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PRISONER'S DILEMMA GAME ON COMPLEX NETWORKS: BETTER COOPERATION OF AFRICAN COUNTRIES

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Abstract. The current study proposes a concept for country-scaled trade institutions how to organize their activities on international market to mutual benefit of all the actors. This concept is based on modelling of trade interconnections as Prisoner's Dilemma and Stag-Hunt games on complex networks, representing interrelated states. The modelling covers three artificial topologies: small world, random, and scale-free and one real network built on state-scale trade flows within African continent. The study focuses on topological factors that promote or hamper cooperation processes. The results show that the initial proportion between cooperators and defectors influences the evolution of cooperation and the overall network effectiveness with difference due to topology. For instance, we found that cooperation evolution on the empirical network for African trade volumes (> $350M) is closer to that of small world network, whereas payoffs on the former might be compared with those on random network.

Keywords: trade relationships, African continent, Prisoner’s dilemma, Stag-Hunt game, complex networks, topologies, cooperators-defectors proportion, cooperation, payoffs.

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ИГРА «ДИЛЕММА ЗАКЛЮЧЕННОГО» В СЛОЖНЫХ СЕТЯХ: УЛУЧШЕНИЕ СОТРУДНИЧЕСТВА АФРИКАНСКИХ СТРАН

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Аннотация. В настоящем исследовании предлагается концепция для торговых институтов странового масштаба, как организовать свою деятельность на международном рынке к взаимной выгоде всех участников. Эта концепция основана на моделировании торговых взаимосвязей в виде дилеммы заключенного и игр «Охота на оленя» в сложных сетях, представляющих взаимосвязанные государства. Моделирование охватывает три искусственные топологии: малый мир, случайный и безмасштабный, и одну реальную сеть, построенную на торговых потоках государственного масштаба на африканском континенте. Исследование сосредоточено на топологических факторах, которые способствуют или препятствуют процессам сотрудничества. Результаты показывают, что первоначальное соотношение между кооператорами и перебежчиками влияет на развитие сотрудничества и общую эффективность сети с различиями в топологии. Например, мы обнаружили, что развитие сотрудничества в сети empirial network для торговли в Африке (> 350 млн. долл.) ближе к развитию сотрудничества в сети small world network, в то время как выгоды в первой из них можно сравнить с доходами в сети random network.

Ключевые слова: торговые отношения, Африканский континент, дилемма заключенного, охота на оленя, сложные сети, топология, соотношение кооператоров и перебежчиков, сотрудничество, выплаты.

Introduction

National economic development depends severely on the strategy the leaders follow in all aspects of governance and in international trade as well. In this regard cooperation and competition environment plays a significant role while achieving necessary goals and creating significant values. It should be noted that cooperation needs in trust among partners. Current study aims to clarify how the structural factors influence and form trust and boost cooperation among countries of African continent in context of international trade. It is envisaged that the results might be useful for policymakers to cohere cooperative networks for serious economic advances in the continent in whole and in each country in particular.

Through an interdisciplinary approach and instruments of contemporary Network Science [1] that integrates complex systems analysis, game theory, and agent-based modeling, this study seeks to contribute to the knowledge of cooperative networks and provide valuable insights for fostering economic integration, regional development, and sustainable trade relationships among African nations.

This study based on fundamentals system analysis supported by Prisoner’s Dilemma game (PDG) and approaches of network science in its complex network treatment [1, 2, 3] formulate methodology for exploration how cooperative strategy of individuals contribute to collective behavior of all the actors in a network and vice versa. Also the research differs classic PDG where payoffs are in terms of punishment and trade process with its payoffs in value discourse and thus considers other insights of PDG, i.e. Stag Hunt game [3].

1. Model

This work boosts a quantitative concept, utilizing agent-based modeling and simulations to analyze cooperation dynamics and network effectiveness within African trade system. The primary focus of the research is directed on modeling the Prisoner's Dilemma -like games on complex networks, which reflect trade transactions of African countries within the continent.

1.1. General methodology

The Prisoner's Dilemma game and its clones (Stag-Hunt, Battle of the Sexes, and Hawk-Dove games) are popular and well-researched mathematical models [4–8]. All PDG-like games include cooperator and defector actors and use Payoff matrices [9] (see Fig. 1, where C = cooperators, D = Defectors, R is the payoff for both players when cooperating (CC outcome), S is the payoff for the cooperator when the cooperator cooperates and the defector defects (CD outcome), T is the payoff for the defector when the cooperator cooperates and the defector defects (DC outcome), P is the payoff for both players when they defect (DD outcome)).

$$\begin{array}{c c} & \begin{matrix} \mathrm{C} & \mathrm{D} \end{matrix} \\ \begin{matrix} \mathrm{C} \\ \mathrm{D} \end{matrix} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array}$$

Fig. 1. Payoff matrix

Classic PDG models contradiction between individual and collective interests, while the Stag-Hunt game seeks for trade-off between risk and cooperation.

1.2. PDG and networks

Cooperation among individuals or entities and corresponding decision-making dynamics on networks induce deep interest in scientific society. In this case each player faces a set of choices, and diverse payoffs depend on the decisions of this very networked players.

Several works were devoted to Prisoner's Dilemma game on complex networks [10–11]. Mostly they focused on behaviour of actors while proportion of cooperators versus defectors changes. Usually small world (SW), random (Erdos-Renyi), ER and scale-free (Barabasi-Albert), BA topologies were used to construct networks for further comparison of impact of structural properties on the PDG processes [12–13]. In [14] memory effect of players was taken into consideration, and it was shown on three topologies (ER, BA, and degree-degree correlation-based ones) that this very effect notably mitigates the structural influence.

1.3. Work model

Complex networks can effectively represent the trade relationships among African countries, capturing the non-uniform nature of these connections.

The proposed model envisaged agent-based simulation techniques for the PDG and Stag-Hunt game on these complex networks. To all agents representing African countries initial strategies (C or D) are assigned. The agents can interact and update their strategies due to the payoffs received while their interactions with network neighbors on previous step.

The data processing, the game per se and the analysis involve:

– data preprocessing: cleaning, formatting, and integrating the data from different sources to ensure consistency and compatibility for analysis;

– node set construction: using the node-level geographical attributes from the "Node data" file;

– network construction: building complex networks representative of African trade relationships (trade volumes) using the “Edge data " file, with nodes representing countries and edges representing trade relasionships;

– agent-based modeling: implementing agent-based models to simulate the Prisoner's Dilemma (PD) and Stag-Hunt games on the constructed networks, using the node-level edge-level attributes;

– simulation experiments: conduct extensive simulations under various scenarios, varying the initial percentage of cooperators, payoff matrices, and network topologies, to explore the cooperation dynamics and network effectiveness;

– statistical analysis: applying statistical techniques to analyze the simulation results, identifying patterns, trends, and relationships between input parameters (e.g., initial proportion of cooperators, network topology) and outcome variables – percentage of cooperators (PC) and payoff per node (PO);

– sensitivity analysis: Performing sensitivity analyses to assess the robustness of the findings and explore the impact of varying model parameters and assumptions on cooperation dynamics and network effectiveness.

2. Data

The data utilized in our research was sourced mostly from publicly available TRADING ECONOMICS resource [15]. The intricate web of African economic ties extends far beyond geographical proximity, shaped by the nuanced interplay of political kinships, resource endowments, and strategic partnerships.

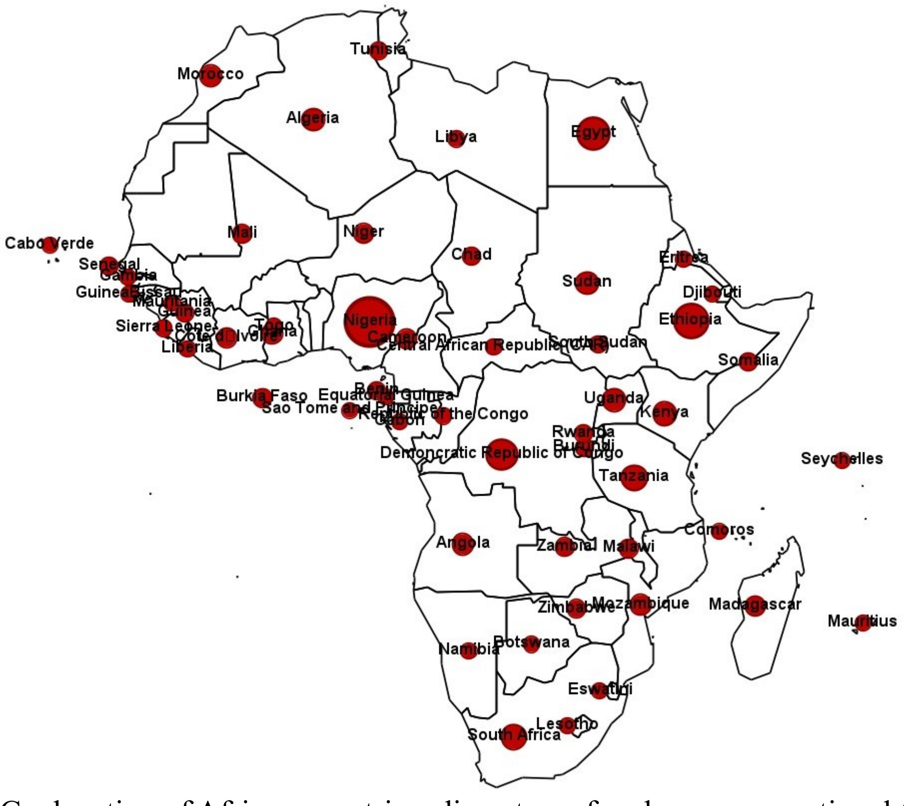

Fig. 2. Geolocation of African countries: diameters of nodes are proportional to country’s populations

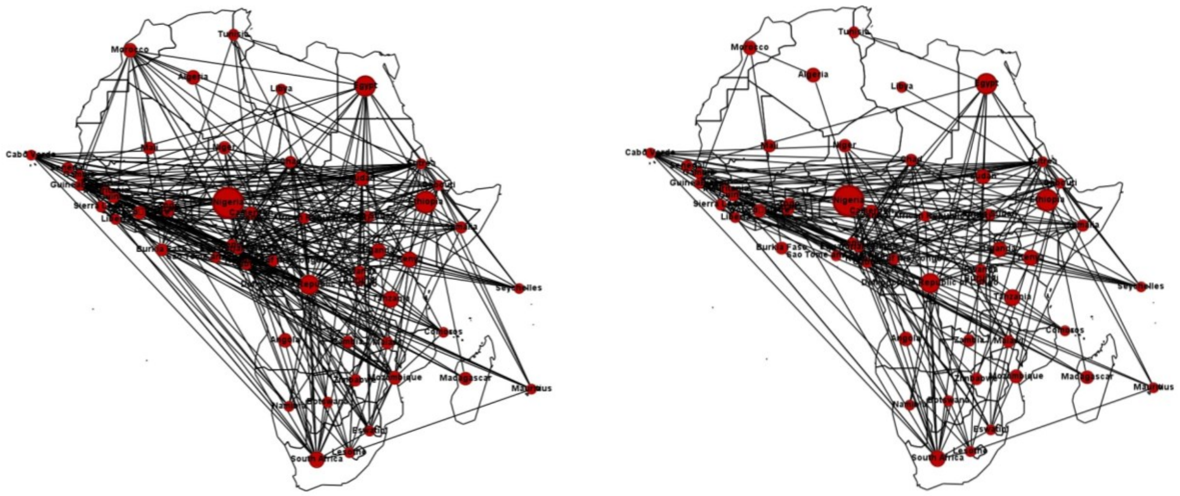

Fig.3. Network of pairwise trade links between African countries for the trade volume threshold: TVT = $160M (left) and TVT = $350M (right)

The data analysis investigates the cooperation dynamics and effectiveness within African trade networks. The primary data sources utilized in this study are the “Node data” and “Edge data” files, which contain information about node attributes and trade relationships for African countries, respectively.

The “Node data” file contains information about individual African countries, such as their labels, geographical coordinates, population, and trade volumes within Africa. These node-level attributes contribute to a more comprehensive understanding of the trade dynamics and potential factors influencing cooperation and network effectiveness.

The “Edge data” file provides details on the trade connections between countries, including the source and target nodes, various trade-related metrics (e.g., Stochastic Multi-Agent Verification, work in process, over_time, incentive, no_of_workers), and the trade volumes between the connected nodes. This data is the basis for constructing the complex networks representing African trade relationships. To compose the network based on the “Edge data” file the thresholds TVT were chosen for trade volumes (links with lesser volume are not taken into account). The “Node data” .csv and “Edge data” .csv files ar imported into popular instrument Gephi [16] to visualize and analyze the data set.

The current version of the GEPHI package 0.10.1 [16] with many plugins and a console for entering commands in the Gython language makes it possible to organize connections between nodes in accordance with the threshold of trading flows between nodes. Nodes are reflected on Fig.2, whereas Fig. 3 portrayed the network edges (threshold value TVT is equal $350M).

This “real” network formed a data set along with “synthetic” ones represented by Kleinberg (K), Erdos-Renyi (ER), Barabasi-Albert (BA) topologies for our further simulation processes.

3. Findings

We realized Prisoner's Dilemma and Stag-Hunt games on cooperation and trade outcomes. Pairwise payoff values were set R = 1, T = 1.5, S = 0, P = 0.1 and R = 1, T = 1.25, S = 0.25, P = 0.75 for these two games, respectively. The analysis reveals that the specific payoff matrices used in the simulations significantly influence the cooperation dynamics and trade effectiveness within African trade networks (TVT = $350M). Certain payoff configurations, characterized by higher rewards for cooperation and lower temptations to defect, tend to promote more stable and robust cooperation over time.

The simulations are conducted under various scenarios, varying the initial proportion of cooperators, the payoff matrices, and the network topologies. The primary outcome variables of interest are the final average percentage of cooperators (PC) and the final average payoff per node (PO), which serve as indicators of cooperation dynamics and network effectiveness, respectively.

The quantitative data obtained from the simulations are analyzed using statistical techniques to identify patterns, trends, and relationships between the input parameters (e.g., the initial proportion of cooperators, network topology) and the outcome variables (PC and PO). The Fig. 4 and Fig. 5 demonstrate preliminary simulation results for the Stag-Hunt game on various network topologies, including Kleinberg (K), Erdos-Renyi (ER), Barabasi-Albert (BA), and the African countries trade network (A). These results include the final average percentage of cooperators (PC) and the final average payoff per node (PO) for different initial proportions of cooperators.

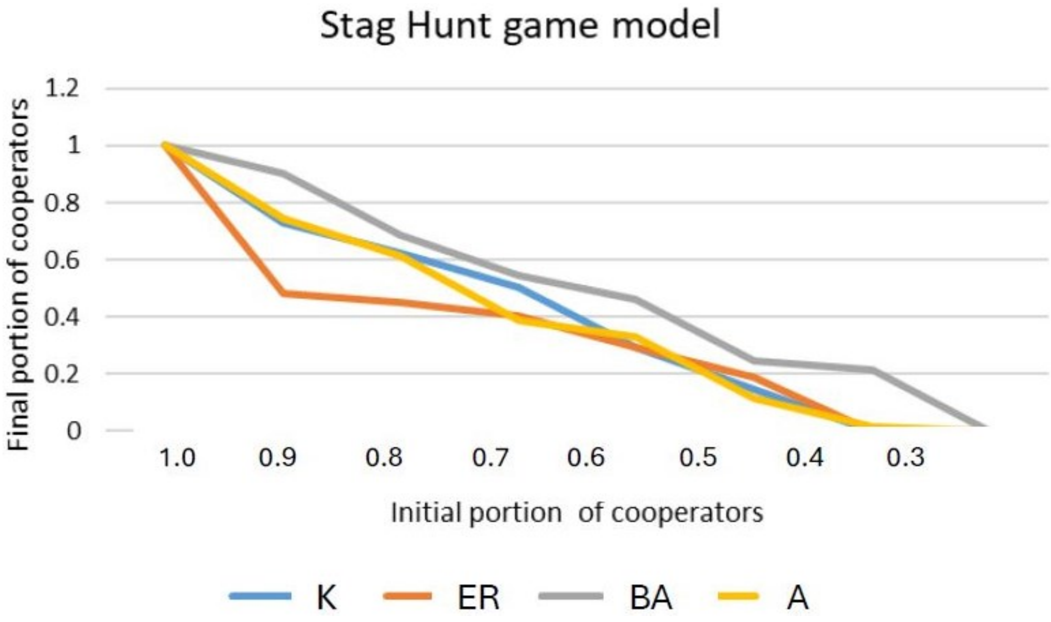

Fig.4. Final average percentage of cooperators vs initial one for different topologies. Number of game iterations is 10

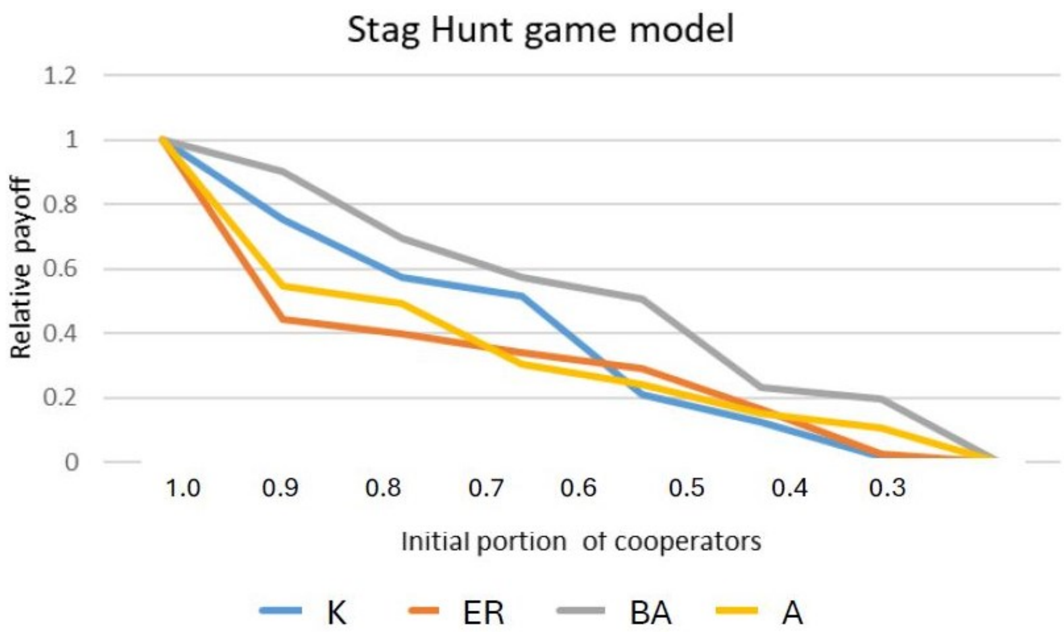

Fig.5. Final average payoff per node vs initial percentage of cooperators for different topologies. Number of game iterations is 10

It should be noted that PC behavior for A topology is close to that for K one, while PO dependence for A structure is more similar to that of ER. Scale-free topology (BA) [17] might be considered as a destination for structural organization of real trade networks which proposes better outcome in terms of cooperation and payoff as well.

By combining agent-based modeling, complex network analysis, and simulations of cooperation games, this study aims to contribute to a deeper understanding of the factors that drive cooperation among African countries in trade networks and the strategies that can be employed to enhance the effectiveness of these cooperative networks.

Interestingly, the node-level attributes, such as population size and trade volumes, play a crucial role in shaping the cooperation behavior of individual nations. Countries with larger populations and higher trade volumes exhibit a greater propensity for cooperative strategies, potentially driven by the desire to maintain stable trade relationships and access larger markets.

Moreover, the trade network might be exposed to rewiring to get better collective and individual effect like that described in [18].

Conclusions

In whole there are three points which might be assessed as novel and specific ones. First, analysis of trade flows between African countries promotes constructing an original network of real trade links of continental scale. Second, the study pays primary attention to Stag Hunt game which is more consistent with trade connections rather than that of PDG. Third, whereas most studies simulated Prisoner’s Dilemma-like game on synthetic networks, e.g. [19] this work comprises into consideration real trade network.

The results show that the initial percentage of cooperators and the network structural properties influence the evolution of cooperation and overall network performance significantly. Higher starting percentage of cooperators generally lead to more effective cooperation and overall payoffs.

The analysis of the key mechanisms governing the dynamics of cooperation on complex networks provides multidimensional scope for proposing practical strategies and policies. These can essentially increase indicators of effectiveness and efficiency for cooperative trade networks interconnecting African countries. Factors such as encouraging starting trends in cooperation, optimizing network structures and adapting incentives to payments become potential regulator to provide sustainable and mutually beneficial trading links.

By integrating elements of game theory with agent-based modeling and complex network analysis, this study suggests an interdisciplinary concept to scrutinizing cooperative networks in real state of affairs.

Moving forward, further research is envisaged to investigate the practical implementation of the prospective strategies and to estimate their consequences on trade volumes, economic integration, and regional development on the African continent, along with counteracting adverse actions and reshaping pertinent networks.

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