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THE DUAL NATURE OF PROBLEMS CONCERNING ECONOMICS OF QUALITY

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ДВОЙСТВЕННЫЙ ХАРАКТЕР ПРОБЛЕМ ЭКОНОМИКИ КАЧЕСТВА

The article dwells upon two main approaches (directions) of optimization problems concerning economics of quality related, on the one hand, to maximizing quality under conditions of limited resources, on the other – minimizing quality costs by restricting the level of quality required. Mathematical interpretations of problems are presented, methods of linear and dynamic programming are considered.
ECONOMICS OF QUALITY; OPTIMIZATION PROBLEMS; MATHEMATICAL INTERPRETATION; OPTIMIZATION OF QUALITY; NATURAL SCARCITY.

Рассматриваются два основных подхода (направления) оптимизационных задач экономики качества, связанных, с одной стороны, с максимизацией качества в условиях ограниченных ресурсов, а с другой – с минимизацией затрат на качество при ограничении на уровень требуемого качества. Представлены математические интерпретации задач. Рассмотрены методы линейного и динамического программирования.
ЭКОНОМИКА КАЧЕСТВА; ОПТИМИЗАЦИОННЫЕ ЗАДАЧИ; МАТЕМАТИЧЕСКАЯ ИНТЕРПРЕТАЦИЯ; ОПТИМИЗАЦИЯ КАЧЕСТВА; ОГРАНИЧЕННОСТЬ РЕСУРСОВ.

One of the most important aspects of economics of quality is finding optimal economic decisions related to quality of various objects [1]. At the same time dual nature of problems concerning economics of quality appears, in quality sphere the necessity to determine the maximum level of quality may arise under conditions of constraints or, on the other hand – finding minimal costs related to reaching quality required.

Optimization problems of economics of quality contain, as a rule, parameters / indicators of quality of products or services as variables. At the same time, to public evaluation of the quality it is usually applied the criterion of «the positive trend» by the principle «the more quality – the better» that determines the necessity for commitment to improving indications of quality of products and services, and corresponds to the all-pervading principle of increasing social utility and quality of living.

Considerably more rarely it is applied the criterion of opposite tendency, namely, the tendency to decreasing, restricting growth or even minimizing quality, though this tendency in certain cases is well economically justified as, in a wide sense, quality limitation causes resource saving.

That kind of «duality» is inherent at the same time to many other fields of economic science, as well. Here an example of financial management of an enterprise / organization may be introduced.

In particular, it is a well-known «Markowitz problem» related to the management of portfolio investments performance. On the one hand, management may be directed to maximizing profitability of assets included in the investment portfolio, on the condition that the risk not to gain the required profit of supportable level will be retained. On the other hand, management may be aimed at minimizing portfolio risk, in particular the risk not to gain the required profit from assets included in the investment portfolio so that assets profitability level will be acceptable.

The dual nature of problems concerning economics of quality drives to differentiate two **principal directions** or two fields in economics of quality, such as:

– The 1-st direction – maximizing quality within limited resources;

– The 2-nd direction – cost minimization within limitation of quality level.

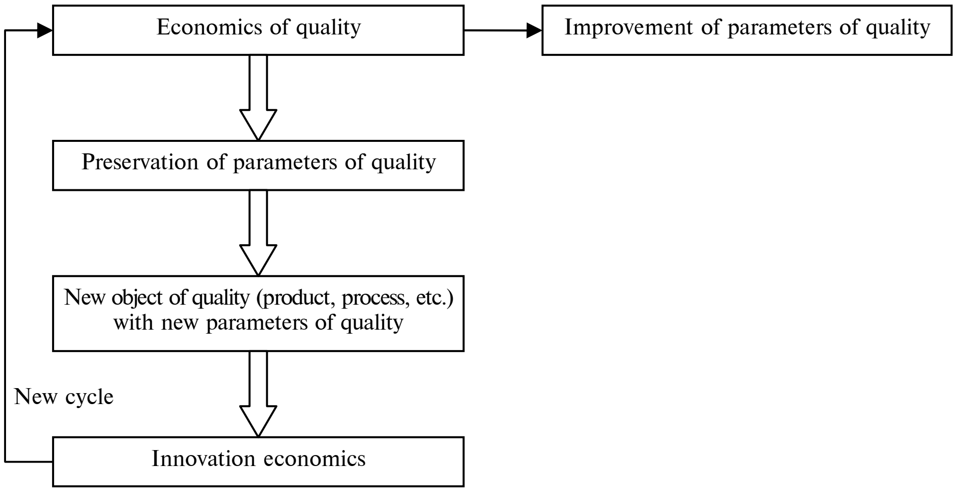

Fig. 1. Interrelation between «economics of quality» and «innovative economy»

Let us consider the content of those directions, that have different benchmarks, mechanisms of costing and results forming and mathematical interpretation.

The first direction of optimization problems of economics of quality reflects an increase (improvement) in parameters / indicators of quality of products / services corresponding to «the positive trend» of quality, according to the principle «the more quality – the better.» At the same time if there are cases when improvement of quality parameters is not necessarily connected to improvement of quality, then they are transformed to standard situation corresponding to «the positive trend».

Costs for achieving improved parameters of quality are mainly the costs of production or that production phase where improved quality is created / implemented. Those costs should not exceed fixed limits of costs. They include also reserves created for compensating the risk of «non-achievement of the required parameters of quality».

The limit of costs can be as well an object of the optimal allocation– one part of reserves can be oriented to compensate risks in the event of their implement, while the other one – to implement programs to reduce probability of «non-achievement of the required parameters of quality». The choice of optimal allocation of costs' limit, i. e. the correlation between directions of costs, can be specified as separate optimization problem of economics of quality being out of scope. The success of quality improvement programs which a propos generally represent engineering, technical, organizational programs, rather than economic content, is achieved with certain probability.

Thus the «chain» of costs / losses is formed, and a series occurs converging to a certain final value of costs / losses. If the series does not converge, then the programs mentioned above ought to be excluded from the optimal plan of probability reduction programs of «non-achievement of the required parameters of quality», they have to be replaced with other programs from «the bank of programs», which forming is not an economic problem.

It is of importance to note that when optimizing programs, aimed at improving parameters / indicators of quality of products / services, it is appeared mutual supplement of «economy of quality» and «innovation economics», as it is shown in Fig. 1. We believe here is exactly the border of the interaction between two important aspects of enterprise management, that, as one may see from the analysis provided, is not neatly defined, and hence requires roll over studies which are beyond the scope of the present research.

Mathematical formulation of the conditions of quality optimization problem of the 1-st direction may be represented by the following formulas (1):

$$\sum_{i=1}^{n} Q_{ij}(P_{ij}) \to \max; \quad \sum_{j=1}^{m} P_{ij} \le L;$$

$$P_{ij} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ Pm \end{bmatrix}, \qquad (1)$$

where m — is the index of products (services) provided, their quality and prices; n — is the index of the number of consumption subjects; Q_{ij} — is the quality of products (services) in conventional units; P_{ij} — is the prices of products (services) in rubles.

Proving the statements above, let us consider quite clear and realistic example of optimization of services' provision program with regard to quality of a travel company. Assume, that a travel agency should provide the service of hotel accommodation for a group of tourists, consisting of two customers, i. e. to provide sleep. Herewith the agency should provide the highest possible quality of services for the group and take into account the limited current budget. The selling prices of the service with regard to their quality are given in Tab. 1.

Table 1

The total of prices and quality of service

Service no.	Price of service (monetary units) P_{ii}	Quality of service Q_{ii}
1	9	3*
2	5	2*
3	3	1*
4	1	–

The quality of service (Q) is determined by the category of hotel (number of stars *), the price of the service (P) depends on it. In this example existing limit of resources $L = 10$ m. e. is spread between two tourists ($n = 2$), the number of quality parameters and, correspondingly, the number of prices (j) is fixed and equal ($m = 4$).

This optimization problem is non-linear as it contains only discrete variables, so the methods used for solving a linear optimization problem can not be applied. Variables of the problem (P) can have only specific values indicated in the problem formulation. For solutions we propose to apply the method of dynamic programming (Bellman method [3]) though for large-scale problems, specific computational procedure should be developed. The quality of service rendered by travel company in this case is defined as «summed-up quality», as the sum of «stars», if we nominally take the star level as absolute utility of each hotel. Such approach is not always possible as it is more correct to assess the quality of services per tourist, however in this case the number of tourists remains constant, so the customary indicator can be used as the target one. Although in general, quality indicators are not always additive (i. e. they can be figured up), and in those cases one has to search an independent decision.

The simplified approach to the solution, that is possible when the allocation of a limited resource is performed between two tourists, is given in Tab. 2.

Table 2

The options of a limited resource / budget allocation between two tourists

The 1-st tourist	The 2-nd tourist	The achieved summed-up quality of service («the sum of utility»)
9 m. e.	1 m. e.	3* + 0* = 3*
5 m. e.	5 m. e.	2* + 2* = 4*
3 m. e.	7 m. e.	1* + 2* = 3*
1 m. e.	9 m. e.	0 + 3* = 3*

From the Tab. 2 one can see that the optimal solution is allocating the limit of costs between tourists equally, so that the maximum quality value 4* is being achieved (an average of 2* per a tourist).

The problem is turned out to be more difficult when the same limit of resources is allocated between, for example, three tourists.

The problem can be solved by the traditional method of dynamic programming (Bellman method [2]).

The sample solutions are presented in calculation Tab. 3–5. The first step of the solution consists of allocating the limit between two tourists – the first and the second one. Each diagonal in the table corresponds to the remainder of the resource limit set for the given option of allocation. As it follows from the Tab. 3, in the case of two tourists the solution corresponds to the one presented above – the limit is spread equally between tourists (5 m. e. per each), at the same time the maximum quality value equaled to «4*» is achieved that corresponds to the previously obtained result. The Tab. 4 shows results of intermediate optimization while the Tab. 5 presents the final results of the solution for the problem of optimal allocation of resources' limit between three tourists.

Table 3

The 1-st step of allocation between tourist 1 and 2

	Tourist 1											
	–	0	1	2	3	4	5	6	7	8	9	10
Tourist 2	0	0	0	0	1	1	2	2	2	2	3	3
	1	0	0	0	1	1	2	2	2	2	3	
	2	0	0	0	1	1	2	2	2	2		
	3	1	1	1	2	2	3	3	3			
	4	1	1	1	2	2	3	3				
	5	2	2	2	3	3	4					
	6	2	2	2	3	3						
	7	2	2	2	3							
	8	2	2	3								
	9	3	3									
	10	3										

Table 4

The intermediate optimal results for tourists 1 and 2

L	10	9	8	7	6	5	4	3	2	1
Q	4	3	3	2	2	1	1	0	0	0

Table 5

The final step of the algorithm

	Tourist 1 + 2											
		0	1	2	3	4	5	6	7	8	9	10
Tourist 3	0	0	0	1	1	1	1	2	2	2	3	4
	1	0			0	1	1	2	2	3		
	2			0	0	2	1	2	2	3		
	3		1	1	1	2	2	3	3			
	4	1	1	1	1	2	2	3				
	5	2	2	2	2	3	3					
	6	2	2	2	2	3						
	7	2	2	2	2							
	8	2	2	2								
	9	3	3									
	10	3										

The solution result is found by the rule determined above – in reverse order towards the course of the solution process. For example according to the Tab. 5, when allocating all limit of resources, the maximum gained value of quality is 3* (the maximum value on the longest diagonal of the table). There are few of those values, so any of them can be chosen. Let us choose the value that corresponds to allocation of 2 resource units to tourist 3 and 8 resource units to tourists 1 + 2, those are distributed among the tourists 1 and 2 in a ratio of 5 resource units to tourist 1 and 3 resource units to tourist 2- that is exactly the optimum allocation providing the maximum indication of quality – 3*.

The second direction of optimization problems concerning economics of quality operates the criterion of minimum costs for increase (improvement) of quality (within the limits of costs) while observing minimum requirements for quality.

In that case, the optimal plan for improving quality will be that of minimum cost for its implementation while observing the required guarantees of quality. This approach can be applied both for products and services.

Mathematical interpretation of the problems of economics of quality related to the second direction may be represented by the expression (2), wherein the choice of product (service) is made in a general way.

Price / quality (*the quality parameters of the product*) *→ minimum* (2)
under the constraint:
the quality parameter of the product ≥
≥ the permissible value of the parameter
for all quality parameters.

Under real conditions, the formulation and solution of the problems above is necessary, for example, when the state order is performed according to which the services are performed for the state. In terms of studying economics of quality, those services can be characterized by parameters (indicators) of price and quality. In that case, there is also objective conformity that the price (as of others) of services is a function of the parameters of its quality by the principle «the better quality of the service is – the more expensive the service is». With forming the plan / portfolio of services within the state order, the requirement to minimize the cost / price of services while observing the required guarantees of their quality is well economically justified.

In analytical way the expression (2) can be simplified and represented as the formulas (3):

$$Z(X_1 ... X_n) \to \min \\ X_i \geq \bar{X}_i, \ \ i = 1...n \tag{3}$$

or in the matrix way (4):

$$Z \cdot X \to \min; \\ X \geq \bar{X}, \tag{4}$$

where $Z(X_1...X_n)$ is function of the cost (price) of the product (service) depending on parameters of its quality; $X_1...X_n$ – quality parameters of the product (service).

The economic substantial formulation of the problem of product quality optimization is as follows: «it is necessary to determine at what minimum cost (price) of the service the required level of parameters that define its quality is achieved». Thus, the criterion for selection of the product (service) from a number of possible alternatives is the minimum of price while observing the quality parameters required.

The optimization task (3), (4) can be considered as the linear one, if we assume that the function of the product (service) cost is of linear nature. In that case, the analytical form of the minimization problem can be represented as follows (5):

$$Z(X_1 ... X_n) = \sum_{i=1}^{n} Z_i X_i \to \min; \\ X_i \geq \bar{X}_i, \ \ i = 1...n. \tag{5}$$

In the formulation above we have to deal with the linear optimization task, its solution can be practically obtained by any of the accepted analytical or approximation methods.

The linear formulation (5) also provides the opportunity for analytical conclusions by means of constructing the inverse (dual) problems [4]. For the considered direct problem of the optimal plan by the level of quality, the problem of determining the optimal quality of the product parameters will be dual in regard to the direct problem. The economic meaning of the dual variable (y) can be characterized as «the price of quality unit» for each quality indicator. The dual problem of optimization implies maximizing the volume of prices related to the quality provided products (services) in totality under condition that the price of obtaining the unit of quality parameter will not exceed the cost of the unit of the quality parameter (indicated as z) – making sense limit of resources.

The analytical form of the dual problem is as follows (6):

$$\sum_{i=1}^{n} y_i \bar{X}_i \to \max; \\ y_i \leq z_i, \ \ i = 1...n. \tag{6}$$

When analyzing the formulation (6) and (1) one may come to the conclusion that problems of economics of quality by two directions are basically interrelated and provide the volume of the optimal level of quality within limited resources.

Main conclusions of the present paper developing theoretical foundation for economics of quality are as follows:

1. In economics of quality when making optimization one may distinguish between two types of optimization problems: either to maximize the value (utility) from the quality under the conditions of constrained resources or minimize costs with limitations for the required level of quality;

2. The problems of economics of quality by two directions are interrelated and provide the solution for finding the optimal level of quality, however for practical purposes one has to apply various formulations of direct problems in each specific case;

3. Generally, the formulation of problems of economic optimization of the quality of products / services in a linear way is based on assumptions being far from reality due to the fact that virtually problems by their economic meaning are integral-valued, so it is more preferable to apply methods of dynamic programming.

The practical implementation of the approaches mentioned is possible when finding the optimal plan for the production of products or provision of services of varying quality.

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