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Analysis of the buckling of spatial truss with cross lattice

Анализ прогиба фермы пространственного покрытия с крестообразной решеткой

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Ключевые слова: деформации; метод индукции; пространственная ферма; покрытие; крестообразная решетка

Abstract. The construction of a beam type spatial truss is proposed. The truss consists of three plane trusses with a cross lattice. The supports of the structure are modeled at the four corner points. The simple analytical dependence of the structure deflection on its size, load and a number of panels has been found for the case of an even number of panels. In the case of an odd number of panels the system is kinematically changeable, which is evident from the zero determinant of the system of equilibrium equations. The system of Maple computer algebra and the method of induction, previously proposed and developed by the author when solving the problems of planar and spatial trusses has been used. A nonmonotonic dependence of the deflection on the number of panels and the expected increase in stiffness at the increased truss height and unexpected decrease in stiffness at an increased base width have been found. The forces in some members of the truss change the sign depending on the parity of the number of panels in half of a span. Asymptotes of the solution are detected. The features of the solution allow optimizing the size of the structure.

Аннотация. Предлагается конструкция пространственной фермы балочного типа, состоящей из трех плоских ферм с крестообразной решеткой. Моделируется опирание конструкции на четыре угловые точки. Для случая четного числа панелей найдена простая аналитическая зависимость прогиба конструкции от ее размеров, нагрузки и числа панелей. При нечетном числе система кинематически изменяема, что явствует из равенства нулю определителя системы уравнений равновесия. Использована система компьютерной математики Марle и метод индукции, ранее предложенный и развитый автором в ряде задач о стержневых плоских и пространственных фермах. Обнаружен немонотонный характер зависимости прогиба от числа панелей, ожидаемое увеличение жесткости при увеличении высоты фермы и неочевидное ее снижение при увеличении ширины основания. Усилия в некоторых стержнях фермы меняют знак в зависимости от четности числа панелей в половине пролета. Найдены асимптотические свойства решения. Выявленные особенности решения позволяют оптимизировать размеры конструкции.

Introduction

In real projects the designs are rarely used as statically determinate systems, as in a statically determinate model of a truss there are rods and a pivot, swivel rods, that probably exist only in theoretical concepts. The model of the rod connections in the truss is sufficiently accurate but significantly simplified. However, the calculation of statically determinate planar and spatial structures has not lost its relevance yet. Firstly, the method of forces can be used for disclosure redundancies, and secondly, it can be successfully used as a test for many approximate methods, including finite element method. In addition, analytical solutions allow us to identify some characteristic features of the system, for example, the cases of the kinematic degeneracy, as it will be shown, in particular, in the present work. The problem of finding a new statically determinate beam structures is studied in [1, 2], some specific schemes of spatial trusses and approaches to their calculation (mainly numerical) are considered in [3-6]. Up-to-date problems of optimization of spatial trusses are discussed in [7-10]. The monograph [11] is devoted to theoretical and experimental study of regular spatial rod systems and the methods of their optimization. Spatial lattice

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structures, composed of a flat truss are studied in [12]. The examples of analytical and numerically-analytical solutions of the problems of elasticity feature in the Maple computer algebra system [13] are given in [14, 15], and the flat ones are given in [16-20].

In the present work, the aim is to obtain a mathematical model of one statically determinate truss and identify its features. To achieve this purpose an algorithm for finding the forces in the bars of the truss in a symbolic form will be built, and a General formula for the deflection of the structure under the action of various loads will be derived. One of the main difficulties in solving this problem is to generalize the formulas to arbitrary number of panels using the method of induction.

The design scheme

The coating consists of three plane trusses with a cross lattice, joined at their long sides (Fig. 1, 2). The side panels have an identical look. We take an even number of panels: n=2k. At the midspan the design contains articulated rod circuit (highlighted in the figure), composed of three rods. The truss rests on the three pillars: spherical hinge, cylindrical, and one vertical rod support. The truss consists of $n_s = 9(n+1)$ elements, including six support rods and $n_u = 3(n+1)$ nodes. Given that in the method of cutting nodes there can be three equilibrium equations in projections for each node, the system of equilibrium equations is close and the construction is statically determinate.

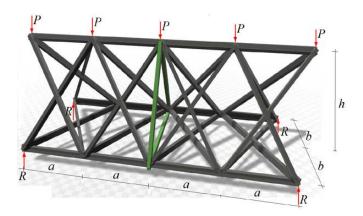


Figure 1.Truss, n = 4

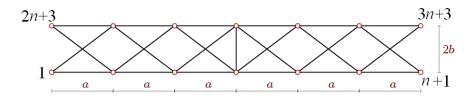


Figure 2. Lower truss belt and the number of supported nodes, n = 6

To set the geometry of the truss, let us introduce the coordinate system with the longitudinal axis x, transverse y and vertical z. We put the original structure in a spherical hinge. To find the guides of the cosines by cutting out nodes we will need their coordinates:

$$\begin{split} \overline{r_i} &= [x_i, y_i, z_i] = [(i-1)a, 0, 0], \\ \overline{r_{i+n+1}} &= [x_i, b, h], \ \overline{r_{i+2n+2}} = [x_i, 2b, 0], \ i = 1, ..., n+1. \end{split}$$

Bearing, the spherical hinge, is modeled by three rods attached to node 1. The coordinates of the ends of the bars fixed on the base, are found as:

$$\overline{r}_{m+1} = [x_1, y_1, z_1 - 1], \quad \overline{r}_{m+2} = [x_1, y_1 - 1, z_1], \quad \overline{r}_{m+3} = [x_1 - 1, y_1, z_1], \quad m = 3n + 3.$$

A cylindrical joint is modeled by two rigid rods fixed in the node number 2n + 3 (Figure 2):

$$\overline{r}_{m+4} = [x_{2n+3}, y_{2n+3}, z_{2n+3} - 1], \overline{r}_{m+5} = [x_{2n+3} - 1, y_{2n+3}, z_{2n+3}].$$

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The vertical support rod attached to the node 3n + 3:

$$\overline{r}_{m+6} = [x_{3n+3}, y_{3n+3}, z_{3n+3} - 1].$$

The order of connection of the rods and nodes is set by the conventional vectors \overline{q}_i , $i=1,...,n_s$ with an arbitrary choice of direction. The chosen direction of the rod-vectors does not affect the force and signs of forces in the rods. The grating vectors (i=1,...,n) have the following form:

$$\begin{split} \overline{q}_i &= [i, i+n+2], & \overline{q}_{i+n} &= [i+1, i+n+1], \\ \overline{q}_{i+2n} &= [i, i+2n+3], & \overline{q}_{i+3n} &= [i+1, i+2n+2], \\ \overline{q}_{i+4n} &= [i+1+n, i+2n+3], & \overline{q}_{i+5n} &= [i+n+2, i+2n+2]. \end{split}$$

The longitudinal bars of the lower and upper belt are calculated as:

$$\overline{q}_{i+6n} = [i, i+1], \ \overline{q}_{i+7n} = [i+n+1, i+n+2], \overline{q}_{i+8n} = [i+2(n+1), i+2n+3].$$

Components of vectors of secondary circuit are calculated as:

$$\overline{q}_{9n+1} = [k+1, n+k+2], \ \overline{q}_{9n+2} = [2n+k+3, n+k+2], \overline{q}_{9n+3} = [2n+k+3, k+1].$$

Support bars are encoded in the vectors:

$$\overline{q}_{9n+3+j} = [1, m+j], \quad j = 1, 2, 3,$$

$$\overline{q}_{9n+6+j} = [2n+3, m+3+j], \quad j = 1, 2,$$

$$\overline{q}_{9n+9} = [m, m+6].$$

Method and Solution

Algorithm of composing the system of equations by the method of cutting out nodes is based on the calculation of the guides of the cosines of the force calculated at the given coordinate, and making the entries in the matrix G. Equilibrium equations are reduced to the system, which we write in matrix form

$$G\overline{S} = \overline{T}^{(j)},$$
 (1)

where \overline{S} – the vector of forces in rods, j=P,1; $\overline{T}^{(P)}$ – vector of loads, $\overline{T}^{(1)}$ – the vector of loading of the system corresponding to a single force.

The components of the vector of loads with numbers 3i-2 correspond to the directions of the forces along the axis x, those with numbers 3i-1 – along the axis y, and with the numbers 3i – along the axis z, with $i=1,\dots,n_u$. For the equations of equilibrium we will require the projection of the rod-vectors on the coordinate axes and their lengths:

$$l_{x,i} = x_{q_{i,1}} - x_{q_{i,2}} , \ l_{y,i} = y_{q_{i,1}} - y_{q_{i,2}} , \ \ l_{z,i} = z_{q_{i,1}} - z_{q_{i,2}} , \ \ l_i = \sqrt{l_{x,i}^2 + l_{y,i}^2 + l_{z,i}^2}$$

The matrix guides of the cosines of $\,G\,$ have the following components:

$$\begin{split} G_{3q_{i,1}-2,i} &= l_{x,i} \ / \ l_i, \ G_{3q_{i,1}-1,i} = l_{y,i} \ / \ l_i, \ G_{3q_{i,1},i} = l_{z,i} \ / \ l_i, \\ G_{3q_{i,2}-2,i} &= -l_{x,i} \ / \ l_i, \ G_{3q_{i,2}-1,i} = -l_{y,i} \ / \ l_i, \ G_{3q_{i,2},i} = -l_{z,i} \ / \ l_i \,. \end{split}$$

The load vector, uniformly distributed on the upper zone, has a form: $T_{3i}^{(P)}=-1$, i=n+2,...,2(n+1). By setting a value of the vertical force on the fourth corner of the truss numbered n+1, which does not have the support, it is possible to simulate more natural and symmetrical loading of the truss, thus avoiding the problem of statically indeterminate truss. In fact, the value

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 $T_{3(n+1)}^{(P)}=(n+1)/4$ is the solution to this problem under the assumption of equal stiffness supports. The remaining components of the vector $\overline{T}^{(P)}$ are equal to zero. To determine forces from a single load applied to the node $n_1=3k+2$, we have the following non-zero component: $T_{3n_1}^{(1)}=1$ and a component $T_{3(n+1)}^{(1)}=-1/4$ which simulates a vertical base in the corner n+1 joint of the lower belt.

The solution of the system of linear equations (1) is found in the symbolic form using Maple algebra: $\overline{S} = G^{-1}\overline{T}$, where $G^{-1} = 1/G$ is the inverse matrix. The results of the program are analytical expressions for the forces in the rods of the truss. For the calculation of deflection we use Maxwell – Mohr's formula:

$$\Delta = \sum_{j=1}^{n_s} \frac{S_j s_j l_j}{EF},$$

where E is the modulus of elasticity of the rods, F — the cross-sectional area of the rods (same for the whole structure), l_j and S_j — the length of the j-th rod and the force in it from the action of a given load, s_j — a single vertical force applied at Midspan in the upper zone. The summation is over all the rods of the truss, except for the reference, which is assumed rigid. Let us now introduce the designation $\tilde{\Delta} = \Delta E F / P$. An induction method to get the relative deflection of the top node in the truss is used:

$$\tilde{\Delta} = \frac{A_k a^3 + B_k b^3 + C_k c^3 + D_k d^3 + Q_k q^3}{16h^2},$$
(2)

where

$$c = \sqrt{a^2 + b^2 + h^2}, \ d = \sqrt{a^2 + 4b^2}, \ q = \sqrt{b^2 + h^2},$$

$$A_k = 5k^4 + (1 + 6(-1)^k)k^2 + (4 + 3(-1)^2)k + 1 - (-1)^k,$$

$$B_k = 8(2k+1), \ C_k = 6k^2 + 4k + 1 - (-1)^k,$$

$$D_k = k(2k+1), \ Q_k = 4(1 + (-1)^k)(k+1).$$

The coefficient A_k is obtained in Maple algebra by the synthesis of the following sequence: 3, 122, 365, 1420, 3007, 6774, 11769, 20984,..., 329584. To get the pattern, it was necessary to calculate a sequence of 16 trusses. To find the general term of the sequence the function $rgf_findrecur$ from the package genfunc was used which is followed by a recurrence equation:

$$A_k = 2A_{k-1} + 2A_{k-2} - 6A_{k-3} + 6A_{k-5} - 2A_{k-6} - 2A_{k-7} + A_{k-8}$$
(3)

Note that this function only works with an even sequence number. The solution of equation (3) was found by rsolve operator. The verification decision will be made on trusses with an arbitrary number of panels with the help of the numerical solution carried out with the same program, but in a numerical mode.

Similarly, when uploading a truss by one concentrated force at the Central node we also get the solution for the deflection of the form (2), but with simpler coefficients:

$$A_k = k(4k^2 + 1 + 4(-1)^k), B = 8, C = 4k, D = k, Q = 4(1 + (-1)^k)$$

In addition, with such a load the length of the sequence number of the truss that detects a pattern, is slightly shorter and equals to 12. It is well known that the time of analytical transformations is noticeably larger than in the numerical calculation, and the number of panels increases approximately in geometric progression with a factor of 1.5 (that is tested empirically). In some cases this decline in the length of the sequence is crucial. Thus, if we calculate the four truss with a sequentially increased number of panels in half the span from 1 to 4 in the analytical form, the time of transformation on the average computer is 8.8 C, for ten trusses this time is equal to 178 C, and for sixteen trusses it takes 1024 or 17 min. This numerical example shows that the inductive method to obtain exact analytical solutions for trusses with a large number of panels can not be replaced by direct calculation of the

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structure by means of symbolic mathematics. Up-to-date computers, with the enhanced processor, just change the time scale of the account, but do not solve the problem of the growth time of transformation with an increasing number of cores.

Analysis

Let us consider the dependence of the deflection of the truss on the number of panels k for a fixed length of half the span L=2ka and a given total load $P_{sum}=(2k+1)P$. The deflection is attributed to the value of the total load $\tilde{\Delta}=\Delta EF/P_{sum}$. The corresponding curves (Fig. 3, dimensions in metres) show the expected results: with the increase of height h increases the rigidity of the structure. The dependence of the deflection of the width of the truss b (Fig. 4) is less obvious. With the increase of the size decreases the stiffness. Much more interesting is the fact that the deflection depends on the number of panels. In this structure, it is non-monotonic, due to "flashing" in terms like $(-1)^k$ of the coefficients of the solution. In addition, if you do not pay attention to the sharp drop of the curve at the beginning of the graph corresponding to unnaturally long panels (a=20m with a truss height h=5m or h=6m), we can see almost linear increase of deflection, which indicates the presence of an inclined asymptote. By the methods of Maple system (operator limit) it is easy to obtain the slope of the asymptotes: $\gamma=(3q^3+8b^3)/(16h^2)$ and the asymptote $\tilde{\Delta}=\Delta_0+\gamma k$ where $\Delta_0=(\rho q^3+5L^3+16b^3)/(32h^2)$. The coefficient ρ depends on the parity of the number of panels: $\rho=1$ for even k and $\rho=9$ for the odd one.

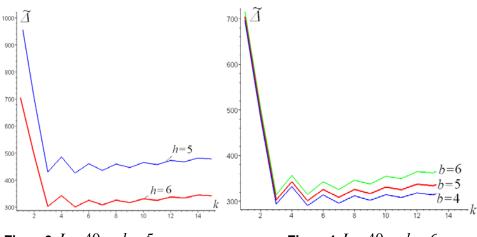


Figure3. L = 40 M, b = 5 M.

Figure 4. $L = 40 \, \text{M}, h = 6 \, \text{M}.$

The solution (2) under any load is easily generalized to the case where the rigidity of the rods of the zones and grids is different. We denote the stiffness of the longitudinal bars bottom and top belt length a by 1, lattice rods of the lower belt length by d-2, side bars – by 3, the rods of the average contour length by b and b and b and b and b and b and b are stiffness of the rods we use the reduced stiffness: b and b are b and b and b and b and b are stiffness of the rods we use the reduced stiffness:

$$\tilde{\Delta} = \frac{A_k \mu_1 a^3 + B_k \mu_4 b^3 + C_k \mu_3 c^3 + D_k \mu_2 d^3 + Q_k \mu_4 q^3}{16h^2}$$

Another interesting and unusual design feature is the sign change of the forces at the secondary terminals of the circuit when the parity of the number of panels is changed. This follows from the analytical solution. So, the force at the bottom rod of the middle loop is the following: $S_{m+3}^{(P)} = (-1)^k (2k+1)b/(2h)$, with the forces in the side bars of the loop: $S_{m+1}^{(P)} = S_{m+2}^{(P)} = -(-1)^k (k+1)q/(2h)$. This fact should be taken into account in the calculation of cores for strength and stability.

In conclusion, let us note one more feature of this design. For the asymmetric version of the truss n = 2k - 1, when a triangular circuit cannot be placed in the plane of symmetry, it turns out that the

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determinant of the matrix of the system of equilibrium equations is equal to zero. It can be missing in numerical analysis, which leads to erroneous solution. A similar effect was previously detected in spatial axisymmetric structures [21] and a lattice flat truss [22].

Results and Discussion

One of the objectives of the present work is to describe the schemes of a statically determinate truss and identify its characteristics. Earlier [18, 21] the spatial patterns of trusses with cross-shaped grating panels were not calculated in the analytical form. The compact and precise formulas for determining the vertical deflection of the truss under the concentrated and uniformly distributed forces across the load balancing nodes are given. It is noted that in the case of one concentrated force, the sequence of solutions made by the method of induction, from which we can deduce the general formula for an arbitrary number of panels, is somewhat shorter than for a distributed load. Unlike in the solutions [3-12] the formulae are obtained due to the different stiffness in the rods of the truss. The effect of the parity of the number of panels in half the span on the signs of forces in the middle path is observed and the degeneracy of the system with an odd number of panels in the span truss is found.

Conclusion

- 1. A mathematical model of spatial statically determinate truss schemes is obtained.
- 2. Formulas for the deflection of the structure for any number of panels are given.
- 3. A critical feature of the studied schemes, degenerated with an odd number of panels is detected.
 - 4. The asymptotic solutions that are not available by numerical calculations [3–12] are found.
- 5. The given algorithm can be applied to develop calculation formulas for statically indeterminate structures of the regular type and for the analysis of inelastic systems.

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