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RELATIVE ORBIT AND ATTITUDE COUPLED CONTROL BASED ON SLIDING-MODE VARIABLE STRUCTURE CONTROL

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Abstract: The problem of relative orbit and attitude coupled control of spacecraft formation flying is studied. In this paper, the coupled attitude and orbit nonlinear dynamic model of six degrees of freedom (6-DOF) is proposed. Considering a variety of space perturbations and model uncertainties, power reaching law sliding-mode variable structure control method is used to develop an orbit and attitude coupling cooperative controller, also the finite time convergence is performed using Lyapunov stability method. Numerical simulations are carried out to verify the validity of the proposed controller. The results show that this controller can achieve synchronization control of the relative orbit and attitude, and it provides good control accuracy and stability.

Keywords: Formation flying; orbit and attitude control; sliding mode variable structure control

Introduction

Recently, formation flying using micro satellites appeared to be a new and promising trend. In aerospace industry, this new technology makes way for new and better applications, such as navigation, remote sensing, electronic reconnaissance, stereo imaging, and etc.

Modeling and control for relative orbit and attitude maneuvers are critical technologies in most spacecraft formation flying missions, and they have received increased attention during the last several years [1–5]. Compared with traditional formation flying control, which usually controls relative orbit and relative attitude separately, this method can take the coupled factors of the relative orbit and relative attitude into consideration. However, these studies do not consider the model uncertainties and external space perturbations and distributions, which cannot be ignored in 6-DOF formation flying control especially for future new formation flying missions where one needs to achieve high accuracy on earth observation and deep space exploration.

In this paper, we address the coupled control for the 6-DOF motion of a follower spacecraft relative to a leader spacecraft using the coupled translation and rotation dynamics. Considering several space perturbations and model uncertainties, power reaching law sliding-mode variable structure control method is employed to achieve coordinated control for formation flying.

Dynamic model

In the following analysis, we denote the leader and follower spacecraft by *l* and *f* subscript respectively. The relative orbit dynamic model can be expressed as [6]

$$\dot{v} + C_t(\dot{f})v + D_t(\dot{f}, \ddot{f}, r_f)p + n_t(r_l, r_f) = F_a + F_d \qquad (1)$$

where $C_t(\dot{f}) = 2\dot{f}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$,

$$D_t(\dot{f}, \ddot{f}, r_f) = \begin{bmatrix} \frac{\mu}{r_f^3} - \dot{f}^2 & -\ddot{f} & 0 \\ \ddot{f} & \frac{\mu}{r_f^3} - \dot{f}^2 & 0 \\ 0 & 0 & \frac{\mu}{r_f^3} \end{bmatrix}, n_t(r_l, r_f) = \mu \begin{bmatrix} \frac{r_l}{r_f^3} - \frac{1}{r_l^2} \\ 0 \\ 0 \end{bmatrix},$$

μ is constant of earth gravitation, F_a is relative orbit control force, F_d is perturbation force due to external effects, such as earth J2 perturbation etc.

The relative attitude quaternion can be expressed as

$$q = q_f \otimes q_l = \begin{bmatrix} \eta_f \eta_l + \varepsilon_f^T \varepsilon_l \\ \eta_l \varepsilon_f - \eta_f \varepsilon_l - S(\varepsilon_f)\varepsilon_l \end{bmatrix} \qquad (2)$$

and relative attitude kinematics equation can be expressed as

$$q = \begin{bmatrix} \dot{\eta} \\ \dot{\varepsilon} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -\varepsilon^T \\ \eta\mathbf{I} + \mathrm{S}(\varepsilon) \end{bmatrix} \omega \tag{3}$$

where $\omega = \omega_{lb,fb}^{fb} = \omega_{i,fb}^{fb} + A_{fl}\omega_{i,lb}^{lb}$ is relative angular velocity between the leader and follower body frame, A_{fl} is the rotation matrix describing rotations from leader spacecraft body frame to follower spacecraft body frame.

Moreover, the dynamics of relative attitude motion is

$$J_f\dot{\omega} + C_r(\omega)\omega + n_r(\omega) = \Gamma_a + \Gamma_d \tag{4}$$

Where $C_r(\omega) = J_f\mathrm{S}\left(A_{fl}\omega_{i,lb}^{lb}\right) - \mathrm{S}\left(J_f(\omega + \omega_{i,lb}^{lb})\right)$ is a skew-symmetric Coriolis-like matrix,$n_r(\omega) = \mathrm{S}\left(A_{fl}\omega_{i,lb}^{lb}\right)J_f A_{fl}\omega_{i,lb}^{lb} - J_f A_{fl} J_l^{-1}\mathrm{S}\left(\omega_{i,lb}^{lb}\right)J_l\omega_{i,lb}^{lb}$ is nonlinear term, and Γ_a, Γ_d are relative attitude control torque and perturbation torque such as gravitational torque, solar perturbation torque, atmosphere drag perturbation, respectively.

Finally, we define the state vectors $x_1 = [p \quad q]^T$, $x_2 = [v \quad \omega]^T$, so the 6-DOF coupled dynamics of formation flying is

$$\dot{x}_1 = \Lambda(x_1)x_2$$

$$M_f\dot{x}_2 = U + W - C\left(\dot{f},\omega\right)x_2 - D\left(\dot{f},\ddot{f},r_f\right)x_1 - n\left(\omega,r_l,r_f\right) \tag{5}$$

where $M_f = \mathrm{diag}([\mathrm{I},J_f])$, $C\left(\dot{f},\omega\right) = \mathrm{diag}([C_t(\dot{f}),C_r(\omega)])$, $n\left(\omega,r_l,r_f\right) = \mathrm{diag}([n_t(r_l,r_f),n_r(\omega)])$, $U = [F_a,F_d]^T$, $W = [\Gamma_a,\Gamma_d]^T$, $\Lambda(x_1) = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \frac{1}{2}\begin{bmatrix} -\varepsilon^T \\ \eta I + \mathrm{S}(\varepsilon) \end{bmatrix} \end{bmatrix}$, $D\left(\dot{f},\ddot{f},r_f\right) = \begin{bmatrix} D\left(\dot{f},\ddot{f},r_f\right) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$

Controller design

The control problem is to design a controller that makes the state x_1 converge to a time-varying smooth trajectory x_d.The desired trajectory can be specified by $x_{d1} = [p_d \quad q_d]^T$, $x_{d2} = [v_d \quad \omega_d]^T$. We define the relative translation error as $\tilde{p} = p - p_d$, the relative rotation error $\tilde{q} = \tilde{\eta} - \tilde{\varepsilon}$ is given by the quaternion product $\tilde{q} = q_d \otimes q$, thus the attitude error dynamics can be written as

$$\dot{q} = T(\tilde{q})\tilde{\omega} = \frac{1}{2}\begin{bmatrix} -\tilde{\varepsilon}^T \\ \tilde{\eta}\mathbf{I} + \mathrm{S}(\tilde{\varepsilon}) \end{bmatrix}\tilde{\omega} \tag{6}$$

Accordingly, the closed-loop system has two equilibrium points in the $(\tilde{q}, \tilde{\omega})$ space which are referred to as the same attitude orientation, namely $\tilde{q}_{\mp} = [\mp 1, \mathbf{0}]^T$. Aiming at minimizing the path length for the desired rotation, for the equilibrium points, we define the relative position and attitude errors as $e_{1\mp} = [\tilde{p}^T, 1 \mp \tilde{\eta}, \tilde{\varepsilon}^T]^T$, $e_2 = x_2 - x_{d2} = [\tilde{v}, \tilde{\omega}]^T = [\mathrm{v} - v_d, \omega - \omega_d]^T$
then

$$\dot{e}_1 = \Lambda_e(e_1)e_2, \Lambda_e(e_1) = \frac{1}{2}\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \begin{matrix} \mathrm{sgn}(\tilde{\eta})\tilde{\varepsilon}^T \\ \tilde{\eta}\mathbf{I} + \mathrm{S}(\tilde{\varepsilon}) \end{matrix} \end{bmatrix} \tilde{\omega} \quad (7)$$

Assuming that the leader spacecraft is in control, which means $\Gamma_{al}^{lb} = -\Gamma_{dl}^{lb}$ and $f_{al} = -f_{dl}$, and according to the sliding-mode control theory, we choose the sliding-mode surface function as

$$s = e_2 + c{\Lambda_e}^T(e_1)e_1 \quad (8)$$

The control method is designed as

$$U = -C(\dot{f}, \omega)x_2 + D(\dot{f}, \ddot{f}, r_f)x_1 + n(\omega, r_l, r_f) + M_f\dot{x}_2 - K_2e_2 - \\ -\lambda|s|^\beta \mathrm{sgn}(s) - K_1{\Lambda_e}^T e_1 \quad (9)$$

where c, K_1, λ is constant positive matrix, and

$$K_2 = M_f c \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \pm\frac{1}{4}(\tilde{\eta}\mathbf{I} + \mathrm{S}(\tilde{\varepsilon})) \end{bmatrix}$$

Proof

We choose the following Lyapunov function

$$V = \frac{1}{2}s^T M_f s + \frac{1}{2}{e_1}^T K_1 e_1 \quad (10)$$

The differentiation of this function along the trajectories of the system results in

$$\dot{V} = s^T M_f \dot{s} + {e_1}^T K_1 \dot{e}_1 = s^T M_f \left(\dot{e}_2 + c\left({\Lambda_e}^T(\dot{e}_1)e_1\right)\right) + \\ {e_1}^T K_1 {\Lambda_e}^T(e_1)e_1 = \\ = -s^T\lambda|s|^\beta \mathrm{sgn}(s) - {e_1}^T\Lambda_e(e_1)c^T K_1{\Lambda_e}^T(e_1)e_1 \quad (11)$$

$\lambda > 0$, $c > 0$, $K_1 > 0$, thus $\dot{V} \le 0$. We can know from (9) that there is only one equilibrium point which can make us sure that $\dot{V} = 0$ among all the point sets $E = \{(s, e)|\dot{V} = 0\}$, and this point is (0, 0), namely *ep*. According to Lasalle's invariant set theory, the system is uniformly asymptotically stable for the equilibrium point *ep*.

Simulations

In this section, numerical simulations are conducted to evaluate the performance of the proposed controller, and $p_{df} = [5\sin(\omega_0 t), 5\cos(\omega_0 t), 0]$km, which means the follower is assumed to be commanded to track around the leader in a circular orbit of radii 5 km, and $\omega_0 = 0{,}011 \mathrm{rad/s}$ means orbital angular velocity. The moment of inertia is $J_f = J_l = \mathrm{diag}([1.1, 1.1, 1.6])\mathrm{kg}m^2$, the initial attitude quaternion is $q_0 = [0.03, 0.19, 0.27, 0.9]$, the initial relative position is $p_0 = [-0.7, 1.8, 0.04]$km, the initial angular velocity is $\omega_{0f} = [0.02, 0.04, 0.05]\mathrm{rad/s}$. Furthermore, the controller parameters are $K_1 = \mathrm{diag}([0.01,\ 0.01,\ 0.01,\ 0.01,\ 0.01,\ 0.01])$, $\beta = 0.3$, $c = \mathrm{diag}([0.4,\ 0.4,\ 0.4,\ 1,\ 1,\ 1])$, $\lambda = \mathrm{diag}([0.02,\ 0.02,\ 0.02,\ 0.02,\ 0.02,\ 0.02])$. Then, simulations have been carried out under the given initial conditions and the designed control system, as shown in figures 1–4.

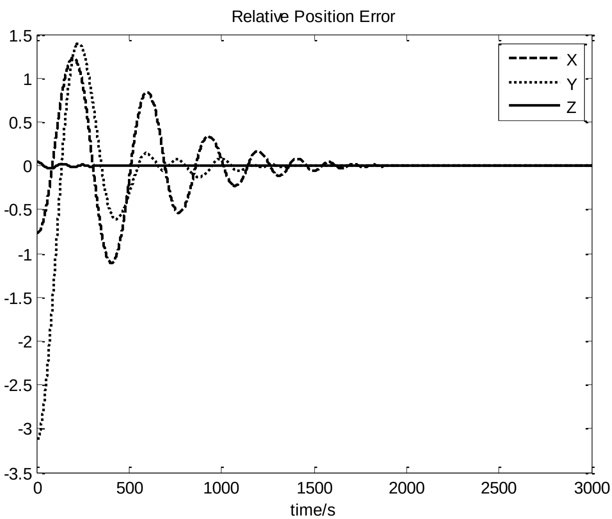

Fig.1. Relative position error.

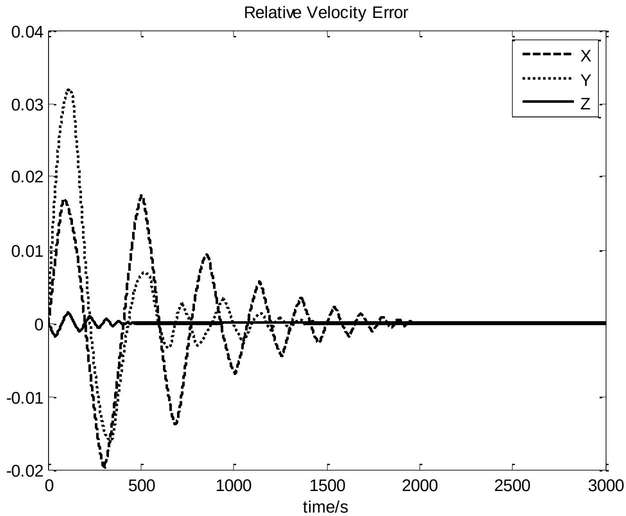

Fig.2. Relative velocity error.

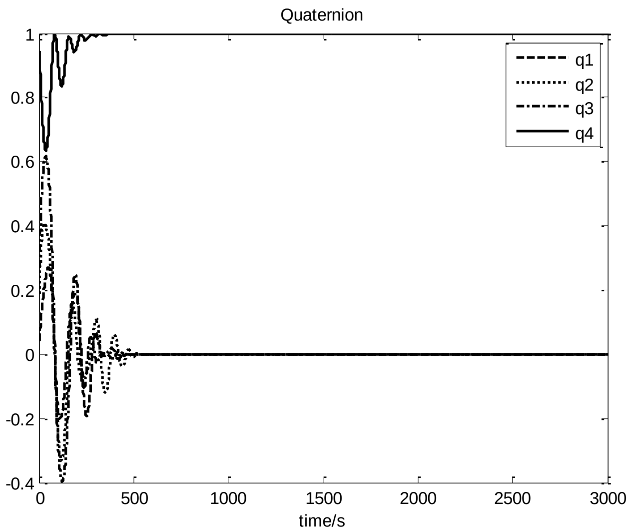

Fig.3. Relative attitude quaternion.

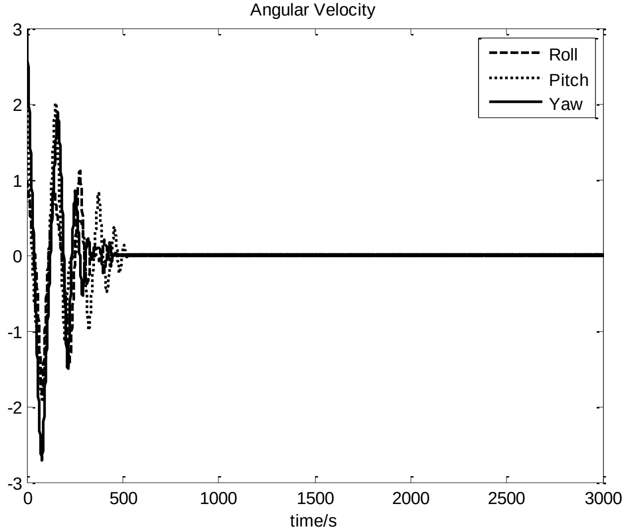

Fig.4. Relative angular velocity.

Table 1. Simulation control errors (3σ)

	Relative position Error (m)	Relative velocity Error (m/s)	Relative angular velocity error (rad/s)
X	0.013	0.003	0.012
Y	0.022	0.002	0.025
Z	0.041	0.005	0.017

The figures 1–4 show that the proposed controller is effective in maneuvering the follower spacecraft into the desired position and attitude and it can achieve the synchronized control of relative orbit and attitude. Table 1 illustrates that the control error of relative orbit and attitude is very small, which means the control system has good accuracy under model uncertainties and external disturbances.

Conclusions

A coupled 6-DOF cooperative control law is proposed on a basis of power reaching law sliding-mode variable structure control theory, in which external perturbations and model uncertainties are considered. As sliding-mode is recursively Lyapunov stable, the closed-loop system stability is achieved. Simulation results demonstrate that control system is able to converge with fine performance.

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A RELATIVE INFLUENCE OF LOW-FREQUENCY AND MICROWAVE RADIO-OPTICAL RESONANCES IN ALKALI ATOMS VAPOR

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Abstract: The paper is devoted to the study of the relative influence of low-frequency and microwave signals of the system of two quantum magnetometers based on simultaneous induction of the spin generator signal on the resonance transition and the M_Z-signal of the edge