\gamma\rho_{\mathrm{NHDE}} + \delta\rho_m).$$

But the models represent current universe only for small values of γ. Therefore, we ignore γ while establishing correspondence between the new holographic dark energy model and the quintessence scalar field model. Quintessence potential is reconstructed which describes the accelerated phase of expansion of the universe.

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