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Combined Method of 3d Analysis for Underground Structures in View of Surrounding Infinite Homogeneous and Inhomogeneous Medium

Пространственные расчеты подземных сооружений с учетом работы окружающего бесконечного массива в однородных и неоднородных областях комбинированным способом

N.V. Chernysheva,
Peter the Great St. Petersburg Polytechnic University, Saint Petersburg, Russia

G.S. Kolosova,
Peter the Great St. Petersburg Polytechnic University, Saint Petersburg, Russia

L.A. Rozin,
Peter the Great St. Petersburg Polytechnic University, Saint Petersburg, Russia

канд. техн. наук, доцент Н. В. Чернышева,
Санкт-Петербургский политехнический университет Петра Великого, Санкт-Петербург, Россия
канд. техн. наук, доцент Г. С. Колосова,
Санкт-Петербургский политехнический университет Петра Великого, Санкт-Петербург, Россия
д-р физ.-мат. наук, профессор Л. А. Розин,
Санкт-Петербургский политехнический университет Петра Великого, Санкт-Петербург, Россия

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Ключевые слова: метод конечных элементов, пространственная задача теории упругости, формула Сомильяны, внешняя задача, бесконечная область, подземные выработки

Abstract. The application of algorithms of the finite element method (FEM) or the boundary element method (BEM) reveals some peculiar properties for a numerical solution of the three-dimensional analysis in infinite domains. Various algorithms offer to avoid such problems at the expense of combining different methods and equations. The algorithm of the 3d analysis developed to solve an external boundary problem by applying the combined method based on incorporating the FEM and Somigliana's integral formula is considered. The algorithm is modified for the case of the interaction of a structure with an inhomogeneous medium. The efficiency of software implementation of both algorithms has been tested. A stress-strain analysis of an inhomogeneous medium with a cavity has been carried out to illustrate the given approach.

Аннотация. Численное решение пространственной задачи теории упругости в бесконечных областях может проводиться в рамках традиционных алгоритмов метода конечных элементов (МКЭ) или метода граничных элементов (МГЭ). Эффективность расчетов может быть повышена за счет использования различных формулировок законов теории упругости и применения упомянутых численных методов в сочетании. Например, возможно построение численного алгоритма решения пространственной задачи теории упругости в бесконечных областях на базе сочетания МКЭ и интегральной формулы Сомильяны. Рассматривается комбинированный алгоритм решения пространственных задач теории упругости в бесконечных областях на базе сочетания МКЭ и формулы Сомильяны. Приводится также модифицированный вариант алгоритма для областей, содержащих физические неоднородности. Обсуждается численная реализация рассмотренного алгоритма и его тестирование для внешних и внутренних пространственных областей.

Introduction

In many cases of practical importance, the analysis of structures considering their interaction with the infinite foundation or the surrounding elastic medium is based on a numerical solution of the three-dimensional external boundary problem [1-4]. The application of algorithms of the finite element method

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(FEM) or the boundary element method (BEM) reveals some peculiar properties for a numerical solution of the three-dimensional analysis in infinite domains. [5-7]. The FEM analysis is performed assuming a reduced domain of the finite size or by the application of special "infinite" elements [8, 9]. Using this approach inevitably arises problems such as the validation of the reduced domain size and the selection of boundary conditions on the external boundary in the first case or complications encountered in the solution of the system of equations in the second case. When the BEM approach application is considered, the analysis of the interaction of a structure and an inhomogeneous medium turns out to be inconvenient, as well as in the case of domains with several cavities. Meanwhile, in practice these problems arise quite rarely.

These specifics of the boundary problem in the case of the infinite domain cause some computational complications and increase the computing cost [10, 11]. All the known algorithms offer to avoid such problems at the expense of combining different methods and equations [12-16, 28, 29, 33, 34]. In the following, the combined method (CM) which represents the algorithm based on combining the FEM facilities, the Somigliana's integral formula is considered [17], and the efficiency of this method is also displayed.

Somigliana's integral formula

The Somigliana's integral formula allows determining the components of the displacement vector $u(\xi) = (u_1(\xi), u_2(\xi), u_3(\xi))^T$ at arbitrary point ξ in a space Ω bounded by a surface S; in the absence of the volume forces:

$$u_j(\xi) = \int_S (t_i(\eta)G_{ij}(\eta,\xi) - F_{ij}(\eta,\xi)u_i(\eta))dS(\eta), \tag{1}$$

where $\xi \in \Omega, \eta \in S$, $i, j = 1,2,3$ are the indexes of Cartesian coordinate axes x_i, $t_i(\eta)$ is the actual force vector and $u_i(\eta)$ is the actual displacement vector at the point η on the surface S; $G_{ij}(\eta,\xi)$ and $F_{ij}(\eta,\xi)$ are the components of force and displacement vectors caused by the unit force acting in the direction x_j at the point ξ (the fundamental solution of Navier's equilibrium equations).

Algorithm of the combined method (CM)

In the following, the algorithm of the 3d analysis to solve an external boundary problem by applying the combined method (CM) is presented.

Let the displacement vector $u(\xi)$ be located in elastic space Ω bounded by a surface S, satisfying at $\xi \in \Omega$ the Navier's equilibrium equations and boundary conditions. The boundary conditions are

$$t_i(\xi) = \sigma_{ij}(\xi)n_i(\xi) = p_i(\xi), \quad \xi \in S_1;\ u_i(\xi) = u_{iS}(\xi), \quad \xi \in S_2;\ S_1 \cup S_2 = S, \tag{2}$$

where $n_i(\xi)$ are components of the unit outer normal on the surface S, $\sigma_{ij}(\xi)$ are the components of the stress tensor, $p_i(\xi)$ and $u_{iS}(\xi)$ are the components of force and displacement vectors.

A sub-space Ω_0 bounded by a surface S_0 is selected in the semi-infinite space Ω (see Fig. 1). The iteration procedure can be performed as follows. The first step in the process is to establish the boundary condition $u_i^{(0)}$ on the surface S_0 (assumed usually $u_i^{(0)} = 0$). Then the boundary problem in the sub-space Ω_0 can be solved by applying the FEM to complete the boundary condition (Eq. 2) on the surface S. This allows using the Somigliana's integral formula (Eq. 1) to determine the first approximation $u_i^{(1)}$.

Generally, having (k-1)-th approximation

$$u_i(\xi) = u_i^{(k-1)}, \quad \xi \in S_0, \tag{3}$$

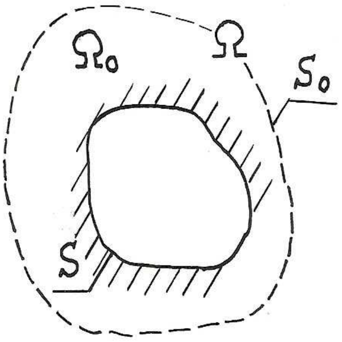

Figure 1. Selection of an analyzed domain in a space Ω

the k-th approximation $u_i^{(k)}(\xi), \xi \in S_0$ can be found by applying the Somigliana's integral formula (Eq. 1), using $u_i^{(k)}(\xi), \xi \in S_1$ and $t_i^{(k)}(\xi), \xi \in S_2$ determined by applying the FEM analysis of the boundary problem in the sub-space Ω_0 with boundary conditions (Eq. 2,3). The process can be terminated when the difference between the successive approximations $u_i^{(k)}(\xi)$ and $u_i^{(k+1)}(\xi)$ $(\xi \in S)$ achieves the required accuracy ε

$$\max_{\xi \in S} \left| \frac{u_i^{(k+1)}(\xi) - u_i^{(k)}(\xi)}{u_i^{(k)}(\xi)} \right| < \varepsilon, \quad i = 1, 2, 3. \tag{4}$$

The following approach is used to integrate the first term in Eq. 1 which contains $t_i(\eta), \eta \in S$. The nodal forces P_i in the nodes ξ^l of a finite element ΔS in the right-hand side of the FEM equations can be treated as some integral characteristics of surface forces $t_i(\eta)$:

$$\sum_{l=1}^{3} P_i(\xi^l) = \int_{\Delta S} t_i(\xi) dS. \tag{5}$$

An approximation is assumed:

$$\int_{\Delta S} t_i(\xi) G_{ij}(\xi, \eta) dS(\xi) \approx \sum_{i=1}^{3} P_i(\xi^l) G_{ij}(\xi^l, \eta). \tag{6}$$

In a strict sense, this formula (Eq. 6) is not a quadrature formula but it provides reasonable accuracy as an alternative of numerical differentiation in the FEM solution.

The developed software based on the implementation of this algorithm has been tested in a series of the model problems. They include external boundary problems with self-balanced and non-self-balanced external loads applied to a sphere-shaped concavity, such as uniform pressure, two self-balanced point forces and one point force applied symmetrically or non-symmetrically. The given problems were also analyzed by applying the FEM in order to compare the results. The FE meshes were different due to the fineness and the number of finite element rows. In general, the accuracy of the CM results even with less number of elements in the FEM modeling was higher than for the FEM. Apart from that, the displacements obtained by the CM for the case of non-self-balanced external loads include the rigid-body components in comparison with those determined by applying the FEM. This feature of the CM may be considered as the opportunity to provide more accurate results in such cases.

In addition, testing of the developed approach was carried out in the case of the Boussinesq problem, i.e. when the point force is applied to a half-space. Since in this case the space Ω has the infinite boundary surface S, it means the integration over the infinite surface in Eq. 1. In the numerical analysis the infinite space was reduced to attain a finite computational space Ω_0 (semi-sphere) with a circular boundary S_*. The radius of the sphere for the CM analysis was taken five times smaller than in the FEM analysis. The accuracy of the CM and the FEM results could be compared due to the availability of the exact solution. The error in the evaluation of the vertical displacement occurred in 4-20% when the CM was applied and in 30-65% for the FEM results (especially at the points of the boundary surface), even when the additional error of the CM caused by the truncation of the range of the integration in Eq. 1 was found. The solution of the Boussinesq problem is frequently used to perform the stress-strain analysis of structures when the foundation displacements and soil subsidence are considered [18-21]. Therefore, the application of the CM might be reasonable to solve similar problems.

Double-boundary algorithm of the combined method (DCM) for an inhomogeneous medium

The algorithm of the combined method was modified for the case of an inhomogeneous medium on the assumption that the inhomogeneous medium may have a substantial influence on the strain-stress state of the structure within a limited surface L. The remaining space Ω_L can be considered as a homogeneous one. A surface S_0 is selected in the space Ω_L (see Fig. 2) and a closed sub-space Ω_0 bounded by surfaces S and S_0 is considered.

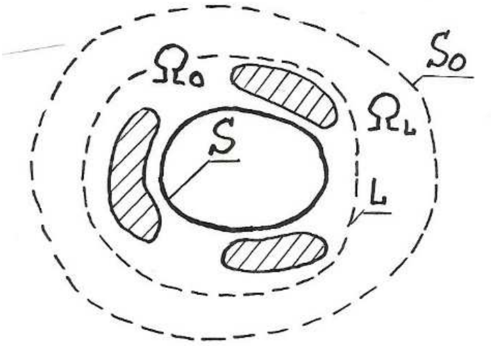

Figure 2. Scheme of the inhomogeneous space selection for analysis

The first step in the process is to establish the boundary condition $u_i^{(0)}$ on the surface S_0 (usually $u_i^{(0)} = 0$). Then the boundary problem in the sub-space Ω_0 can be solved by applying the FEM to define the boundary condition (Eq. 2) on the surface L. The domain of the integration in Eq. 1 is the boundary of a homogeneous region, consequently it will be the surface L instead of the surface S. Then the boundary problem in the sub-space Ω_0 is solved by applying the FEM to define the second approximation $u_i^{(2)}$ on the surface L and so far. It is a double-boundary algorithm of the combined method (DCM).

In Eq.1 the values of stress and strain in the modified algorithm are calculated by the FEM approximately, while in the original algorithm the part of them is defined by Eq.2. That is why, designing the FE mesh has to be more careful to decrease the error.

The developed software based on the implementation of the DCM algorithm has been tested in stress-strain analysis of a sphere-shaped cavity surrounded by a spherical layer with physical properties different from those in the surrounding space under the internal uniform pressure. The dimension of the analyzed domain for the CM analysis was 3 times less in comparison with that for the FEM analysis. The accuracy of the CM and the FEM results was comparable. Both methods resulted in partly underestimated stresses and strains compared to the results of an exact solution due to insufficient dimensions of the considered domain.

Strain analysis of a rock massif with a cavity on the boundary of rock layers

The underground non-reinforced caverns subjected to an internal pressure or any other loads are often used in construction and mining, for gas or oil storages or for waste dumping. The large-sized cavern is often located in an inhomogeneous medium because of the complicated structure of the rock formation. The analysis of the stability of such caverns should be made taking into account the specifics of the layered rock structure [22-24, 35, 36]. Also for the land surface subsidence prediction and risk analysis of buildings, it is necessary to estimate the deformation of the rock mass caused by the presence of underground caverns. Consequently, the ground subsidence and horizontal displacement under building foundation are the important topics for the analysis [25-27, 30-32, 37-40].

For example, an underground closed cylindrical cavern subjected to internal pressure on the boundary of two rock layers was selected; a complete cohesion of the layers on the boundary was assumed (Fig. 3).

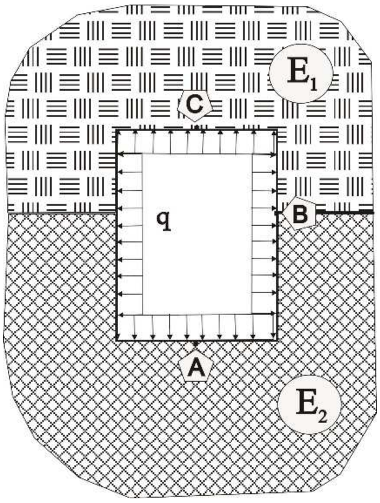

Figure 3. Underground cavern in an inhomogeneous rock massif

The layers' properties are characterized by the Young's modules, E_1 and E_2, with E_2 varied from E_1 to $10\ E_1$. The variety may be caused by the variability of rock crumbling even in a homogeneous massif. The influence of the relationship of Young's modules E_1/E_2 on the stress-strain state of the massif was evaluated. The calculations were carried out by applying the DCM as well as the FEM to verify results. The modeled area for the DCM was two times smaller than that for the FEM. The DCM FE model had two layers where the first one Ω_L consisted of two different materials and the second one ($\Omega_0 \backslash \Omega_L$) had assumed average Young's modulus E_3. The FEM model had four layers consisting of two different materials. The intrinsic stress state of the massif caused by the weight of an upper rock was accounted for by the method of deleting loads. For this purpose the FEM analysis of the massif without the cavern under the action of vertical and horizontal mass forces corresponding to the depth of location was performed. Thus, the loads on the surface S to be deleted were determined as nodal forces and then equivalent forces opposite in sign to above deleted forces were used to define the boundary conditions.

The safety factors $K_\sigma = \sigma_u^*/\sigma$ were calculated in the inner layers of finite elements around the cavern. There $\sigma_u^* = 0.75\sigma_u$ is the weakened rock compressive strength diminished with respect to the standard rock compressive strength σ_u. As might be expected, the tangential stress σ_θ was the highest on the side surface of the cavern ($K_{\sigma_\theta} = 1.09 \div 1.43$) as well as the vertical stress σ_z on the upper and the bottom surfaces ($K_{\sigma_z} = 1.15 \div 1.71$ for the upper surface and $K_{\sigma_z} = 1.05 \div 1.15$ for the bottom one). The values of the safety factor for the sound layer with modulus E_2 were smaller than those for the weak one. They decreased accordingly the ratio E_1/E_2. Therefore, there are reasonable grounds to conclude that the volumes where $K_\sigma < 1.2$ should be reinforced.

Fig. 4, 5 illustrate displacements u_z at the points A, C and u_ρ at the point B compared with corresponding displacements in a homogeneous massif with Young's modulus E_1 for point A or with modulus E_2 for point C as functions of the ratio E_1/E_2. Fig. 4 shows that displacement u_{zC} of the point C in the weak rock volume is almost unaffected by increasing modulus E_2, while displacement u_{zA} of the point A in the sound rock volume decreases the irrespective of the changes of modulus E_2. The radial displacement $u_{\rho B}$ of the point B on the contact surface is smaller than that in a homogeneous massif with Young's modulus E_1. The displacement $u_{\rho B}$ is larger than that in a homogeneous massif with Young's modulus E_2 if $E_1/E_2 > 0.3$ because of the weak rock dominant influence. In case of $E_1/E_2 < 0.3$ displacement $u_{\rho B}$ decreases because of the strong rock influence.

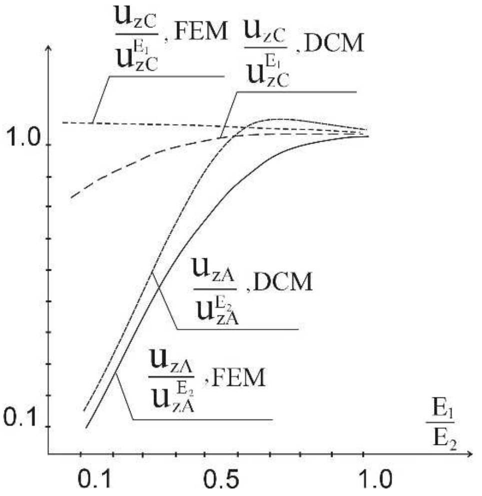

Figure 4. Vertical displacements of the upper and bottom surfaces of the cavern

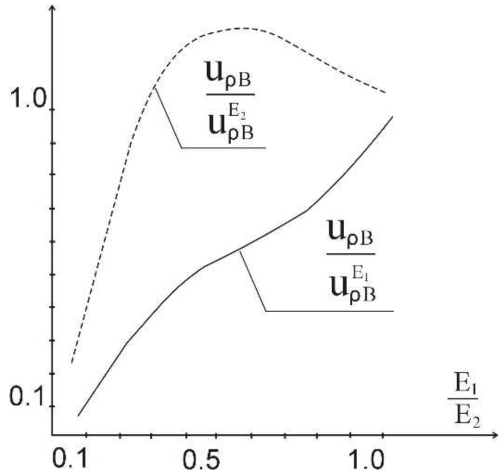

Figure 5. Radial displacement of the point on the cohesion surface

As is evident from the examples above (see Fig. 4, 5), all the results of the FEM and DCM analyses are mostly the same. So far, the stress-strain state of the massif can be estimated by applying the DCM. Besides, the considered domain and the number of finite elements necessary for the DCM analysis were far smaller than those for the FEM analysis.

Summary

The results of the performed analysis serve as evidence of the practicality of the method under the given study. The accuracy of the obtained results is mostly the same as the accuracy of the FEM' results even though a reduced computational space and number of finite elements are smaller when they are compared.

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Natalia Chernysheva,
89817713236; chernat0000@mail.ru

Galina Kolosova,
+7(921)5661321; gala.kolosova2013@yandex.ru

Leonid Rozin,
5526087; smitu@cef.spbstu.ru

Наталия Вячеславовна Чернышева,
89817713236; эл. почта: chernat0000@mail.ru

Галина Сергеевна Колосова,
+7(921)5661321; эл. почта:
gala.kolosova2013@yandex.ru
Леонид Александрович Розин,
5526087; эл. почта: smitu@cef.spbstu.ru

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