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ANALYTICAL MODELLING OF LUBRICANT LAYER TRIBOSYSTEMS USING THE METHOD OF DIMENSIONS

Abstract

This study proposes various analytical models for estimating the wear intensity of a friction pair operating in a lubricant medium functionalized with a nanoscale powder additive. The analytical expressions are received using the method of dimensions. It is shown that applying this method allows for the consideration of basic dependencies for estimating the influence of several friction parameters.

Keywords: analytical model, method of dimensions, surfactant, wear.

Introduction

Optimization of lubricants for tribological systems and parameters such as coefficient of friction and wear are important in many industrial applications, ranging from industrial and automotive applications to biomedical devices.

Despite advances in tribological materials science and lubricant development technology, challenges remain in accurately predicting and controlling the magnitude of wear in complex tribological systems. Our study is an attempt to approach the construction of basic dependencies for tribological systems based on dimensional method approaches.

The main purpose of this study is to develop analytical models to estimate the wear rate of friction pairs operating in lubricating media containing nanoscale powder additives.

Several authors have successfully developed mathematical models to describe both specific instances of tribosystem operation and generalized theoretical frameworks [1-9]. Modeling a tribosystem becomes even more challenging when it includes a lubricating layer that contains functional components such as additives, thickeners, and additives [10-12]. Real lubricants typically have a heterogeneous structure and are multi-component systems. Numerous studies have reviewed existing models for tribology, proposed analytical models to describe additional friction-related parameters, such as acoustic emissions. [13-15].

Tribological phenomena are challenging to describe using formal physical and mathematical laws due to their multifactorial nature. One way to address this issue is by applying the method of dimensions.

The basis for applying the method of dimensions in our problem is the statement that the wear of friction surfaces (in its stationary state) can be represented as a set of independent variables.

In our study, we apply two fundamental theorems of dimensional analysis: the theorem on the dimensionality of quantities in the system of basic dimensions of mechanics and Buckingham's theorem on finding the number of dimensionless complexes.

Materials and methods

To estimate the wear rate of a friction pair, we will use two theorems of the method: the dimensionality theorem and Buckingham's π-theorem. Application of the first theorem requires constructing a list of quantities that can affect wear and expressing these quantities in a system of basic dimensions.

Buckingham's theorem states that if there is a physically meaningful equation involving a certain number n of physical variables, then this equation can be rewritten in terms of a set of p=n-k dimensionless parameters constructed from the original variables.

Results and discussion

Let us define the basic (fundamental) dimensions: Mass [M], Length [L], Time [T]. Let us express the variables in terms of fundamental dimensions. We will write each variable in terms of its dimension.

Table 1 – Dimensions of basic variables

Wear intensity **W**: [L]3[T]-1.	Hardness **H**: [M][L]-1[T]-2
Load **F**: [M][L][T]-2	Modulus of elasticity: **E**: [M][L]−1[T]−2
Velocity **V**: [L][T]-1	Cylinder diameter: [L]
Viscosity of lubricant **μ**: [M][L]-1[T]-1	Surfactant concentration: **Cs** [L]-3
Density of lubricant **ρ**: [M][L]-3	Nanoparticles concentration **Cn**: [L]-3

The basic analytical wear and tear model may look as follows:

$$W = \frac{FV}{H} f\left(\frac{\rho V^2}{E}, \frac{FH}{\mu^2 V^2}, \frac{H}{E}, \frac{F}{E\ \ D^2}, C_s D^3, \frac{C_s}{C_n}\right) \tag{1}$$

Let us include a function f in equation (1), the nature of which should be determined experimentally (linear, quadratic, etc.).

One or another variant of the criterion dependence is chosen depending on the the target objective of the research: e.g. minimizing friction or wear, minimizing friction or wear, increasing scuff resistance, etc. This approach allows with the help of relatively simple models to carry out optimization studies of complex tribosystems, containing multicomponent lubricating media.

Considering the hypothesis that the wear rate is independent of the viscosity of the lubricating medium, the viscosity is included in a single dimensionless complex. Since we hypothesize that the wear rate W is independent of the viscosity of the medium, this complex is automatically excluded from the general relation. The functional dependence in this case takes the form:

$$W = \frac{FV}{H} f\left(\frac{\rho V^2}{E}, \frac{H}{E}, \frac{F}{E \cdot D^2}, C_s\ D^3, \frac{C_s}{C_n}\right) \tag{2}$$

Some sources (including the Archard model) state that the wear rate is inversely proportional to the hardness of materials. In equation (2) this dependence is already present in the first factor. Therefore, the H/E complex should be removed from the function:

$$W = \frac{FV}{H} f\left(\frac{\rho V^2}{E}, \frac{F}{ED^2}, C_s\ D^3, \frac{C_s}{C_n}\right) \tag{3}$$

If we also assume that the wear rate is proportional to the load to the first degree, then in expression (3) the second π-complex should be deleted, since there already exists a multiplier with F to degree 1 in front of f:

$$W = \frac{FV}{H} f\left(C_s \cdot D^3, \frac{C_s}{C_n}, \frac{\rho V^2}{E}\right) \tag{4}$$

Then the proportionality factor in the Archard equation, as we can see from this expression, depends on surface geometry, additive concentration, elastic properties of materials, lubricant density and speed.

In the following, to show the additional capabilities of this modeling method, we will not follow Archard in the context of velocity- and load-dependent wear rates.

Let us assume that the friction mode is such that it does not depend on the elastic properties of the bodies, but is determined only by their hardness. Therefore, the entire complex $\frac{\rho V^2}{E}$ will not exist in the function f:

$$W = \frac{FV}{H} f\left(C_s \cdot D^3, \frac{C_s}{C_n} \right) \tag{5}$$

Taking into account the elastic properties of the bodies, we will consider a step dependence, which is quite often used to describe experimental data in tribology, as another variant of the model. Let us assume that $W = \frac{1}{E^n}$. In this case, expression (4) will take the form:

$$W = \frac{FV}{H} \cdot \frac{\rho^n \cdot V^{2n}}{E^n} f\left(C_s \cdot D^3, \frac{C_s}{C_n} \right) \tag{6}$$

Transforming expression (6) with speed included, we obtain:

$$W = \frac{F \cdot \rho^n \cdot V^{2n+1}}{H \cdot E^n} f\left(C_s \cdot D^3, \frac{C_s}{C_n} \right) \tag{7}$$

Proceeding from the fact that W is proportional to the velocity V in a positive degree, it follows that $2n + 1 > 0$, thus $n > -\frac{1}{2}$.

To consider different variants of the behavior of function W for different n, it is necessary to distinguish between two ranges: negative values: $-\frac{1}{2} < n < 0$ and positive values: $n > 0$.

For the subsequent application of the obtained models, we will take the main parameters of the friction pair and lubricant as standard. The basic information is shown in Table 2.

Table 2 – Main parameters of the model

Hardness **H**:	285 MPa (for alloy steel 40X)
Modulus of elasticity **E**:	211 GPa (for alloy steel 40X at temperature 100 °C)
Cylinder diameter **D**:	0.1 m
Surfactant concentration $\mathbf{C_s}$:	0.25
Nanoparticles concentration $\mathbf{C_n}$:	0.5
Density of lubricant **ρ**:	900 kg/m^3

Let us provide characteristic dependences of W depending on different indices n for the region $n \le 0$ (Fig. 1).

In the case when $n = 0$, the expression takes the form:

$$W = \frac{F \cdot V}{H} f\left(C_s \cdot D^3, \frac{C_s}{C_n} \right) \tag{8}$$

For this situation it is possible to construct characteristic dependences W(V) for the case of variation of parameter F (Fig.2).

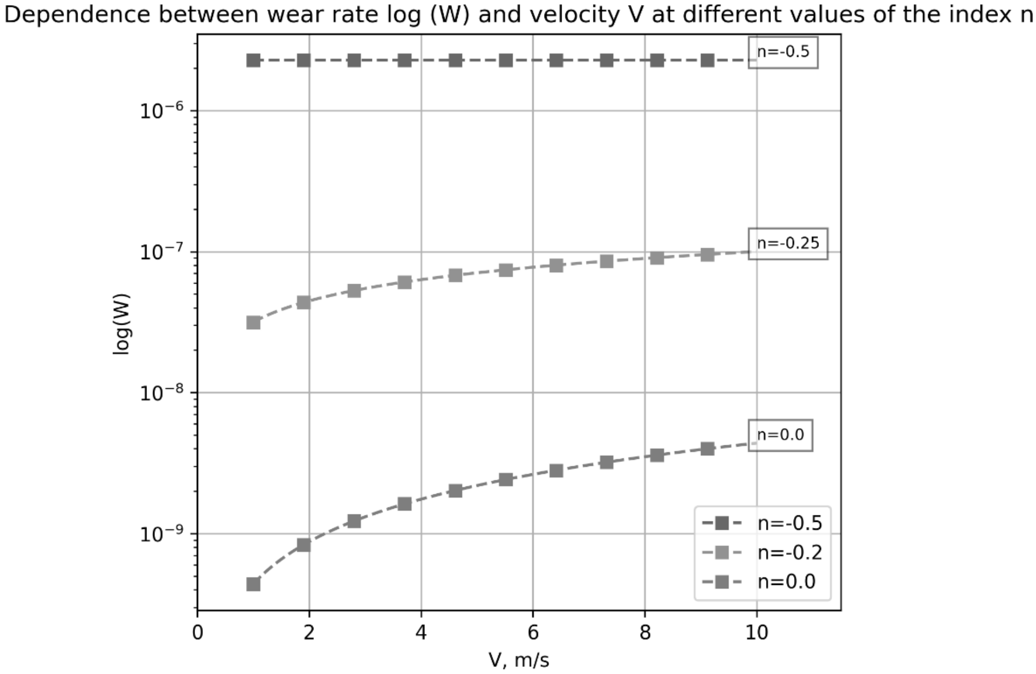

Figure 1: Dependence between the logarithm of wear rate log(W) and velocity V, m/s for the case of negative values of n and zero value of n

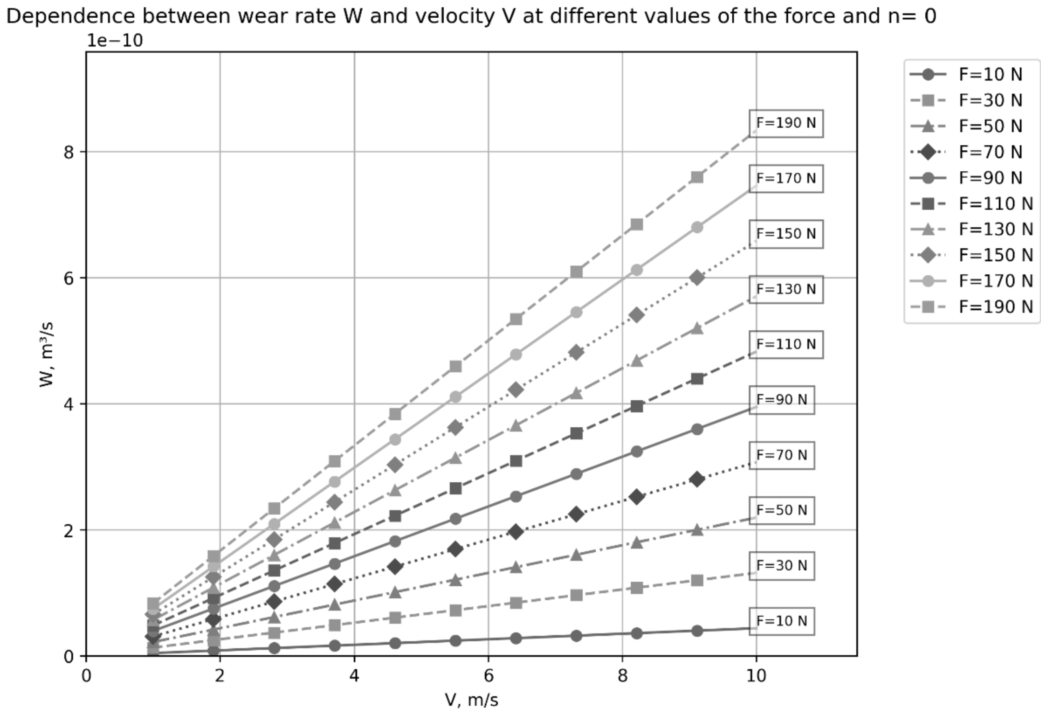

Figure 2: Dependence between wear rate W and velocity V, m/s for case n=0 and variation of load force in the range from 10 N to 190 N

Thus, taking $n = -\frac{1}{4}$, the following dependence can be obtained from equation (7):

$$W = \frac{F \cdot \sqrt{V} \cdot \sqrt[4]{E}}{H \cdot \sqrt[4]{\rho}} f\left(C_s \cdot D^3, \frac{C_s}{C_n}\right) \quad (9)$$

Graphically, the formula (9) for the variation of the load parameter F is shown in Figure 3.

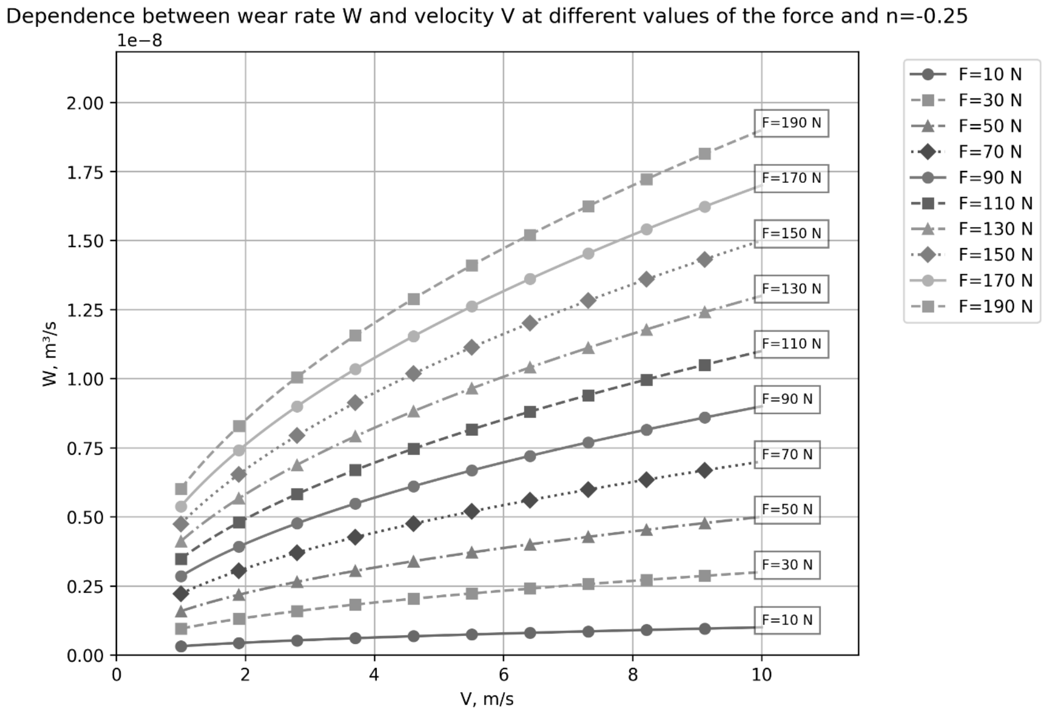

Figure 3: Dependence between wear rate W and velocity V, m/s for the case n=-0.25 and load force variation in the range from 10 N to 190 N

For positive values of the index n it is also possible to plot characteristic dependencies as presented in Fig. 4.

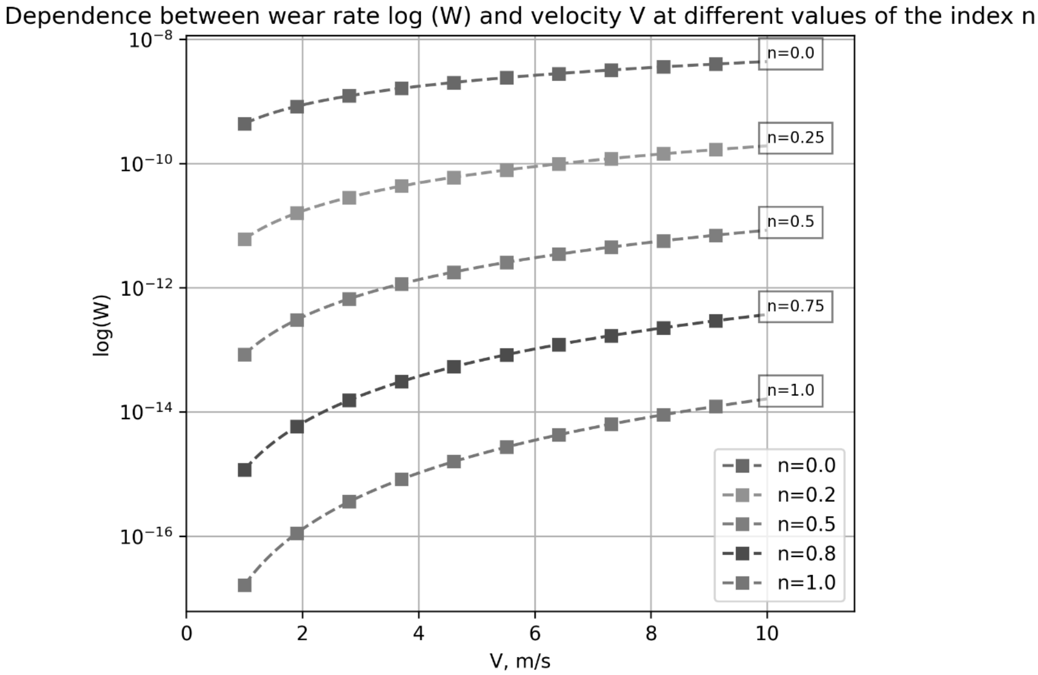

Figure 4: Dependence between the logarithm of wear rate log(W) and velocity V, m/s for the case of positive indices n

It is possible to identify the most representative situations. Thus, in the case when $n = 1$, we can write the following:

$$W = \frac{F \cdot \rho \cdot V^3}{H \cdot E} f\left(C_s \cdot D^3, \frac{C_s}{C_n}\right) \quad (10)$$

It follows from expression (10) that at some friction modes a cubic law of wear on velocity can be observed. Graphical representation of such dependences is shown in Fig. 5.

It should be noted that Expression (10) is obtained in a standard way: we leave the second complex in the function unchanged and transform the first one by multiplying it by the second one. In this case, we can formulate various rational hypotheses about the expected results of experiments. Let us provide two examples of hypotheses regarding the relationship between the concentration of triboactive additives and the wear rate in a tribosystem with a lubricating layer.

Let us consider characteristic cases.

Hypothesis 1: let the wear rate decrease linearly with increasing surfactant concentration:

$$W = W_0\ kC_s \tag{11}$$

where in formula (11), k is some dimensional coefficient. Then the expression for the intensity of wear will take the form:

$$W = W_0\left(1 - \frac{f\left(C_n\ D^3\right)C_s}{C_n}\right) \tag{12}$$

where f is some function of the dimensionless complex.

Hypothesis 2: the wear rate decreases also directly proportional to the concentration of chemically active additive. Then the function f will uniquely take the form:

$$f\left(C_n D^3\right) = a\left(C_n D^3\right)^2 \tag{13}$$

where a in formula (13) is some dimensionless constant. Then the expression for the wear rate will take the form:

$$W = W_0\left(1 - aC_n C_s D^6\right) \tag{14}$$

Using these examples, we can observe that the construction of similarity formulas is a rather flexible procedure that allows us to “adjust” the model to the peculiarities of a particular tribological object.

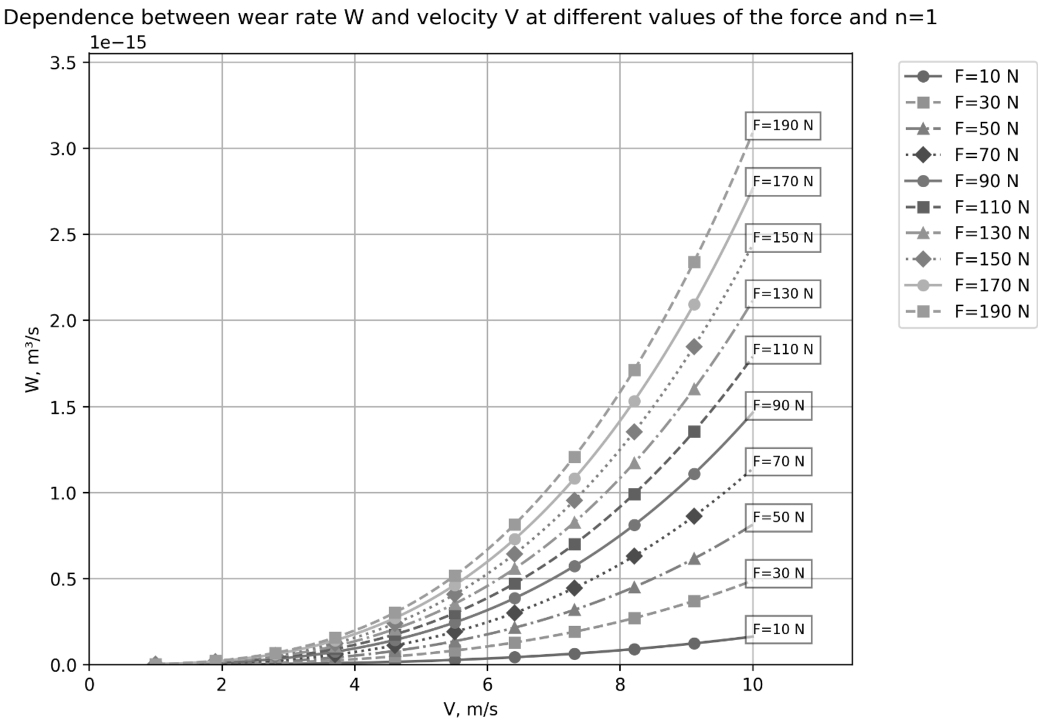

Figure 5: Dependence between wear rate W and velocity V, m/s for case n=0 and variation of load force in the range from 10 N to 190 N

Conclusion

Analytical models for estimating wear rate that account for a wide range of parameters have been proposed. The application of the method of dimensions and the creation of similarity models provide new opportunities for studying these objects, particularly regarding the role of the composition, structure, and properties of lubricating layers.

REFERENCES

1. Regularities of sliding friction of grey cast iron bodies in lubricating media depending on the sliding speed / A. D. Breki, D. V. Zimin, S. G. Chulkin [et al.] // Chebyshevskii Sbornik. - 2022. - Vol. 23. - № 5. - P. 198-205.

2. Levit G.A., Lurie B.G. 1965, "Investigation of the mechanism of mixed friction in the guides of metal-cutting machine tools", Theory of friction and wear, Acad. sciences of the USSR. Scientific advice on friction and lubrication. 1965. pp. 250-254.

3. Tichy J. A. Review of solid mechanics in tribology / J. A. Tichy, D. M. Meyer // International Journal of Solids and Structures. - 2000. - Vol. 37. - № 1-2. - P. 391-400.

4. Voitov A. V. Mathematical model of running-in of tribosystems under conditions of boundary lubrication. Part 1. Development of a mathematical model / A. V. Voitov // Problems of Tribology. - 2023. - Vol. 28. - № 1/107. - P. 25-33.

5. Berardo A. A model for hierarchical anisotropic friction, adhesion and wear / A. Berardo, N. M. Pugno // Tribology International. - 2020. - Vol. 152. - P. 106549.

6. Study of the identification model of tribological interaction of friction couples / G. M. Ismailov, A. E. Tyurin, A. N. Gavrilin [et al.] // IOP Conference Series: Materials Science and Engineering. - 2020. - Vol. 919. - № 2. - P. 022056.

7. Yadav Y. K. Prediction of erosive wear - A novel mathematical model / Y. K. Yadav, R. Sehgal, Siddhartha // Tribology International. - 2024. - Vol. 191. - P. 109133.

8. Grzelczyk D. Wear Processes in a Mechanical Friction Clutch: Theoretical, Numerical, and Experimental Studies / D. Grzelczyk, J. Awrejcewicz // Mathematical Problems in Engineering. - 2015. - Vol. 2015. - P. 1-28.

9. Argatov I. I. A Simple Model for the Wear Accumulation in Partial Slip Hertzian Contact / I. I. Argatov, J. W. Bae, Y. S. Chai // International Journal of Applied Mechanics. – 2020. – Vol. 12. – № 07. – P. 2050074.

10. Chang L. A Mathematical Model for the Mixed Lubrication of Non-Conformable Contacts With Asperity Friction, Plastic Deformation, Flash Temperature, and Tribo-Chemistry / L. Chang, Y.-R. Jeng // Journal of Tribology. - 2014. - Vol. 136. - № 2.

11. Mathematical modeling for nonlinear dynamic mixed friction behaviors of novel coupled bearing lubricated with low-viscosity fluid / J. Cai, G. Xiang, S. Li [et al.] // Physics of Fluids. - 2022. - Vol. 34. - № 9. - P. 093612.

12. Study on Wear Model and Adhesive Wear Mechanism of Brass under Boundary Lubrication / L. Liu, C. Yang, J. Zhou [et al.] // Protection of Metals and Physical Chemistry of Surfaces. - 2021. - Vol. 57. - № 2. - P. 367-373.

13. Guy B. A Survey on Mathematical Aspects of Lubrication Problems / B. Guy, V. Carlos // Bol. Soc. Esp. Mat. Apl. - 2007. - Vol. 39. - № September. - P. 31-74.

14. Taylor R. I. Rough Surface Contact Modelling—A Review / R. I. Taylor // Lubricants. - 2022. - Vol. 10. - № 5. - P. 98.

15. Modeling and simulation in tribology across scales: An overview / A. I. Vakis, V. A. Yastrebov, J. Scheibert [et al.] // Tribology International. - 2018. - Vol. 125. - P. 169-199.