Вопросы, затронутые в настоящей работе, а также другие, относящиеся к теории подобия и размерности, достаточно полно отражены в указанных ниже книгах. В частности, большое число конкретных примеров использования этой теории в задачах механики, тепло- и электроэнергетики приведено в [3, 5, 6].

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НАХОЖДЕНИЕ РАВНОВЕСИЯ ПО НЭШУ ДЛЯ ОДНОЙ АКТУАРНОЙ ЗАДАЧИ

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Аннотация. В данной статье рассматривается подход к решению линейных теоретико-игровых моделей на примере процесса взаимодействия страховых компаний по линии бизнеса ОСАГО. В качестве принципа оптимальности в игре используется локальное равновесие по Нэшу, для определения которого задача сводится к задаче линейного программирования, оптимальный план для которой находится с помощью адаптивного метода Р. Габасова. Рассмотренный алгоритм реализован в среде MATLAB, для численной реализации использованы статистические данные.

Ключевые слова: оптимальное управление, адаптивный метод, метод Р. Габасова, равновесие по Нэшу, актуарная математика, численные методы, теория игр.

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NASH EQUILIBRIUM POINT IN ONE ACTUARIAL PROBLEM

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Abstract. In this paper, we construct a non-cooperative game-theoretic model describing the interaction process of insurance companies in the CMTPL business line. As an optimality principle in the game, we use a local Nash equilibrium [1], to define Nash equilibrium the method of reduction to the linear programming problem and the adaptive method by R. Gabasov [2] are used. The corresponding algorithm is implemented in MATLAB, statistical data is used for numerical simulation.

Keywords: optimal control, adaptive method, Gabasov's approach, Nash equilibrium, actuarial mathematics, numerical methods, game theory.

Introduction

In this paper, we discuss an approach to solving linear game-theoretic problems using the adaptive method. The considered approach is based on reducing the game-theoretic problem to two related linear programming problems, the optimal plan is constructed using the adaptive method. Let us consider the mathematical formulation of the problem.

1. Problem statement

We consider a non-cooperative game with two players in normal form. The strategy for the first player is $x \in R^n$, for the second player is $y \in R^m$. The payoff functions for the first player and for the second are

$$f_1(x,y) = c_1^1 x + c_2^1 y\,,\ f_2(x,y) = c_1^2 x + c_2^2 y$$

consequently.

The set of strategies for the first player is

$$X(y) = \left\{ x \in R^n : \mathrm{A}x = \mathrm{d} - \mathrm{B}y, \mathrm{l}_{1*} \le x \le \mathrm{l}_1^* \right\},$$

for the second player is

$$Y(x) = \left\{ y \in R^m : \mathrm{B}y = \mathrm{d} - \mathrm{A}x, \mathrm{l}_{2*} \le y \le \mathrm{l}_2^* \right\}.$$

The pair (x^*, y^*), $x^* \in X(y)$, $y^* \in Y(x)$ is a local Nash equilibrium if the

$$f_1(x^*, y^*) = \max_{x \in X(y^*)} f_1(x, y^*)\,,\ f_2(x^*, y^*) = \max_{y \in Y(x^*)} f_2(x^*, y)\,.$$

following equations are true.

Also, we can consider this game-theoretic problem like two linear programming problems.

Let the vectors be such that $y = y(x) \in Y(x)$ and $x = x(y) \in X(y)$, then we get following system

$$c_1^2 x + c_2^2 y(x) = \max_{y \in Y(x)} (c_1^2 x + c_2^2 y).$$

$$c_1^1 x(y) + c_2^1 y = \max_{x \in X(y)} (c_1^1 x + c_2^1 y).$$

The vectors $y^* = y^*(x^*)$, $x^* = x^*(y^*)$ are local Nash equilibrium if the if the following equations are true

$$f_1(x^*, y^*) = \max f_1(x, y(x)), x \in X(y(x)),$$

$$f_2(x^*, y^*) = \max f_2(x(y), y), y \in Y(x(y)).$$

For solving interval linear programming problem, we use the Adaptive method, which has many advantages over the classical simplex-method. We consider the details of the algorithm and features of the adaptive method by R. Gabasov [2].

1.1. Adaptive method

To find the optimal plan in linear programming problems, the simplex-method is usually used, but in this paper, we will consider an alternative method.

The adaptive method does not require the introduction of new variables and an increase of the dimension of the problem, besides the adaptive method can use any points from a set of plans, not only vertices. The algorithm of the adaptive method at each iteration the direction and length of the step along this direction are selected according to certain rules.

We use the direct algorithm of the adaptive method [2] for solving the interval linear programming problem

$$F^T U \to \max_U,$$
$$T_* \le KU \le T^*, \qquad (1)$$
$$U_* \le U \le U^*.$$

Where F, U, U_*, U^* are n-dimension vectors, T_*, T^* are m-dimension vectors, K is m*n–dimension matrix.

The adaptive method algorithm consists of several steps.

0 step First of all, you need to choose the initial plan U and support $K_0 = (I_0, J_0)$.

Let $I_n = I \setminus I_0$ and $J_n = J \setminus J_0$.

1 step Calculation of potential vectors ξ and estimates Δ:

$$\xi^T(I_0) = F^T(J_0) K^{-1}(J_0, I_0), \quad \xi(I_n) = 0, \qquad (2)$$

$$\Delta(J_n) = F(J_n) - K^T(I_0, J_n)\xi(I_0), \qquad \Delta(J_0) = 0. \qquad (3)$$

$$U_j = \begin{cases} U_*, \text{if } \Delta(\text{j})<0; \\ U^*, \text{if } \Delta(\text{j})>0; \\ \in \left[U_*, U^*\right], \text{if } \Delta(\text{j})=0, \text{ j} \in \text{J}_n; \end{cases} \tag{4}$$

$$K(i,J)U = \begin{cases} T_*, \text{if } \xi(\text{j})<0; \\ T^*, \text{if } \xi(\text{j})>0; \\ \in \left[T_*, T^*\right], \text{if } \xi(\text{j})=0, \text{ i} \in \text{I}_0. \end{cases} \tag{5}$$

2 step Verification of the optimality criterion:

In the case of fulfilling the criterion of optimality of execution, the desired solution is found and the algorithm should be completed. If it is not fulfilled, then you need to write out the index $i_0 \in I_0$ or $j_0 \in J_0$, which the optimality criterion is violated and go to the next step.

3 step Construction of the direction of the plan change:

If at **step 2** the optimality criterion (4) – (5) is violated for some index j_0, then we assume:

$$l(j_0) = \text{sgn}(\Delta(j_0)), \;\; l(J_n \setminus j_0) = 0,$$
$$l(J_0) = -K^{-1}(J_0, I_0)K(I_0, j_0)l(j_0).$$

If at **step 2** the optimality criterion (4) – (5) is violated at some index i_0, then we assume:

$$l(J_n) = 0, \quad \psi(i_0) = \text{sgn}(\xi(i_0)), \quad \psi(I_0 \setminus i_0) = 0,$$
$$l(J_0) = -K^{-1}(J_0, I_0)\psi(I_0).$$

4 step Calculation of the maximum step along the direction:

If at **step 2** the optimality criterion (4) – (5) is violated for some index j_0, then $\lambda(i_0) = \infty$ and if it is violated with the index i_0, then $\lambda(j_0) = \infty$.

$$\lambda^0 = \min(\lambda(k), k \in J_0 \cup j_0 \cup I_n \cup i_0),$$

$$\lambda(k) = \begin{cases} (U^*(k) - U(k))/l(k), \text{if } l(\text{k})>0; \\ (U_*(k) - U(k))/l(k), \text{if } l(\text{k})<0; \\ \infty, \text{if } l(\text{k})=0, \text{ k} \in \text{J}_0 \cup \text{j}_0; \end{cases}$$

$$\lambda(k) = \begin{cases} (\omega^*(k) - K(k,J)U)/K(k,J)l, \text{if } K(k,J)l>0; \\ (\omega_*(k) - K(k,J)U)/K(k,J)l, \text{if } K(k,J)l<0; \\ \infty, \text{if } K(k,J)l=0, \; k \in I_n \cup i_0. \end{cases}$$

If $\lambda^0 = \infty$, then the objective function is unlimited and the algorithm needs to be completed.

5 step Calculation of a new plan:

$$\widehat{U} = U + \lambda^0 l.$$

6 step Support replacement:

If $\lambda^0 = \lambda(k^*)$, where $k \in J_0 \cup I_n$, then the support needs to be replaced.

One of the four cases is possible:

1) If on step 2 was index $j_0 \in J_n$, $k^* = j^* \in J_0$, then

$$\widehat{I_0} = I_0, \quad \widehat{J_0} = (J_0 \setminus j^*) \cup j_0.$$

2) If on step 2 was index $j_0 \in J_n$, $k^* = i^* \in I_n$, then

$$\widehat{I_0} = I_0 \cup i_0, \quad \widehat{J_0} = J_0 \cup j_0.$$

3) If on step 2 was index $i_0 \in I_0$, $k^* = j^* \in J_0$, then

$$\widehat{I_0} = (I_0 \setminus i_0) \cup i^*, \ \widehat{J_0} = J_0.$$

4) If on step 2 was index $i_0 \in I_0$, $k^* = i^* \in I_n$, then

$$\widehat{I_0} = I_0 \setminus i_0, \quad \widehat{J_0} = J_0 \setminus j^*.$$

go to **1 step**.

2. Actuarial Problem

Motor third party vehicle insurance is compulsory insurance and it covers civil legal liability for damage caused to the third party by driving the motor vehicle, which means that this will not be paid by a driver, but by the insurance company which issued the policy. In Russia, it's called CMTPL (rus. «ОСАГО») – Compulsory Motor Third Party Liability.

CMTPL insurance allows the shifting of financial responsibility for the property damage caused by you or to the health of the person to the insurance company. In CMTPL offended policyholder can apply to his own insurance company to settle the loss. After that, Company pays for policyholder his loss and then sends this loss to the company of guilty policyholder through Clearing house to receive compensation.

The process of exchanging the requirements evolves in the following way: if an insured event occurs with two participants, the victim can apply not only to the insurance company where he is a client, in this case, the insurance company, which the victim turned directly, pays the losses to the victim. Later on, the insurance company should demand compensation from the company of the culprit.

Russian insurance market uses the Belgian system [3] for the process of exchange the requirements. During the settlement session (one week), insurance companies exchange requirements, and based on the settlement session results, the average is calculated and all claims are refunded according to it. Since the requirements have different sums, the calculating process of the average sum of all session requirements can be adjusted to obtain a large profit during the session. Let us consider more detail about the process of exchange the requirements between insurance companies in the market.

2.1. Belgian system

Direct compensation for losses called “Belgian system” and is regulated by the RAMI (Russian Association of Motor Insurers).

Clearing session – a period of time when insurance companies submit losses to the clearing center (equal to one calendar week).

The process of regulating requirements on the market occurs according to the following rules:

• During one clearing session (week), all insurance companies send requirements to each other;

• After the end of the week, RAMI divides all requirements into groups according to the region of the owner and the type of vehicle (30 groups);

• In each group, the average value is calculated (Fix values);

• On Tuesday, the average (Fix value) is paid back for each submitted claim.

This system has following problems: for many small companies their sum of losses much bigger, then average on the market, that is why these companies can’t get all compensation and have bad financial result, besides all insurance companies in Russia are obliged to insure customers coming from the RAMI system who have a negative insurance history and bring big losses, these losses almost always give a negative financial result. This article discusses the mathematical approach to the process of setting requirements, to improve the financial result of the companies.

2.2. Mathematical problem statement

Let us consider a game-theoretic model describing the process of direct compensation of losses between insurance companies. Players are the insurance companies that make decisions on claiming losses during one settlement session; therefore, we consider a static game. For simplicity, we consider one session and two insurance companies or static two-player game.

Let us consider the problem as a problem of game theory with two players. Company A is the first player and company B is the second player.

Every company has a set of requirements:

$$N = \left\{ \xi_i : 0 \le \xi_i \le 400000, i = \overline{1, n} \right\},$$

$$M = \left\{ \eta_j : 0 \le \eta_j \le 400000, j = \overline{1, m} \right\}.$$

Players’ strategy sets are finite sets of vectors where the components are the requirements that are sent by the insurance companies:

$$X = \left\{ x = (x_1, ..., x_i, ..., x_{n1})^T, d_{1*} \le x \le d_1^*, x_i \in N, n_1 \in \overline{0, n} \right\},$$

$$Y = \left\{ y = (y_1, ..., y_j, ..., y_{m1})^T, d_{2*} \le y \le d_2^*, y_j \in M, m_1 \in \overline{0, m} \right\},$$

where x_i, y_j are the requirements of companies A and B respectively.

Payoff functions of players have the form

$$f_1(x,y)=\sum_{i=1}^{n_1}(p-x_i),\ f_2(x,y)=\sum_{j=1}^{m_1}(p-y_j),$$

where $p=\frac{1}{n_1+m_1}(\sum_{i=1}^{n_1}x_i+\sum_{j=1}^{m_1}y_j)$ is the average value of all claims requirements.

2.3. Adaptive method for actuarial problem

The payoff functions of players are linear.

$$f_1(x,y)=c_1^1x+c_2^1y,\ f_2(x,y)=c_1^2x+c_2^2y,$$

where

$$c_1^1=-\frac{m_1}{n_1+m_1}e_{n_1},\qquad c_2^1=\frac{n_1}{n_1+m_1}e_{m_1},$$

$$c_1^2=\frac{m_1}{n_1+m_1}e_{n_1},\qquad c_2^2=-\frac{n_1}{n_1+m_1}e_{m_1},$$

and e_t is unit vector *1*t*.

Vector (x_i,y_j) is Nash equilibrium point if the following equations

$$f_1(x^*,y^*)=\max_{x\in X}f_1(x,y^*),\qquad f_2(x^*,y^*)=\max_{y\in Y}f_2(x^*,y)$$

are true.

We consider the actuarial game-theoretic problem as two linear programming problems.

If $x\in X$ is some given vector, then $y=y(x)$ is the optimal plan of the second player such that

$$c_1^2x+c_2^2y(x)=\max_{y\in Y}(c_1^2x+c_2^2y),\tag{6}$$

By analogy $y\in Y$ and $x=x(y)$ is the optimal plan of the first player

$$c_1^1x(y)+c_2^1y=\max_{x\in X}(c_1^1x+c_2^1y).\tag{7}$$

The vectors $x^*\in X$, $y^*=y^*(x^*)$, $x^*=x^*(y^*)$, $y^*\in Y$ is Nash equilibrium points, if the following equations are true

$$f_1(x^*,y^*)=\max f_1(x,y(x)),x\in X,\tag{8}$$

$$f_2(x^*,y^*)=\max f_2(x(y),y),y\in Y.\tag{9}$$

To find the equilibrium point we should solve one of the linear programming problems (6) – (7), find $y(x)\in Y$ or $x(y)\in X$, and then solve the corresponding problem (8) – (9).

2.4. Algorithm

Let us consider the steps of the algorithm of the adaptive method

0 Step Set initial value x

1 Step Find Δ_x from the auxiliary system

$$c_1^1 \Delta x \to \max,$$
$$d_{1*} - x \le \Delta x \le d_1^* - x. \quad (10)$$

2 Step Find $\overline{y}$

If $c_2^1 \ge 0,$ then $y = d_{2*}$, else $y = d_2^*$.

3 Step

If $\Delta x \ne 0,$ then $\overline{x} = x + \Delta x \to$ **1 Step**

If $\Delta x = 0,$ then $\overline{x} = x,\ \overline{y} = y$.

3. Numerical implementation

For a numerical experiment, consider a few examples with the real statistical data.

3.1. First example

Suppose that companies send only one requirement, then $n_1 = 1$, $m_1 = 1$.

Sets of requirements (in thous. rub.) are

$$N = \{100, 200, 400\},\ \ M = \{50, 100\}.$$

Let's construct a payoff matrix (Table 1).

Nash equilibrium is $(x_1, y_1): \overline{\upsilon} = \underline{\upsilon} = (-25, 25)$.

Table 1

The payoff matrix for the first example

	y_1	y_2
x_1	**-25\25**	0\0
x_2	-75\75	-50\50
x_3	-175\175	-150\150

By the Adaptive method initial $x = d_1^*$. The adaptive method finds the equilibrium point in 2 steps.

$$\Delta x(1) = -300,\ \Delta x(2) = 0.$$

The Nash equilibrium point is (x_1, y_1), the same as in the payoff matrix (see Tab.1).

3.2. Second example

Suppose that companies send two requirements, then $n_1 = 2$, $m_1 = 2$.

Sets of requirements (in thous. rub.) are

$$N = \{50, 200, 400, 400\},\ M = \{100, 250, 400\}.$$

Then we get the following vectors

$$x_1 = (50, 200)^T,\ x_2 = (50, 200)^T,\ x_3 = (200, 400)^T,\ x_4 = (400, 400)^T,$$

$$y_1 = (100, 250)^T,\ y_2 = (100, 400)^T,\ y_3 = (250, 400)^T,$$

$$d_{1*} = (50, 200)^T,\ d_1^* = (400, 400)^T,\ d_{2*} = (100, 250)^T,\ d_2^* = (250, 400)^T.$$

Let us construct a payoff matrix (Fig. 1).

Nash equilibrium is (x_1, y_1): $\overline{\upsilon} = \underline{\upsilon} = (50, -50)$

By the Adaptive method initial $x = d_{1*}$. The Nash equilibrium point by the adaptive method is (x_1, y_1), which corresponds to the payoff matrix (see Tab. 2).

3.3. Third example

Suppose that n = 500, m = 600, n_1 = 120, m_1 = 150.

Nash equilibrium by the Adaptive method is (x_1, y_1) (see Tab. 3).

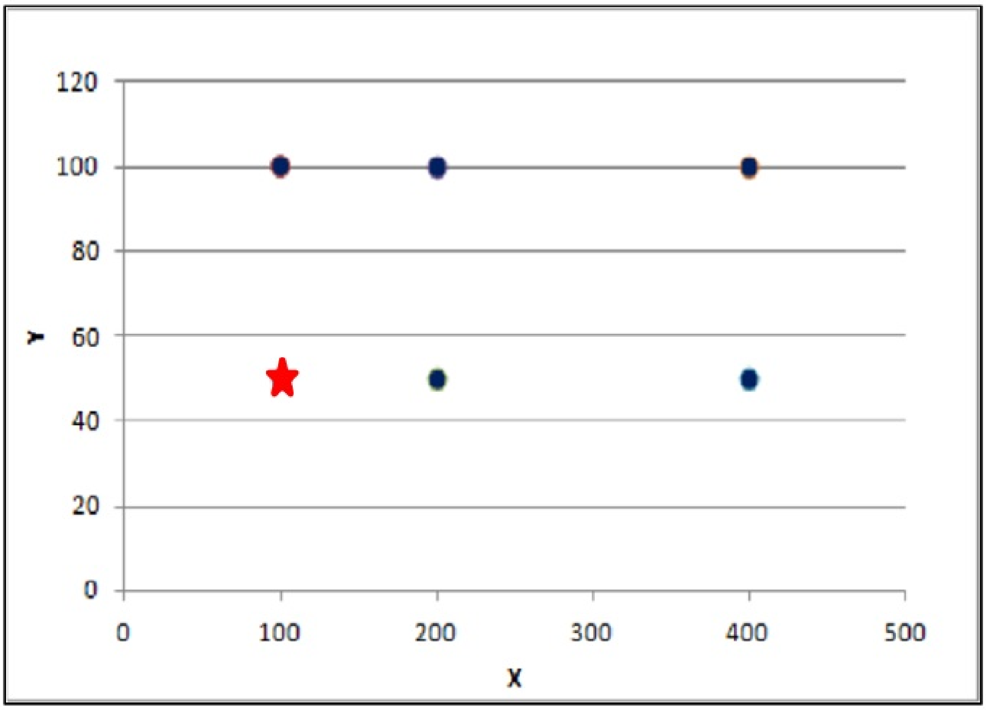

Fig 1. The strategies for the first example

Conclusion

Using the actuarial problem as an example, we considered the use of the adaptive method for finding the local Nash equilibrium for linear game-theoretic models. To implement the considered algorithm, a software package was developed in the MATLAB, several numerical examples of different dimensions were considered, and for each example, the obtained equilibrium points are given. The results obtained in this paper reflect the results obtained using expert evaluations, but in real life, the model has a larger number of players and a more complex set of parameters. Future studies will continue to expand this approach to the case of a game of *n*-players and dynamics games.

Table 2

The payoff matrix for the second example

	y_1	y_2	y_3
x_1	**50\-50**	125\-125	200\-200
x_2	-50\50	25\-25	100\-100
x_3	-125\125	-50\50	-25\25
x_4	-225\225	-150\150	-75\75

Table 3

The optimal strategies for the third example

X	1	1	2	5	6	7	7	8	8	9	12	13	14	14	14	15	15	16	17	18
Y	0	1	1	2	2	2	3	3	3	4	4	4	4	7	9	10	10	10	10	10
X	18	18	18	19	20	20	21	22	22	23	23	24	25	25	26	29	29	30	30	32
Y	11	12	13	13	15	16	16	17	17	17	18	19	19	20	20	22	23	25	26	26
X	33	33	36	36	37	38	39	39	39	40	41	42	42	42	44	44	45	50	50	51
Y	27	28	30	31	31	32	33	33	33	33	34	35	35	36	36	37	38	38	38	39
X	51	52	52	54	54	59	59	60	61	61	63	64	64	65	66	69	69	70	71	72
Y	40	40	41	42	43	45	46	46	47	47	48	49	51	51	52	52	53	54	54	54
X	73	74	74	76	76	77	78	78	80	81	82	83	84	84	86	86	86	87	88	89
Y	55	55	56	56	57	57	59	61	61	62	63	64	64	64	65	66	67	68	68	68
X	90	91	91	91	91	92	92	92	93	93	94	95	96	97	97	97	98	99	100	101
Y	69	69	69	69	69	70	70	70	71	72	74	74	74	74	75	76	76	76	77	78
X																				
Y	78	79	79	80	80	82	83	83	83	83	84	86	87	87	87	88	88	91	92	92
X																				
Y	92	93	93	94	94	95	95	96	97	98										

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ОПРЕДЕЛЕНИЕ ОТНОСИТЕЛЬНОЙ ДОЛИ СВОБОДНОГО ГАЗА В ПОТОКЕ ПРОДУКЦИИ НЕФТЯНЫХ СКВАЖИН

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Аннотация. В нефтяной промышленности, и особенно в нефтяных скважинах, необходимо иметь возможность проводить непрерывные производственные замеры, чтобы управлять скважиной наилучшим образом. В данной работе представлен анализ данных для бесконтактного метода измерения газовой фракции в двухфазных нефтяных потоках продукции скважин на основе прямого и рассеянного гамма-излучения. Метод основан на классификации интервалов наблюдения двухфазного потока с использованием статистических оценок. Особенность оценок информативности получена с использованием расхождения Кульбака-Лейблера, которая выявила влияние активности источника излучения, эффективности детектора, а также роли рассеянного гамма-излучения для выявления моментов отсутствия свободного газа в эксплуатационном потоке, что существенно влияет на измерения. Кроме того, приведена формула для оценки газовой фракции, присутствующей в нефтегазовом потоке.