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COMPARATIVE ANALYSIS OF HYBRID NEURAL NETWORK AND MULTILAYER MODELING OF A CIRCULAR MEMBRANE DEFLECTION UNDER A LOAD LOCATED ASYMMETRICALLY TO ITS CENTER

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Abstract. This article is devoted to the problem of a hybrid approach in modelling, which combines methods based on mathematical physics equations and data-driven methods. The issue of choosing a hybrid model for circular membrane deflection under a load is considered. To build models, the Laplace equation inaccurately describing the object and measurement data of sufficiently high accuracy are used. With the help of cross-validation methods, an algorithmic comparison of the generalising ability of a multilayer model, a physics informed neural network model and a classical approach is made. The results obtained allow us to recommend neural network and multilayer methods for modelling objects when a sufficiently accurate classical description using a boundary value problem is unknown or excessively difficult and additional information is available in the form of measurement results. Multilayer methods are preferable in case of shortage of data or its dynamic nature, if a compact adaptive model is needed, including for use in embedded systems and digital twins.

Keywords: hybrid models, circular membrane deflection, Laplace equation, PINN, multilayer model, physics-based architecture

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СРАВНИТЕЛЬНЫЙ АНАЛИЗ ГИБРИДНЫХ НЕЙРОСЕТЕВЫХ И МНОГОСЛОЙНЫХ МОДЕЛЕЙ ПРОГИБА КРУГЛОЙ МЕМБРАНЫ ПОД ДЕЙСТВИЕМ ГРУЗА, РАСПОЛОЖЕННОГО АСИММЕТРИЧНО ОТНОСИТЕЛЬНО ЕЕ ЦЕНТРА

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Аннотация. Данная статья посвящена проблеме гибридного подхода в моделировании, при котором соединяются методы основанные уравнениях математической физики и методы, управляемые данными. Рассматривается проблема выбора гибридной модели для задачи о прогибе круглой мембраны на тканевой основе под действием груза. Для построения моделей используется уравнение Лапласа, неточно описывающее объект, и данные измерений достаточно высокой точности. С помощью методов скользящего контроля произведено алгоритмическое сравнение обобщающей способности многослойной модели, построенной с помощью аналитической модификации классических численных методов, физически информированной нейросетевой модели и классического подхода. Полученные результаты позволяют рекомендовать нейросетевой и многослойный методы при моделировании объектов, для которых неизвестно или избыточно сложно достаточно точное классическое описание с помощью граничной задачи для дифференциальных уравнений и имеется дополнительная информация в виде результатов измерений. Многослойные модели предпочтительны в случае нехватки или динамических данных, при необходимости компактной адаптивной модели, в том числе, для использования во встроенных системах и цифровых двойниках.

Ключевые слова: гибридное моделирование, прогиб круглой мембраны, уравнение Лапласа, физически информированные нейронные сети, многослойная модель, основанная на физике архитектура

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Introduction

Simultaneously with the development of new technologies [1–2] and the widespread implementation of digital twins [3–4] there arises the problem of modelling real objects to meet the requirements of the modern world. An incomplete list of factors that must be included in the state-of-the-art paradigm of modelling is the following: the speed and cost of constructing models, their adaptability, the use of equations that describe an object inaccurately, the absence of conditions allowing the application of classical modelling methods, the possibility of using additional heterogeneous information, etc.

This article is devoted to the problem of hybrid approach to modelling when methods based on physics of an object (equations) and data-driven methods are combined.

Obviously, neither of these two types of modelling based on physics or data is perfect for using in digital twins. The hybrid approach is often applied in two situations: firstly, in modelling dynamical systems when the data is used to update the model and forecast, for example, in [7–8], and, secondly, in solving problems involving multifidelity [9–11].

The problem under consideration belongs to the second case. The differential equation is regarded as low-fidelity data and the measurements – as high-fidelity data. We propose an original approach to constructing models that combines multilayer methods [12], which are our analytical modification of the known numerical methods of solving differential equations and data-driven methods. The resulting hybrid multilayer model is compared with two other models constructed by means of a classical method and the common neural network approach, better known as the physics-informed neural network (PINN). Multilayer method as the method of building a model with an architecture based on physics and PINN as a hybrid method are regarded in the context of a new paradigm of modelling in work [13]. To compare models, a cross-validation allowing the estimation of different models from an algorithmic point of view [14] is used.

All types of models constructed can be used for computational modelling the objects of a similar nature after the parameter adjustment according to the actual measurement data.

Materials and Methods

Let us consider the task of approximating a deflection function for a circular membrane with a fixed edge under a load placed at a distance from the center and without overlapping it. The domain under the load is circular too. As a differential equation describing the real object, we use the Laplace equation for small deviations of a membrane under a load [15]. This equation obvious reflects the behavior of the membrane with low fidelity. In the process of the experiment, quite a big deflection described by more complicated equations is observed. These equations have a bigger number of parameters and require more information about physical properties of a membrane. As high-fidelity data, we use measurements obtained during the experiment.

The experiment consisted of the following. After placing a load with radius r_0 = 3.95 cm and weight 300 g on the membrane with radius R = 50 cm at a distance 10 cm from a membrane center N = 24 measurements of membrane deflection were taken in points around the load. In order to make high-fidelity measurements a coordinate mesh was drawn and markups have been made every 10 cm. Experimental setup was wooden, a membrane material was elastane. All measurements were made by means of a laser level.

Depending on the method used to construct the membrane deflection model, the Laplace equation was formulated in Cartesian or polar coordinates (Fig. 1). In the first case, we have an equation of the form

$$h''_{xx} + h''_{yy} = 0 \tag{1}$$

in the domain $D = \left\{(x, y) \middle| (x+a)^2 + y^2 \geq r_0^2,\ x^2 + y^2 \leq R^2\right\}$ (eccentric ring), where $R > a + r_0 > 2r_0$, R is a radius of membrane, r_0 is a radius of the area under the load, a is a distance from a membrane center to a center of the loaded area.

The boundary value problem for equation (1) corresponds to zero deflection at the fixed edge of the membrane

$$h(x, y) = 0 \text{ at } \left\{x^2 + y^2 = R^2\right\},$$

the other conditions are unknown. Thus, we have an incorrect problem that we solve using additional measurement data.

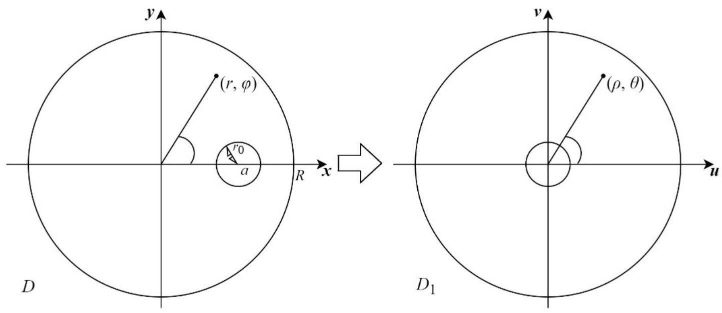

Fig. 1. Cartesian and polar coordinates for non-symmetric and symmetric formulations of the problem of finding the deflection of the membrane connected by a conformal mapping

If an equation of the form (1) is enough to apply the PINN, then other methods require some transformations. Using the conformal mapping [16], equation (1) in the domain D is reduced to the Laplace $h''_{uu} + h''_{vv} = 0$ equation in the symmetric closed domain D_1 (concentric ring).

Then in polar coordinates (ρ, θ), $u = \rho\cos\theta$, $v = \rho\sin\theta$, we obtain the Laplace equation

$$\rho\frac{\partial}{\partial\rho}\left(\rho\frac{\partial h}{\partial\rho}\right)+\frac{\partial^2 h}{\partial\theta^2}=0 \quad \text{or} \quad h''_{\rho\rho}+\frac{1}{\rho}h'_{\rho}+\frac{1}{\rho^2}h''_{\theta\theta}=0. \tag{2}$$

The first ("classic") method is as follows. The solution is written as a series [17] which we approximate by its partial sum

$$h(\rho,\theta)=\sum_{n=1}^{m}\left(\rho^n-\frac{1}{\rho^n}\right)\left(A_n\cos n\theta+B_n\sin n\theta\right)+D_0\ln\frac{1}{\rho}, \tag{3}$$

where the coefficients A_n, B_n, D_0 are found from the boundary conditions and the measurement data by the least squares method. The relationship between the polar coordinates (r, φ) and (ρ, θ) [16] allows us to obtain the final expression for a solution directly for the original domain D.

A PINN is considered the second model. Parameters of a neural network with one hidden layer $h(x,y,c_i,\mathbf{a}_i)=\sum_{i=1}^{N}c_i v(x,y,\mathbf{a}_i)$ and a radial basis function (a Gaussian function) $\exp\left(-b\left((x-d_1)^2+\right.\right.$ $\left.\left.+(y-d_2)^2\right)\right)$ are adjusted during training, namely, minimising the loss function $J = J_1 + \delta_1 J_2 + \delta_2 J_3$. Here, summands J_1, J_2, J_3 correspond to the quadratic errors of satisfying the neural network solution to the Laplace equation, boundary conditions and experimental data. The hyperparameters δ_1, $\delta_2 > 0$ reflect the contribution of the corresponding terms to the loss function after the initial random initialisation of the PINN weights. This method is described in more detail in [16].

A new approach to solving the problem of modelling the deflection of a membrane under a load is the multilayer method [12] based on the modification of classical numerical methods such as Runge–Kutta, Euler and others. This technique leads to the multilayer functional approximations of the solution of the Cauchy problem for a system of ordinary differential equations

$$\mathbf{y}'=\mathbf{f}(x,\mathbf{y}_0),$$

$$\mathbf{y}(x_0)=\mathbf{y}_0,\quad x\in R,\quad \mathbf{y}_0\in R^n. \tag{4}$$

The interval $[x_0, x]$ is divided in a certain way into n parts $x_0 < x_1 < \ldots < x_n = x$ with corresponding steps h_k. Then the recurrence formula is used n times

$$\mathbf{y}_{k+1} = \mathbf{F}\left(\mathbf{f}, h_k, x_k, \mathbf{y}_{k+1}, \mathbf{y}_k, \mathbf{y}_{k-1}, \ldots, \mathbf{y}_0\right), \tag{5}$$

stipulated by specific classical numerical methods. The final expression y_n is regarded as an analytical function $y(x)$ of an end of the interval and is called a multilayer solution of the Cauchy problem. n determines the number of layers of the model and the length of the interval where the resulting solution satisfies the known estimates for the basis classical method and is selected in accordance with the required accuracy and permissible complexity of calculations.

We return to equation (2) in polar coordinates given in the symmetric domain D_1 which is obtained from the original domain using a conformal mapping, and the unknown function of deflection $h(\rho, \theta)$.

Denote $\rho \dfrac{\partial h}{\partial \rho} = z$ and we reduce equation (3) to a normal system of differential equations with respect to variable ρ:

$$\begin{cases} \dfrac{\partial h}{\partial \rho} = \dfrac{z}{\rho}, \\ \dfrac{\partial z}{\partial \rho} = -\dfrac{1}{\rho} \dfrac{\partial^2 h}{\partial \theta^2}. \end{cases} \tag{6}$$

We apply our modification [12] of the implicit Euler method with one layer, that is, the operator F in expression (6) has the form

$$F\left(f, h, x_{k+1}, \boldsymbol{y}_k, \boldsymbol{y}_{k+1}\right) = \boldsymbol{y}_k + hf\left(x_{k+1}, \boldsymbol{y}_{k+1}\right),$$

and starting at some point selected further. We get the expressions:

$$\begin{cases} h_1 = h_0(\theta) + \dfrac{\rho - R_0}{\rho} z_1, \\ z_1 = z_0(\theta) - \dfrac{\rho - R_0}{\rho} \dfrac{\partial^2 h_1}{\partial \theta^2}. \end{cases}$$

Given that $h_0(\theta) = h_1\big|_{\rho = R_0}$, $z_0(\theta) = z_1\big|_{\rho = R_0}$, the solution takes the form

$$h_1 = h_0(\theta) + \frac{\rho - R_0}{\rho} z_0(\theta) - \left(\frac{\rho - R_0}{\rho}\right)^2 \frac{\partial^2 h_1}{\partial \theta^2}. \tag{7}$$

Let us find $z_0(\theta)$. The edge of the membrane is fixed so $h_1\big|_{\rho = R} = 0$ and from equation (7)

$$h_0(\theta) + \frac{R - R_0}{R} z_0(\theta) = 0.$$

Expressing $z_0(\theta)$ and substituting it into equation (7), we obtain

$$\left(\frac{\rho - R_0}{\rho}\right)^2 \frac{\partial^2 h_1}{\partial \theta^2} + h_1 = h_0(\theta) \frac{R_0}{\rho} \frac{R - \rho}{R - R_0}. \quad (8)$$

The functions h_1, h_0 can be written as:

$$h_1(\rho, \theta) = \sum_{n=0}^{+\infty} a_n(\rho) \cos n\theta, \quad h_0(\rho, \theta) = \sum_{n=0}^{+\infty} A_n(\rho) \cos n\theta$$

and replace the series with partial sums

$$h_1(\rho, \theta) = \sum_{n=0}^{m} a_n(\rho) \cos n\theta, \quad h_0(\rho, \theta) = \sum_{n=0}^{m} A_n(\rho) \cos n\theta \quad (9)$$

Substitute expressions (9) into equation (8) and find the coefficients

$$a_n(\rho) = \frac{A_n \alpha(\rho)}{1 - \left(\frac{\rho - R_0}{\rho}\right)^2 n^2},$$

where $\alpha(\rho) = \frac{R_0}{\rho} \frac{R - \rho}{R - R_0}$, R_0, A_n are parameters that are found for the approximations of functions by expression (9) from the measurement data by means of the least squares method. In this case, the deflection values measured in the experiment are used directly and formulas corresponding to the conformal mapping in the form

$$\rho = \rho(r, \varphi) = k \sqrt{\frac{4a^2 r^2 + 4arQ_- \cos\varphi + Q_-^2}{4a^2 r^2 + 4arQ_+ \cos\varphi + Q_+^2}} \quad \text{and} \quad \mathrm{tg}\theta = \frac{r\sqrt{\left(R^2 + a^2 - r_0^2\right)^2 - 4R^2 a^2} \sin\varphi}{a\left(R^2 + r^2\right) + r\left(R^2 + a^2 - r_0^2\right)\cos\varphi}.$$

Computation results

Using the data measured during the experiment described in the previous section two multilayer models (9) with different numbers of terms in the final sum were constructed. To estimate the obtained models and compare them with PINN model and "classical" solution, the method of cross-validation was utilised [14, 18]. A relatively small number $N = 24$ of measurements used for training and calculating model parameters allows calculating estimates for all partitions of the sample into training and test settings in the case of leave-one-out cross-validations. In addition, the ability of models to generalise is tested for the first time using the cross-validation method for a smaller volume of the training set.

We performed 40 calculation variants for each type of the model constructed using the "classical" method, neural network, and multilayer methods respectively. In experiments No. 1–24, the test sample consists of 1 test point, the remaining 23 points are a training sample; in experiments No. 25–40, training samples contain 16 randomly selected points and the remaining 8 points make up a test sample.

Thus, a total of 120 models were built, 24 "classical" analytical solutions of the form (4) among them. For each model, the unknown coefficients of approximation of the series (4) by the first summand (the sum of the first order) were calculated by the least squares method for 23 points. The maximum deviation is no more than 0.31 cm, the mean square deviation averaged over all variants for the training and test samples is 0.13 cm.

A solution was also obtained based on two terms of the partial sum of the series (5). A similar error distribution was observed in the solution as in the calculation by the first-order method. The maximum deviation was no more than 0.32 cm, the standard deviation averaged over the variants for the training and test sample points was 0.13 cm and 0.16 cm, respectively. A small increase in the standard deviation can be associated with an increase in instability in the load domain.

At the same time, the condition number of a matrix of the system for finding the coefficients of the solution (5) based on the l_2-norm, grew to 200. While for the approximation of a series by one term, it ranged from 6 to 10 for different variants of training samples.

To train 24 PINN models, the same 23 points were used as in the calculations with the "classical" method. The maximum deviation was no more than 0.22 cm, the mean square deviation averaged over all variants for the training and test sample of points was 0.03 cm and 0.07 cm. It turned out that when using PINN, the maximum deviation of the calculation results from the experimental data was about a third less, the standard deviation was 2 times less than when using the "classical" method.

For each of the 24 partitions of the measurement data, multilayer models for finite sums (9) of the first and second order were constructed for the first time. For multilayer models of the first type, the maximum deviation is no more than 0.31 cm, the calculated mean square deviation for the training and test sample of points was 0.13 cm and 0.12 cm respectively.

For the sum of the second order the maximum deviation of the multilayer model from the test samples of experimental data decreased and was 0.28 cm, the calculated mean square error for the training and test samples decreased slightly and was 0.1 cm and 0.11 cm, respectively. In general, the results of calculations by the multilayer method practically coincided in accuracy with the results obtained using the "classical" method but the solution was more stable in the sense of maximum error for individual objects.

For all models, the maximum deviations of the calculated data from the experimental data at the points of the training and test samples, $ERR_{\max}^{learn}$ and $ERR_{\max}^{test}$, the maximum standard errors at the points of the training and test samples, $MSE_{\max}^{learn}$ and $MSE_{\max}^{test}$, the standard deviations of both samples averaged over all calculations, the so-called averaged regularity criterion, $\langle MSE \rangle^{learn}$ and $\langle MSE \rangle^{test}$, are presented in Table 1. Table 2 shows the number of adjusted parameters m for each method.

Thus, the PINN model allows us to get a minimum error in all indicators for leave-one-out exhaustive cross-validation. For a profound comparison, we analyzed the results of leave-8-out cross-validation for sixteen random partitions of a set of measurement data.

Table 1

The maximum and standard deviation of the calculated values of the models on the training and test sample for leave-one-out exhaustive cross-validation, cm

Method	$ERR_{\max}^{learn}$	$ERR_{\max}^{test}$	$MSE_{\max}^{learn}$	$\langle MSE \rangle^{learn}$	$\langle MSE \rangle^{test}$
"Classical", the first order sum	0.32	0.43	0.14	0.13	0.13
"Classical", the second order sum	0.32	0.47	0.13	0.13	0.16
PINN	0.15	0.22	0.06	0.03	0.07
Multilayer, the first order sum	0.28	0.31	0.13	0.13	0.12
Multilayer, the second order sum	0.29	0.3	0.12	0.11	0.1

The results of calculations using a training set with 16 points are presented in Table 3. It contains the same estimates as Table 1. While maintaining the magnitude of estimates for training sets, the “classical” approach shows a weak ability to generalise on a test sample. A similar difference in estimates compared to the case of a larger training set is observed for a neural network model.

We can see that for a multilayer model the maximum deviations and root-mean-square estimates on both the training sample and the test sets differ little from those observed in the calculations for 23 points. Therefore, 16 points are sufficient to build the model, and the resulting solution does not depend on the sample and maintains its generalising ability.

Table 2

The number of adjusted parameters for each method

Method	m
"Classical", the first order sum	3
"Classical", the second order sum	5
PINN	48
Multilayer, the first order sum	3
Multilayer, the second order sum	4

Note that the estimates presented in Table 3 for test sets allow considering the quality of generalisation for all three models and avoiding optimistic underestimation of the error for a more complex PINN solution containing the maximum number of adjusted parameters (48 parameters in total, that is, 4 parameters with 12 neurons).

Table 3

Maximum and standard deviation in the case of the training (16 points) and test (8 points) samples, cm

Method	$ERR_{\max}^{learn}$	$ERR_{\max}^{test}$	$MSE_{\max}^{learn}$	$MSE_{\max}^{test}$	$\langle MSE\rangle^{learn}$	$\langle MSE\rangle^{test}$
"Classical", the first order sum	0.32	0.58	0.15	0.24	0.13	0.16
"Classical", the second order sum	0.35	0.98	0.14	0.44	0.12	0.24
PINN	0.15	0.43	0.05	0.24	0.02	0.1
Multilayer, the first order sum	0.3	0.44	0.14	0.21	0.12	0.16
Multilayer, the second order sum	0.29	0.4	0.13	0.2	0.11	0.15

For visual comparison of the obtained models, 3D images of some models were constructed. Fig. 2 on the left shows the dependence of the deflection of the membrane on the coordinate for one of the calculations using the "classical" method, on the right – the difference between the result of numerical calculations using the same model and experimental data. The results for a PINN model are shown in Fig. 3. It can be seen that the shape of the membrane deflection became more natural, close to the physical one observed in the experiment. An example of a solution obtained by the method of multilayer approximations is shown in Fig. 4.

Conclusion

In this paper, a new multilayer approach to solving the incorrectly posed problem of finding the analytical function of deflection of a circular membrane under the action of a load placed far enough from

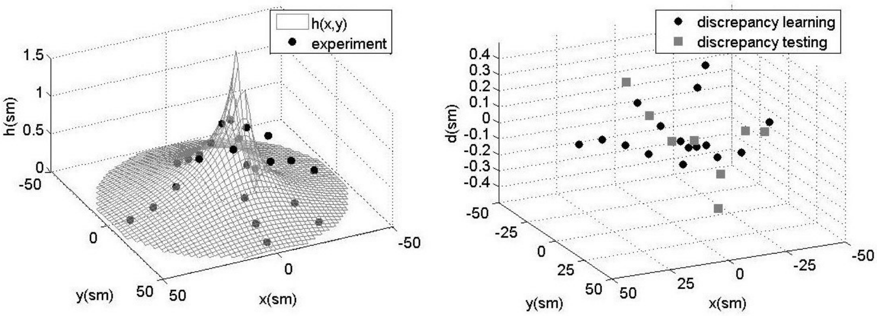

Fig. 2. The model of membrane deflection and deviation from experimental data. "Classical" method, the first order sum

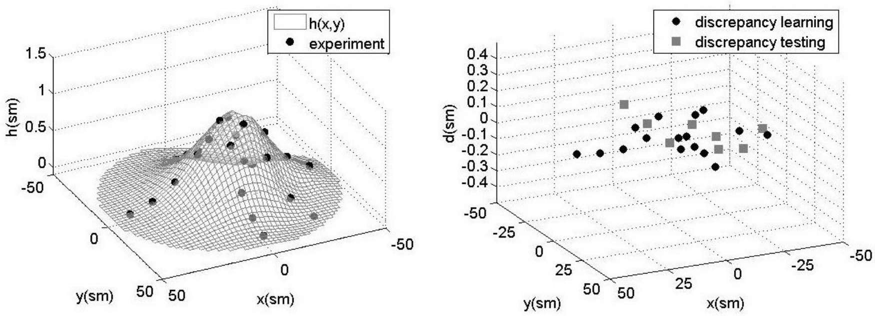

Fig. 3. The model of membrane deflection and deviation from experimental data. PINN model

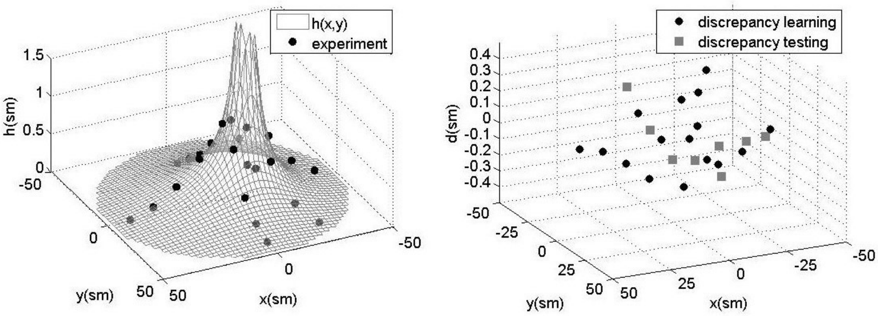

Fig. 4. The model of membrane deflection and deviation from experimental data. Multilayer method, the first order sum

its center is considered. Instead of one of the boundary conditions, it is proposed to use measurement data obtained on a full-scale experiment. An algorithmic comparison has been made using the cross-validation method of a multilayer model with the basis implicit Euler method with a physics informed neural network model and a classical approach using the representation of the solution of the Laplace equation in polar coordinates through a trigonometric series.

As a result of the application of these techniques, 120 semi-empirical mathematical models were constructed using a relatively small set of experimental data that set the amount of deflection at each point of the membrane surface. With the help of the estimates obtained during the cross-validation, the generalising abilities of all models for the problem under consideration were analysed for different volumes of training samples. Based on the conducted research, it can be concluded that PINN and multilayer methods should be applied to modelling objects when a sufficiently accurate classical description using a boundary value problem for a differential equation (a system of differential equations) is unknown or excessively difficult and additional information is available in the form of measurement results. At the same time, PINN models are more effective in a situation with many measurements, and the accuracy requirements are quite high. It is advisable to apply analytical modifications of numerical methods when there are few measurements or they arrive dynamically and a compact adaptive model is required, for example, in embedded systems.

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