УДК 303.732
doi:10.18720/SPBPU/2/id24-450

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FUZZY-DUAL PARAMETER SENSITIVITY ANALYSIS OF ECOSYSTEMS

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Abstract. This short paper considers the mathematical modelling of ecosystems in the perspective of their control. It is shown that the adopted mathematical models to design control policies are oversimplified, leading to few successful practical achievements. It appears that a reason for this is that many studies have adopted models of ecosystems which are composed of finite-dimensional ordinary differential equations with lumped parameters while their nature is essentially distributed. However, since the pioneer work of Alfred Lotka and Vito Volterra about prey-predator models, a wide variety of mathematical formalisms have been employed to study the dynamics of ecosystems. The present paper focuses on the sensitivity analysis of ecosystems through dynamical models with lumped parameters since this is a prerequisite for the design of control systems which may expect certain efficiency.

The adopted formalism is an original one and is based on the use of the fuzzy-dual numbers, functions and models. In this paper it is shown how this approach allows to tackle the uncertainty of parameters with respect to sensitivity analysis in a parsimonious way, when considering its limited data requirements and resulting computational burden. After introducing the essentials of the proposed formalism, a general approach is developed to generate dynamically uncertainty bounds for the state of an ecosystem which result from the parametric uncertainty of its model. The proposed method is illustrated in the case of the classical prey-predator ecosystem.

Keywords: ecosystems, mathematical models, lumped parameters, fuzzy-dual representation, sensitivity analysis, prey-predator ecosystems.

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НЕЧЕТКИЙ ДУАЛЬНЫЙ ПАРАМЕТРИЧЕСКИЙ АНАЛИЗ ЧУВСТВИТЕЛЬНОСТИ ЭКОСИСТЕМ

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Аннотация. В статье рассматривается математическое моделирование экосистем, с точки зрения управления ими. Показано, что принятые математические модели для разработки политики управления чрезмерно упрощены, что приводит к немногочисленным успешным практическим результатам. По-видимому, причина этого заключается в том, что во многих исследованиях были приняты модели экосистем, которые состоят из конечномерных обыкновенных дифференциальных уравнений с сосредоточенными параметрами, в то время как их природа, по существу, распределенная. Однако со времен работы Альфреда Лотки и Вито Вольтерры о моделях «жертва-хищник» для изучения динамики экосистем использовался широкий спектр приёмов математической формализации. Настоящая статья посвящена анализу чувствительности экосистем с помощью динамических моделей с сосредоточенными параметрами, поскольку это является необходимым условием для проектирования систем управления, от которых можно ожидать определенной эффективности. Используемый способ формализации является оригинальным и основан на применении нечетких дуальных чисел, функций и моделей. В статье показано, как этот подход позволяет минимизировать неопределенность параметров в отношении анализа чувствительности, учитывая ограниченные требования к данным и, как следствие, вычислительную нагрузку. После представления основ предложенного способа формализации, разработан общий подход для динамического определения границ неопределенности состояния экосистемы, которые являются результатом параметрической неопределенности ее модели. Предлагаемый метод проиллюстрирован на примере классической экосистемы «жертва-хищник».

Ключевые слова: экосистемы, математические модели, сосредоточенные параметры, нечетко-двойственное представление, анализ чувствительности, экосистемы «жертва-хищник».

Introduction

Since the industrial revolution of the 19th century, human activity has transformed the environment through its exacerbated exploitation of fossil or living natural resources, through its accelerated energy consumption and through the multiple resulting pollutions aggressively attacking the ecosystems which constitute our environment. To deal with this situation by stopping this catastrophic spiral for humanity and to ensure the long-term sustainability of the planet, an environmental policy at several levels in space (from global to local) and in the time (from long term to short term), has seemed necessary. However, the first stages of these efforts invariably resulted in the postponement of deadlines that seemed irrevocable. Beyond the possible lack of political will, these repeated failures were also due to the fact that at the level of individuals or groups of individuals, it was not possible to implement sufficiently effective management actions over the ecosystems in which they move.

Sectors of Mathematics and Engineering have developed quantitative tools, in general models, intended to represent the response of numerous ecosystems to different policies [1]. However, the results actually obtained were often different from what was expected. The decision support methods developed over the last decades to try to effectively manage ecosystems such as populations' dynamics, epidemics' dynamics, spatial ecology and climate change, can be divided into two:

– first, those establishing tactical decisions and which generally fall under Operational Research [2];

– those establishing decisions influencing the short-term evolution of ecosystems and which relate to Automation and more particularly to Control Theory [3].

Nevertheless, it appeared that the application of control theory to ecosystems presented numerous difficulties: approximate representation of the internal dynamics of the system, insufficient consideration of the interactions of the ecosystem with its natural environment, technical and economic insufficiency of sensors and actuators available to practically guarantee observability and controllability.

In this paper is at first considered the issue of modelling ecosystems for their control, then the sensitivity issue is discussed through the adoption of the fuzzy-dual formalism to represent parameter uncertainty. This formalism is briefly introduced, and the proposed approach is displayed. This approach is illustrated using the classical model of a prey-predator ecosystem.

1. Mathematical modelling of ecosystems

Ecosystems involve complex dynamic processes over a given spatial domain which is in general continuous. Many theories allow to represent these ecosystems by state models with distributed parameters described by partial differential equations of infinite dimension.

A general equation for these ecosystems is given by:

$$\frac{\partial x(t,z)}{\partial t} = f\big(x(t,z)\big) + g(t,z) \cdot u(t,z), \tag{1}$$

where t is the time between $t \in T =]0, t_F[$, with t_F being adopted the time horizon for the study, $z \in \Omega \subset R^3$ is a position in the considered spatial domain Ω of boundary $\partial\Omega$, $x \in R^n$ is the state vector composed of the characteristic variables of the ecosystem with values over Ω and T, $u \in R^m$ is the control vector acting over Ω and T, f and g are in general differential operators, n and m are integer numbers. The initial situation of the ecosystem is given by:

$$x(0,z), \;\; z \in \Omega. \tag{2}$$

Depending of the nature and configuration of the ecosystem, boundary conditions may be present:

$$l\big(x(t,z)\big) = 0, \;\; z \in \partial\Omega, \tag{3}$$

where l is also a differential operator.

Many confined industrial processes are described by such type of equations [4]. Knowledge of their state throughout the system is obtained by using batteries of sensors which promote the spatial discretization of the control problem. Various complex control techniques (non-linear control, adaptive control, distributed control) [5] then make it possible to develop control orders implemented by actuators distributed in or around the process.

In the case of ecosystems, detailed knowledge of the state of the system is not possible given the scope of the task and possibly external conditions contrary to the realization of precise measurements. Also on the actuator side, the options to act on the system are often very limited: local, one-off or of a static nature. Leaving aside the perfect non-homogeneity of ecosystems (variable density of a component according to the position, irregular distribution os species in an ecosystem) many mathematical models working on variables describing in a macroscopic way the dynamic evolution of the ecosystem has been developed. This has led to the adoption of systems with lumped parameters which are described by finite-dimensional ordinary differential equations such as:

$$\frac{dx}{dt} = f(x,u,p,t) \quad \text{with } x(0) = x_0, \tag{4}$$

where f is now a continuous function of class C^∞, $x \in R^n$ is the state vector composed of variables characterizing over T at the macroscopic level (total value or mean value) the ecosystem, $u \in R^s$ is the control vector acting over T, p is the vector of the lumped parameters.

An example of this class of models is the Lotka-Volterra predator-prey autonomous (no entry) model [6] such as:

$$\frac{dX_1}{dt} = (a - bX_2)X_1 \quad \text{with } X_1(0) = X_{10}, \tag{5}$$

$$\frac{dX_2}{dt} = (cX_1 - d)X_2 \quad \text{with } X_2(0) = X_{20}, \tag{6}$$

where $X_1(t)$ is the total amount of preys at time t over the whole ecosystem (a pond, a forest, an island) and $X_2(t)$ is the total amount of predators at time t over the considered ecosystem. Comparing (5) and (6) with (4), we get $x(t) = \left(X_1(t), X_2(t)\right)'$ and $p = (a,b,c,d)'$. The interpretation of the vector of lumped parameters is here such as a is the growth rate of the preys in the absence of predators, b is the prey death impact factor, c is the birth rate impact factor and d is the death rate of predators in the absence of preys.

2. Fuzzy dual numbers, functions and systems

The formalism of fuzzy-dual numbers [7] has been proposed recently to diminish the computational burden when dealing with uncertainty in decision problems. A fuzzy dual number is written $r + \varepsilon u$, where r is the deterministic part and u is an uncertainty bound and ε is the uncertainty unit with $\varepsilon^2 = 0$. A fuzzy dual number $r + \varepsilon u$ is attached to a triangular symmetrical fuzzy set of mean value r and of half base width u. There addition and multiplications are such as:

$$(r_1 + \varepsilon u_1) + (r_2 + \varepsilon u_2) = (r_1 + r_2) + \varepsilon(u_1 + u_2), \tag{7}$$

$$(r_1 + \varepsilon u_1)(r_2 + \varepsilon u_2) = r_1 r_2 + \varepsilon(r_1 u_2 + r_2 u_1). \tag{8}$$

The fuzzy-dual representation of uncertainty has to cope with uncertain mathematical programming problems through fuzzy dual dynamic programming [8], while in the field of optimal control of uncertain dynamical systems, fuzzy dual necessary optimality conditions have been established [9]. A fuzzy dual function $\tilde{f}$ associated to a base function f with $f \in C^\infty$ is such as:

$$\tilde{f}(x + \varepsilon y) = f(x) + \varepsilon f_x(x) y. \tag{9}$$

Relation (9) means that $\tilde{\boldsymbol{f}}$ represents any first order approximation of $f(x+\delta x)$ around x for $|\delta x| \leq |y|$. Figure 1 proposes a scalar view of a fuzzy-dual function where its fuzzy-dual input is related with the fuzzy-dual output of the considered function ($\mu(\ )$ are fuzzy membership functions):

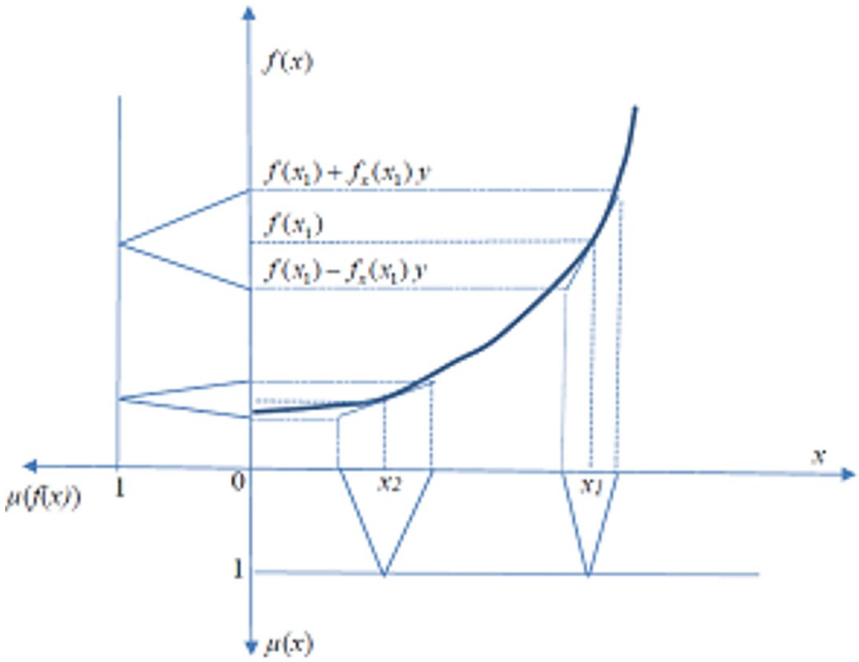

Fig. 1. Fuzzy-dual function with associated membership functions

Here it is considered that the state dynamics of the process are given by:

$$\frac{dz}{dt} = \tilde{f}(z,u,t), \tag{10}$$

where $\tilde{f}$ represents a fuzzy-dual function attached to a base function f which is of class C^{∞} with respect to z and u where z is a fuzzy-dual variable written $z = x + \varepsilon y$ with $x(0) = x_0$ and $y(0) = 0$.

Introducing the Jacobian matrix:

$$f_x(x,u,t) = \left[\left(\backslash\partial f_i/\partial z_j\right)_{x,u,t}\right] \tag{11}$$

the fuzzy-dual state equation (10) can be rewritten in R^{2n} as:

$$\frac{dx}{dt} = f(x,u,t) \quad \text{with } x(0) = x_0 \tag{12}$$

and

$$\frac{dy}{dt} = f_x(x,u,t)y \quad \text{with } y(0) = 0. \tag{13}$$

The uncertainty about the future value of x can originate in its initial value and in the dynamics of the process.

3. Fuzzy dual parametric sensibility analysis

Here it is supposed that an ecosystem obeys to the dynamics given by:

$$\frac{dz}{dt} = \tilde{f}(z, u, t, p), \tag{14}$$

where p is a vector of J_p parameters such as f is also of class C^{∞} with respect to him.

Here the case in which the uncertainty in the dynamics manifests itself at the level of the model parameter values, is considered for sensitivity analysis purposes. Since a detailled probabilistic distribution of these values is in general unavailable, it is assumed that an expected interval of values is available and a fuzzy-dual representation is adopted for their values:

$$p = \bar{p} + \varepsilon \Delta p. \tag{15}$$

Introducing as additional state variable Δp with state equation $\Delta \dot{p} = 0$, the augmented fuzzy dual state equation can be written now as:

$$\frac{dx}{dt} = f(x, u, y, \bar{p}) \quad \text{with } x(0) = x_0, \tag{17}$$

$$\frac{dy}{dt} = f_x(x, u, t, \bar{p})y + f_p(x, u, t, \bar{p})\Delta p \quad \text{with } y(0) = 0. \tag{18}$$

Considering the uncertainty of the state of the system at time t_F ($t_F > 0$), the solution of equation (13) can be written under the form:

$$y(t_F) = M(t_F, u, \bar{p})\Delta p, \tag{19}$$

where the values of $M(t_F, u, \bar{p})$ are computed numerically. Let now $m_{ij}(t_F)$ be the i^{th} row and j^{th} column component of this matrix, the uncertainty interval for the i^{th} state of the original system at time t_F can be computed. Since at t_F:

$$y_i^+(t_F) = \sum_{j=1}^{J_p} \left| m_{ij}(t_F) \right| \frac{\Delta p_i}{2} \quad \text{and} \quad y_i^-(t_F) = -\sum_{j=1}^{J_p} \left| m_{ij}(t_F) \right| \frac{\Delta p_i}{2} \tag{20}$$

we have:

$$y_i(t_F) = \sum_{j=1}^{J_p} \left| m_{ij}(t_F) \right| \Delta p_i \tag{21}$$

and a fuzzy-dual representation of the state of the system at time t_F is given by:

$$z(t_F) = x(t_F) + \varepsilon y_i(t_F), \tag{22}$$

where $x(t_F)$ is the solution of equation (17). In the case of the previous Lotka–Volterra predator-prey model given by the equations (5) and (6), the vector of parameters is given by: $p = (a, b, c, d)'$ while equation (18) is written here:

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} \bar{a} - \bar{b}X_2 & -\bar{b}X_1 \\ \bar{c}X_2 & \bar{c}X_1 - \bar{d} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} X_1\Delta a - X_1X_2\Delta b \\ X_1X_2\Delta c - X_2\Delta d \end{bmatrix}. \tag{23}$$

The nominal non-zero equilibrium of this model is given by $X_{1e} = \bar{d}/\bar{c}$ and $X_{2e} = \bar{a}/\bar{b}$ and a direct fuzzification of this result gives the fuzzy-dual expression for the equilibrium state:

$$\tilde{X}_{1e} = \frac{\bar{d}}{\bar{c}} + \varepsilon \frac{\bar{c}\Delta d - \bar{d}\Delta c}{\bar{c}^2} \text{ and } \tilde{X}_{2e} = \frac{\bar{a}}{\bar{b}} + \varepsilon \frac{\bar{b}\Delta a - \bar{a}\Delta b}{\bar{b}^2}.$$

Observe that these uncertainties lead to the equilibrium of system (23), allowing to conclude on the consistency of the proposed approach.

Conclusion

This paper has considered first the issue of modelling the dynamics of ecosystems for their control, leading to the consideration of differential models with lumped parameters. To tackle the sensitivity analysis of this class of models, the fuzzy-dual formalism has been adopted to represent in a parsimonious way the uncertainty of the values of these parameters. The fuzzydual formalism has been briefly introduced and then the proposed approach has been displayed. There, the dynamics of the uncertainty of the states of an ecosystem are established and have allowed to compute all along their evolution, uncertainty bounds for these states.

This approach is illustrated using the classical model of a prey-predator ecosystem, from its initial state, up to its equilibrium state.

This study should be completed by extensive numerical simulation studies to display the effectiveness of the proposed sensitivity analysis approach. Also, the application of this formalism should be extended to the analysis of the effects of parameter uncertainty over the predicted stability of ecosystems.

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