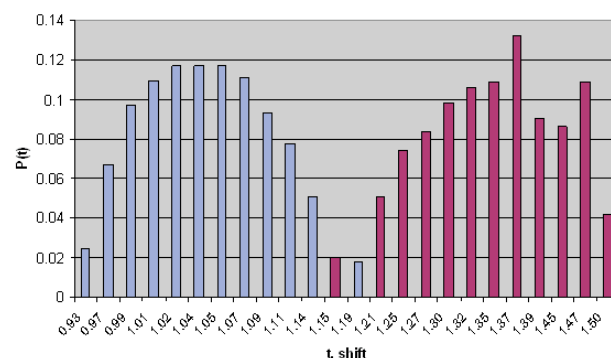
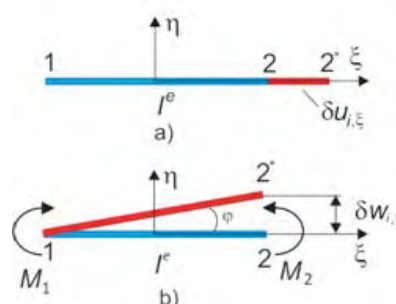
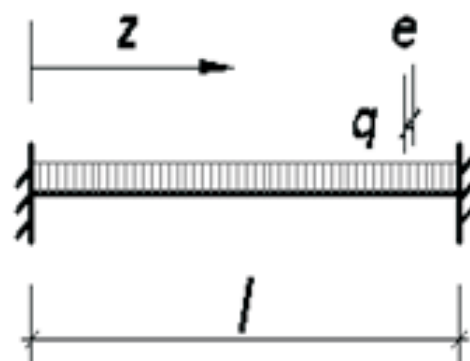
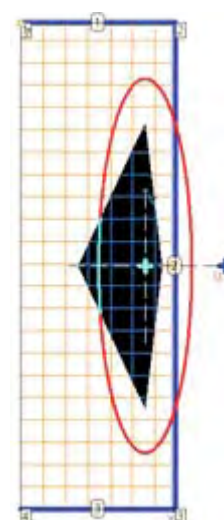
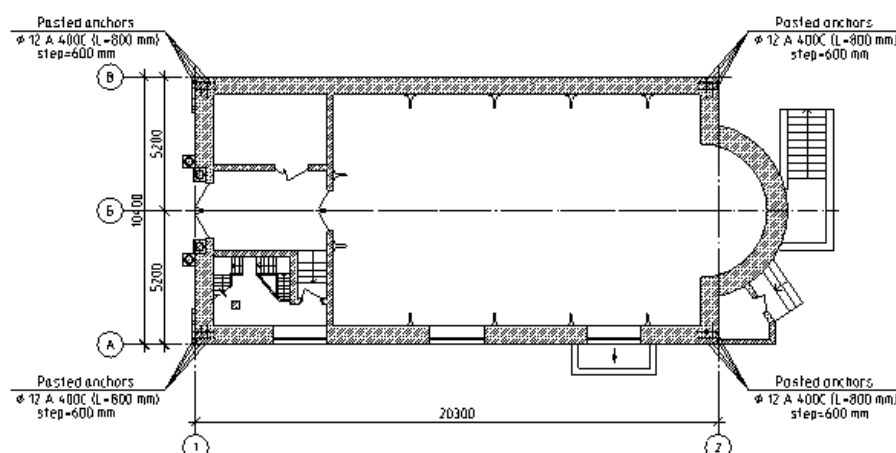


Concrete class B10 Concrete class B15 Concrete class B20



method PERT Universal method





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<b>БС-01</b>	«Безопасность и качество выполнения геодезических, подготовительных и земляных работ, устройства оснований и фундаментов»	1,2,3,5
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## The load-bearing capacity of hanging piles by the strength criterion of a pile or soil material

## Несущая способность висячих свай по критерию прочности материала сваи или грунта

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**Key words:** pile foundation; load-bearing capacity; probability theory; possibility theory; the combined methods of reliability assessment; buildings; construction; civil engineering

**Ключевые слова:** свайный фундамент; несущая способность; теория вероятности; теория возможности; комбинированные методы оценки надежности; сооружения; строительство; гражданское строительство

**Abstract.** The article describes various methods of assessment of load-bearing capacity (reliability) of a pile as a main element of pile foundation. It is shown that deterministic, probabilistic and possibilistic methods have a number of advantages and limitations. An actual task is to develop new approaches to assessment of foundations load-bearing capacity. The combined method providing the optimal assessment according to the given examples is developed in the article. Some features of the proposed methods are in the probability and possibility theories application to account uncertainty or incompleteness of initial data in quantifying the reliability of a pile. Presented in the article methods for a quantitative assessment of single piles reliability can be used for more complex computational models, including multielement pile foundations and more complex models of soil foundations. These methods have not been applied to piles reliability research so far and the article pre-sented is a pioneering one and has no analogues known to the authors.

**Аннотация.** В статье рассмотрены различные методы оценки несущей способности (надежности) свай как основного элемента конструкции свайного фундамента. Показано, что детерминированный, вероятностный и возможностный методы имеют ряд преимуществ и ограничений. Актуальной является задача развития новых подходов к оценке несущей способности фундаментов. В статье развит комбинированный метод, дающий оптимальную оценку, что продемонстрировано на примерах. Особенность предлагаемого метода заключена в совместном применении теорий вероятности и возможности для учета неполноты или неопределенности исходных данных при количественной оценке надежности свай. Представленные в статье методики количественной оценки надежности одиночных свай могут быть использованы для более сложных расчетных моделей, в том числе многоэлементных свайных фундаментов и более сложных моделей грунтовых оснований.

### Introduction

One of the main indicators characterizing the properties of "a pile-soil environment" system is load-bearing capacity, i.e., property of the pile to resist external load in the absence of further punching. In design practice to determine the load-bearing capacity various methods based on the use of test and experimental data are applied in the deterministic setting. Estimates of load-bearing capacity are made

Иванова Т.В., Альберт И.У., Кауфман Б.Д., Шульман С.Г. Несущая способность висячих свай по критерию прочности материала сваи или грунта // Инженерно-строительный журнал. 2016. № 7(67). С. 3–12.

using a number of empirical coefficients input into the calculations to consider the working conditions of the pile, immersion techniques, the strength characteristics of the pile material, the pile grillage structure [1–4, 9, 10, 15, 17, 22, 26, 29]. Some examples of such assessments, based on the static probing results, are set out in [27, 28]. The significant scatter of input data can dramatically affect the results obtained, which can be refined by methods described below.

An actual task is to develop new approaches to assessment of foundations load-bearing capacity. The combined method providing the optimal assessment according to the given examples is developed in the article.

Some features of the proposed methods are in the probability and possibility theories application to account uncertainty or incompleteness of initial data in quantifying the reliability of a pile.

In the works of other authors on this subject [2–4, 9, 10, 26] there is no possibility of taking into account the incompleteness and uncertainty of initial data when assessing the load-bearing capacity of a pile. In this paper this problem is solved by applying the theory of probability and possibility theory to take account of uncertainty or incompleteness of initial data in quantifying the reliability of a pile.

## Methods

A condition providing the required load-bearing capacity of a pile by the method of limit states is expressed in the form of inequality:

$$N_d < \frac{1}{k_1} F_d, \quad (1)$$

where  $N_d$  is the axial load on the pile,  $F_d$  – ultimate bearing capacity,  $k_1$  – safety factor.

Obviously, the result of substituting in the expression (1) known values of the axial load on the pile and ultimate bearing capacity allows us to formulate one of the two alternative, mutually exclusive judgments about the reliability of the pile. Thus the basic feature of the initial data – their random nature is ignored; both as regards the parameters of the external action and the characteristics of soil and the pile material. This can lead to the wrong conclusion on the reliability of the pile. A more reasonable estimate of reliability which takes into account the random nature of the original data and has a quantitative expression is a technique based on a probabilistic approach, possibility theory, the theory of fuzzy sets as well as their combinations [6–8, 10–14, 19–21, 23, 24].

As a simple example of the use of such methods, consider a pile reliability quantitative assessment based on the strength criteria for piles of any type and based on the bearing capacity of soil foundation in the case of a friction pile.

### *The assessment of a pile load-bearing capacity by the strength criterion for the pile material*

In the deterministic approach the condition of providing the necessary strength of a pile material is as follows [9]:

$$\frac{1}{k_1} (R_b A_b + R_s A_s) > N_d, \quad (2)$$

where  $R_b, R_s$  are the resistance of concrete and reinforcement accordingly,

$A_b, A_s$  – cross-sectional areas of the pile and the reinforcement,

$N_d$  – axial load.

If the parameters  $R_b, R_s, N_d$  are considered as normally distributed random variables with known probability characteristics – mathematical expectations  $m(N_d), m(R_b), m(R_s)$  and the standard deviations  $\sigma(R_b), \sigma(R_s), \sigma(N_d)$ , the probability of failure-less operation of the pile  $P$  is determined by the relation

$$P(F < 0) = \frac{1}{2} [1 + \Phi(-m(F)/\sigma(F))], \quad (3)$$

where:  $\Phi$  – the function of normal distribution (Laplace),

$$F = N_d - (R_b A_b + R_s A_s) \quad (4)$$

In general, when dealing with such problems function  $F$  is non-linear and the reliability assessment is carried out by various methods (linearization, statistical tests, etc.). In this example the function of random parameters  $F$  is linear of  $R_b, R_s, N_d$  and has a normal distribution and its characteristics are determined in accordance with the rules laid down in the guidelines on probability theory for linear functions of normally distributed variables:

$$m(F) = mN_d - [m(R_b A_b) + m(R_s A_s)], \quad (5)$$

$$\sigma(F) = \sqrt{\sigma^2(N_d) + \sigma^2(A_b R_b) + \sigma^2(R_s A_s)} \quad (6)$$

By setting different values of the mathematical expectations of axial load  $m(N_d)$  we can obtain the probability of failure – free operation of the pile by material strength criterion.

It is known, however, that the correct use of probabilistic methods is associated with the presence of sufficiently complete statistical information on the basic random variables included in the mathematical models of limit states and loads. The lack of sufficient initial statistical information is the main reason of the emergence of the need to look for other (non-probabilistic) methods for uncertainties account. Since the 60s of the last century the various theories for the formal description of uncertainties began to develop intensively. In particular, the possibility theory developed by American mathematician Zadeh [12] and its further development in Dubois and Prada work [7, 13] were fairly widespread.

### *The result of its application is the interval estimate of reliability.*

Note that interval estimations can be obtained on the basis of a probabilistic approach as well, but the interval resulting from this analysis quite roughly estimates the required probability in most cases [8, 14].

Also a variety of combined techniques that allow the most complete account of the available information on the parameters are proposed. V.S. Utkin and N.S. Galaeva [25] proposed a method in which, depending on the availability of information, some variables are considered as random (in terms of probability theory) and others – as fuzzy ones (in terms of possibilities theory). For example, if  $S$  is a fuzzy variable and  $R$  is a random variable changing according to the normal distribution with the density function

$$\rho_R(x) = \frac{1}{\sqrt{2\pi}\sigma_R} \exp\left[-\frac{(x-m_R)^2}{2\sigma_R^2}\right], \quad (7)$$

then the expressions for the lower  $\underline{P}$  and upper  $\bar{P}$  boundaries of the reliability interval look like

$$\begin{aligned} \underline{P} &= \int_{a_S}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_R} \exp\left[-\frac{(x-m_R)^2}{2\sigma_R^2}\right] \left\{1 - \exp\left[-\frac{(x-a_S)^2}{b_S^2}\right]\right\} dx, \text{ with } \underline{P}_S(x) = 0 \text{ at } x \leq a_S; \\ \bar{P} &= \int_0^{a_S} \frac{1}{\sqrt{2\pi}\sigma_R} \exp\left[-\frac{(x-m_R)^2}{2\sigma_R^2}\right] \left\{\exp\left[-\frac{(x-a_S)^2}{b_S^2}\right]\right\} dx + \int_{a_S}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_R} \exp\left[-\frac{(x-m_R)^2}{2\sigma_R^2}\right] dx, \end{aligned} \quad (8)$$

where  $a_S$  – an average value and  $b_S$  – a fuzziness factor are the parameters of fuzzy value  $S$ .

## **Results and Discussion**

The estimations of the probability of failure-free operation of a pile based on the strength condition and made by the probabilistic and combined methods are discussed below. When using the combined

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method the pile bearing capacity was considered a random variable, the load – a fuzzy variable. The comparison of the results of two approaches is given in Table 1.

The evaluation of piles reliability with regard to the specific construction site does not introduce any fundamental changes in the technique and procedure of calculation under review.

As the initial data we will use the data of the example given in [4]:

the pile length – 12 m

the cross section is a square which side is 0.35 m

concrete class (options) – B10, B15, B20

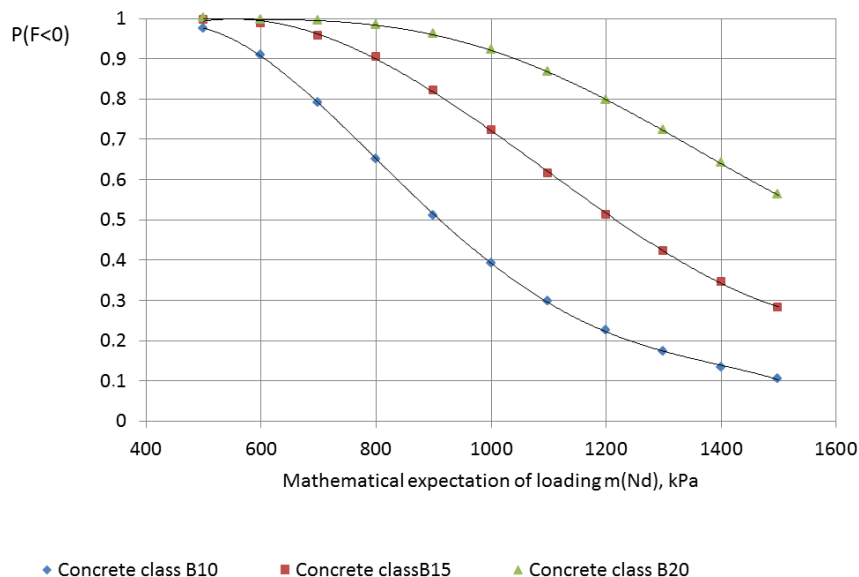
reinforcement class – A1

cross sectional area,  $m^2$  – 0.1225

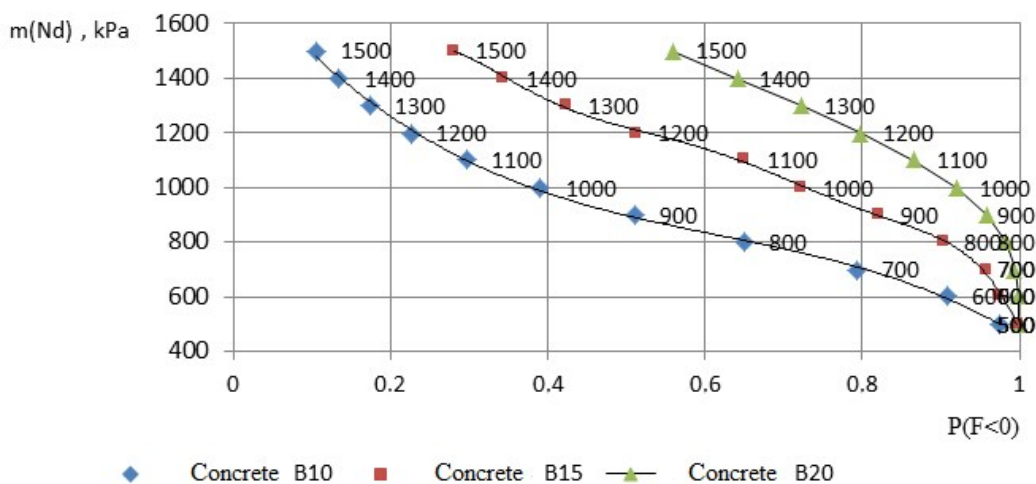
axial load variants (mean values), kPa – 500–1500

reinforcement cross-sectional area  $m^2$  –  $8.042 \cdot 10^{-4}$ .

The calculation results are shown in Figures 1 and 2 in the form of dependences of probability of ensuring pile strength on the axial load average value (mathematical expectation) for the three types of concrete. The comparison of the results of probabilistic and combined approaches is given in Table 1.



**Figure 1. The probability of ensuring the strength of the pile by the material strength criteria (reinforced concrete). Reinforcement class A1, the number of reinforcing bars 4**



**Figure 2. A pile strength probability as a function of an average value of axial loading**

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**Table 1. Probabilistic and combined estimations of failure-free pile operation according to strength criterion**

$N_d, \text{kPa}$	Concrete B 10		Concrete B 15		Concrete B 20	
	Prob.	Comb.	Prob.	Comb.	Prob.	Comb.
500	0.971	$0.909 \div 1$	0.997	$0.990 \div 1$	1	$0.999 \div 1$
600	0.901	$0.720 \div 0.998$	0.987	$0.955 \div 1$	0.998	$0.994 \div 1$
800	0.648	$0.226 \div 0.970$	0.903	$0.718 \div 0.997$	0.981	$0.943 \div 1$
1000	0.390	$0.025 \div 0.853$	0.720	$0.340 \div 0.980$	0.918	$0.769 \div 0.997$
1200	0.230	$0.0 \div 0.644$	0.510	$0.094 \div 0.922$	0.794	$0.486 \div 0.988$
1400	0.140	$0.0 \div 0.442$	0.340	$0.015 \div 0.804$	0.641	$0.228 \div 0.962$

***The probability of ensuring required load-bearing capacity of a friction pile.***

It is known that the piles are classified on the basis of interaction with the soil environment into two groups:

- pile-pillars that rely on rocky ground, and perceive the external axial load due to front resistance of the rock;
- friction piles – resistance consists of two components:

a) resistance forces arising due to the friction between the outer surface of the pile and the soil environment and

b) front resistance force due to the interaction between the lower end of the pile and the soil environment [9]. The values of coefficients  $R, f_i$  as the functions of the depth of a pile penetration, soil types and some physicomechanical characteristics are presented in the same tables and in graphs in Figures 3, 4.

The method of estimation of reliability by the ground in a deterministic setting is based on the inequality:

$$F = N_d - \frac{1}{k_1} (R * A + u \cdot (\sum_i h_i f_i)) < 0, \quad (9)$$

where  $F$  – reliability function,

$N_d$  – the axial load on the pile,

$R$  – front resistance coefficient,

$A$  – cross sectional area of the pile,

$u$  – the cross-sectional perimeter,

$h_i$  – the depth of the  $i$ -th layer,

$f_i$  – lateral resistance coefficient of the  $i$ -th soil layer,

$k_1$  – safety factor.

The procedure for quantitative assessment of bearing capacity by the ground is carried out in the same sequence as in the previous example. Function  $F$  which determines the bearing capacity is linear regarding random parameters  $N_d, R, f_i$ . Assuming that all the random variables are normally distributed with known probability characteristics (mathematical expectations and dispersions) the expressions for them can be written as follows:

$$m(F) = m(N_d) - [(m(R * A) + u_1 h_1 m(f_1))] \quad (10)$$

$$(\sigma_F^2) = \sigma_{N_d}^2 + A^2 \cdot (\sigma_R^2) + (uh)^2 \cdot \sigma_f^2 + 2 \cdot r_{Af} \cdot uh \cdot \sigma_A \cdot \sigma_f, \quad (11)$$

where  $\sigma_{N_d}, \sigma_R, \sigma_f$  — standards for random variables,  $r_{Rf}$  — correlation coefficient for random variables  $R, f$ .

To estimate the degree of relation between random variables  $R, f$  in Figure 5 there is a dependence  $f(R)$ . You can see from the graph that the coupling equation with a sufficient degree of accuracy can be approximated by a linear dependence; therefore, a normalized correlation coefficient may be set equal to one.

To simplify the calculation the soil foundation is assumed to be homogeneous:  $i = 1$ ; in the case of inhomogeneous foundation the sum of terms of the form  $u \cdot h_i \cdot f_i$  should be calculated.

The calculation results are shown in Figure 6, which shows two dependencies: the probability of ensuring the bearing capacity by the pile material and by the foundation soil.

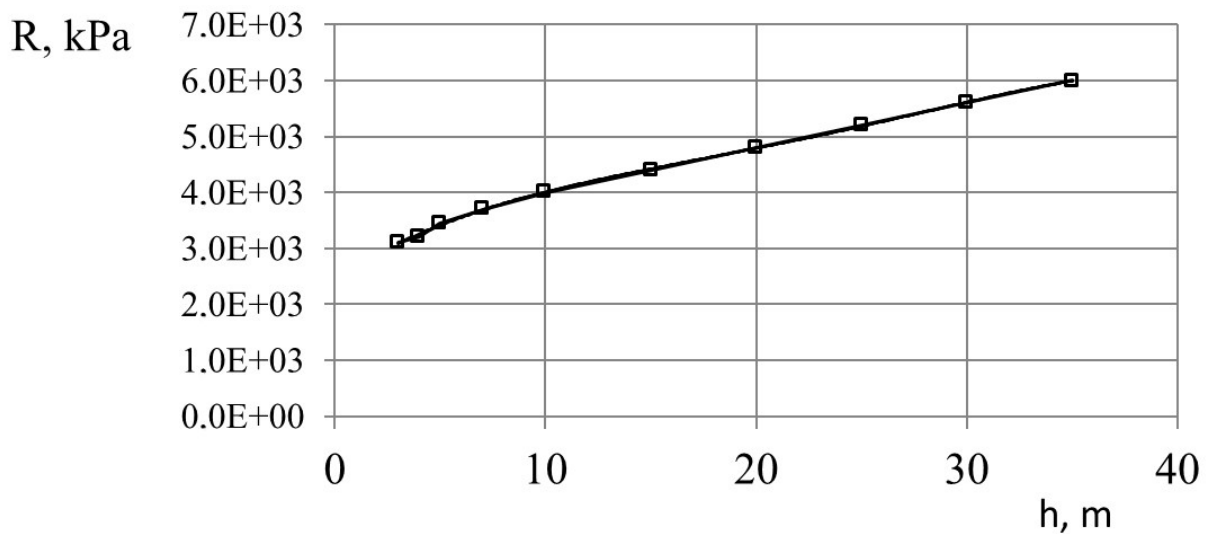


Figure 3. The dependence of the front resistance coefficient of the immersion depth of the pile

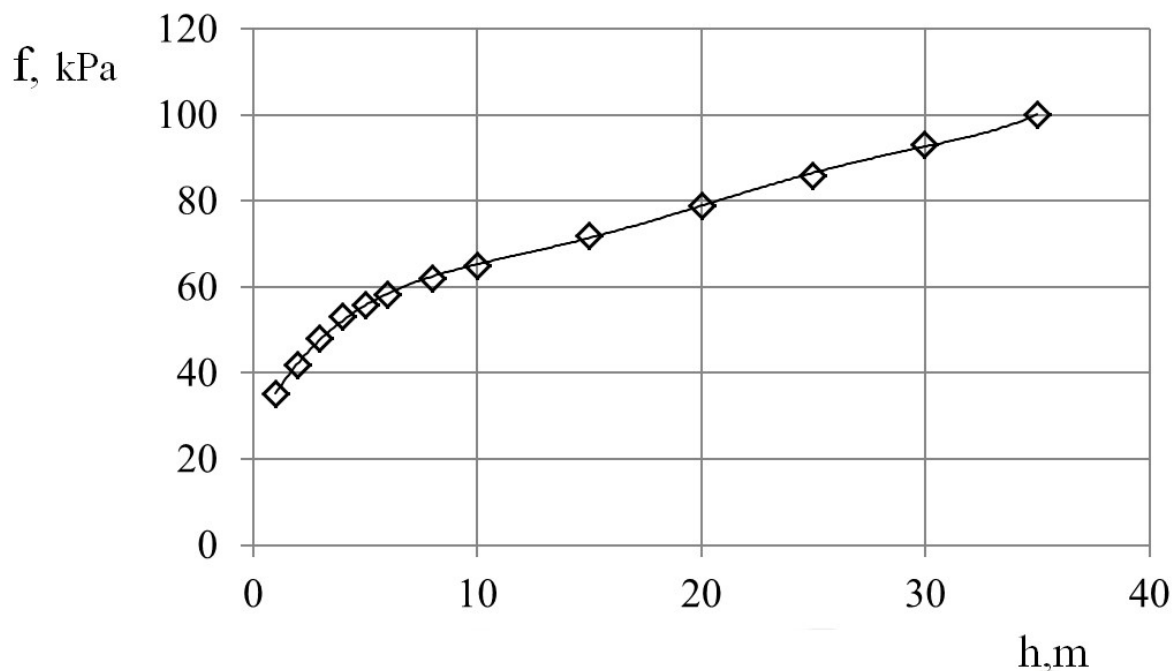


Figure 4. The dependence of the lateral resistance coefficient of the immersion depth of the pile

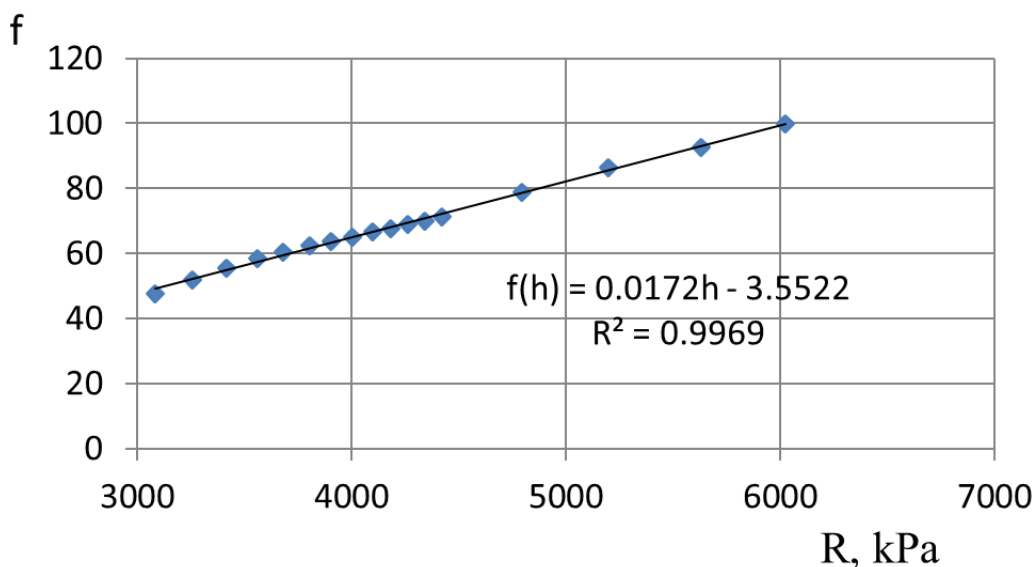


Figure 5. The relation between random variables

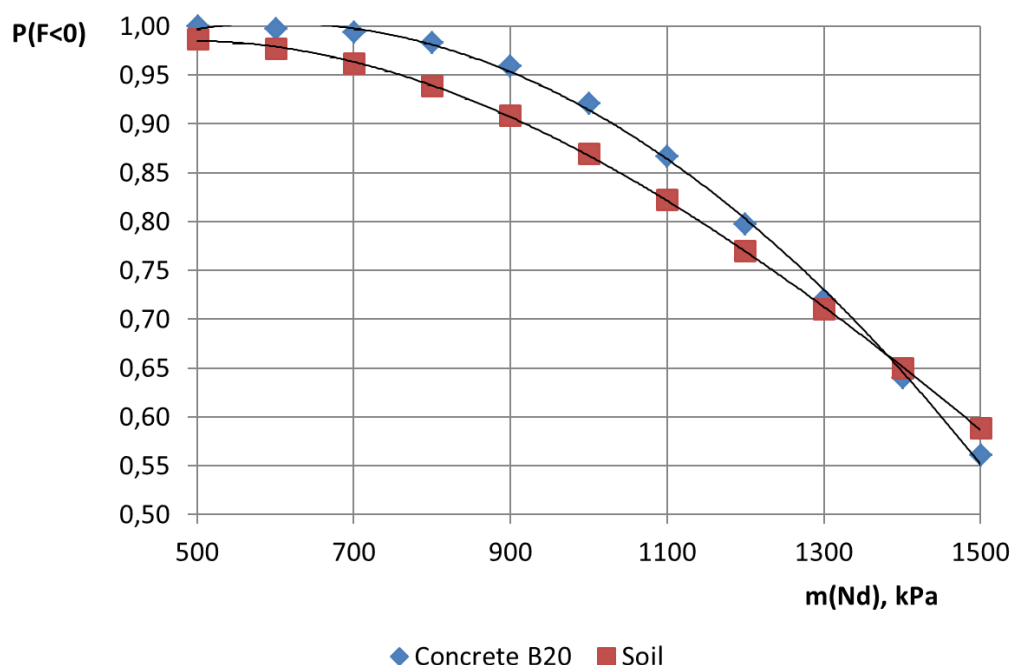


Figure 6. The probabilities of ensuring a friction pile bearing capacity by the ground and by the pile material

The graph in Figure 6 shows the dependences of the probability of ensuring a given level of reliability by pile material strength and by soil material. The graph shows that with the load increase the probability of reliable operation of the pile decreases both by the soil strength criterion and by the pile material strength criterion.

Table 2 shows the results of estimation by the foundation soil for a failure – free pile operation in two cases – in the first case loading and soil characteristics are represented by random variables and point probability estimation is given and in the second case loading is a fuzzy variable, and ground characteristics are random ones, i.e. a combined method giving the interval estimation is used. In all cases, the probabilistic estimations lie within the intervals.

**Table 2. Probabilistic and combined estimations of failure-free pile operation based on the foundation soil**

$N_d, \text{kPa}$	$P \text{ Prob}$	$P \text{ Comb.}$
500	0.987	$0.975 \div 0.996$
600	0.977	$0.953 \div 0.993$
800	0.938	$0.871 \div 0.985$
1000	0.869	$0.726 \div 0.970$
1200	0.770	$0.538 \div 0.944$
1400	0.65	$0.348 \div 0.905$

The comparison of pile load-bearing capacity calculations by the pile material and foundation soil allows for the following conclusion:

- from the two variants of load-bearing capacity estimations the bearing capacity by the soil is less favorable (with the exception of the medium axial loads of more than 1,400 kPa)
- the final load-bearing capacity estimation at adopted initial data is taken minimal.

The proposed method allows a quantitative assessment of the load-bearing capacity and reliability of a pile with incomplete information about the characteristics of the soil parameters and loads, as well as the considerable variation in the original data values. In the works of other authors such a possibility was not available until now.

The deterministic approach used by other authors' results in a qualitative assessment only. The correct application of probabilistic methods giving a point result cannot always be realized due to lack of the initial information. In this situation the authors use the combined method in which the parameters of the problem are presented as fuzzy values [7, 12] and which gives a reliable interval assessment (Table 2).

## Conclusion

Thus, in this article the technique of quantitative assessment of piles reliability under incomplete initial information about the parameters of mathematical models is presented.

Presented in the article methods for a quantitative assessment of single piles reliability can be used for more complex computational models, including multielement pile foundations and more complex models of soil foundations.

These methods have not been applied to piles reliability research so far and the article presented is a pioneering one and has no analogues known to the authors.

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## Organo-mineral additives based on oil and gas complex waste to building materials

### Органоминеральные добавки к строительным материалам на основе отходов газовой и нефтяной промышленности

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**Key words:** buildings; organo-mineral additives; oil-contaminated waste; spent sorbents; waste utilization; ecological safety; secondary resources; expanded clay

**Ключевые слова:** здания; органоминеральные добавки; нефтесодержащие отходы; отработанные сорбенты; утилизация отходов; экологическая безопасность; вторичные ресурсы; керамзит

**Abstract.** The use of waste and products of their utilization as secondary raw materials is promising and rational way for their disposal. The addition of waste to mixtures provides either high-quality products or improvement some of the physical and mechanical characteristics. Organo-mineral additives are produced by oil-contaminated waste and spent sorbents utilization meet environmental safety requirements and are suitable for use as an additive in building materials, for example, expanded clay and asphalt mixtures. The aim of the paper is to develop technology for producing organo-mineral additives that is aimed at the elimination of environmental pollution with wastes, their involvement into resource circulation and ensures rational nature management with reducing the consumption of reagents and receiving high quality ecologically safety products. It is established the minimum necessary quantity of quicklime (calcium oxide reagent) required to transfer viscous sludge in bulk and obtain organo-mineral additives. In order to achieve the most important technical parameters of building materials in particular expanded clay – a bulk density and strength, it is used various additives. Application of the complex additives can increase the swelling clay factor up to 3 times and get the required strength of concrete block with less energy consumption, which significantly reduces the cost of the products. It is carried out the research to develop formulations of puddle clay with organo-mineral additives and choose burning conditions for preparing expanded clay. It's established the optimal temperatures and a thermal presintering for producing products that meet the requirements of the standards, with minimal energy consumption. The impact of the percentage of organo-mineral additive on the coefficient of clay swelling is determined. In the process of burning poor swelling clay with using organo-mineral additive at temperature 1050 °C constructive quality factor is increased by 70–97 % in comparison with the expanded clay obtained from raw materials without additives. Proposed technology for production of expanded clay with organo-mineral additives allows getting high-quality environmentally friendly products and disposing in its composition oil and gasing complex waste, involving them into resource management, and satisfies best available techniques due to encouraging re-use of waste.

**Аннотация.** Использование отходов и продуктов их переработки в качестве вторичного сырья является перспективным и рациональным способом их утилизации. Добавление отходов в смеси обеспечивает и высокое качество продуктов, и улучшение некоторых физико-механических характеристик. Органоминеральные добавки, получаемые при обезвреживании отработанных сорбентов и нефтесодержащих отходов, отвечают требованиям экологической безопасности и пригодны для использования в качестве добавки в строительные материалы, например, керамзит и асфальтобетонные смеси. Целью работы является разработка технологии получения органоминеральных добавок, которая направлена на ликвидацию загрязнения окружающей среды отходами, их вовлечение в ресурсооборот и обеспечение рационального природопользования через уменьшение потребления реагентов и получение высококачественной экологически безопасной продукции. Установлено минимальное необходимое количество негашеной извести (реагента оксида кальция), требуемое для перевода вязкотекучего шлама в сыпучее и получения органоминеральных добавок. Для достижения наиболее важных технических характеристик строительных материалов, в частности, керамзита – насыпной плотности и прочности, применяют Литвинова Т.А. Органоминеральные добавки к строительным материалам на основе отходов газовой и нефтяной промышленности // Инженерно-строительный журнал. 2016. № 7(67). С. 13–21.

различные добавки. Органические добавки улучшают вспучивание глины и снижают насыпную плотность керамзита. Кремнеземсодержащие добавки помогают увеличить прочность готового продукта. Применение комплексных добавок способствует увеличению коэффициента вспучивания глины до 3х раз и получению требуемой прочности бетонного блока с меньшим потреблением энергии, что значительно снижает себестоимость продукции. Было проведено исследование по разработке составов глинистого теста с органоминеральными добавками и выбору режима обжига для получения керамзита. Были установлены оптимальные температуры и режим предварительной термоподготовки для производства продукта, отвечающего требованиям стандартов, с минимальным потреблением энергии. Определено влияние процентного содержания органоминеральной добавки на коэффициент вспучивания глины. В процессе обжига слабовспучивающейся глины с использованием органоминеральной добавки при температуре 1050 °С коэффициент конструктивного качества увеличивается на 70–97 % по сравнению с керамзитом, полученным из сырья без добавок. Таким образом, предлагаемая технология для производства керамзита с органоминеральными добавками позволяет получить высококачественные экологически безопасные продукты и утилизировать в своем составе отходы нефтегазового комплекса, вовлекая их в ресурсооборот, и отвечает требованиям наилучших доступных технологий, стимулируя повторное использование отходов.

### *Introduction*

The current situation in the field of formation, accumulation and disposal of oil and gas complex waste leads to dangerous contamination of all components of the environment – surface water and groundwater, vegetative ground cover, air, as well as unsustainable use of natural resources, significant economic damage and poses a real threat to health current and future generations of the country. At the enterprises of the oil and gas industry oily waste are generated during the construction of oil and gas wells, commercial exploitation, transportation and processing of oil and gas, treatment of waste water containing oil products and cleaning of tanks and other equipment. The accumulation of large masses of waste is due to objectively existing level of technology for the processing of raw materials and lack of its complex use.

For the oil industry it is characterized formation of oil-contaminated liquid and solid waste - waste water and sludge. Most of the refinery waste is oil sludge generated in the wastewater treatment process; scales and deposits on equipments. Output of oil sludge is up to 10 kg per 1 ton of crude oil processed [1]. Among the solid waste of oil refining let us note the spent adsorbents used for wastewater treatment and spent catalysts used in the catalytic oil processing. Considering the gas industry, there is spent adsorbent formed by the dehydration of natural gas. Every year during the extraction of fuel and energy minerals it's generated about 2000 million tons of oil sludge, including in Krasnodar Territory – to 12000 tons [2].

The use of oil-containing waste as secondary raw materials is one of the most efficient methods for their treatment and disposal. Oil sludge has been used in road construction, building materials, fuel industry, oil and gas industry. Adding sludge in mixtures allows obtaining high-quality products that comply with regulatory requirements, to improve some of the physical and mechanical characteristics similar products. Areas of oil sludge use are chosen both by technological, technical and sanitary requirements for products, as well as on performance standards for raw materials. Suitability oil waste as technogenic raw materials is determined during their complex analysis.

Thus, it is topical and essential the development of effective methods of oil and gas complex waste disposal and the best available techniques of their utilization for the elimination of air, water, soil pollution by waste of hazard classes II to III and environmental remediation. Along with this it is important resource-using waste and products of their disposal as complex additives in building materials, involving at the same time waste into resource management.

The possibility of using oil sludge in the production of building materials are determined by the type of the products (brick, concrete block, aggregates) and the role of waste in the process (burn-out additive or softener).

Oil-contaminated waste is widely used as the organic binder in the manufacture of waterproofing materials. Using the oil waste can not only reduce the consumption of bitumen or oil, but also obtain materials with high physical and mechanical properties [4–7].

It is worth noticing an insulating material [8] that can be used in the disposal of toxic industrial waste of hazard classes III to IV, including municipal solid waste. The material contains clay (10–60 mass.%), sediment of slaked lime or sludge of chemical water purification in quality of used lime

waste material (15–40 mass.%), bottom, floating oil-slime or soil polluted with mineral oils in quality of oil-slime (25–50 mass.%).

It is used oil waste solidification techniques that allow to get products in the form of blocks for its use as structural elements in the construction and waterproofing of landfills for the disposal of waste. Method of detoxifying petroleum-containing wastes includes adding 10-% aqueous emulsion of waterproofing liquid and resultant mixture is solidified by mixing with cement. The quantity of oil waste in the composition ranges from 20.8 to 41.6 mass.% [9].

In research papers [10–14] it is suggested the use of oil sludge for production of expanded clay. According to the mineral composition of oil sludge waste are similar to the components of the raw mixture, and according to the fractional composition of the organic part they are similar to expansion admixture.

In order to improve the quality of expanded clay it is commonly used various additives. Organic additives can improve distension clay, resulting in a reduction in the bulk density of expanded clay. Silica-containing supplements help to increase the strength of the finished product due to the saturation of silicon ions and increase the proportion of vitreous component. It is used organo-mineral and alkaline additives to reduce the burning temperature. Powdering the pellets with refractory powders surface, in particular limestone, ground quartz sand, gypsum causes blistering expansion slot. Application of the complex additives can increase the swelling clay factor of up to 3 times and get the required strength of concrete block with less energy consumption which significantly reduces the cost of the product.

At the Department of oil and gas technology it is developed technologies of oil-contaminated waste disposal by reagent method with introducing quicklime and adsorbing additives based on industrial waste, including oil and gas industry (waste siliceous adsorbents: silica gels, ODM-2F, diatomite, C-sorbents, the products of pyrolysis of used tires, rice husk) [15–23]. The composition of waste utilization products is suitable for use as complex additives in building materials, for example, in the production of expanded clay and asphalt concrete [24–29]. Patented technologies take into account the requirements for BAT [30, 31]. One of the criteria for inclusion to the BAT is encouraging re-use of waste. Application of BAT in the oil and gas industry is a comprehensive solution to the problem of waste management, including the transition to energy-efficient, resource-saving technologies to the improvement of the environment and citizens' health. Using BAT in the field of waste management will eliminate the environmental pollution by waste [32–34].

The composition of oil utilization products by reagent method includes calcium oxide, silicon oxide and conversion products, calcium hydroxide, encapsulated hydrocarbons from oil-sludge and organic compounds from the spent siliceous sorbents. Consequently, those products are organo-mineral additives and in fact are complex additives for increasing the coefficient of clay swelling, reduction the bulk density and maintaining granule strength. The introduction of neutralized oil sludge makes it possible to mix clay mass without contamination of working area, eliminating the release of hydrocarbons and other harmful substances from waste. Organic components stimulate porization of expanded clay and intensify processes occurring during the clay swelling including the transition clay into pyroplastic state. By reducing the bulk density of expanded clay it is important to ensure an optionally strength because increasing the pore size causes a sharp decline in the strength of the pellets due to the reduction of wall thickness and then enhance stress concentrations. Not only volume and pore size affect on the strength of porous materials. The structure and composition of the solid phase component of the vitreous play a great role. The strength of the vitreous increases by the saturation of aluminum and silicon ions. Therefore, silica-containing additives, including the siliceous spent sorbents, help to increase the strength of the finished product.

Following the research the objectives of this study are to develop technology for producing organo-mineral additives for the elimination of environmental pollution with wastes, to involve wastes into resource circulation and to ensure rational nature management with reducing the consumption of reagents and to receive high quality ecologically safety products.

## Methods

Method of obtaining organo-mineral additive to building materials consists in mixing oil-contaminated sludge with quicklime, preliminarily milled to a finely dispersed condition, and exhausted silica gel, which represents the gas industry waste product at the stage of natural gas drying, with the following introduction of water. As for exhausted silica gel it contains more than 90 mass % silicon oxide, 2.4 mass % coke deposits, 3.6 mass % organic components and the remaining metal oxide [35–36], and

it's used as the hydraulic additive in manufacture of waterproof concretes on the basis of calseal-puzzolane binding agent [37–40].

The necessary quantity of water for slaking is determined stoichiometrically including water, present in the oil-contaminated sludge, and wateradsorption of exhausted silica gel. The obtained organo-mineral additive is exposed until the process of calciumsilicate structure formation is finished. First, the quantity of oil components in the oil-contaminated sludge is determined, which is then used to calculate the necessary quantity of quicklime (1).

$$y = (0.023x - 0.001) \cdot m, \quad (1)$$

where  $y$  – necessary quantity of quicklime, kg;

$x$  – quantity of oil components in the oil-contaminated sludge, mass. %;

$m$  – quantity of oil-contaminated sludge, kg;

0.023 – empirically determined coefficient;

0.001 – empirically determined coefficient.

The obtained data are used to calculate the necessary quantity of exhausted silica gel (2).

$$z = 1.1 \cdot y, \quad (2)$$

where  $z$  – necessary quantity of exhausted silica gel, kg;

1.1 – empirically determined coefficient, taking into account the quantity of silicon oxide in exhausted silica gel.

Before mixing powdered quicklime it's treated with waterproofing additive in the form of a solid technical fat, heated to a temperature of 28–40 °C, taken in an amount calculated by equation 3:

$$n = 0.05 \cdot y, \quad (3)$$

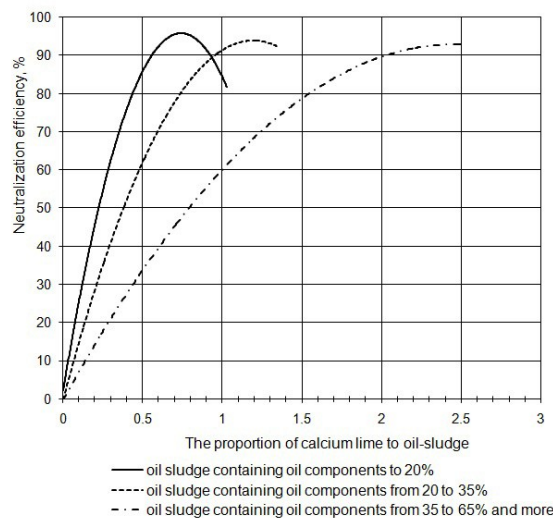
where  $n$  – necessary quantity of waterproofing additive, kg;

0.05 – empirically determined coefficient.

The obtained by this method organo-mineral additive meets ecological safety requirements and is suitable for use as a complex additive in construction materials.

## Results and Discussion

For the rational disposal of oil-contaminated waste and production effective organo-mineral additives to building materials it's important to determine necessary quantity of quicklime taking into account oil components in waste. The dependence of oil-sludge neutralization efficiency from quantity of quicklime (Fig. 1) is obtained by experimental research of the effectiveness of oily waste neutralization with different quantity of organic components.



**Figure 1. The dependence of oil-sludge neutralization efficiency from quantity of quicklime**

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It is established the minimum necessary quantity of quicklime (calcium oxide reagent) required to transfer viscous sludge in bulk and obtain organo-mineral additives. To achieve neutralization efficiency of 85–95 % the proportion of calcium lime to oil-sludge varies widely the average from 0.4 to 1.7 and higher. Thus further increase quicklime leads to decrease in the efficiency of neutralization, which is associated with an excess of the reagent.

With a minimum quantity basic reagent (quicklime) environmental safety of organo-mineral additives is achieved by increasing the quantity of silica component in neutralizing composition – exhausted silica gel, the main part of which is silicon oxide.

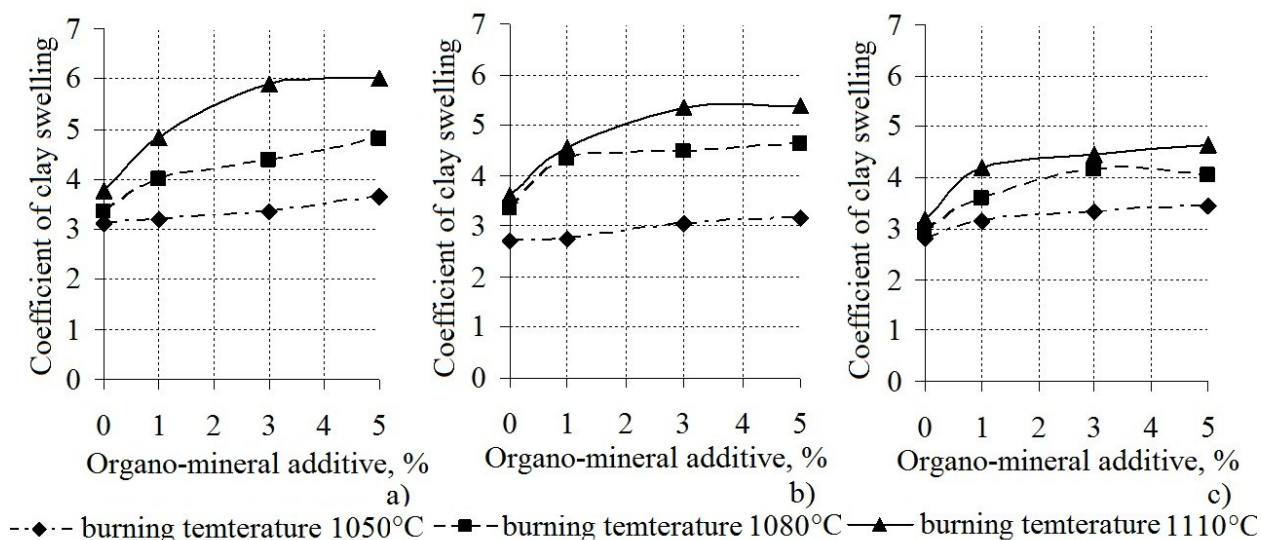
So, considering the composition of organo-mineral additive it is a complex additive, for example, for increasing the coefficient of clay swelling, reduction the bulk density and maintaining granule strength.

Formulation of puddle clay for preparing expanded clay with additives deals with the selection the optimum percentage of additive in the raw mix to achieve the requirements of standards for the products. It is carried out the research to develop formulations of puddle clay with organo-mineral additives and choose burning conditions for preparing expanded clay.

Method of preparing expanded clay involves mixing clay raw material, an additive and water, granulation of the obtained mixture, drying and burning. Mixing is carried out in two steps. At the first step clay raw material is mixed with the additive (1–5 mass %). At the second step water is added in an amount sufficient for obtaining a mixture with water content of 18–20 %. In accordance with the composition of the raw materials it's characterized by quantity of silicon oxide 67.84 % and quantity of organic components 0.40 %, so it refers to moderately ductile raw materials. With adding organo-mineral additive in quantity from 1 to 5 mass. % the clay composition will be included in the regulated Russian technical specifications TU 21-0284739-12-90 value.

Selecting the mode of burning raw granules is to establish the optimal temperatures and a thermal pre-sintering for producing products that meet the requirements of the standards, with minimal energy consumption. Test results of obtained expanded clay samples showed that optimum temperature conditions are rapid and gradual thermal treatment at temperatures of 200, 300, 400 °C with the addition of 1–3 % OMD in the burning temperature range from 1050 to 1110 °C.

The impact of the percentage of organo-mineral additive on the coefficient of clay swelling is expressed by the following relationship (Fig. 2).



**Figure 2. The impact of the percentage of organo-mineral additive on the coefficient of clay swelling at various burning temperatures and rapid thermal treatment at 200 °C (a), 300 °C (b), 400 °C (c)**

In the range of raw materials burning from 1050 to 1110 °C adding organo-mineral additive in quantity 1 mass. % increases the coefficient of swelling from 3 to 40 % in comparison with pure raw materials. Adding organo-mineral additive in quantity 3 mass. % increases the coefficient of swelling from 5 to 45 % and in quantity 5 mass. % – from 12 to 50 %. However, the use 5 % of organo-mineral additive limits the narrow swelling range in 30 °C and surface glazing at a lower temperature 1110 °C due to the increase the organic components in the clay composition.

Thus, in the process of burning poor swelling clay with using organo-mineral additive at temperature 1050 °C constructive quality factor is increased by 70–97 % in comparison with the expanded clay obtained from raw materials without additives. Therefore, developed method is energy-saving, environmentally friendly and technologically appropriate technology of expanded clay production.

## Conclusions

1. The research results can be used in the oil and gas industry to utilize the accumulated and annually produced at the enterprises technogenic waste for minimization environmental pollution, while reducing air, water and soil pollution by waste of hazard classes II to III; as well as in the building industry, reducing the consumption of natural resources and energy.

2. The developed technology for production of expanded clay with organo-mineral additive provides preparing high-quality environmentally friendly expanded clay and disposed in its composition oily waste and meets BAT requirements.

3. The use of complex additives OMD increases coefficient of clay swelling up to 3 times and obtaining the required strength of expanded clay with lower energy consumption, which significantly reduces the cost of the product due to the use raw materials of poor quality.

4. It is established the optimal temperatures and a thermal pre-sintering for producing products that meet the requirements of the standards, with minimal energy consumption.

5. The impact of the percentage of organo-mineral additive on the coefficient of clay swelling is determined. In the process of burning poor swelling clay with using organo-mineral additive at temperature 1050 °C constructive quality factor is increased by 70–97% in comparison with the expanded clay obtained from raw materials without additives.

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## Exergoeconomic model of a central air conditioning system

### Эксергоэкономическая модель центральной системы кондиционирования воздуха

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**Key words:** energy efficiency; air conditioning; thermodynamic analysis; buildings; construction; exergoeconomic model; civil engineering

**Ключевые слова:** энергоэффективность; кондиционирование воздуха; термодинамический анализ; здания; сооружение; эксергоэкономическая модель; гражданское строительство

**Abstract.** The article considers the issues of energy saving in central air conditioning systems by means of their operation parameters optimization, based on the exergoeconomic (thermoeconomic) approach. The necessity of joint consideration thermodynamic and economic factors of system operation is identified. Literature review in the field of study is submitted. There is the schematic diagram of the central air conditioning system provided for which the exergoeconomic model is created. Necessary assumptions are stated. Exergy economic model of central air conditioning system is shown in graphical form. The model is presented as separate zones, connected in-series. Basic expressions of the exergoeconomic model are stated. The expressions allow solving the problem of energy consumption minimization using the Lagrange's method of undetermined multipliers. Expression of a lagrangian for a problem of optimization of parameters of the functioning of the central air conditioning system is received. The performance control laws of separate zones of the air conditioning system providing minimal energy consumption during its operation are offered in a general view. As a conclusion possibility of considerable energy consumption decreases during operation of the air conditioning system.

**Аннотация.** В статье рассмотрены вопросы энергосбережения в центральных системах кондиционирования воздуха путем оптимизации параметров их функционирования на основе метода эксергоэкономики (термоэкономики). Выявлена и обоснована необходимость совместного учета и термодинамических, и экономических факторов функционирования систем. Представлен обзор литературы в области исследований. Приведена принципиальная схема центральной системы кондиционирования воздуха, для которой создается эксергоэкономическая модель. Сформулированы необходимые допущения. Разработана эксергоэкономическая модель центральной системы кондиционирования воздуха в графическом виде. Модель представлена в виде отдельных зон, соединенных между собой последовательно. Приведены основные аналитические выражения модели, позволяющие решить задачу минимизации энергопотребления с использованием метода неопределенных множителей Лагранжа. Получено выражение лагранжиана для задачи оптимизации параметров функционирования центральной системы кондиционирования воздуха. Предложены в общем виде законы управления производительностью отдельных элементов системы кондиционирования воздуха, обеспечивающие минимальное значение энергетических затрат при ее функционировании. Сделан вывод о возможности значительного снижения энергетических затрат в процессе работы системы кондиционирования воздуха.

### Introduction

Central air conditioning system (ACS) is designed to create and maintain the conditions most favorable for human activity, the normal functioning and safety of technological equipment and materials in public buildings and in the technological areas. Central ACS consists of the following main elements: a central air conditioner, power cooling (chiller, cooling machine with water recycling system, etc.), heating

source, the air ducts [1]. Their structure and behavior are defined by a scheme of heat and humidity air treatment.

Central ACS are extremely energy-intensive facilities, their individual elements are connected with each other and with customers processes of energy-mass-transfer. Operating costs of ACS can achieve 60–70 % of the operating costs of the building. Because of the inherent features of the central ACS, there are significant losses of heat and cold during system operation, resulting in increased energy consumption. It should be noted that more and more air cooling is required not only during the warm period but during the transition and cold periods.

Therefore, at present it is necessary to improve both elements of ACS and their operation modes to reduce energy consumption required for their normal operation.

The aim of research is to form an approach to ACS optimization, which let optimize operating parameters and decrease energy consumption during ACS operation. In order to achieve the aim some particular problems were solved. They were choice of research method, formalization of energy consumption processes in ACS and development of laws of performance control of individual elements of ACS, the laws are needed for required parameters maintenance.

### *Literature review*

Energy saving and improvement of energy efficiency of the central ACS can be implemented with [2–5]:

- formation and adoption of a more rational volumetric-planning solutions, construction and design measures to reduce heat exchange of buildings with the environment,
- use of more efficient equipment with ACS and their elements,
- technology and automatic control systems improvement in ACS,
- use of ACS circuit solutions to dispose of waste heat to the needs of air conditioning and ventilation.

In many studies the determination of optimal operation parameters of the energy conversion systems is done with the use of exergy approach [6–13]. In the majority of articles the ACS improvement is done on the exergy efficiency basis [14–16]. However, not always the system which is optimal in thermodynamic terms is optimal in economic terms. Thermoeconomic approach allows taking into consideration simultaneously both thermodynamic and economic factors of ACS operation when optimizing the system [17, 18]. The term "exergoeconomic" is used more often than "thermoeconomic" nowadays [19–24].

The main thing in exergoeconomic approach is application of thermodynamic function of exergy, which defines is the goal of system operation achieved or not, for assessment of changes occurring in the energy conversion system [25–28].

### *Description of the research*

When using the method of exergoeconomic the author describes the changes with the main flow of exergy for the operation of the system with a given capacity. In case of ACS system operation with a given capacity is to obtain the necessary exergy of conditioned air. In the course of this analysis not only the exergy losses, which are occurring in transmission and transformation of energy in individual elements of ACS, but also economic costs which are associated with elements of ACS operation are reviewed and considered.

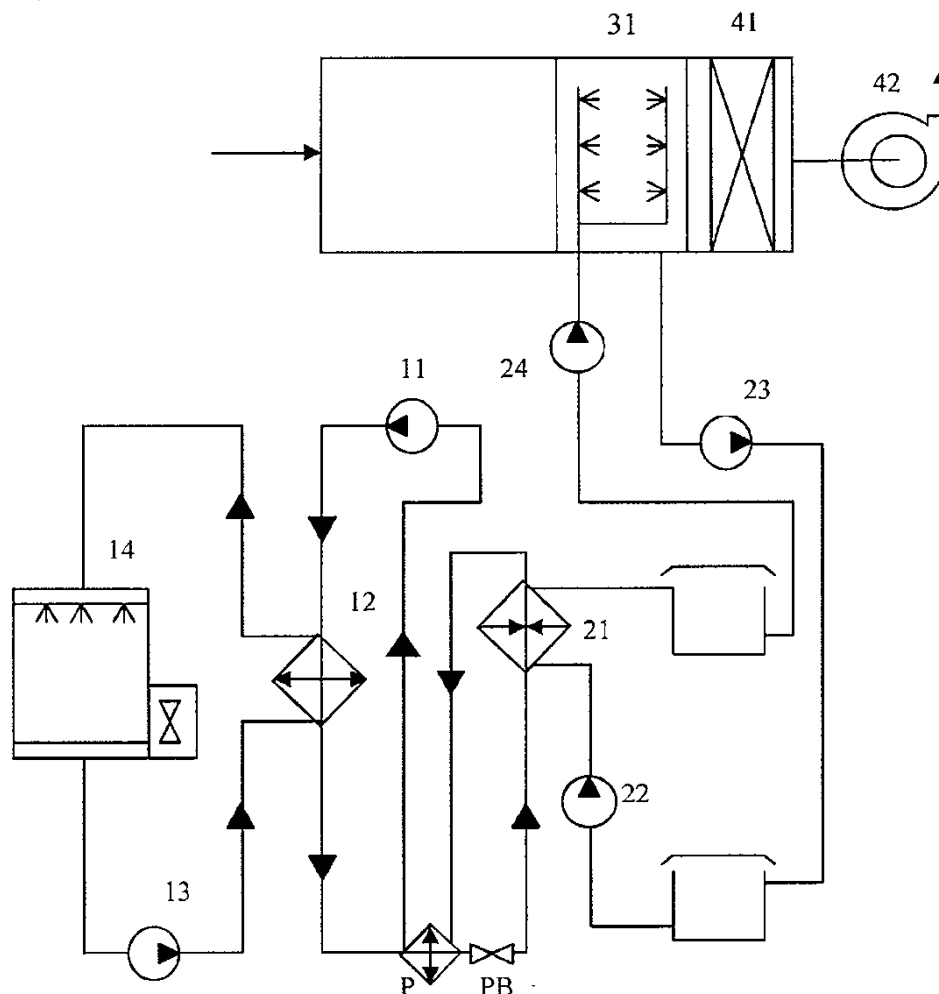
Both exergy losses and economic costs lead to the increase in the unit cost of flow exergy as it moves from the point of input of exergy in the energy conversion system to the flow output from the system. Taking into consideration that the value of exergy of conditioned air for the analyzed ACS is known, therefore, conditions ensuring minimum exergy price of conditioned air have to be determined to optimize the operation parameters of ACS.

On a substantial level the optimization problem can be formulated as follows: find the minimum energy operational cost of the ACS producing certain amount of conditioned air of a given quality. To solve this problem exergoeconomic model of ACS is developed in the article.

Exergoeconomic approach analysis of changes of the main exergy flow which provides system operation with a given capacity allows us to represent exergoeconomic model of the central ACS as

several separate zones. These zones are connected in series. Each zone includes a group of elements of ACS with relative autonomy within the system. Such linearization of the technological scheme of ACS simplifies further calculations due to the taking out of consideration of individual technological ties, without affecting the system power consumption.

Schematic diagram of central ACS for which the exergy model is developed is shown in Figure 1. Figure 1 shows the following elements of ACS: 11 – compressor with an electric motor, 12 – condenser, 13 – pump with an electric motor for supplying cooling water to the condenser, 14 – fan motor on the cooling tower, RV – regulating valve, R – regenerator, 21 – evaporator, 22, 23, 24 – respectively, pumps with electric motors for supplying coolant from the tank of warm water into the evaporator, from the spray chamber into the tank of warm water, from the tank to the chilled water into the spray chamber, 31 – spray chamber, 41 – after heater, 42 – fan with an electric motor to supply air to the consumers.



**Figure 1. Schematic diagram of the considered central ACS**

Taking into consideration necessity of energy consumption decrease, energy costs of ACS operation are used as target function while developing exergoeconomic model. This is due to the fact that energy costs which are directly related to the thermodynamic characteristics of the system include costs of all matter and energy flows that go to the considered ACS through exergy.

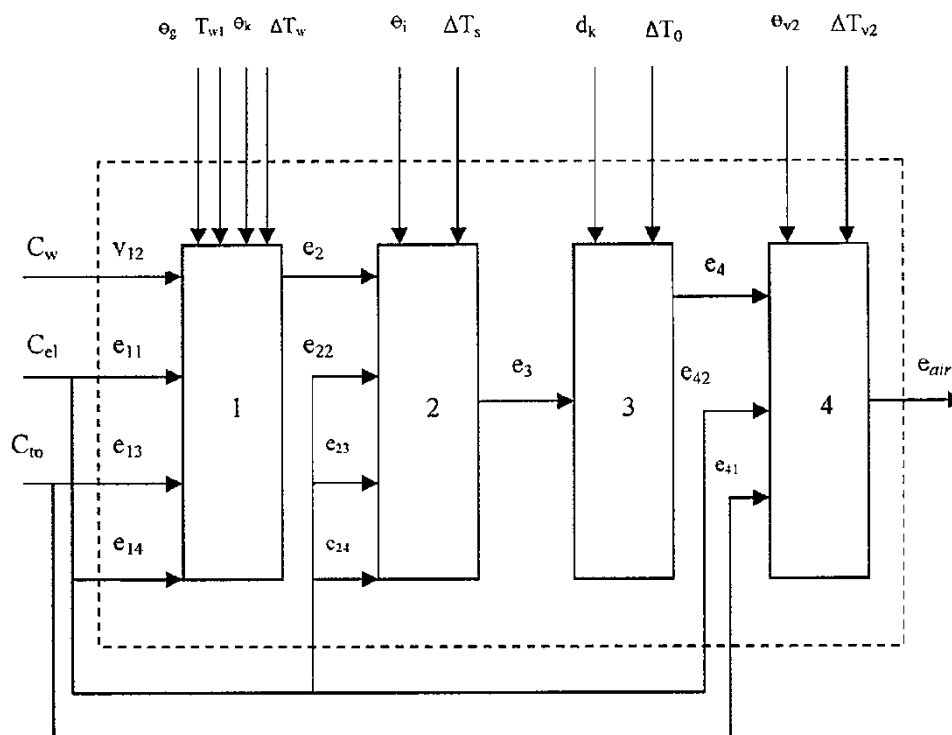
To simplify the resulting expressions exergoeconomic model of ACS formulated the following assumptions:

- 1) change of pressure loss in pipelines and air ducts during transportation of the heat transfer agent, air is not taken into account. Pressure losses in pipelines, ducts and heat exchangers ACS are considered constant and do not dependent on mode of operation;
- 2) exergy losses occurring in pipelines and ducts because of the heat exchange with the environment, are considered constant, independent of the mode of operation of the ACS;

- 3) heat exchange between working fluid of the refrigeration machines (the refrigerant) and environment occurring in the compressor and heat exchanger via their external surface, washed by the air is not taken into consideration;
- 4) overheat of absorbable vapor in the compressor and subcooling of the liquid working fluid (refrigerant) flowing to the expansion valve are not optimized. It is believed that steam superheating is caused by the rules of safe operation of refrigerating machines;
- 5) heating of the air in the fan and the heating pumped water in the pumps is not taken into consideration;
- 6) the optimum parameter of the air in the air-conditioned rooms are characterized by the point on the I-D diagram;
- 7) characteristics of air in the working area of the premises and settings of the outgoing air are the same.

Formulated assumptions have almost no influence on the accuracy of definition of the energy consumption rate. Their impact is estimated at a rate of 1–1.5 %.

Taking into consideration the starting positions and assumptions made exergoeconomic model of ACS is presented in the form of four series-connected zones, shown in Figure 2.



**Figure 2. Exergoeconomic model of ACS**

Exergy is supplied through the control surface of the model from an external source to various areas:  $e_{11}$ ,  $e_{13}$ ,  $e_{14}$ ,  $e_{22}$ ,  $e_{23}$ ,  $e_{24}$ ,  $e_{42}$  – to drive motors of compressor, pump cooling water, cooling tower fan, pumps of intermediate heat transfer medium, the air conditioner fan. The price of exergy supplied from an external source of electrical energy, is known and equals  $C_{el}$ . Cooling water is supplied from an external source, flow rate of which equals  $v_{12}$ , price –  $C_w$ . Exergy for heating air in after heater (thermal energy)  $e_{41}$  is supplied from an external source with the price of  $C_{tp}$ .

Thermal processes are essential in the operation of ACS. Therefore, the optimized variables are the variables that allow developing exergoeconomic model and relatively easy determining the temperature conditions of technological processes in ACS. These variables are following: temperature difference in the cooling tower  $\Theta_g$ , condenser  $\Theta_k$ , evaporator  $\Theta_i$ , after heater  $\Theta_{v2}$ , the water temperature at the inlet of the condenser  $T_{w1}$ , and changes the temperature of cooling water in the condenser  $\Delta T_w$ , intermediate refrigerant in the evaporator  $\Delta T_s$  after heater  $\Delta T_{v2}$ , process air temperature difference and outdoor air  $\Delta T_o$ , the moisture content of the air treated in the chamber irrigation  $d_k$ .

Taking into consideration the assumptions made and the adopted notation the value of the energy cost, including costs for electric and thermal energy, as well as costs for water circuit with a cooling tower is determined by the dependencies:

$$S_{en} = [C_{el} \cdot (e_{11} + e_{13} + e_{22} + e_{23} + e_{24} + e_{42}) + C_w \cdot v_{12} + C_{tp} \cdot e_{41}] \tau, \quad (1)$$

where:  $\tau$  – the time work of ACS.

Consumption of electric energy to drive the compressor, pumps, fans, water consumption and thermal energy depend on the operation mode of the ACS, and therefore, on the temperature pressures in heat exchangers, intervals of change of heat carrier temperature and moisture content processed in the spray chamber. Therefore, the right side of expression (1) is a function of the selected optimization variables. Hence, energy consumption is a function of several variables, its extreme value is determined by the condition of equality to zero of partial derivatives of an energy consumption function of optimized variables.

$$\begin{aligned} \partial S_{en} / \partial \theta_q &= 0; & \partial S_{en} / \partial \Delta T_w &= 0; \\ \partial S_{en} / \partial \theta_k &= 0; & \partial S_{en} / \partial \Delta T_s &= 0; \\ \partial S_{en} / \partial \theta_i &= 0; & \partial S_{en} / \partial \Delta T_{v2} &= 0; \\ \partial S_{en} / \partial \theta_{v2} &= 0; & \partial S_{en} / \partial \Delta T_0 &= 0; \\ \partial S_{en} / \partial \theta_{w1} &= 0; & \partial S_{en} / \partial \Delta d_k &= 0. \end{aligned} \quad (2)$$

It can be applied in case when all the optimized variables are independent and the problem is reduced to the determination of the absolute extremum. In practice, these variables are linked, which makes analytical description of the relationships between all the optimized variables extremely difficult. Exergoeconomic method simplifies this task.

The idea of the exergoeconomic model of ACS as a number of series-connected zones allows expressing the exergy, supplied to each of the zones, in the form of functional dependencies from the flow exergy, leaving the reporting zone, and affecting this zone of optimized variables.

Then the amount of exergy, supplied to the various elements of ACS from an external source  $e_j$  (Figure 2), and volumetric flow rate of the cooling medium (water), used for discharging the heat of condensation  $v_{12}$ , in general terms are described as follows:

$$\begin{aligned} e_{11} &= E_{11} (e_2, \theta_q, T_{w1}, \theta_k, \Delta T_w), \\ e_{13} &= E_{13} (e_2, \theta_q, T_{w1}, \theta_k, \Delta T_w), \\ e_{12} &= E_{12} (e_2, \theta_q, T_{w1}, \theta_k, \Delta T_w), \\ e_{22} &= E_{22} (e_3, \theta_i, \Delta T_s), \\ e_{23} &= E_{23} (e_3, \theta_i, \Delta T_s), \\ e_{24} &= E_{24} (e_3, \theta_i, \Delta T_s), \\ e_{41} &= E_{41} (e_{air}, \theta_{v2}, \Delta T_{v2}), \\ e_{42} &= E_{42} (e_{air}, \theta_{v2}, \Delta T_{v2}), \end{aligned} \quad (3)$$

where:  $e_j$  – the amount of exergy,  $E_j$  – the function describing its variation.

Equations included in the system (3) belong to different zones of exergoeconomic model, zones are connected with the main exergy flow. Exergy flow connecting separate zones is presented as functional dependence on exergy flow leaving the zone and optimized variables affecting the considered zone:

$$\begin{aligned} e_2 &= E_2 (e_3, \theta_i, \Delta T_s), \\ e_3 &= E_3 (e_4, d_k, \Delta T_0), \\ e_4 &= E_4 (e_{air}, \theta_{v2}, \Delta T_{v2}). \end{aligned} \quad (4)$$

The links between optimized variables leads to a consideration of the problem of minimization of the energy consumption as the optimization problem of several variables function in the presence of equality constraints (equations), i.e. as the problem of finding a conditional extremum. One of the most effective ways to solve problems associated with finding the conditional extremum is Lagrange's method of undetermined multipliers. The application of Lagrange's method of undetermined multipliers allows to transform and reduce the problem of finding the conditional extremum of the original function of energy consumption (1) to the problem of finding the unconditional extremum (a minimum) a new function – the Lagrangian [17].

The Lagrangian expression for the problem of optimization of operation parameters of ACS, given the systems of equations (3) and (4) is written as follows:

$$L = \{C_{el}[E_{11}(e_2, \theta_q, T_{w1}, \theta_k, \Delta T_w) + E_{13}(e_2, \theta_q, T_{w1}, \theta_k, \Delta T_w) + E_{22}(e_3, \theta_i, \Delta T_s) + E_{23}(e_3, \theta_i, \Delta T_s) + E_{24}(e_3, \theta_i, \Delta T_s) + E_{42}(e_{air}, \theta_{v2}, \Delta T_{v2})] + C_w V_{12}(e_2, \theta_q, T_{w1}, \theta_k, \Delta T_w) + C_w E_{41}(e_{air}, \theta_{v2}, \Delta T_{v2}) + \lambda_2 [E_2(e_3, \theta_i, \Delta T_s) - e_2] + \lambda_3 [E_3(e_4, d_k, \Delta T_0) - e_3] + \lambda_4 [E_4(e_{air}, \theta_{v2}, \Delta T_{v2}) - e_4]\} \tau. \quad (5)$$

To find the conditions of the extremum partial derivatives from the Lagrangian (5) over all the variables (as optimized and additional equations imposed by communication) should be taken and set equal to zero. Partial derivatives for the exergy flows connecting separate zones of exergoeconomic model  $e_j$ , allow determining values of the Lagrange multipliers  $\lambda_j$ . For example, Partial derivative to  $e_2$  has the following form:

$$\partial L / \partial e_2 = \tau \partial / \partial e_2 [C_{el}(E_{11} + E_{13}) + C_w V_{12} - \lambda_2 e_2] = 0; \quad (6)$$

because  $\tau \neq 0$ , then the value of the derivative is zero. Whence it follows that:

$$\lambda_2 = \partial / \partial e_2 [C_{el}(E_{11} + E_{13}) + C_w V_{12}]. \quad (7)$$

Similarly the expressions for the multipliers  $\lambda_3$  and  $\lambda_4$  can be obtained.

The derivative of (5) to the optimized variables allows us to obtain the expression represented by the system (8).

$$\begin{aligned} \partial L / \partial \theta_q &= C_{el}(E_{11} + E_{13}) + C_w V_{12} = 0; \\ \partial L / \partial T_{w1} &= C_{el}(E_{11} + E_{13}) + C_w V_{12} = 0; \\ \partial L / \partial \theta_k &= C_{el}(E_{11} + E_{13}) + C_w V_{12} = 0; \\ \partial L / \partial T_w &= C_{el}(E_{11} + E_{13}) + C_w V_{12} = 0; \\ \partial L / \partial \theta_i &= C_{el}(E_{22} + E_{23} + E_{24}) + \lambda_2 E_2 = 0; \\ \partial L / \partial \Delta T_s &= C_{el}(E_{22} + E_{23} + E_{24}) + \lambda_2 E_2 = 0; \\ \partial L / \partial d_k &= \lambda_3 E_3 = 0; \\ \partial L / \partial \Delta T_0 &= \lambda_3 E_3 = 0; \\ \partial L / \partial \theta_{v2} &= C_{el} E_{42} + C_{tp} E_{41} + \lambda_4 E_4 = 0; \\ \partial L / \partial \Delta T_{v2} &= C_{el} E_{42} + C_{tp} E_{41} + \lambda_4 E_4 = 0. \end{aligned} \quad (8)$$

The system of equations (8) establishes a relationship between the energy dissipation and energy consumption in every zone of exergoeconomic model for certain values of economic indicators  $C_{el}$ ,  $C_{tp}$ ,  $C_w$ ,  $\lambda_2$ ,  $\lambda_3$ ,  $\lambda_4$ . The Lagrange multipliers  $\lambda_2$ ,  $\lambda_3$ ,  $\lambda_4$  in general case represent the price per unit of exergy leaving each zone of exergoeconomic model.

The solution of system (8) taking into consideration equations (7) and similar equations for  $\lambda_3$  and  $\lambda_4$  allows determining the necessary conditions for finding the minimum of Lagrangian (5). To solve equations (7) and (8) expressions (3) and (4), written in general form, must be submitted in the form of deployed analytical expressions describing the processes occurring in the separate elements of ACS. In the description of these processes the characteristics of heat and humidity treatment of air, depending on

the spray chamber type and the presence of the heat recovery operational peculiarities of the cooling machine depending on the type of the used compressor (piston, rotary, centrifugal or screw) and the applied refrigerant are taken into consideration. For example, the heat load on the condenser of the cooling machine is determined by empirical formulas I.S. Badyl'kes [29]. It also should be considered that the characteristic mode of operation of ACS is the continuous load change due to the influence of external disturbing factors, which are climatic conditions – the outdoor air temperature  $T_{os}$  and the moisture content of outdoor air  $d_{os}$ .

Obtained by optimizing the mode of operation of ACS values of operating parameters are used in determining the optimal laws of performance control of individual elements of ACS, minimizing energy consumption. The laws of performance control of individual elements of ACS represent the dependence on the values of different medium of temperature and moisture content of outdoor air, for example:

$$\begin{aligned}v_{12} &= V_{12} (T_{os}, d_{os}), \\v_{14} &= V_{14} (T_{os}, d_{os}), \\v_{22} &= V_{22} (T_{os}, d_{os}),\end{aligned}\tag{9}$$

where:  $v_{12}$  – water flow in the system of water recycling,  $v_{14}$  – the airflow of cooling tower fans,  $v_{22}$  – consumption of intermediate refrigerant through the evaporator of the cooling machine.

Obtained laws of performance control allow to formulate proposals for the development of requirements for the automatic control system based on programmable logic controllers that provides a ACS operation in energy saving modes [30]. Such automatic control system takes into account all the parameters of ACS required for high-speed control and maintainance of energy efficient operation modes. Implementation of the resulting control laws is possible by using variable frequency drives for fans, pumps, valves, which receive control signals from programmable logic controllers.

Approach to exergoeconomic analysis and optimization of the central ACS proposed in the article lets determine the optimal values of operational parameters of ACS and to achieve a reduction of the energy consumption by 5–6 % in the process of their work.

## Conclusions

1. The exergoeconomic method is discussed in relation to optimization of the functioning of the central ACS.
2. Expressions allowing to solve the problem of minimizing the energy consumption of the central ACS using the method of uncertain Lagrange multipliers are given in general form.
3. The proposed approach can be used for optimization of ACS with other circuit solutions.
4. The laws of performance control of individual elements of the ACS that allows developing of automatic control system of ACS based on programmable logic controllers are given in general form.

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## Seismic stability of the restored architectural monument

### Сейсмостойкость отреставрированного памятника архитектуры

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Республика Крым, Россия***Key words:** architectural monument; seismic;  
authenticity; strengthening; pasted anchor**Ключевые слова:** памятник архитектуры;  
сейсмический; аутентичность; усиление;  
вклеенный анкер

**Abstract.** The conservation of monument of architecture of XIX century – the Church of St. Archangel Michael in Sevastopol is discussed in the article. The analysis of the results of the survey of the building is developed and recommendations to strengthen the supporting structures of the church are worked out. The use of traditional methods for enhancing structural and seismic reinforcement of buildings with walls of masonry leads to the inevitable loss of the original facade or interior of the temple. To enhance the analysis and design of earthquake resistant walls pasted anchors performed experimental research collaboration adhesive bonding steel anchors in the stone elements from limestone Krymbalsk deposits. The proposed measures to strengthen allow performing repair and restoration work, without breaking the historical reliability of the facades. Comprehensive solutions for strengthening and ensuring of an acceptable level of safe operation in seismic areas are developed.

**Аннотация.** В статье обсуждается сохранение памятника архитектуры XIX века – церкви Святого Архангела Михаила в Севастополе. Произведён анализ результатов обследования здания, разработаны рекомендации по усилению несущих конструкций церкви. Применение традиционных способов усиления несущих конструкций и повышения сейсмостойкости зданий со стенами из каменной кладки приведет к неизбежной утрате изначальных фасадов или внутреннего убранства храма. Для расчета и конструирования, повышающих сейсмостойкость стен, вклеиваемых анкеров выполнено экспериментальное исследование совместной работы клеевого соединения стальных анкеров в каменных элементах из известняков Крымбальского месторождения. Предлагаемые меры по укреплению позволяют выполнять ремонтно-реставрационные работы, не нарушая исторической достоверности фасадов. Разработаны комплексные решения для укрепления и обеспечения приемлемого уровня безопасной эксплуатации в сейсмических районах.

### *Historical information*

The Church of St. Michael the Archangel in the city of Sevastopol (Fig. 1) [1] was constructed in 1849 on the initiative of the chief commander of the Black Sea Fleet – Admiral Mikhail Petrovich Lazarev. Author of the project is not exactly known, but it most likely colonel Fonderveyde, creator of the Cathedral of St. Nicholas, St. Michael's Church because it was the Chapel.



**Figure 1. The Church of St. Michael the Archangel in the city of Sevastopol. Historical views**

During the 1st defense of Sevastopol garrison church was the main church of the city. Funeral service was conducted there in honour of admirals Kornilov, Istomin, Nakhimov [2].

August 2, 1855 in the church building hit out a bomb, which made it great destruction. The building was restored in 1857 at the expense of the contractor Ivan Krasil'nikov. In 1889 the church became the regimental church of the 50th Regiment Bialystok Infantry.

In the 1920s, in the building there was the Reading Room named after the French Communist Martyr. From 1931 Home Health Education was situated there. After World War II the building housed a small temple hall of the House of officers of the fleet. Since 1969 the hall number 8 of the Black Sea Fleet Museum is situated here (Fig. 2).

The building is rectangular in plan. Portal is in the form of niches in one another, covered by arches that rest on columns (two on each side) with Byzantine capitals. Archivolt is carved on the left and right of the entrance there are medallions with rosettes. The walls and columns are built up of small masonry blocks from Krymbalsk deposit (Inkerman).



**Figure 2. Hall number 8 Black Sea Fleet Museum (Church of St. Michael the Archangel) in Sevastopol is directly adjacent to the House of officers of the Black Sea Fleet. 2012**

Two-tier iconostasis with artistic icons made in Odessa by master Dmitri Pashin according drawing, drawn up at the District Engineer's Office by architect Maas. "Interior walls were made by designer Rafael Izellow according to drawing and the direction of the architect Maas with ceiling painted in the colour of the air. Furthermore, it was gilded two rods on the ceiling cornice Fritz "a la grec", with a width of 8

inches, with the width of the tape 1 inch" (Business Engineering Office of Simferopol distance to restore the Church of St. Michael the Archangel in 1870, No. 22).

### *Results of the survey*

The building of the hall number 8 Black Sea Fleet Museum of Russian Federation is located not far from the sea in the south-central part of Sevastopol on Lenin street No. 11, on one of the slopes of the central hill of the city [3].

The construction site is located in the climatic region with the following characteristics [4]: weight of snow cover – 82 kg/m<sup>2</sup>; to wind pressure – 46 kg/m<sup>2</sup>. The depth of soil freezing is 0.8 m. The plot on which the building is located, refers to the area from 8 – point the calculated seismicity map A [5, 6].

One-story building with a basement is rectangular in plan with dimensions 20.3 x 10.4 m. From the eastern facade was attached apse projecting from the plane of the eastern lateral walls 2.2 m. The height of the basement is 2.35 m, height of the first floor – 7.5 m. The height of the interior space of the apse is 6.0 m.

Foundations under the walls are of the tape rubble. The strength of limestone rocks compression corresponds brands 75–100. Fragmentary masonry made of stones stronger brands up to M 300–400. The width of the sole foundation is not less than 1.2 m, which is sufficient for such building. There is no deformation of the building associated with irregular deflections of foundation so the technical status of the bases is satisfactory. It should be noted that the lack of waterproofing basements require particularly careful drainage from the walls of the building during its operation.

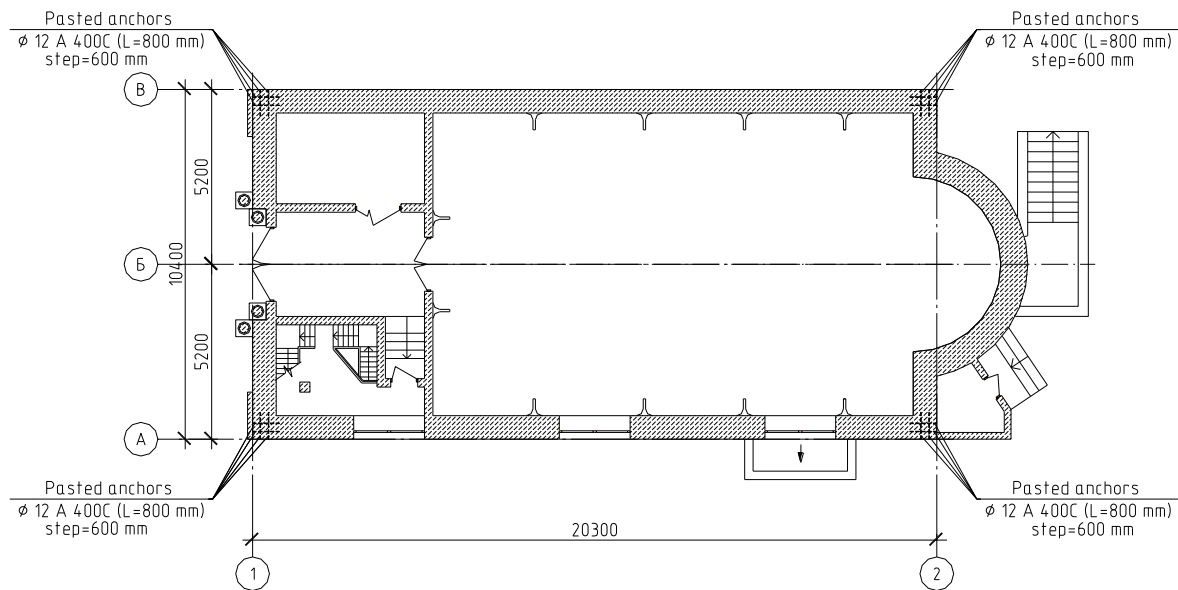
Located in the basement boilers serving the adjacent building of the House of Naval Officers and practically does not work, is a serious threat to the monument of architecture of the XIX century in the case of industrial accident or leakage communications. In addition, when you run this equipment in the basement of the building, there are vibration exposure transmitted to the building and reflects negatively on their condition. It is advisable to this energy-intensive and inefficient equipment obsolete sample to be dismantled or brought outside the building of the monument of architecture of XIX century.

Stone columns section 750 x 750 mm located in the basement of the building are made of sawed limestone rock M75 are used as intermediate support beams hardwood floor basement. Technical condition is not suitable for further use without amplification. It is necessary to strengthen it by means of outside steel cage of angular steel.

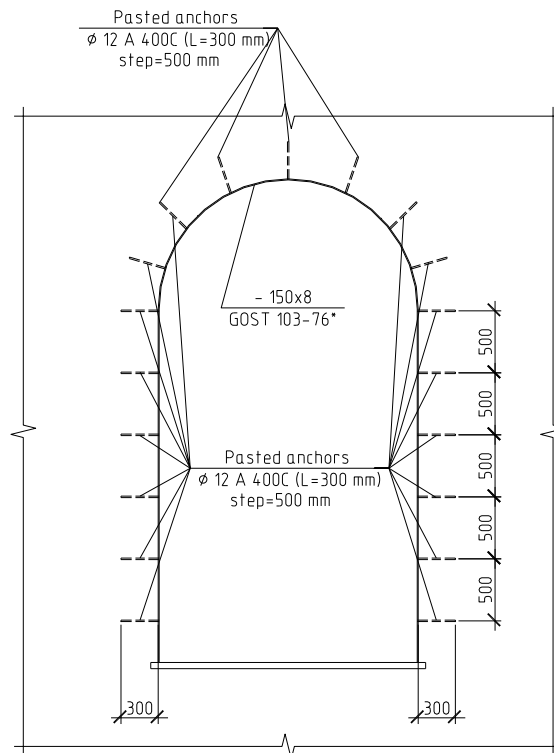
Overlap of basement is made of wooden structures. Excessive deflections of wooden supporting structures overlap led to the beginning of the destruction of parquet flooring. Elements of overlap are susceptible defeat beetle carpenter. There are traces of a fire in the basement of the building which appeared in charring bearing wooden beams. Pairing beams with stone supports are made without any insulating spacers. Technical condition above the basement is defined as not suitable for further use without strengthening. In this connection, it is recommended to limit the service tour groups in the hall of the museum. Taken into account the actual state of the hardwood floor, it does not seem effective to strengthen it. Expedient removal and device of new overlap are used.

The walls of the building are made of stone masonry from Krymbalsk deposit. Compressive strength of the stone reaches the M 75–M 100. Because of the poor state of the roof and no functioning system water drainage from the roof there is damage of the limestone rocks by defrost. Need a local repair of damaged areas using grids and strengthening complex building mortars. In the cornice area of the walls the disclosure of masonry joints is observed. They need to be repaired by injecting masonry strengthening of specific products [7]. The technical condition of the walls is satisfactory.

In order to improve the building seismic resistance it is advisable to strengthen the outer corners of the building hidden by staging pasted anchors of reinforcing steel  $\varnothing 12$  A 500 C length of 800 mm in mutually perpendicular directions. Be sure to mask their butts embedded indent by fragments of facade stone masonry (Fig. 3). Consolidation work arched bridges three windows of the southern wall (Fig. 4) by steel strip with pasted anchors (performed secretly unchanged appearance of the historic facade). Existing doorway in the south wall facade is converted into the window, according to the original appearance of the building.



**Figure 3. Scheme of strengthening the intersection exterior walls with help of pasted anchors of the Church St. Archangel Michael**



**Figure 4. Amplification circuit arched window openings of the southern wall of the building by staging hidden pasted anchors**

Reinforced belt section height 150 mm must be arranged on top of the walls of the main building. it is anchored with glued rods into three levels of height masonry.

The overlap of the first floor was made from metal and wooden trusses on wooden beams using board and plaster. The ceiling of the exhibition hall is damaged with cracks everywhere. The technical condition of the overlap is not suitable for further use without amplification. Given the seismic danger of territory and the simultaneous determination of the potential in the building of a large number of people, it is advisable to replace the heavy wooden ceiling on the modern lightweight suspension systems with efficient light insulation.

On the inner surface of the north wall there is a number of cracks in the stucco layer. All of these areas must be free from loose elements and repaired with the use of reinforcing mesh.

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The roof is made with the use of steel and wooden trusses. Steel trusses are in satisfactory condition. Two wooden trusses are in poor condition; need to replace them with new – metal.

Wooden elements of the roof are subject to constant local leakage and are technically able not suitable for further use. Bearing wooden elements on masonry gables are made without their fix. When you replace sections of the roof it is necessary to arrange the concrete belt height of 150 mm with the statement pasted into the body of masonry anchors and equipment issues for secure pairing with the elements of the roof.

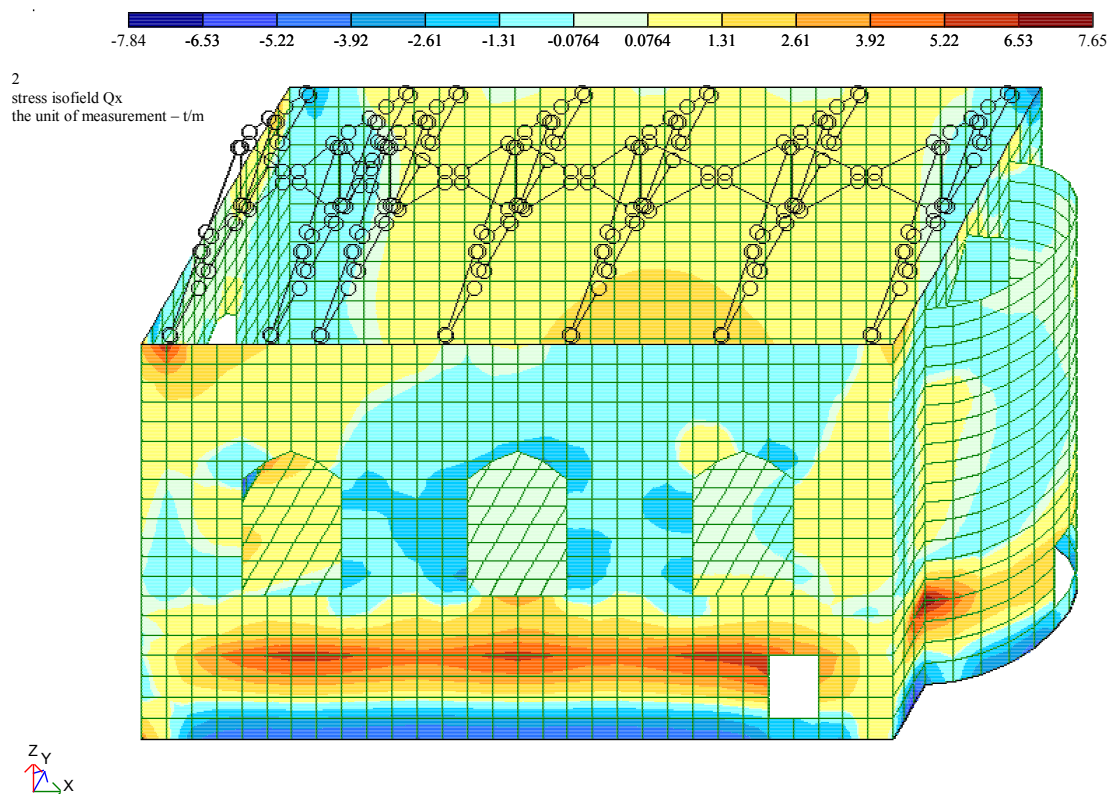
Bolting suspensions longitudinal joists of exposition hall corrode. Revision of all elements of the compounds is needed when replacing the elements of overlap.

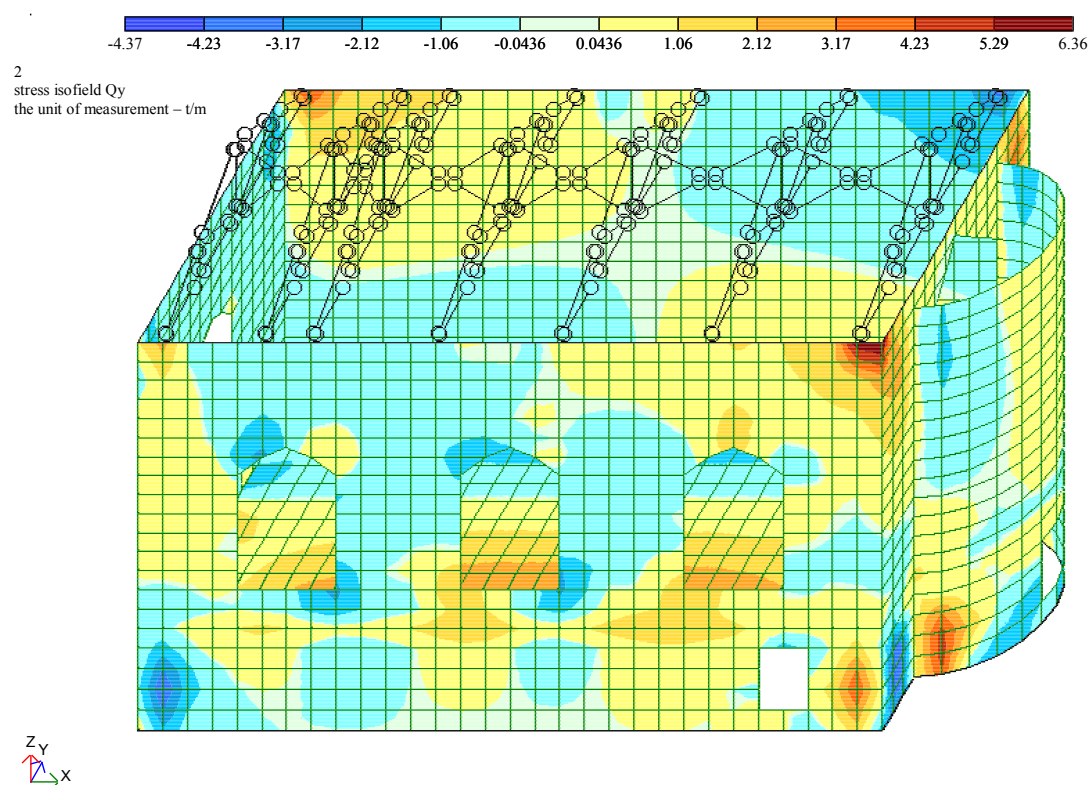
It is necessary to set new metal girders instead of the dismantled wooden trusses and near gables. It is necessary to arrange vertical connections on farms, as well as the necessary horizontal linkages to the lower and upper chord. You need to enable gables through embedded parts in reinforced concrete (arranged) belt in the overall spatial zones of the new system load-bearing structures of the roof. Roofing replaced with new unit system water drainage from the roof, including the roof of the apse.

Dome is recommended to restore the original version of the dome at the western front wall as shown in Figure 1. Supporting the dome to implement the newly arranged by metal girders. Dome must be performed with metal-plating carcass of vertical walls with light modern materials.

### *The results of calculation*

Spatial settlement building is made taking into account the recommended device of metal roof trusses connected by a common system with concrete belts on top of the walls (Fig. 5). Characteristics of materials adopted in accordance with the results of the survey. In the load on the farm is considered advisable to reconstruct the dome of a metal frame, the vertical part of which is trimmed with light facade materials.





**Figure 5. The contour plots of the critical types of stresses in the walls of the building**

Existing metal girders may be allowed to continue operating without amplification. Their carrying capacity is sufficient for the perception of the newly arranged the roof with the new sheet metal ceilings of gypsum boards with effective easy insulation.

It is recommended to restore the historical authenticity of the building with three chimneys on the roof, which should be used for ventilation. Heating and ventilation of the building it is recommended to carry out the forced injection through the use of ventilation ducts. Air heating system combined with ventilation to provide a uniform temperature throughout the volume of the serviced apartments [8].

## Conclusions

1. Supporting structures do not meet modern requirements for seismic stability of buildings [5, 6, 9–14]. Hardwood floor basement is in technical condition, not fit for further use. The absence of concrete vertical elements strengthening masonry walls and seismic belts makes use of the building for the mass people stay in the liturgy is very problematic.
2. The use of traditional methods for enhancing structural and seismic reinforcement of buildings [15–19] with walls of masonry lead [20] to the inevitable loss of the original facade or interior of the temple.
3. Proposed by the authors [21–23] the hidden designs of reinforcement of bearing masonry structures glued installation of anchors, opening up new opportunities preserve the authenticity of the Church of St. Michael the Archangel in the city of Sevastopol.
4. To enhance the analysis and design of earthquake resistant walls pasted anchors performed experimental research collaboration adhesive bonding steel anchors in the stone elements from limestone Krymbalsk deposits (Inkerman) 1840's. For steel anchors is widely used reinforcing steel  $\varnothing 12$  A 500 C and anchor mixture – Ceresit CX 5. Exhaustion of the bearing capacity of the adhesive joint with a minimum depth of 300 mm sealing is achieved with loads of 2.5–3.1 ton. With further increase in load is pulling the anchor stone of the sample (size 200 x 200 x 400 mm) and was accompanied by his split from the main transverse tensile stresses. Split stones occurred at loads greater than 15–20 % load bearing capacity of the exhaustion of the adhesive joint (by limiting the displacement of the anchor relative to an end surface of the stone – 0.4 mm).
5. Given the almost complete absence of adhesion stones Krymbalsk deposit (Inkerman) Crimea with lime-sandy and sandy-clay masonry mortar employed in the first half of the nineteenth century, as well as a sufficiently high strength of these stones compression (reaching 75–100 kg/cm<sup>2</sup>) – Consolidation of masonry anchors hidden most loaded horizontal impact, with a potential earthquake zones of the

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building can increase its seismic resistance preserving the historical aura of the architectural appearance.

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## Stress finite element models for determining the frequencies of free oscillations

### Определение частот свободных колебаний методом конечных элементов в напряжениях

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**Key words:** finite element method in stresses; free vibration frequency; lower limit values; principle of virtual displacements

**Ключевые слова:** метод конечных элементов в напряжениях; частоты свободных колебаний; нижняя граница значений; принцип возможных перемещений

**Abstract.** Development of calculation methods, allowing determining the lower limits frequencies of free oscillations is actual. Such methods, in combination with the decisions by the method of finite element in displacements, will let to assess the accuracy of the frequencies of free oscillations calculated values. The frequencies of free oscillations constant cross section rods with different supports of the ends are calculating by the stress finite element analysis. The proposed method is based on combination of the additional potential energy functional and the virtual displacements principle. The last is used to construct the equilibrium equations. The solution reduces to finding the minimum of the additional energy functional with constraints in form of the linear algebraic system equilibrium equations. The equilibrium equations, taking into account inertia forces, are writing for the finite element mesh nodes in the directions of coordinate axes. Using the Lagrange multipliers the equilibrium equations are included in the functional. The Lagrange multipliers are the nodes displacements values. Considered two variants of bending moment's approximation on the finite element field: linear and piecewise constant. The free oscillations forms are represented as polygonal lines. According to the proposed method the first three frequencies of free oscillations were defined for constant cross-section rods with different supports of the ends. The calculated values of the frequencies were compared with the exact values. In comparison with the method of finite elements in displacements, it is shown that the proposed method allows to get the opposite bound values for the frequencies of free oscillations.

**Аннотация.** Актуальной является разработка метода расчета конструкций, позволяющего определять нижние границы частот свободных колебаний. Такие методы, в сочетании с решениями по методу конечных элементов в перемещениях, позволят дать оценку точности вычисленных значений частот свободных колебаний. В работе предлагается решение задачи определения частот свободных колебаний стержней постоянного сечения с различными опорами на концах методом конечных элементов в напряжениях. Методика основывается на сочетании функционала дополнительной энергии и принципа возможных перемещений, используемого для построения уравнений равновесия. После дискретизации предметной области, решение задачи сводится к поиску минимума функционала дополнительной энергии деформации при наличии ограничений в виде системы линейных алгебраических уравнений равновесия. Уравнения равновесия, с учетом сил инерции, составляются для узлов сетки конечных элементов по направлениям осей координат и включаются в функционал при помощи множителей Лагранжа, которыми являются величины перемещений узлов. Рассматриваются два варианта аппроксимации изгибающих моментов по области конечного элемента: линейная и кусочно-постоянная. Форма свободных колебаний представляется в виде ломанной линии. По предложенной методике были выполнены расчеты первых трех частот свободных колебаний для стержней постоянного сечения с различными вариантами опирания концов. Выполнено сравнение расчетных величин первых трех частот свободных колебаний с точными значениями. Показано, что предлагаемый метод расчета позволяет получить противоположную, по сравнению с методом конечных элементов в перемещениях, границу значений для частот свободных колебаний.

## Introduction

Currently, the most common method for determining the frequencies of free oscillations is the finite element displacement analysis [1–12], less often used mixed [13–20] or analytical methods [21–24]. In the article [25] for determining the frequency of free oscillations is used the method of boundary elements. It is known that the method of finite element in displacements gives the more "stiffer" solution and consequently higher values of the free oscillation frequencies compared to accurate values. Displacements which were received by mixed or hybrid methods are approached to the exact values both from below and above. Analytical methods do not have a generality of numerical methods and are applied for a limited group of problems. Actual problem is to develop a method to determine the upper boundary of displacements or low boundary of free oscillations frequencies. This possibility is provided by the functional of additional energy, since the solutions, based on it, are more "flexible", compared with the exact values [1, 2]. But the direct use of this functional is complicated by necessity using the approximating functions that satisfy the differential equations of equilibrium [1]. If the nodal unknowns are self-equilibrated forces, then their choice has difficulties for variety of finite elements. Moreover, these systems should not include the forces to provide a statically determinate structure fixing. If the decision to use the stress function was reached, also there are difficulties to their choice and providing static boundary conditions [2].

In [3] compares the values of the frequencies of the cantilever beams free oscillations, obtained using the ANSYS, and experimentally measured values. To simulate flexural and torsional vibrations used fine mesh flat finite elements. Nonlinear free vibrations of curved shallow shells are investigated using hierarchical finite element in [5]. The decision considers the geometric nonlinearity, shear and rotational inertia. The first and higher frequencies were calculated and the high accuracy of the proposed finite element was shown.

Using the flat joint finite elements for modeling the free beam vibrations is offered in [6]. This approach allows us to consider the effect of shear deformations on the values of the frequencies of free oscillations. Free nonlinear oscillations rods considering shear deformations are considered in [7]. The solution is built using finite elements with independent linear approximations of the longitudinal and transverse displacements, and rotation angles. Hamilton's principle is used for the motion equations. The paper presents the numerical results, which show the effect of vibration amplitude and shear deformation on values of the frequencies of free vibrations of beams with different support conditions.

Precision triangular element, considering the shear deformations, is presented in [12] for bending plates. When the stiffness matrix is formed, the finite element is divided into three triangles. The proposed element allows us to calculate the frequencies of free vibrations with high precision. In [13] isoparametric quadrilateral finite element with eight nodes is used to study the free oscillations of thin and thick plates. It examines the different schemes of construction mass matrices and examples of the calculation of rectangular plates. It is concluded that the use of the diagonal mass matrix provides a high accuracy of frequencies. Various forms of the equations of the rods dynamics and significance accurate determination of the free oscillations frequencies spectrum were shown in [26, 27]. The articles [28–33] devoted to research of the various variational formulations for deciding nonlinear and thin-wall rods systems by finite element method in displacements.

In [16] as the nodal unknowns of finite element method are accepted forces, bending and torsional moments, corresponding deformations and curvatures. Distribution of forces to finite element area is taken to be linear. The system of equations consists of the equilibrium equations of nodes, which are formed by means of the principle of possible displacements and strain compatibility equations, which are written using the static-geometric analogy. The number of unknowns is reduced using the known relationship between stresses (moments) and strains (curvatures). There are examples of calculating the bending plates, shallow shells and plane elasticity problems.

Solution of the free vibrations of a rectangular, clamped along the contour plate using the expansion of displacements in Taylor, McLaren's series is presented in [21]. The article compares the frequencies of free oscillations of plates with different aspect ratio, obtained using the proposed method with those of other authors.

Thus, the main part of work is aimed at improving the method of finite elements in displacements. Some work focused on the development of finite element methods for some constructions in the form of the method of forces or stresses. Still actual is problem of constructing solutions by finite element method in stresses, which will have the same universality, as the finite element method in displacement, and will allow obtaining of alternative solutions abroad.

## Methods

In [34–38], based on the additional energy functional and the principle of virtual displacements, were built solving the building structures problems by using the stress finite element analysis. Using for the approximations of stresses (forces) in the field of finite element constant or piecewise constant functions we will get the upper border of displacements. In general, the solution of the problem reduces to finding the minimum of the additional energy functional (1) in the presence of limitations in the form of equilibrium equations of nodes (2).

$$\Pi^c = U^* + V^* = \frac{1}{2} \int \{\sigma\}^T [E]^{-1} \{\sigma\} d\Omega - \int \{T\}^T \{\bar{\Delta}\} dS \rightarrow \min, \quad (1)$$

$$\begin{aligned} \{C_{i,x}\}^T \{\bar{\sigma}_i\} + \bar{P}_{i,x} &= 0, & i \in E_x, \\ \{C_{i,y}\}^T \{\bar{\sigma}_i\} + \bar{P}_{i,y} &= 0, & i \in E_y, \\ \{C_{i,z}\}^T \{\bar{\sigma}_i\} + \bar{P}_{i,z} &= 0, & i \in E_z. \end{aligned} \quad (2)$$

where  $U^*$  – additional energy of the strains,  $V^*$  – potential boundary forces corresponding to the specified displacements [1];  $\{\bar{\sigma}_i\}$  – vector of unknown node stresses (forces) of finite elements adjacent to the node  $i$ ;  $E_x, E_y, E_z$  – sets of nodes that have free displacements along the axes  $X, Y$  и  $Z$  respectively;  $\{\bar{\Delta}\}$  – vector given displacements of nodes;  $\{T\}$  – vector boundary forces;  $S$  – boundary surface, on which the displacement nodes are given;  $\{C_{i,x}\}, \{C_{i,y}\}, \{C_{i,z}\}$  – vectors, whose elements are the coefficients (multipliers) of the unknown node stresses (forces) of finite elements adjacent to the node  $i$ ;  $\bar{P}_{i,x}, \bar{P}_{i,y}, \bar{P}_{i,z}$  – external loads potential corresponding to the virtual unit displacements of the node  $i$  along axes  $x, y, z$  respectively. The equilibrium equations (2) are formed by means of the principle of virtual displacements for all admissible displacements of nodes along the coordinate axes. If node united more than two finite elements, then you must add the equations of equilibrium moments about the axes for this node. Below, we will view only straight rods, so the equilibrium equations of moments are not considered.

To go to the problem unconstrained minimization, we use the method of Lagrange multipliers. Then the advanced additional energy functional takes the following form:

Consider the process of constructing resolving equations on the example of a flat rod system (Fig. 1).

$$\Pi_c^u = U^* + V^* + \sum_{j=x,y,z} \sum_{i \in E_j} u_{i,j} \left( \{C_{i,j}\}^T \{\bar{\sigma}_i\} + \bar{P}_{i,j} \right) \rightarrow \min, \quad (3)$$

where  $u_{i,j}$  – the actual displacement of the node  $i$  towards  $j$ , which is the Lagrange multiplier for the corresponding equilibrium equation. When using functional (3) there is no need to use a stress field, which satisfy the differential equations of equilibrium, as required by the principle of minimum additional energy. The equilibrium equations will be carried out in discrete sense – in the form of the equilibrium equations of the finite element mesh nodes.

Consider the process of constructing resolving equations on the example of a flat rod system (Fig. 1).

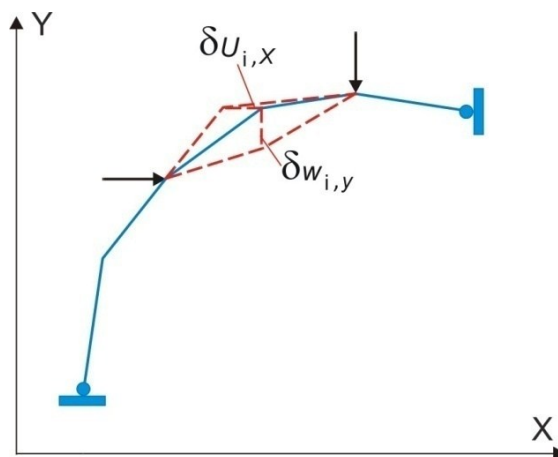
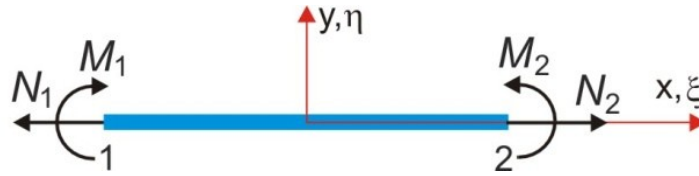


Figure 1. Flat rod system. Virtual displacements of the node

The longitudinal forces and bending moments are the nodal unknowns  $\{\bar{\sigma}_i\} = \begin{Bmatrix} N_i \\ M_i \end{Bmatrix}$  (Fig. 2).



**Figure 2. The rod finite element unknowns**

Consider the linear approximations of forces and moments in the field of finite element

$$N = \sum_{i=1}^2 L_i N_i, \quad M = \sum_{i=1}^2 L_i M_i, \quad (4)$$

where:

$$L_i = \frac{(1+\xi_i\xi)}{2}, \quad \xi = \frac{2x}{l^e}, \quad i = 1, 2, \quad (5)$$

where  $l^e$  – length of the finite element.  $L_i$  – linear function-forms, recorded by using the relative coordinate  $\xi$ . The  $\xi, x$  coordinates are measured from the center of the finite element. Thus,  $\xi_1 = -1$  and  $\xi_2 = 1$ . For the constant cross section rod, the additional energy is the sum of two integrals

$$U_e^* = \frac{1}{2} \int_0^{l^e} \frac{N(x)^2 dx}{EA^e} + \frac{1}{2} \int_0^{l^e} \frac{M(x)^2 dx}{EI^e}, \quad (6)$$

where  $E$  – modulus of elasticity;  $A^e$  – cross-sectional area;  $I^e$  – cross-sectional moment of inertia.

Denote the vector of nodal unknowns for finite element  $\{\bar{\sigma}^e\}^T = (N_1 \ M_1 \ N_2 \ M_2)$ . Then, after integration we get that

$$U_e^* = \frac{1}{2} \{\bar{\sigma}^e\}^T [B^e] \{\bar{\sigma}^e\}, \quad (7)$$

where:

$$[B^e] = \begin{bmatrix} \frac{l^e}{3EA^e} & 0 & \frac{l^e}{6EA^e} & 0 \\ 0 & \frac{l^e}{3EI^e} & 0 & \frac{l^e}{6EI^e} \\ \frac{l^e}{6EA^e} & 0 & \frac{l^e}{3EA^e} & 0 \\ 0 & \frac{l^e}{6EI^e} & 0 & \frac{l^e}{3EI^e} \end{bmatrix}. \quad (8)$$

If the approximations of forces and moments are piecewise constant functions, then

$$[B^e] = \begin{bmatrix} \frac{l^e}{2EA^e} & 0 & 0 & 0 \\ 0 & \frac{l^e}{2EI^e} & 0 & 0 \\ 0 & 0 & \frac{l^e}{2EA^e} & 0 \\ 0 & 0 & 0 & \frac{l^e}{2EI^e} \end{bmatrix}. \quad (9)$$

Global additional energy matrix  $[B^e]$  for the whole system is formed from a matrix  $[B]$  for each finite element, in accordance with global indexes of unknowns.

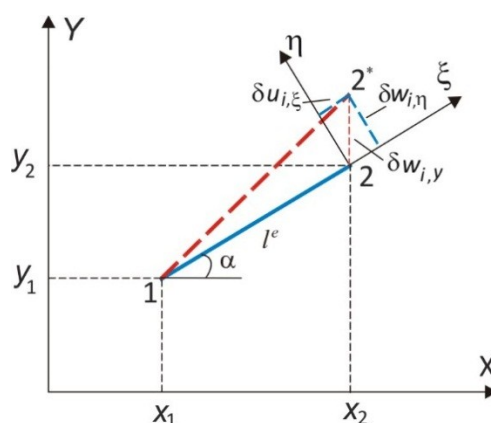


Figure 3. Local and global coordinate system

To form the equilibrium equations of nodes is introduced local coordinate system  $\xi O \eta$ , associated with the considered finite element (Fig. 3). The possible displacements of node  $\delta \bar{u}_{i,x}$ ,  $\delta \bar{w}_{i,y}$  along the global axes X and Y can be represented as the geometric sum of the displacements  $\delta \bar{u}_{i,\xi}$ ,  $\delta \bar{w}_{i,\eta}$  along the local axes. Possible displacement along axis Y causes the following displacements:

$$\delta \bar{u}_{i,\xi} = \delta \bar{w}_{i,y} \sin \alpha, \quad \delta \bar{w}_{i,\eta} = \delta \bar{w}_{i,y} \cos \alpha. \quad (10)$$

Displacement  $\delta \bar{u}_{i,x}$  can be replaced by the following possible displacements along the axes of the local coordinate system:

$$\begin{aligned} \delta \bar{u}_{i,\xi} &= \delta \bar{u}_{i,x} \cos \alpha, & \delta \bar{w}_{i,\eta} &= -\delta \bar{u}_{i,x} \sin \alpha, \\ \cos \alpha &= \frac{x_2 - x_1}{l^e}, & \sin \alpha &= \frac{y_2 - y_1}{l^e}. \end{aligned} \quad (11)$$

Displacements of points of the finite elements, abutting to this node, are linear functions (Fig. 4a). Therefore, displacement causes the constant deformation

$$\delta \varepsilon = \frac{\delta \bar{u}_{i,\xi}}{l^e}. \quad (12)$$

Then the energy deformation of the finite element in view of (4) is

$$\delta \bar{U}_{i,\xi}^e = \int_0^{l^e} N(x) \delta \varepsilon dx = (N_1 + N_2) \frac{\delta \bar{u}_{i,\xi}}{2}. \quad (13)$$

If displacement of the node is directed along the normal  $\delta \bar{w}_{i,\eta}$ , then the finite element moved as rigid body (Fig. 4b), so that the axis curvature of the rod is zero. In this case, the deformations energy equal the work of nodal moments:

$$\delta \bar{U}_{i,\eta}^e = (M_2 - M_1) \varphi = \frac{(M_2 - M_1)}{l^e} \delta \bar{w}_{2,\eta}. \quad (14)$$

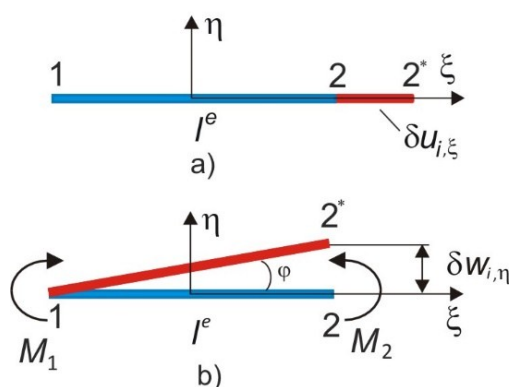


Figure 4. Possible displacements in the local coordinate system

In general, the expression (14) can be written in relative coordinates of the nodes.

$$\delta \bar{U}_{i,\eta}^e = \frac{\delta \bar{w}_{i,\eta} \xi_i}{l^e} (\xi_1 M_1 + \xi_2 M_2) = \frac{\delta \bar{w}_{i,\eta} \xi_i}{l^e} \sum_{J=1}^2 \xi_J M_J. \quad (15)$$

The relative coordinates of the nodes have the following values:  $\xi_1 = -1, \xi_2 = 1$ . Using the expressions (13), (15) and (10), the energy of deformation from possible displacement of the node along the global X-axis of coordinates is written in the following form:

$$\delta \bar{U}_{i,x}^e = \frac{\cos \alpha}{2} (N_1 + N_2) - \frac{\xi_i \sin \alpha}{l^e} (\xi_1 M_1 + \xi_2 M_2). \quad (16)$$

In vector notation

$$\delta \bar{U}_{i,x}^e = \{C_{i,x}^e\}^T \{\bar{\sigma}^e\}, \quad (17)$$

Equation (17) can be expressed by the vector unknowns of all finite elements, adjacent to the node.

$$\delta \bar{U}_{i,x}^e = \{C_{i,x}^{eg}\}^T \{\bar{\sigma}_i\}. \quad (18)$$

The elements of the vector (18) are moved in a pre-zeroed global vector  $\{C_{i,x}^{eg}\}$  in accordance with global numeration of unknowns. The size of the vector  $\{C_{i,x}^{eg}\}$  is equal to the number of nodal unknowns. For all finite elements, adjacent to the node  $i$

$$\delta \bar{U}_{i,x} = \sum_e \{C_{i,x}^{eg}\}^T \{\bar{\sigma}_i\} = \{C_{i,x}\}^T \{\bar{\sigma}_i\}. \quad (19)$$

The potential of external forces at possible unit displacement along the X-axis

$$\delta \bar{V}_{i,x} = \bar{P}_{i,x} = P_{i,x} + \frac{1}{2} \sum_e q_x^e l^e, \quad (20)$$

where:  $P_{i,x}$  – projection of the force on the X-axis;  $q_x^e$  – evenly distributed load for the finite element, adjacent to the node  $i$ . Equilibrium equation of the node along the X axis will be have the standard form

$$\{C_{i,x}\}^T \{\bar{\sigma}_i\} + \bar{P}_{i,x} = 0. \quad (21)$$

For possible displacement of the node  $\delta \bar{w}_{i,y} = 1$  along the Y-axis

$$\delta \bar{U}_{i,y}^e = \frac{\sin \alpha}{2} (N_1 + N_2) + \frac{\xi_i \cos \alpha}{l^e} (\xi_1 M_1 + \xi_2 M_2). \quad (22)$$

In vector notation

$$\delta \bar{U}_{i,y}^e = \{C_{i,y}^e\}^T \{\bar{\sigma}^e\}, \quad (23)$$

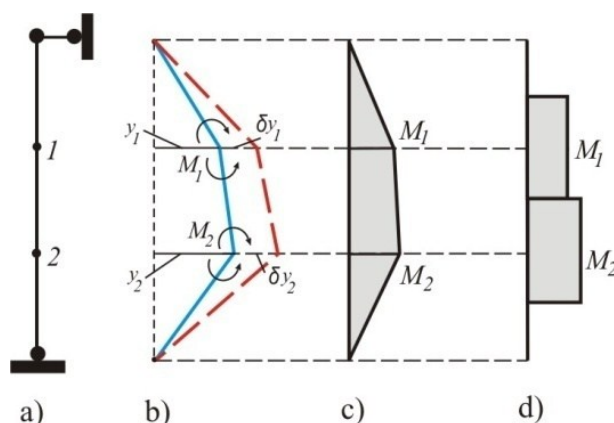
$$\{C_{i,y}^e\} = \left\{ \begin{array}{c} \frac{\sin \alpha}{2} \\ \frac{\xi_i \xi_1 \cos \alpha}{l^e} \\ \frac{\sin \alpha}{2} \\ \frac{\xi_i \xi_2 \cos \alpha}{l^e} \end{array} \right\}. \quad (24)$$

The potential of external forces at possible unit displacement along the Y-axis

$$\delta \bar{V}_{i,y} = \bar{P}_{i,y} = P_{i,y} + \frac{1}{2} \sum_e q_y^e l^e, \quad (25)$$

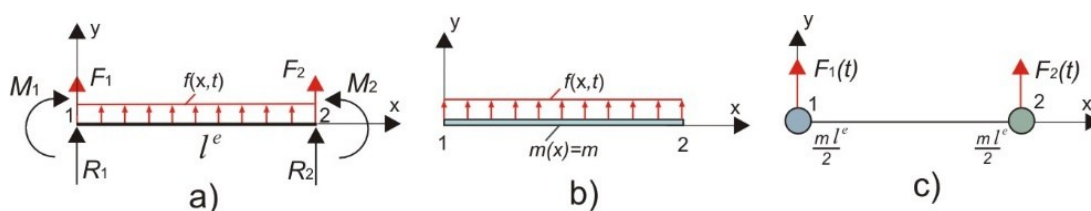
Equilibrium equation of the node along the Y-axis will be like (22).

Consider the free oscillations of constant cross-section rods. To illustrate the method simply supported rod, divided into three finite elements, is showed in Figure 5. The nodal displacements and bending moments will be unknowns.



**Figure 5. Simply supported rod: a) finite elements; b) a form of free oscillations; c) linear approximation of moments; d) piecewise constant approximation of moments**

The length of the finite element –  $l^e$ , bending stiffness –  $EI$ , the mass per unit length of rod –  $m$ . In Figure 6, for any finite element, the following notation:  $M_1, M_2$  – the internal bending moments;  $R_1, R_2$  – nodal reactions corresponding bending moments;  $F_1, F_2$  – nodal reactions, corresponding inertia forces  $f(x, t)$ . The longitudinal vibrations are absent.



**Figure 6. The finite element: a) internal forces and nodal reactions; b) dynamic finite element model with distributed mass; c) dynamic finite element model with concentrated masses**

Using the principle of virtual displacements or equations of equilibrium moments, nodal reactions  $R_1, R_2$  can be expressed through the nodal bending moments. For example, reaction  $R_2$  is the derivative of the virtual deformations energy  $\delta \bar{U}_{i,\eta}^e$  by virtual displacement  $\delta \bar{w}_{2,\eta}$  (see (14)). Expression for the finite element nodal forces in the matrix form is written as

$$\begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = \begin{bmatrix} \frac{1}{l^e} & -\frac{1}{l^e} \\ -\frac{1}{l^e} & \frac{1}{l^e} \end{bmatrix} \begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix} \text{ or } \{R^e\} = [A^e]\{M^e\}. \quad (26)$$

Matrix  $[A^e]$  is static equations matrix. Vector  $\{R^e\}$  is vector of unknown nodal bending moments for finite element. The form of the rod oscillations is approximated by broken line (Fig. 5b). In the field of finite element, displacements of points are represented by a linear function. Thus, the distributed forces of inertia are determined by expression (28):

$$f(x, t) = -m\ddot{y} = m\omega^2 \left( y_1 \left( 1 - \frac{x}{l^e} \right) + y_2 \frac{x}{l^e} \right) \sin(\omega t). \quad (27)$$

Then, excluding  $\sin(\omega t)$ , we obtain nodal reactions corresponding distributed inertia forces

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} \frac{\omega^2 m l^e}{3} & \frac{\omega^2 m l^e}{6} \\ \frac{\omega^2 m l^e}{6} & \frac{\omega^2 m l^e}{3} \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} \text{ or } \{F^e\} = [B^e] \{y^e\}. \quad (28)$$

If the mass concentrated in the nodes, we obtain a diagonal matrix

$$[B^e] = \begin{bmatrix} \frac{\omega^2 m l^e}{2} & 0 \\ 0 & \frac{\omega^2 m l^e}{2} \end{bmatrix}. \quad (29)$$

Additional energy of the finite element strains is expressed as the integral

$$\Pi^{*e} = \frac{1}{2} \int_0^l \frac{M(x)^2}{EI} dx. \quad (30)$$

After integration, we obtain the following matrix expression:

$$\Pi^{*e} = \frac{1}{2} \{M^e\}^T [D^e] \{M^e\}, \quad (31)$$

Flexibility matrix  $[D^e]$  is written in two versions –  $[D^e]_{lin}$  for linear (Fig. 5c) and  $[D^e]_{pic}$  for piecewise constant (Fig. 5d) approximations of the moments:

$$[D^e]_{lin} = \begin{bmatrix} \frac{l}{3EI} & \frac{l}{6EI} \\ \frac{l}{6EI} & \frac{l}{3EI} \end{bmatrix}, [D^e]_{pic} = \begin{bmatrix} \frac{l}{2EI} & 0 \\ 0 & \frac{l}{2EI} \end{bmatrix}. \quad (32)$$

By forming from local vectors and matrices (with index  $e$ ) according to the unknown's numeration, we obtain similar global matrix and vector for the whole system. Expressing through them functional (3) we get:

$$\Pi_c^u = \frac{1}{2} \{M\}^T [D] \{M\} + \{y\}^T ([A] \{M\} + \{F\}) \rightarrow \min. \quad (33)$$

By equating to zero the derivatives of the (34) along the vectors  $\{M\}$  and  $\{y\}$ , we obtain

$$[D] \{M\} + [A]^T \{y\} = 0, \quad (34)$$

$$[A] \{M\} + \{F\} = 0. \quad (35)$$

The vector of inertia forces expressed through the global matrix  $[B]$ , then the system of linear homogeneous equations takes the following form:

$$\begin{cases} [D] \{M\} + [A]^T \{y\} = 0, \\ [A] \{M\} + [B] \{y\} = 0. \end{cases} \quad (36)$$

By equating to zero the determinant of the system (36), we find the frequencies of the free oscillations  $\omega$ . The number of unknowns of the system (36) can be reduced. To do this, we express the vector  $\{M\}$  from the first matrix equation in system (36) and substitute it in the second equation. Then we will get the system of equations (37). The system (38) is obtained by expression vector  $\{y\}$  from the second matrix equation in (36).

$$(-[A][D]^{-1}[A]^T + [B]) \{y\} = 0, \quad (37)$$

$$(-[A]^T[B]^{-1}[A] + [D]) \{M\} = 0. \quad (38)$$

The system (37) have advantage, if we will use the piecewise constant approximation of moments along length of the finite element. Then the matrix  $[D]$  consists of the matrices  $[D^e]_{pic}$  and the matrix  $[D]^{-1}$  elements are calculated analytically. If we used only concentrated masses, then matrix  $[B]$  will take the diagonal form. In this case, it is easier to use the system of equations (38). If matrices  $[D]$  and  $[B]$  does not have the diagonal form, the frequency must be determined by the system (36), which is Tyukalov Yu.Ya. Stress finite element models for determining the frequencies of free oscillations. *Magazine of Civil Engineering*. 2016. No. 7. Pp. 39–54. doi: 10.5862/MCE.67.5

bigger than the system (37) or (38). If we calculate the frequency by means an iterative process, it would require more computing operations.

### Results and Discussion

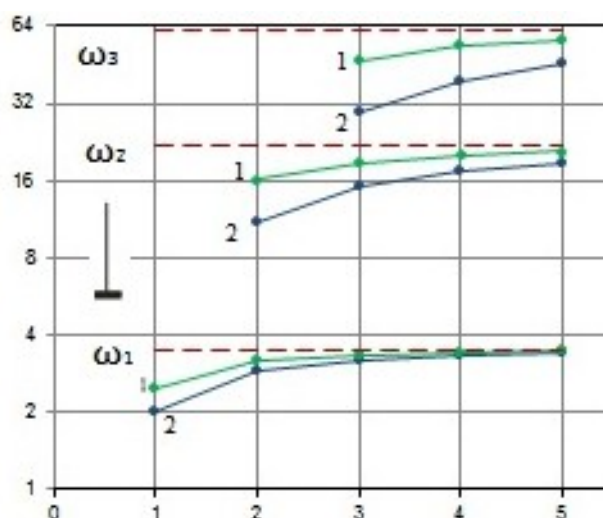
To evaluate the accuracy determination of the frequencies by the method proposed above, we have been calculated first three frequencies of the free oscillations for constant cross-section rods with different supports of the ends. To simplify the analysis length, bending stiffness and mass per meter of the rods was taken equal to unity. In the calculations was used Mathcad 14.0 program.

Graphics in Figures 7–14 show comparison of the calculated values of the first three frequencies of free oscillations with the exact values. The first version of the figures (green line – 1) corresponds to a linear approximation moments along length of the finite element, the second – a piecewise constant moments (blue line – 2).

The results show, that by using the concentrated masses (Fig. 7–10) frequencies calculation values are approached to exact values from below, when we use both linear and piecewise constant approximation of the moments.

If the mass of the rod is evenly distributed (Fig. 11–14) and moments are piecewise constants, the frequencies are approached to exact values from below. When using linear approximation moments – the calculated frequencies are approached to exact values from above, as in the case of using the finite element method in displacement.

The proposed method of calculation allows you to get the opposite bound values for the frequencies of free oscillations, in comparison with the finite element analysis in displacements. This requires use the concentrated masses and linear or piecewise constant approximation of the moments. Also, we can use the distributed mass in combined with piecewise constant approximation of the moments.



**Figure 7. The frequency of free oscillations of a cantilever rod depending on the number of grid nodes. The mass of the rod concentrated at the nodes**

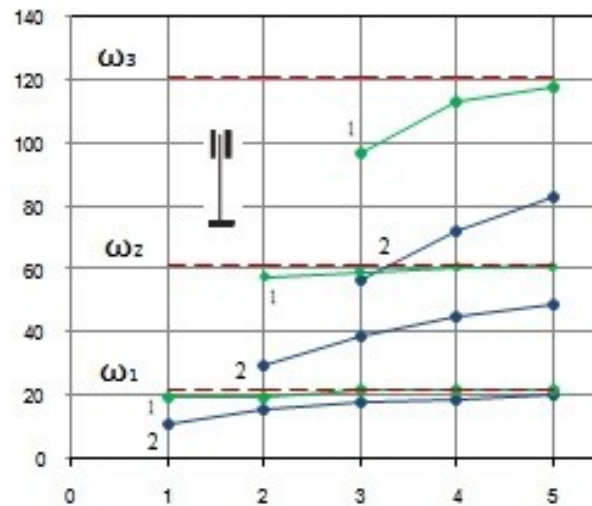


Figure 8. The frequencies of free oscillations of a clamped rod depending on the number of grid nodes. The mass of the rod concentrated at the nodes

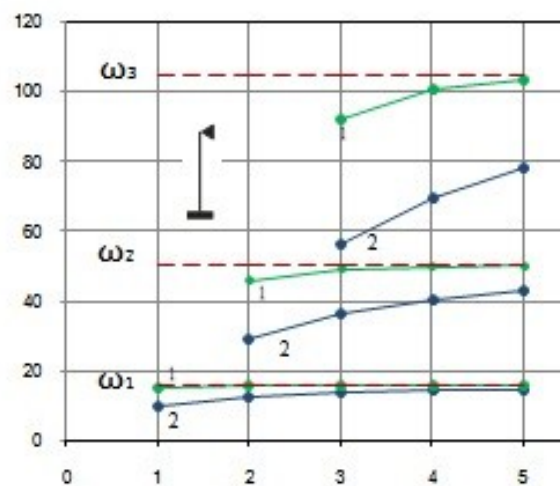


Figure 9. The frequencies of free oscillations of a rod with one clamped and one hinge supports depending on the number of grid nodes. The mass of the rod is concentrated at the nodes

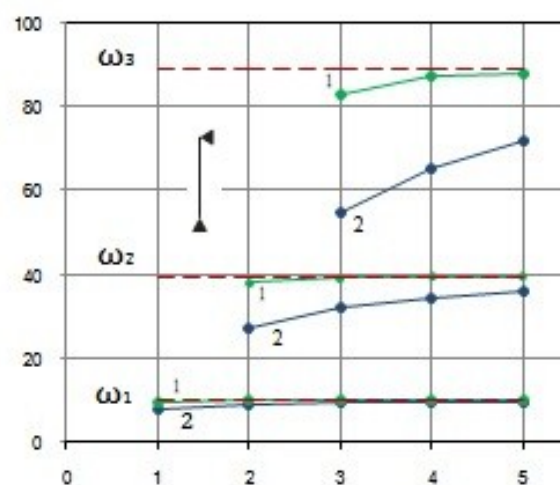


Figure 10. The frequency of free oscillations of simply supported rod depending on the number of grid nodes. The mass of the rod is concentrated at the nodes

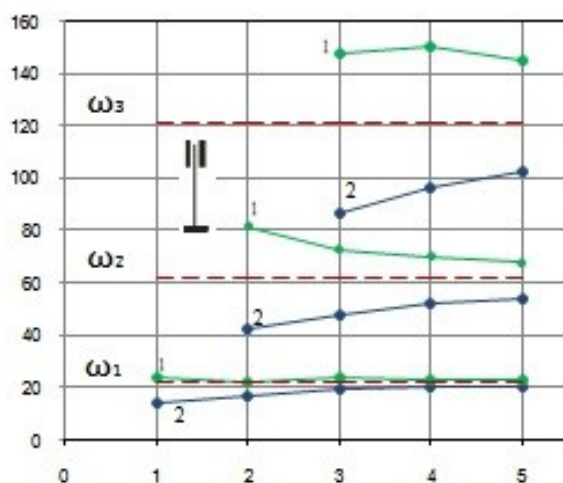


Figure 11. Frequencies of free oscillations of the clamped rod, depending on the number of grid nodes. The mass of the rod is evenly distributed

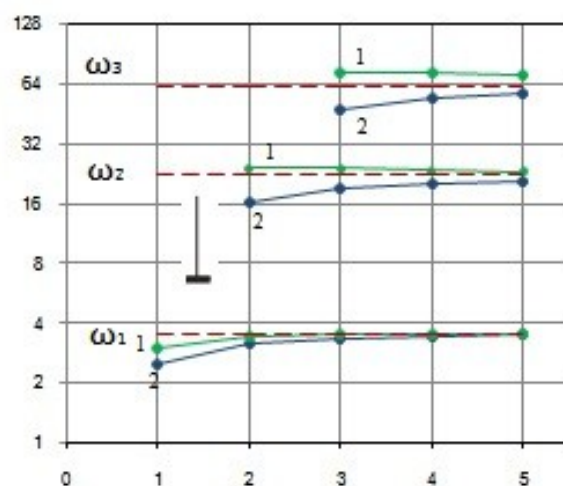


Figure 12. The frequency of free oscillations of a cantilever rod depending on the number of grid nodes. The mass of the rod is evenly distributed

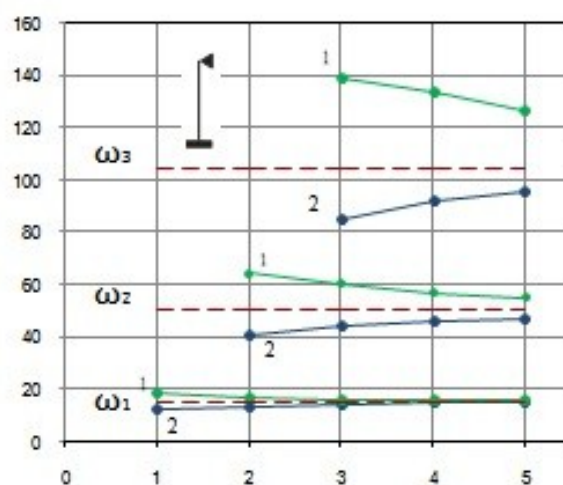
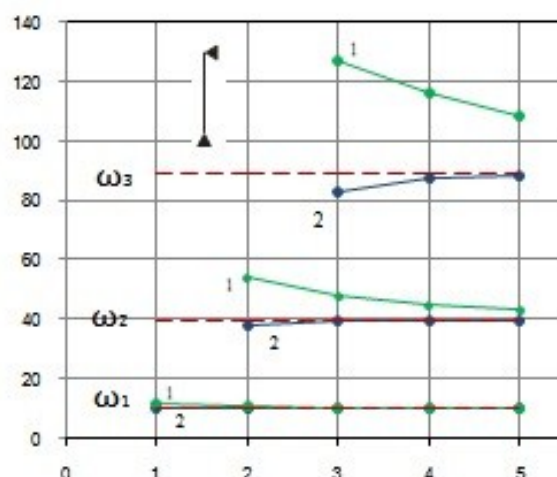


Figure 13. The frequency of free oscillations of a rod with one clamped and one hinged supports depending on the number of grid nodes. The mass of the rod is evenly distributed



**Figure 14. The frequency of free oscillations of hinged rod depending on the number of grid nodes. The mass of the rod is evenly distributed**

Tables 1–4 show the calculations results and their accuracy estimates that were received by using the concentrated masses and linear approximation of the moments. Table 5 show the calculation results accuracy estimates. Number of grid nodes is equal five. In Table 5 rods are designated as: 1 – cantilever rod; 2 – clamped rod; 3 – rod with one clamped and one hinged supports; 4 – hinged rod.

The above data in Tables 1–4 show, that by crushing grid the results accuracy is increasing quite quickly. For cantilever beam, the calculating error is reducing from 30.2 % to 1.8 % for the first frequency, from 26.2 to 5.9 for the second frequency and from 23.8 to 9.3 for the third frequency. Obviously, for a more accurate calculation of the second and third frequencies values is required more nodes. For other supports of the rods, the accuracy of the calculated frequency values is substantially higher and it is satisfactory for all three frequencies even at five nodes.

In Table 5 the accuracy estimates of the calculated results in percentage for the grid with five nodes are shown. The highest precision is achieved by using a linear approximation of the bending moments and concentrated masses. Less accurate results are obtained by using a piecewise constant approximation points and distributed masses. More nodes of the grid are required to achieve high accuracy of the results using piecewise constant approximations of moments and concentrated masses, but in this case, as in the previous two, calculated values of the frequencies are approaching to the exact values from below. When using linear approximation moments and distributed mass, the calculated frequencies are approached to exact values from above and are required more nodes for achieve high accuracy of the results.

**Table 1. The results for cantilever rod (Fig. 7 green line – 1)**

Number of grid nodes	Exact value $\omega_1 = 3.516 \text{ rad/sec}$		Exact value $\omega_2 = 22.035 \text{ rad/sec}$		Exact value $\omega_3 = 61.701 \text{ rad/sec}$	
	Calculated values	Error %	Calculated values	Error %	Calculated values	Error %
1	2.449	30.3	–	–	–	–
2	3.156	10.2	16.258	26.2	–	–
3	3.346	4.8	18.886	14.3	47.028	23.8
4	3.418	2.8	20.090	8.8	53.202	13.8
5	3.453	1.8	20.734	5.9	55.953	9.3

**Table 2. The results for clamped rod (Fig. 8 green line – 1)**

Number of grid nodes	Exact value $\omega_1 = 22.373$ rad/sec		Exact value $\omega_2 = 61.67$ rad/sec		Exact value $\omega_3 = 120.903$ rad/sec	
	Calculated values	Error %	Calculated values	Error %	Calculated values	Error %
1	19.596	12.4	–	–	–	–
2	20.069	10.3	57.65	6.5	–	–
3	22.302	0.3	59.20	4.0	97.40	19.4
4	22.350	0.1	60.90	1.25	113.12	6.4
5	22.364	0.04	61.39	0.45	118.01	2.4

**Table 3. The results for rod with one clamped and one hinge supports (Fig. 9 green line – 1)**

Number of grid nodes	Exact value $\omega_1 = 15.418$ rad/sec		Exact value $\omega_2 = 49.964$ rad/sec		Exact value $\omega_3 = 104.248$ rad/sec	
	Calculated values	Error %	Calculated values	Error %	Calculated values	Error %
1	14.832	3.8	–	–	–	–
2	15.349	0.45	45.632	8.7	–	–
3	15.402	0.10	49.054	1.8	91.53	12.2
4	15.412	0.04	49.683	0.56	100.43	3.7
5	15.416	0.01	49.851	0.23	102.82	1.4

**Table 4. The results for simply supported rod (Fig. 10 green line – 1)**

Number of grid nodes	Exact value $\omega_1 = 9.869$ rad/sec		Exact value $\omega_2 = 39.478$ rad/sec		Exact value $\omega_3 = 88.826$ rad/sec	
	Calculated values	Error %	Calculated values	Error %	Calculated values	Error %
1	9.798	0.7	–	–	–	–
2	9.859	0.1	38.184	3.3	–	–
3	9.867	0.02	39.192	0.7	83.21	6.3
4	9.868	0.01	39.381	0.25	87.18	1.8
5	9.869	0	39.436	0.1	88.18	0.7

**Table 5. Accuracy estimate of the calculated results in percentage**

$\omega$	Rod	M – linear m – distributed	M – linear m – concentrated	M – piecewise constant m – distributed	M – piecewise constant m – concentrated
$\omega_1$	1	0.3	1.8	1.8	3.2
	2	-2.7	0.04	7.8	10.1
	3	-2.6	0.01	3.5	5.9
	4	-2.3	0	0	2.3
$\omega_2$	1	-4.0	5.9	6.0	14.4
	2	-9.8	0.45	12.3	20.5
	3	-9.8	0.23	6.0	14.3
	4	-9.4	0.1	0.1	8.8
$\omega_3$	1	-13.2	9.3	9.3	26.5
	2	-19.7	2.4	15.6	31.0
	3	-21.0	1.4	8.5	25.2
	4	-21.6	0.7	0.7	18.9

## Conclusions

1. The proposed method of calculation allows getting the opposite bound values (bottom values) for the frequencies of free oscillations, in comparison with the finite element analysis in displacements.
2. The highest precision is achieved by using a linear approximation of the bending moments and concentrated masses. For grid with five nodes the maximal calculating error is 1.8 % – for first frequency, 5.9 % – for second frequency and 9.3 % – for third frequency. The calculated values of the frequencies are approaching to the exact values from below.
3. Use the concentrated masses and linear or piecewise constant approximation of the moments or the distributed mass in combined with piecewise constant approximation of the moments, the calculated values of the frequencies are approaching to the exact values from below.
4. Using linear approximation moments and distributed mass, the calculated frequencies are approached to exact values from above. For grid with five nodes the maximal calculating error is 2.7 % – for first frequency, 9.8 % – for second frequency and 21.6 % – for third frequency.

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## Non-uniform torsion of thin-walled open-section multi-span beams

### Стесненное кручение многопролетных тонкостенных балок открытого профиля

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**Key words:** structural mechanics; light steel thin-walled structures; bending moment; bimoment; multi-span continuous thin-walled beams; frame system; beam deformation; normal stress; coefficient of proportionality; method by B.P.E. Clapeyron

**Ключевые слова:** энергоэффективность; коэффициент теплотехнической однородности; геометрический параметр; сопротивление теплопередаче; ограждающая конструкция здания; температурное поле; теплота; термограмма; здание; строительство

**Abstract.** In the article a calculation of the bending moment and bimoment for different occasions single- and multi-span continuous thin-walled channel section beams using B.P.E. Clapeyron and N.L. Kuzmin methods have been made. Article considers the cases of uniformly distributed loading with the eccentricity relative to the center line of the bend sections of one-, two-, three- and five-span beams with spans 2.5 meters long and 5 meters. Ratio between the bending moment and bimoment in feature points and proportionality coefficients between the bending moment and bimoment has been calculated experimentally. Article suggests a formula that allows to determine bimoment, if only the bending moment and the eccentricity of applied load are given.

**Аннотация.** В статье выполнен расчет изгибающего момента и бимоента для разных случаев одно- и многопролетных неразрезных тонкостенных балок швеллерового сечения с использованием методов Б.П.Э. Клапейрона и Н.Л. Кузьмина. Рассмотрены случаи загрузки равномерно распределенной нагрузкой с эксцентриситетом относительно линии центров изгиба сечений одно-, двух-, трех- и пятипролетной балок с пролетами: длиной 2,5 метров и 5 метров. Посчитаны отношения величин изгибающего момента и бимоента в характерных точках пролетов и определены коэффициенты пропорциональности между изгибающим моментом и бимоментом. Предложена формула, позволяющая определить бимомент, зная лишь изгибающий момент и эксцентриситет приложения нагрузки.

### Introduction

At the turn of XXI century in all over the earth metallurgy industry is quite actively developing. Today there is widespread application of metal structures in civil engineering. The main advantages of steel structures are universality, strength and, at the same time, lightness. Moreover, these constructions are pre-fabricated, that is very important in construction.

The most profitable and effective way to construction of prefabricated structures is to use the system of light steel thin-walled structures (LSTS below), and also thermal insulation, facing and vapor sealing.

However, such a significant feature as lightness, can effect on the ability of constriction's bearing strength. So, testing and analysis of bearing strength of the product are necessary (especially for open sections, such as I-profiles and Z-profiles, angle and channel profiles).

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LSTS can be used as an economic variant in low-rise housing construction.

Despite the widespread application of LSTS, there are significant weaknesses in the standard and methodological bases of LSTS calculation.

To solve these problems, it is necessary to refer to the fundamental science – structural mechanics.

Cold-rolled thermoprofile and light thin galvanized sheet beams are the main elements of LSTS. Beams are used to build the frame of the building, and for the installation of individual elements.

The special shape of the profile ensures high durability of the product and makes on the thermal profile walls slots reduce heat loss and avoids the emergence of "cold bridges". [1]

In construction practice bending is the most common type of strain, which is more typical for beam structures. In most cases, share force occurs with the bending moment in the beams and that bending is called cross.

Normal stresses dominate in cross bend of thin-walled beams. It determines the bearing capacity of the beam.

The normal stress is a summation of longitudinal force acting, bending moments of two planes and bimoment acting.

Bimoment – additional force characteristic. It is self-balanced factor and it can not be found from equilibrium conditions of clipping part. Bimoment characterizes the changes, introduced in linear zones stress distribution of distortion of cross-section. It is a pair of bending moments applied in opposite directions, or the four forces, the action direction of two is not the same as the rest.

V.Z. Vlasov made a huge technical contribution to the thin beams theory development. He is considered as the founder of the theory. He suggested the formula of bimoment calculation for thin-walled metal-roll. His theory was developed and continued throughout the twentieth century by dozens of Russian scientists. S.P. Tymoshenko made a big contribution to the development of the thin-walled beams stability theory. Engineer E.N. Popova engaged the problem of accounting embossments in thin-walled profiles. [1]

A.R. Tusnin and M. Prokich examined the combined effect of bending moment and bimoment on I-profiles. They have given recommendations for determination their load-bearing capacity with the development of plastic deformations. [2]

N.L. Kuzmin, P.A. Lucash, I.E. Mileikovskii engaged the methods of bimoment and stresses calculation for multi-span beams, but they did not define the link between bimoment and bending moment. [3]

D.A. Trubina, L.A. Kononov, A.A. Kaur, E.D. Pichugin, D.A. Abdulaev in their article [5] examined various methods of thin-walled structures behaviour, and also determined the dependence of local stability of the cross-section of the various parameters and found the degree of influence on the intermediate amplifiers LPA wall.

V. Rybakov and O.S. Gamayunova in their article [6] provide an overview of the characteristics and problems of the stress-strain state (SSS) analysis of the thin-walled frame structures elements of open and closed profiles, and also some specific effects of their work, such as warping of the cross-section and the "bimomental" normal stress distribution over the cross section.

M.K. Bronzova, N.I. Vatin, M.R. Garifullin in their article [7] describe a new design of frame buildings using the skeleton of LSTS monolithic foam concrete.

M.R. Garifullin, N.I. Vatin considered domestic and foreign publications relating to cold-formed thin-walled profile, which works on a bend. [8]

T.V. Nazmeeva in her article [9] provides numerical and experimental study of bearing capacity of racks of various lengths from cold-formed C-profile solid and perforated section in order to obtain reliable methodology for their calculation.

A.S. Sinelnikov made a numerical analysis of the expanded stretching profile based on the finite element method, and conducted experimental and analytical studies. [10, 11]

In their work [13] V.V. Lalin, V.A. Rybakov have developed a numerical method for the calculation of thin-walled rod systems semi-shearing and unshearing theories calculation. [13]

Pavlenko A.D., Rybakov V.A., Pikht A.V., Mikhailov E.S. Non-uniform torsion of thin-walled open-section multi-span beams. *Magazine of Civil Engineering*. 2016. No. 7. Pp. 55–69. doi: 10.5862/MCE.67.6

According to the researches [13–16] in thin-walled bar structures, which are subjected by bending torsion, component of normal stresses from bimoment can exceed component from bending moment and tangent strain effects on stress strain behavior small by the side of normal stresses. [4]

V.V. Lalin, V.A. Rybakov, S. Morozov in their publication reviewed test problems of non-uniform torsion of thin-walled beam with different boundary conditions. In the terms of the search static force factor in a non-uniform torsion were examined: bimoment, sectorial torque and moment of pure torsion. [17]

The article [18] considers the technology of multi-storey buildings construction using LSTS, and also offers the possibility of constructive solutions on the basis of existing projects. Article offers a comparison of LSTS with other materials used as cladding elements in buildings with high altitude. The main advantages and disadvantages of the application of light steel thin-walled structures in high-rise construction were showed in the issue.

The article [20] of A.R. Tusnin shows the features of calculation and design of thin-walled structures, which are widely used in construction. The numerical method of calculation of bar beam systems is considered.

In [21] free vibration analysis of thin-walled channel section beam including warping effects has been carried out using Finite element method and compared with experimentally measured results.

The article [22] discusses the dynamics of a composite thin-walled box-beam built into a rigid hub and performing combined motion of rotation and in-plane translation.

The article [23] shows that for the calculation of thin-walled structures considering restrained torsion can be successfully applied finite element method.

In [24] the crushing behavior of circular aluminum thin-walled tubes with functionally graded thickness (FGT) is investigated experimentally.

The lateral stability of bending non-prismatic thin walled beams is calculated using orthogonal Chebyshev series in the work [25].

This paper [26] investigates the effects of fatigue material data and finite element types on accuracy of residual life assessments under high cycle fatigue conditions.

The article [27] presents an experimental study on the behaviors and modes of failure of square thin-walled steel tubed RC columns subjected to concentrically axial load applied directly to the RC core.

The purpose of this paper [28] is to resolve aforesaid problems and find out a kind of new technics substitute for present forming method.

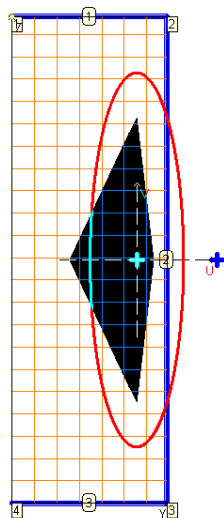
According to available sources, bimoment is calculated by different formulas, depending on the type of design schemes. The disadvantage is in cumbersome formulas of the application of a hyperbolic sine and cosine. [4]

The goal of this article is to obtain a design proportionality coefficient (maximum) value of the bending moment and bimoment for multi-span beams with equal span, loaded with a uniformly distributed load equipartition, which will facilitate the difficult calculation of bimoment.

The objectives of the article – a function definition (diagrams) of the bending moment in the statically indeterminate single- and multi-span beams, including the method of B.P.E. Klaperyon and N.L. Kuzmin.

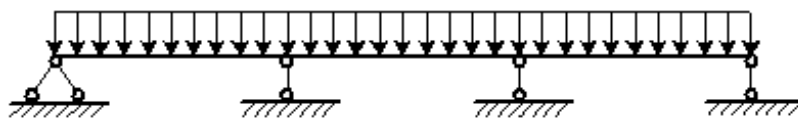
## Methods

Assume channel section of beam (Figure 1) with dimensions: 70 mm – width of a channel, 220 mm – height:



**Figure 1. Cross-section of the beam under consideration in the program TONUS (SCAD Office)**

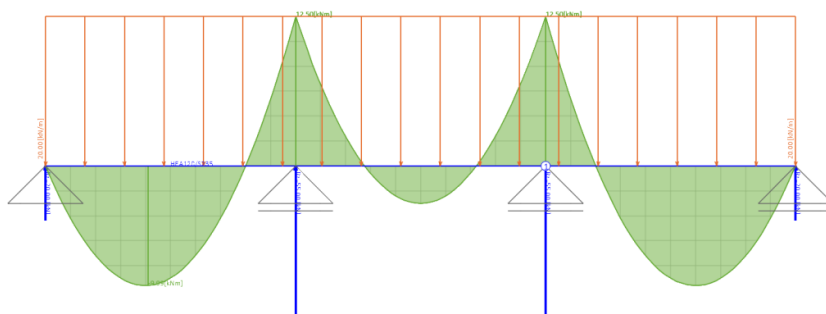
Consider the calculation of bending moments and bimoment for the case of three-span beams (Figure 2), loaded with a uniformly distributed load  $q = 20 \text{ kN/m}$ , the length of each span  $l = 2.5 \text{ m}$ .



**Figure 2. Design model**

*Define of values of bending moment, using the method of B.P.E. Clapeyron [4]:*

The value of the bending moment in the middle supports will be found with the equation of three moments.



**Figure 3. The bending moment (kN·m) for the three-span beam**

Finding bimoment is divided into four steps.

Consider bimoment value for each span of the uniformly distributed load, using (1) according to [3]:

$$B_w = \frac{qe}{k^2} \left[ 1 - \frac{\text{chk}(\frac{l}{2} - z)}{\text{ch} \frac{kl_1}{2}} \right], \quad (1)$$

where  $z$  – horizontal coordinate of span varies from 0 to 2.5 m, the eccentricity  $e$  is calculated by the formula (2):

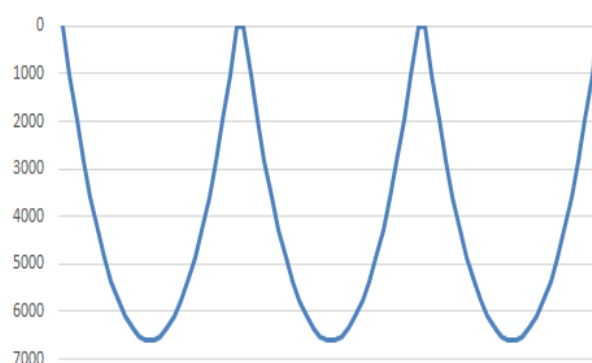
$$e = \frac{b}{3} + \alpha, \quad (2)$$

where  $\frac{b}{3}$  – accepted position in the load center from the origin application center of wall channel gravity [1],  $b$  – the width of the channel,  $\alpha$  – coordinate of the bending channel beams center found using a satellite program “TONUS” (SCAD Office). [19]

Flexural-torsional characteristics of response section  $k$  will be found with the formula (3):

$$k = \sqrt{\frac{GI_d}{EI_w}}, \quad (3)$$

where the shear modulus steel is  $G = 79.3 \cdot 10^9$  Pa, elastic modulus steel is  $E = 2 \cdot 10^{11}$  Pa; torsional inertia moment  $I_d = 0.095 \text{ cm}^4$  and sectorial inertia moment  $I_w = 2647.298 \text{ cm}^6$ . Geometric characteristics are defined in the "TONUS" program [19].



**Figure 4. Diagram of bimoment ( $\text{kN} \cdot \text{m}^2$ ) from uniformly distributed load with eccentricity in each span**

*Define values of bimoment  $B_1$  on supports, with using formula of three bimoments (4) [3]:*

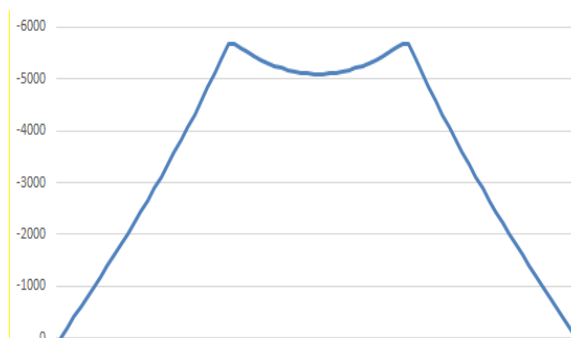
$$B_{n-1}l_n\psi_n + 2B_n(l_n\varphi_n + l_{n+1}\varphi_{n+1}) + B_{n+1}l_{n+1}\psi_{n+1} = -6EJ_\omega(\bar{\theta}_{n+1}^{(\text{left})} - \bar{\theta}_{n+1}^{(\text{right})}), \quad (4)$$

where  $\psi_n$  and  $\varphi_n$  – values, determined with interpolation method from table [3, p. 64],  $n$  – number of supports,  $\theta_{n+1}$  – rotation angles on anchorages, determined with formula (5):

$$\theta = -\frac{qe}{k^3EJ_\omega} \left( \frac{kl}{2} - \frac{\text{sh} \frac{kl}{2}}{\text{ch} \frac{kl}{2}} \right), \quad (5)$$

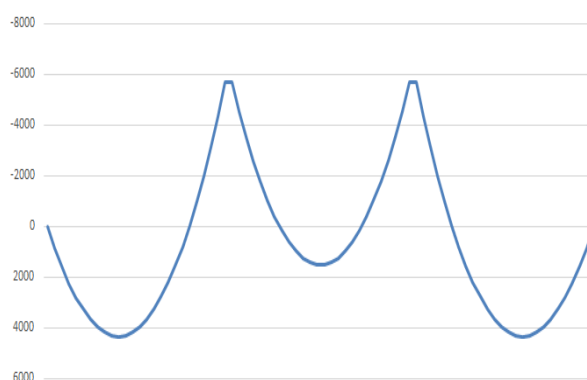
Define bimoments for each span from determined bimoment  $B_1$ , applied in the middle supports by formula (6):

$$B_{\frac{1}{2}} = B_1 \frac{1}{2 \text{ch} \frac{kl_1}{2}}, \quad (6)$$



**Figure 5. Diagram of bimoment (kN·m<sup>2</sup>) from concentrated bimoments on anchorages**

Conclusive diagram of bimoments frame as algebraical sum of couple diagrams (Figures 4 and 5)



**Figure 6. Conclusive diagram of bimoments (kN·m<sup>2</sup>)**

### *Comparison of bending moment diagrams and bimoment diagrams*

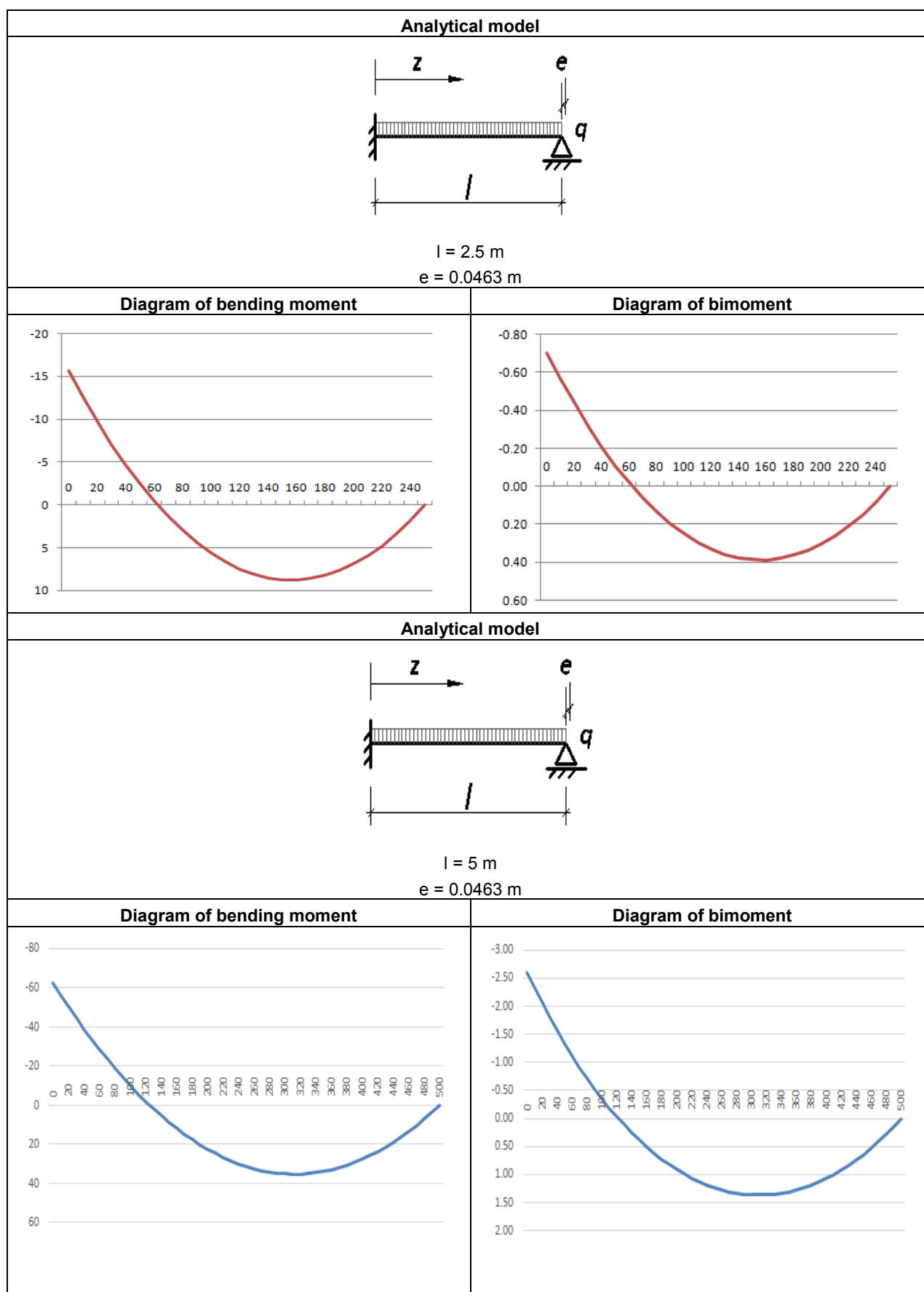
In the same way calculations were made for others modifications of analytical models and span's lengths. Analytical models and totals of calculations (diagram of bending moment and bimoment) were summarized in the Table 1.

As is seen from Figures 3–6, diagrams of bending moment and bimoment have similar configuration.

Suppose, that coversine of bimoment it is possible to express by coversine of bending moment using the proportionality coefficient.

To prove it, compare diagrams of bending moment and bimoment.

**Table 1. Diagrams of bending moment and bimoment for different single-span analytical models**



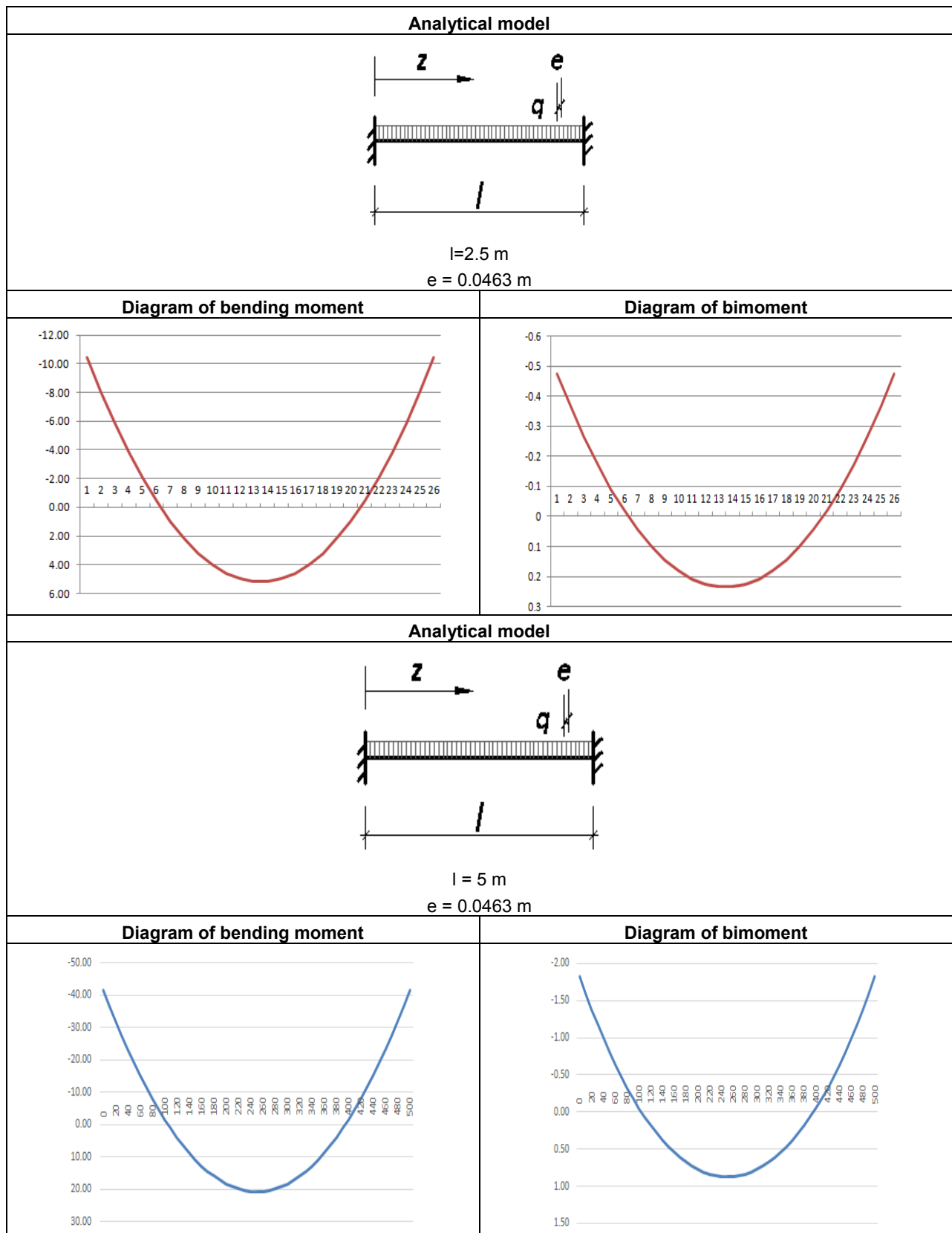
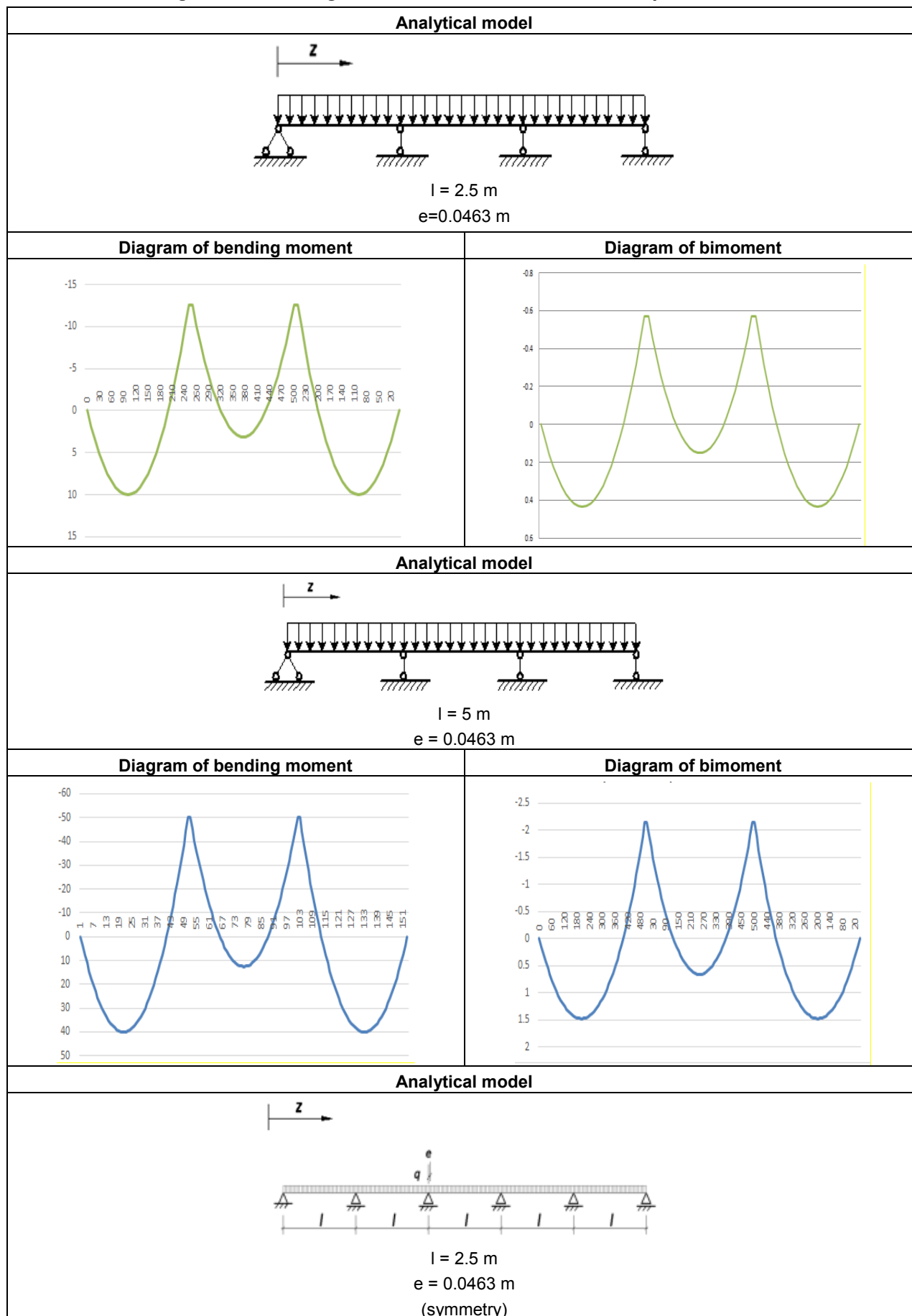


Table 2. Diagrams of bending moment and bimoment for multi-span beams



Павленко А.Д., Рыбаков В.А., Пихт А.В., Михайлов Е.С. Стесненное кручение многопролетных тонкостенных балок открытого профиля // Инженерно-строительный журнал. 2016. № 7(67). С. 55–69.

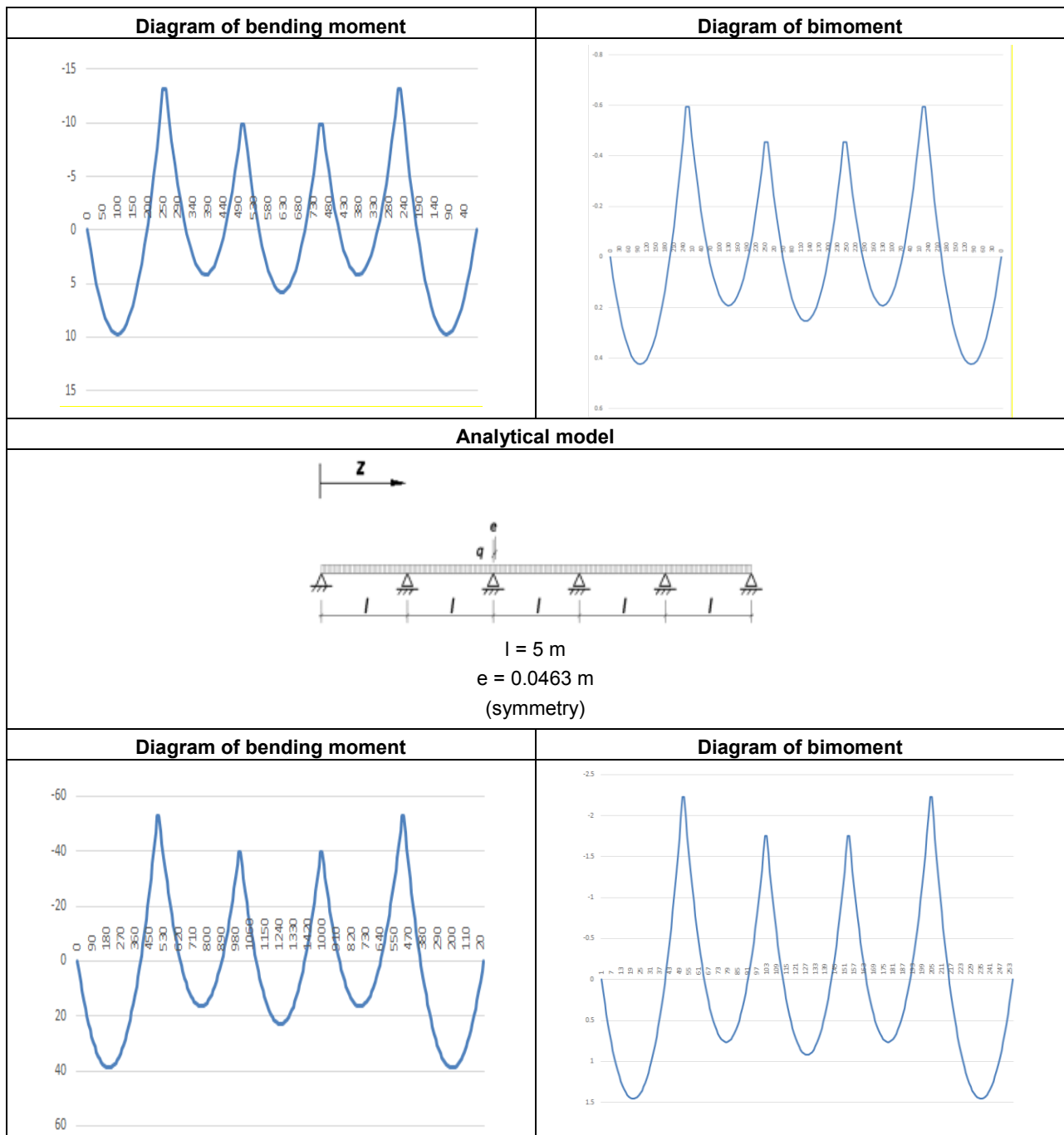
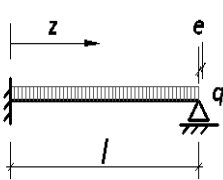
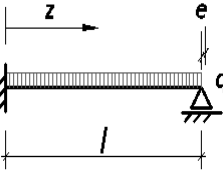
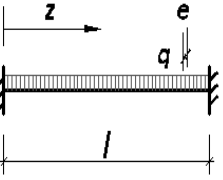
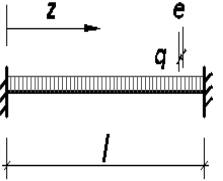
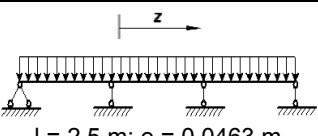
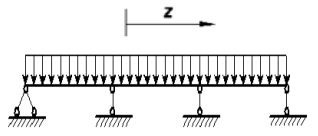


Table 3. Values of bending moment and bimoment for single-span beams

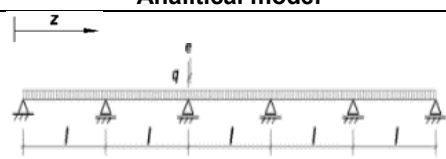
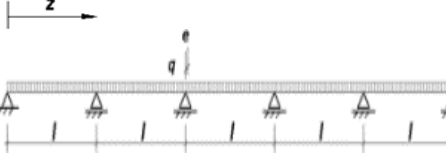
<div> <div>Bending moment (bimoment)</div> <div>Analitical model</div> </div>	<div> <div>Moment value in fixed support (kN·m)</div> <div>Bimoment value in fixed support (kN·m<sup>2</sup>)</div> <div>Moment and bimoment ratio (m<sup>-1</sup>)</div> </div>	<div> <div>Moment value in the middle of span (kN·m)</div> <div>Bimoment value in the middle of span (kN·m<sup>2</sup>)</div> <div>Moment and bimoment ratio (m<sup>-1</sup>)</div> </div>	<div> <div>Moment value in point of local extremum (kN·m)</div> <div>Bimoment value in point of local extremum (kN·m<sup>2</sup>)</div> <div>Moment and bimoment ratio (m<sup>-1</sup>)</div> </div>
<div>  <p><math>l = 2.5 \text{ m}; e = 0.0463 \text{ m}</math></p> </div>	-15.63	7.81	8.78 ( $z = 1.6 \text{ m}$ )
	-0.70	0.346	0.39 ( $z = 1.6 \text{ m}$ )
	22.23	22.56	22.64 ( $z = 1.6 \text{ m}$ )
	<b>0.967</b>	<b>0.957</b>	<b>0.959</b>
<div>  <p><math>l = 5 \text{ m}; e = 0.0463 \text{ m}</math></p> </div>	-62.50	31.25	35.15 ( $z = 3.1 \text{ m}$ )
	-2.60	1.23	1.36 ( $z = 3.1 \text{ m}$ )
	24.05	25.45	25.77 ( $z = 3.1 \text{ m}$ )
	<b>0.898</b>	<b>0.850</b>	<b>0.914</b>
<div> <div>Bending moment (bimoment)</div> <div>Analitical model</div> </div>	<div> <div>Moment value in fixed support at the left (kN·m)</div> <div>Bimoment value in fixed support at the left (kN·m<sup>2</sup>)</div> <div>Moment and bimoment ratio (m<sup>-1</sup>)</div> </div>	<div> <div>Moment value in the middle of span (kN·m)</div> <div>Bimoment value in the middle of span (kN·m<sup>2</sup>)</div> <div>Moment and bimoment ratio (m<sup>-1</sup>)</div> </div>	<div> <div>Moment value in fixed support at the right (kN·m)</div> <div>Bimoment value in fixed support at the right (kN·m<sup>2</sup>)</div> <div>Moment and bimoment ratio (m<sup>-1</sup>)</div> </div>
<div>  <p><math>l = 2.5 \text{ m}; e = 0.0463 \text{ m}</math></p> </div>	-10.42	5.21	-10.42
	-0.48	0.24	-0.48
	21.94	22.17	21.94
	<b>0.995</b>	<b>0.995</b>	<b>0.995</b>
<div>  <p><math>l = 5 \text{ m}; e = 0.0463 \text{ m}</math></p> </div>	-41.67	20.83	-41.67
	-1.82	0.87	-1.82
	22.85	23.86	22.85
	<b>0.943</b>	<b>0.902</b>	<b>0.943</b>

Analysis for similarity coversines bending moment and bimoment for multispan beams view values in feature points (maximum values; values in middle of spans and values on intermediate supports).

**Table 4. Values of bending moment and bimoment for three-span beams**

<b>z</b>	<b>z = 1.0 m</b>	<b>z = 1.25 m</b>	<b>z = 2.5 m</b>	<b>z = 3.75 m</b>	<b>z = 5.0 m</b>	<b>z = 6.25 m</b>	<b>z = 6.5 m</b>
 $l = 2.5 \text{ m}; e = 0.0463 \text{ m}$	10.00	9.38	-12.50	3.13	-12.50	9.38	10.00
	0.43	0.41	-0.57	0.15	-0.57	0.41	0.43
Moment and bimoment ratio	23.01	23.03	22.02	20.56	22.02	23.03	23.01
Coefficient value k	<b>0.929</b>	<b>0.944</b>	<b>0.985</b>	<b>1.035</b>	<b>0.985</b>	<b>0.944</b>	<b>0.929</b>
<b>z</b>	<b>z = 2.0 m</b>	<b>z = 2.5 m</b>	<b>z = 5.0 m</b>	<b>z = 7.5 m</b>	<b>z = 10.0 m</b>	<b>z = 12.5 m</b>	<b>z = 13.0 m</b>
 $l = 5 \text{ m}; e = 0.0463 \text{ m}$	40.00	37.50	-50.00	12.50	-50.00	37.50	40.00
	1.48	1.38	-2.14	0.66	-2.14	1.38	1.48
Moment and bimoment ratio	27.09	27.15	23.32	19.05	23.32	27.15	27.09
Coefficient value k	<b>0.799</b>	<b>0.795</b>	<b>0.924</b>	<b>1.140</b>	<b>0.924</b>	<b>0.795</b>	<b>0.799</b>

**Table 5. Values of bending moment and bimoment for five-span beams**

<b>Analitical model</b>						
 $l = 2.5 \text{ m}$ $e = 0.0463 \text{ m}$ (symmetry)						
<b>z = 1.0 m</b>	<b>z = 1.25 m</b>	<b>z = 2.5 m</b>	<b>z = 3.75 m</b>	<b>z = 3.8 m</b>	<b>z = 5.0 m</b>	<b>z = 6.25 m</b>
9.74	9.05	-13.16	4.11	4.17	-9.87	5.76
0.43	0.39	-0.59	0.19	0.19	-0.45	0.25
<b>Moment and bimoment ratio</b>						
22.91	22.91	22.13	21.51	21.63	21.74	22.65
<b>Coefficient value k</b>						
0.954	0.931	0.968	0.998	0.984	0.985	0.937
<b>Analitical model</b>						
 $l = 5 \text{ m};$ $e = 0.0463 \text{ m}$ (symmetry)						
<b>z=2.0 m</b>	<b>z=2.25 m</b>	<b>z=5.0 m</b>	<b>z=7.5 m</b>	<b>z=7.6 m</b>	<b>z=10.0 m</b>	<b>z=12.5 m</b>
29.47	36.18	-52.65	16.43	16.59	-39.50	23.00
1.12	1.35	-2.23	0.76	0.76	-1.75	0.92
<b>Moment and bimoment ratio</b>						
26.41	26.73	23.64	21.60	21.70	22.52	24.99
<b>Coefficient value k</b>						
0.821	0.806	0.915	0.999	0.989	0.957	0.864

Analogous research was made by A.M.Sergei in his master's thesis [4]. It was considered an approximation of the theoretical relationships between bimoment and bending moment in a general way. He offered number of coefficients and tables that allows framing a diagram of bimoment by using the diagrams of the bending moment. However, from the practical point of view, the proposed communication continues to be complex and time-consuming for routine engineering tasks.

## Conclusions

1. Values of bending moments and bimoments were determined for typically using analytical models and span's lengths.
2. It was established, that bimoment value is directly-proportional of bending moments values for the same spans and cross-sections; proportionality factor is eccentricity  $e$  of load application.
3. It was established, that ratio values of bending moment and bimoment change from 0.929 to 1.035 – for span with length 2.5 metres, from 0.795 to 1.14 – for span with length 5 metres in case of single eccentricity for typically widespread spans (2.5 m...5 m).
4. It was recommended the dependence:

$$B_w = M \cdot e \cdot k, \quad (7)$$

where  $k$  – proportionality factor, determined according to the Tables 2–4.

The set dependence allows determining bimoment value quickly in feature points of the span, knowing bending moment, eccentricity of load application, and also coefficient  $k$ , given from tables.

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## Universal method for calculation of reliable completion times

### Универсальный метод вычисления достоверных сроков реализации проекта

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**Key words:** civil engineering; construction management; project scheduling; critical path method

**Ключевые слова:** управление инвестиционно-строительными проектами; расписание проекта; метод критического пути

**Abstract.** Usually the actual duration of construction projects significantly exceeds the scheduled durations. The reason is the lack of realistic scheduling. This paper presents a universal method for computing the reliable completion time for a construction project with network plans that have imprecise durations. The study is based on the consecutive comparison and calculation of the time distributions for the fulfilment of construction events. The general applicability of the algorithm is demonstrated. The method was used for the calculation of a more realistic time span for the construction of a road. The network schedule is formed on the basis of the flow sheet. A comparison was drawn between the proposed method and traditional techniques. The mean duration of the technological process calculated by a universal method is 30% more than for a known critical path method. It is confirmed that the traditional method of calculating the time to complete a project is almost always shorter. The method and algorithm presented in this study used Microsoft Excel. The calculations lasted some seconds. Implementation of the universal method will allow for the determination of a more precise duration for the performance of complex works at the planning stage. The suggested methodology can be recommended for use by construction project managers.

**Аннотация.** Обычно фактическая продолжительность строительных проектов значительно превышает запланированные сроки. Причина – отсутствие инструмента достоверного планирования. В статье представлена универсальная методика для вычисления надежного времени завершения строительных проектов на основе сетевых графиков с неопределенными продолжительностями работ. Исследование основано на последовательном сравнении и вычислении распределений сроков свершений совместных событий графика. Продемонстрирована общая применимость алгоритма. Метод использовался для вычисления достоверных сроков сооружения участка дороги, для чего на основе технологической карты был сформирован соответствующий сетевой график. Сравнение между предложенным подходом и традиционными методами показало, что средняя продолжительность технологического процесса, вычисленного универсальным методом, на 30 % больше, чем для известного метода критического пути. Подтверждено, что традиционный метод критического пути существенно занижает сроки реализации проекта. Метод и соответствующий алгоритм реализован в среде Microsoft Excel. Вычисления продлились несколько секунд. Внедрение универсального метода позволит определять достоверные сроки выполнения комплексов работ при планировании проектов. Предложенный подход может быть рекомендован для использования руководителями строительных проектов.

### Introduction

The analysis of the current state of the theory and practice of scheduling illustrates the lack of realistic scheduling.

Therefore, the actual duration of various construction projects significantly exceeds the planned ones [1–6].

The reason for the significant difference between planned and actual construction durations is, primarily, the impact on the works due to numerous stochastic factors [7]), which can be evaluated using probabilistic estimation [8–12].

The second reason is insufficiently reliable traditional PERT (Program Evaluation and Review Technique) method. The PERT method is generally intended for the calculation of schedules that have certain structures set by unambiguous technological processes. The activity time spans are assumed to follow a general Beta distribution [13–17].

There have been a number of studies [18, 19] that consider project scheduling and PERT using fuzzy logic for scheduling under uncertainty. Fuzzy logic has been proposed as an alternative approach for quantifying uncertainty related to project activity duration.

The traditional PERT method uses only the activity time means to calculate the critical path, reducing the stochastic model to a deterministic model. In PERT, three time estimates are required for each activity. The time estimates represent a pessimistic time, an optimistic time, and a most likely time for duration of the activity.

The method assumes that the sum of the mean completion times of activities on the critical path is normally distributed. This allows the calculation of the probability of completing the project within a given time period. A single critical path is thus calculated and relied upon, where in reality, there may be numerous possible critical paths that exist. For a large network plan the probability that any given path could be the critical path may be very small. PERT method yields results which are biased high. The construction project manager is thus grossly misled into thinking his chances are very good, when in reality they are very poor. If network has multiple parallel paths with relatively equal means, PERT calculations will be considerably biased [20]). As a result, the time to complete a project calculated by the traditional PERT method is almost always too short [21]).

Similar results have also been observed when using the technique of crashing PERT [22]). Completion times with the PERT method are much shorter than completion times calculated with the Monte Carlo method [15, 23, 24].

## Methods

The problem of calculating a realistic duration for the completion of events is formulated as follows:

Activities  $i-j$  and  $i-k$  are performed in parallel (Fig. 1). Their late completion time defines the duration for the fulfilment of an event  $m$  ( $t^m$ ).

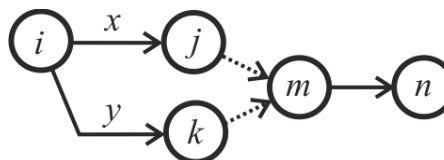


Figure 1. The scheme for the calculation of  $t^m$

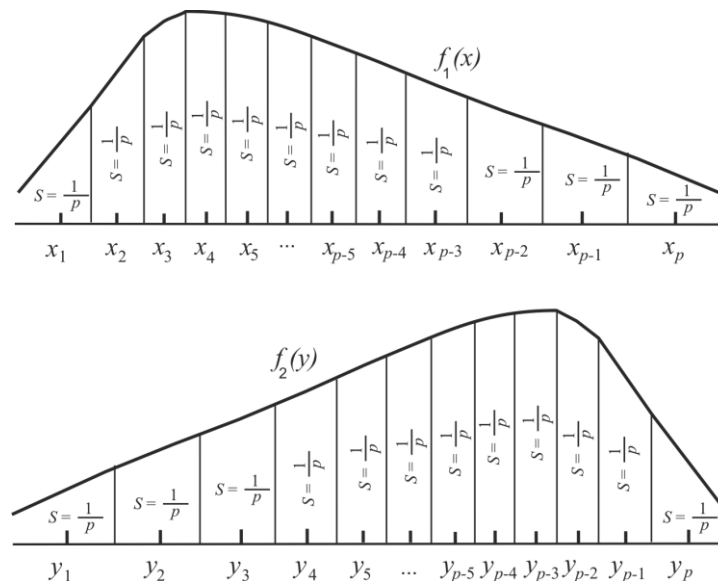
Durations of performance of works of  $i-j$  and  $i-k$  are independent random variables ( $X$  and  $Y$ ). The durations are characterized by distributions  $f_1(x)$  and  $f_2(y)$ .

The distribution of the function  $Z$  in Equation 1 is established as:

$$Z = \max(X, Y). \quad (1)$$

The problem is solved by using a universal DCI method (division, comparison, integration). The method is based on consecutive computing procedures listed as follows:

1. Parameters of the initial event of  $i$  are established. For the first event, ( $t_i = 0$ ).
2. The total area under a density curve is equal to 1 and divided by  $p$ , the number equal areas:  $\left(S = \frac{1}{p}\right)$ . The centre of gravity of each area is defined. These points will define  $p$  as equiprobable values of duration  $(x_1, x_2, \dots, x_{p-1}, x_p; y_1, y_2, \dots, y_{p-1}, y_p)$  (Fig. 2).



**Figure 2. Distributions of activities (*i-j*, *i-k*)**

3. Equiprobable durations for the fulfilment of events of *j* are defined as:

$$\begin{aligned} t_1^j &= t^i + x_1; \\ t_2^j &= t^i + x_2; \\ &\dots \\ t_{p-1}^j &= t^i + x_{p-1}; \\ t_p^j &= t^i + x_p. \end{aligned} \quad (2)$$

4. Equiprobable durations for the fulfilment of events of *k* are defined as:

$$\begin{aligned} t_1^k &= t^i + y_1; \\ t_2^k &= t^i + y_2; \\ &\dots \\ t_{p-1}^k &= t^i + y_{p-1}; \\ t_p^k &= t^i + y_p. \end{aligned} \quad (3)$$

5. Completion times for event *m* are calculated. To this end, equiprobable durations for the fulfilment of an event *j* are compared in pairs to the equiprobable durations for the fulfilment of an event *k*. In each pair, the maximum value is chosen.

$$\begin{aligned} t_{1,r}^m &= \max(t_1^j; t_r^k), r = 1 \dots p; \\ t_{2,r}^m &= \max(t_2^j; t_r^k), r = 1 \dots p; \\ &\dots \\ t_{p-1,r}^m &= \max(t_{p-1}^j; t_r^k), r = 1 \dots p; \\ t_{p,r}^m &= \max(t_p^j; t_r^k), r = 1 \dots p. \end{aligned} \quad (4)$$

The ascending numerical series is a result formed by  $p^2$  of the equiprobable values of the durations for the fulfilment of an event *m* ( $t_1^m, t_2^m, \dots, t_s^m, \dots, t_{p^2-1}^m, t_{p^2}^m$ ),  $t_{s-1}^m \leq t_s^m, s = 1 \dots p^2$ .

6. This series is consistently divided by *p* segments for the purpose of further calculation reduction.

Each segment includes  $p$  of values  $(t_1^m, \dots, t_p^m; t_{p+1}^m, \dots, t_{2p}^m; \dots; t_{(p-1)p}^m, \dots, t_{p^2}^m)$ . The integrated series  $(t_1^m, t_2^m, \dots, t_{p-1}^m, t_p^m)$  is formed by the mean values of each segment.

7. The equiprobable values of the duration of  $m$ - $n$   $(t_1^{mn}, t_2^{mn}, \dots, t_p^{mn})$  are defined (similar to 2).

8. The equiprobable durations for the fulfilment of events  $n$  are defined as,

$$\begin{aligned} t_{11}^n &= t_1^m + t_1^{mn}; t_{12}^n = t_1^m + t_2^{mn}; \dots; t_{1p}^n = t_1^m + t_p^{mn}; \\ t_{21}^n &= t_2^m + t_1^{mn}; t_{22}^n = t_2^m + t_2^{mn}; \dots; t_{2p}^n = t_2^m + t_p^{mn}; \\ &\dots \\ t_{(p-1)1}^n &= t_{p-1}^m + t_1^{mn}; t_{(p-1)2}^n = t_{p-1}^m + t_2^{mn}; \dots; t_{(p-1)p}^n = t_{p-1}^m + t_p^{mn}; \\ t_{p1}^n &= t_p^m + t_1^{mn}; t_{p2}^n = t_p^m + t_2^{mn}; \dots; t_{pp}^n = t_p^m + t_p^{mn}. \end{aligned} \quad (5)$$

The ascending numerical series is a result formed by  $p^2$  of the equiprobable values of the durations for the fulfilment of an event  $n$   $(t_1^n, t_2^n, \dots, t_s^n, \dots, t_{p^2-1}^n, t_{p^2}^n)$ ,  $t_{s-1}^n \leq t_s^n, s = 1 \dots p^2$ .

9. This series is consistently divided by  $p$  segments for the purpose of further calculation reduction.

Each segment includes  $p$  of values  $(t_1^n, \dots, t_p^n; t_{p+1}^n, \dots, t_{2p}^n; \dots; t_{(p-1)p}^n, \dots, t_{p^2}^n)$ . The integrated series  $(t_1^n, t_2^n, \dots, t_{p-1}^n, t_p^n)$  is formed by the mean values of each segment.

Calculations for two parallel works of equal duration and standard normal distribution [25] (for  $p = 2, 4, 8, 10, 12, 14, 16, 32, 64, 128, 256, 512$ ) showed the following result:

The true mean value of the duration for the performance of an event  $m$  did not exceed 0.56427 ( $\mu^* \leq 0.56427$ ). At  $p = 12$ ,  $\mu^{12} = 0.54025$ . The deviation from the true value ( $\mu^*$ ) did not exceed 5 %. This indicates that this method is suitable for practical calculations of a network at  $p = 12$  (Table 1).

**Table 1. Calculated equiprobable values for the fulfilment of event  $m$**

Parameters		$x$											
		-1.7317	-1.1503	-0.8122	-0.5485	-0.3186	-0.1046	0.1046	0.3186	0.5485	0.8122	1.1503	1.7317
		144 equiprobable values for the fulfilment of event $m$											
$y$	-1.7317	-1.7317	-1.1503	-0.8122	-0.5485	-0.3186	-0.1046	0.1046	0.3186	0.5485	0.8122	1.1503	1.7317
	-1.1503	-1.1503	-1.1503	-0.8122	-0.5485	-0.3186	-0.1046	0.1046	0.3186	0.5485	0.8122	1.1503	1.7317
	-0.8122	-0.8122	-0.8122	-0.8122	-0.5485	-0.3186	-0.1046	0.1046	0.3186	0.5485	0.8122	1.1503	1.7317
	-0.5485	-0.5485	-0.5485	-0.5485	-0.5485	-0.3186	-0.1046	0.1046	0.3186	0.5485	0.8122	1.1503	1.7317
	-0.3186	-0.3186	-0.3186	-0.3186	-0.3186	-0.3186	-0.1046	0.1046	0.3186	0.5485	0.8122	1.1503	1.7317
	-0.1046	-0.1046	-0.1046	-0.1046	-0.1046	-0.1046	-0.1046	0.1046	0.3186	0.5485	0.8122	1.1503	1.7317
	0.1046	0.1046	0.1046	0.1046	0.1046	0.1046	0.1046	0.1046	0.3186	0.5485	0.8122	1.1503	1.7317
	0.3186	0.3186	0.3186	0.3186	0.3186	0.3186	0.3186	0.3186	0.3186	0.5485	0.8122	1.1503	1.7317
	0.5485	0.5485	0.5485	0.5485	0.5485	0.5485	0.5485	0.5485	0.5485	0.5485	0.8122	1.1503	1.7317
	0.8122	0.8122	0.8122	0.8122	0.8122	0.8122	0.8122	0.8122	0.8122	0.8122	0.8122	1.1503	1.7317
	1.1503	1.1503	1.1503	1.1503	1.1503	1.1503	1.1503	1.1503	1.1503	1.1503	1.1503	1.1503	1.7317
	1.7317	1.7317	1.7317	1.7317	1.7317	1.7317	1.7317	1.7317	1.7317	1.7317	1.7317	1.7317	1.7317
Integrated series. Mean = 0.54025		-0.907	-0.395	-0.122	0.105	0.301	0.472	0.614	0.812	1.038	1.15	1.683	1.732

The stated method used Microsoft Excel.

The distribution of parameters  $X$  ( $Y$ ),  $Z$  (for events  $j$  ( $k$ ),  $m$ ) is presented in Figure 3.

Density was defined as the difference of adjacent equiprobable values (at half values for extreme intervals).

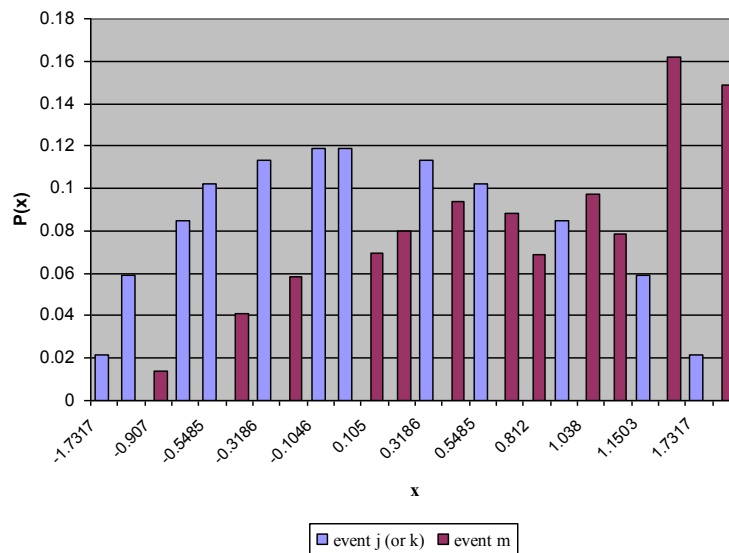


Figure 3. Distribution of parameters  $X$  ( $Y$ ),  $Z$  (for events  $j$  ( $k$ ),  $m$ )

Calculations show a shift of the maximum density to the maximum values  $Z$ .

## Results and Discussion

The universal method presented in this paper was used to calculate a more realistic duration for the construction of a banquette of the road.

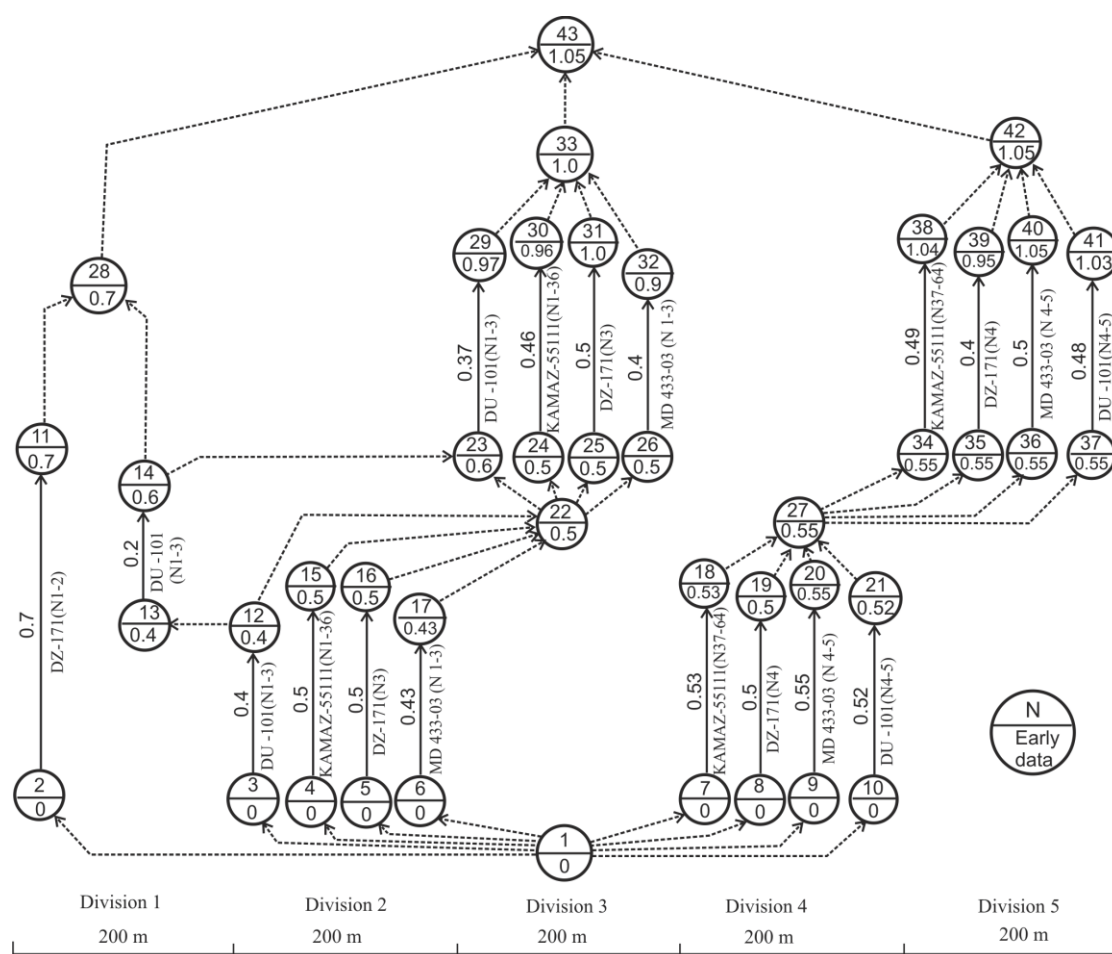
The technology and the organization of works are defined in the flow sheet [26]. According to the flow sheet the work process is performed in five divisions during one shift (Fig. 4).

# division	1	2	3	4	5
Processes	1. Paring 2. Consolidation of the basis of an banquette	1. Soil transportation bay dump trucks for the first layer of an banquette 2. Leveling of soil in an banquette the bulldozer 3. Dumping of soil 4. Consolidation of the first layer of an banquette	1. Soil transportation bay dump trucks for the second layer of an banquette 2. Leveling of soil in an banquette the bulldozer 3. Dumping of soil 4. Consolidation of the second layer of an banquette	1. Soil transportation bay dump trucks for the third layer of an banquette 2. Consolidation of the third layer of an banquette 3. Dumping of soil 4. Consolidation of the third layer of an banquette	1. Soil transportation bay dump trucks for the upper layer of an banquette 2. Consolidation of the upper layer of an banquette 3. Dumping of soil 4. Consolidation of the upper layer of an banquette
Length, m	200	200	200	200	200
Machines, standard time (shift)	1. Bulldozer DZ-171 N1-2 (0.7) 2. Tamping roller DU-101 N1-3 (0.2)	Complex 1 1. Dump truck KAMAZ-55111 N1-36 (0.5) 2. Bulldozer DZ-171 N3 (0.5) 3. Sprinkler MD 433-03 N1-3 (0.43) 4. Tamping roller DU-101 N1-3 (0.4)	Complex 1 1. Dump truck KAMAZ-55111 N1-36 (0.46) 2. Bulldozer DZ-171 N3 (0.5) 3. Sprinkler MD 433-03 N1-3 (0.4) 4. Tamping roller DU-101 N1-3 (0.37)	Complex 2 1. Dump truck KAMAZ-55111 N37-65 (0.53) 2. Bulldozer DZ-171 N4 (0.5) 3. Sprinkler MD 433-03 N4-5 (0.55) 4. Tamping roller DU-101 N4-5 (0.52)	Complex 2 1. Dump truck KAMAZ-55111 N37-65 (0.49) 2. Bulldozer DZ-171 N4 (0.4) 3. Sprinkler MD 433-03 N4-5 (0.5) 4. Tamping roller DU-101 N4-5 (0.48)
Schedule	7 6 5 4 3 2 1	7 6 5 4 3 2 1	7 6 5 4 3 2 1	7 6 5 4 3 2 1	7 6 5 4 3 2 1
Direction of flow works	←				

Figure 4. Fragment of flow sheet for the construction of a banquette

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The network schedule was composed on the basis of the flow sheet (Fig. 5).



**Figure 5. Network schedule**

The critical path (activities 1-9-20-27-36-40-42-43) is equal to 1.05 shifts.

The stochastic parameters of the equipment [10] are presented in Table 2.

**Table 2. Stochastic parameters of equipment**

Machines	12 equiprobable values standard time, shift												$\bar{t}$
Bulldozers	0.644	0.719	0.772	0.817	0.862	0.909	0.960	1.018	1.089	1.184	1.332	1.698	1.00
Tamping rollers	0.655	0.729	0.780	0.826	0.870	0.915	0.964	1.020	1.088	1.178	1.318	1.656	1.00
Dump rucks	0.691	0.761	0.809	0.851	0.891	0.932	0.976	1.026	1.085	1.161	1.277	1.541	1.00
Sprinklers	0.843	0.887	0.915	0.939	0.959	0.979	1.001	1.023	1.048	1.078	1.120	1.200	1.00

The stochastic parameters of the activities network schedule are presented in Table 3.

**Table 3.** Stochastic parameters of processes of construction of banquette

# activity	Processes	12 equiprobable values time, shift												$\bar{t}$
	Division 1													
2–11	1.ParingDZ-171 N1-2	0.451	0.503	0.540	0.572	0.603	0.636	0.672	0.712	0.762	0.828	0.932	1.188	0.700
13–14	2. Consolidation of the basis DU-101 N1-3	0.131	0.146	0.156	0.165	0.174	0.183	0.193	0.204	0.218	0.236	0.264	0.331	0.200
	Division 2													
4–15	1. Soil transportation KAMAZ-55111 N1-36	0.346	0.381	0.405	0.426	0.446	0.466	0.488	0.513	0.543	0.581	0.639	0.771	0.500
5–16	2.Leveling of soil DZ-171 N3	0.322	0.359	0.386	0.408	0.431	0.454	0.480	0.509	0.544	0.592	0.666	0.849	0.500
6–17	3. Dumping of soil MD 433-03 N1-3	0.362	0.381	0.393	0.404	0.413	0.421	0.430	0.440	0.450	0.464	0.482	0.516	0.430
3–12	4. Consolidation of the first layer DU-101 N1-3	0.262	0.292	0.312	0.330	0.348	0.366	0.386	0.408	0.435	0.471	0.527	0.662	0.400
	Division 3													
24–30	1. Soil transportation KAMAZ-55111 N1-36	0.318	0.350	0.372	0.391	0.410	0.429	0.449	0.472	0.499	0.534	0.587	0.709	0.460
25–31	2.Leveling of soil DZ-171 N3	0.322	0.359	0.386	0.408	0.431	0.454	0.480	0.509	0.544	0.592	0.666	0.849	0.500
26–32	3. Dumping of soil MD 433-03 N1-3	0.337	0.355	0.366	0.375	0.384	0.392	0.400	0.409	0.419	0.431	0.448	0.480	0.400
23–29	4. Consolidation of the second layer DU-101 N1-3	0.242	0.270	0.289	0.306	0.322	0.339	0.357	0.377	0.403	0.436	0.488	0.613	0.370
	Division 4													
7–18	1. Soil transportation KAMAZ-55111 N37-64	0.366	0.403	0.429	0.451	0.472	0.494	0.517	0.544	0.575	0.615	0.677	0.817	0.530
8–19	2.Leveling of soil DZ-171 N4	0.322	0.359	0.386	0.408	0.431	0.454	0.480	0.509	0.544	0.592	0.666	0.849	0.500
9–20	3. Dumping of soil MD 433-03 N4-5	0.464	0.488	0.503	0.516	0.528	0.539	0.551	0.563	0.576	0.593	0.616	0.660	0.550
10–21	4. Consolidation of the third layer DU-101 N4-5	0.341	0.379	0.406	0.430	0.452	0.476	0.501	0.530	0.566	0.613	0.685	0.861	0.520
	Division 5													
34–38	1. Soil transportation KAMAZ-55111 N37-64	0.339	0.373	0.396	0.417	0.437	0.457	0.478	0.503	0.532	0.569	0.626	0.755	0.490
35–39	2.Leveling of soil DZ-171 N4	0.258	0.288	0.309	0.327	0.345	0.364	0.384	0.407	0.435	0.474	0.533	0.679	0.400
36–40	3. Dumping of soil MD 433-03 N4-5	0.422	0.444	0.457	0.469	0.480	0.490	0.500	0.511	0.524	0.539	0.560	0.600	0.500
37–41	4. Consolidation of the upper layer DU-101 N4-5	0.314	0.350	0.374	0.396	0.418	0.439	0.463	0.490	0.522	0.565	0.633	0.795	0.480

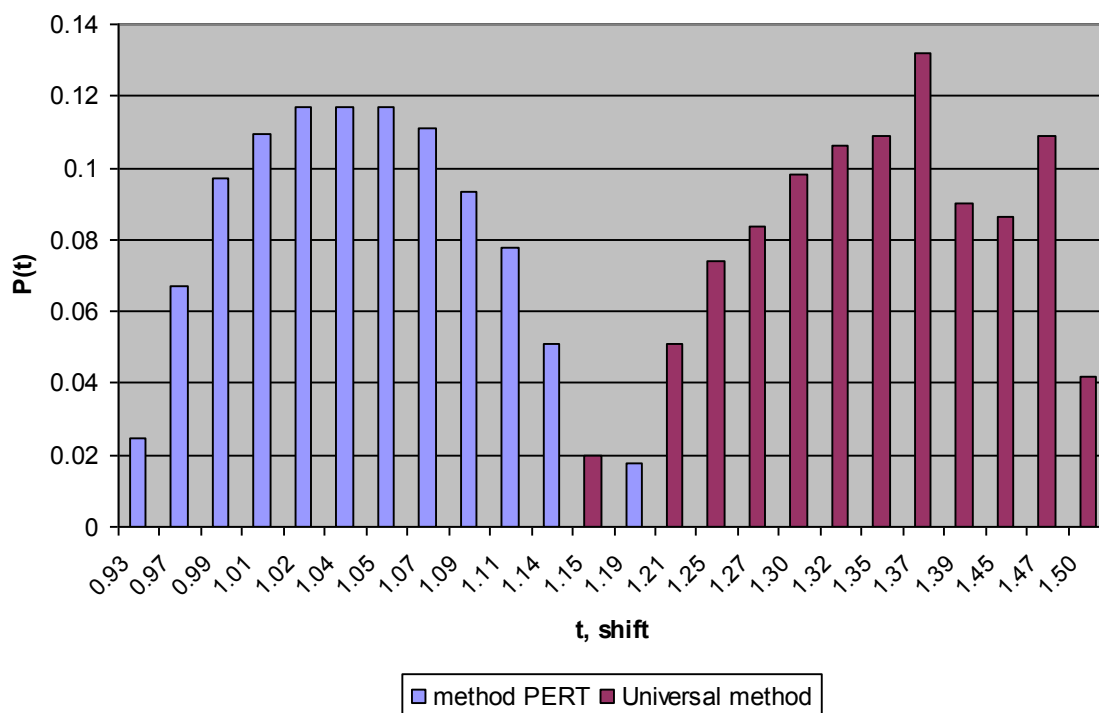
The stochastic parameters of the events network schedule were calculated by the universal method (Table 4).

Kalugin Yu.B. Universal method for calculation of reliable completion times. *Magazine of Civil Engineering*. 2016. No. 7. Pp. 70–80. doi: 10.5862/MCE.67.7

Table 4. Stochastic parameters of events

# an of event	12 equiprobable values time, shift												P(t)= 0.25	P(t)= 0.5	P(t)= 0.75
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.451	0.503	0.540	0.572	0.603	0.636	0.672	0.712	0.762	0.828	0.841	0.880	0.556	0.667	0.795
12	0.262	0.292	0.312	0.330	0.348	0.366	0.386	0.408	0.435	0.471	0.479	0.501	0.321	0.383	0.453
13	0.262	0.292	0.312	0.330	0.348	0.366	0.386	0.408	0.435	0.471	0.479	0.501	0.321	0.383	0.453
14	0.433	0.473	0.500	0.523	0.544	0.567	0.591	0.617	0.648	0.687	0.702	0.728	0.511	0.584	0.667
15	0.346	0.381	0.405	0.426	0.446	0.466	0.488	0.513	0.543	0.581	0.593	0.617	0.416	0.484	0.562
16	0.322	0.359	0.386	0.408	0.431	0.454	0.480	0.509	0.544	0.592	0.601	0.629	0.397	0.476	0.568
17	0.362	0.381	0.393	0.404	0.413	0.421	0.430	0.440	0.450	0.464	0.473	0.484	0.399	0.426	0.457
18	0.366	0.403	0.429	0.451	0.472	0.494	0.517	0.544	0.575	0.615	0.628	0.654	0.440	0.512	0.595
19	0.322	0.359	0.386	0.408	0.431	0.454	0.480	0.509	0.544	0.592	0.601	0.629	0.397	0.476	0.568
20	0.464	0.488	0.503	0.516	0.528	0.539	0.551	0.563	0.576	0.593	0.605	0.618	0.510	0.545	0.585
21	0.341	0.379	0.406	0.430	0.452	0.476	0.501	0.530	0.566	0.613	0.623	0.651	0.418	0.497	0.590
22	0.426	0.460	0.483	0.506	0.524	0.542	0.577	0.611	0.650	0.665	0.689	0.716	0.495	0.571	0.657
23	0.485	0.530	0.556	0.584	0.611	0.638	0.654	0.676	0.730	0.760	0.779	0.808	0.570	0.651	0.745
24	0.426	0.460	0.483	0.506	0.524	0.542	0.577	0.611	0.650	0.665	0.689	0.716	0.495	0.571	0.657
25	0.426	0.460	0.483	0.506	0.524	0.542	0.577	0.611	0.650	0.665	0.689	0.716	0.495	0.571	0.657
26	0.426	0.460	0.483	0.506	0.524	0.542	0.577	0.611	0.650	0.665	0.689	0.716	0.495	0.571	0.657
27	0.505	0.536	0.560	0.581	0.599	0.616	0.644	0.666	0.682	0.785	0.760	0.786	0.571	0.643	0.734
28	0.512	0.564	0.600	0.630	0.660	0.687	0.725	0.758	0.812	0.861	0.880	0.916	0.615	0.717	0.836
29	0.793	0.859	0.899	0.932	0.965	0.998	1.032	1.072	1.115	1.159	1.192	1.230	0.915	1.020	1.137
30	0.808	0.869	0.908	0.943	0.975	1.009	1.043	1.083	1.129	1.184	1.209	1.248	0.926	1.034	1.157
31	0.817	0.884	0.927	0.967	1.003	1.040	1.080	1.123	1.175	1.239	1.265	1.308	0.947	1.069	1.207
32	0.807	0.850	0.878	0.902	0.925	0.951	0.980	1.011	1.043	1.087	1.103	1.132	0.890	0.973	1.065
33	0.990	1.056	1.098	1.137	1.164	1.194	1.230	1.249	1.295	1.325	1.365	1.400	1.118	1.209	1.310
34	0.505	0.536	0.560	0.581	0.599	0.616	0.644	0.666	0.682	0.785	0.760	0.786	0.571	0.643	0.734
35	0.505	0.536	0.560	0.581	0.599	0.616	0.644	0.666	0.682	0.785	0.760	0.786	0.571	0.643	0.734
36	0.505	0.536	0.560	0.581	0.599	0.616	0.644	0.666	0.682	0.785	0.760	0.786	0.571	0.643	0.734
37	0.505	0.536	0.560	0.581	0.599	0.616	0.644	0.666	0.682	0.785	0.760	0.786	0.571	0.643	0.734
38	0.908	0.970	1.008	1.040	1.072	1.108	1.144	1.187	1.233	1.283	1.311	1.350	1.024	1.134	1.258
39	0.823	0.882	0.918	0.950	0.980	1.014	1.051	1.095	1.142	1.193	1.217	1.256	0.934	1.043	1.167
40	0.976	1.022	1.051	1.076	1.098	1.121	1.145	1.173	1.216	1.276	1.279	1.308	1.063	1.145	1.246
41	0.885	0.948	0.988	1.022	1.056	1.092	1.133	1.175	1.223	1.278	1.306	1.347	1.005	1.121	1.250
42	1.082	1.151	1.187	1.222	1.256	1.282	1.312	1.341	1.366	1.391	1.437	1.470	1.204	1.291	1.378
43	1.145	1.206	1.245	1.273	1.303	1.324	1.349	1.369	1.386	1.450	1.468	1.497	1.259	1.335	1.418

The distribution of completion times for the 43rd event is presented in Figure 6.



**Figure 6. Distribution of completion times (for the 43rd event)**

The duration of the calculations lasted 3–4 s.

Using a known critical path method, the mean duration was equal to 1.05 shifts, while the mean duration was equal to 1.335 shifts with the universal method. With a probability of 0.75, construction works was scheduled to end after 1.418 shifts

## Conclusions

1. The actual duration of various construction projects significantly exceeds the scheduled durations. The reason for the significant difference between planned and actual durations is, primarily, the impact of the numerous stochastic factors on the works. Additionally, the traditional PERT method uses a single critical path. As a result, the PERT method of calculating the time to complete a project is almost always too short.

The construction project manager is thus grossly misled into thinking his chances are very good.

2. The universal method to calculate an accurate duration for the fulfilment of events is presented. The basics of this universal DCI method (division, comparison, integration) include:

- The total area under a density curve (for parallel activities with a joint event) divides into  $p$  equal areas. The centres of gravity of each area will define  $p$  as equiprobable values of duration;
- Equiprobable completion times of one activity are compared in pairs to completion times of another activity. In each pair the maximum value is chosen. The ascending numerical series as a result is formed by the  $p^2$  of the equiprobable values of the duration for the fulfilment of an event;
- This series is consistently divided by  $p$  segments for the purpose of further calculation reduction. The integrated series is formed by the mean value of each segment.

Calculations for two parallel works of equal duration and standard normal distribution showed that this method is suitable for practical calculations of a network at  $p=12$

The method presented in this study used Microsoft Excel.

3. The universal DCI method was used to calculate a realistic duration for the construction of a banquet of the road. The technology and the organization of works are defined in the flow sheet. According to the flow sheet, the works will be performed in five divisions during 1.05 shifts. With a traditional PERT method, the mean duration is also equal to 1.05 shifts. Using the universal DCI method, Kalugin Yu.B. Universal method for calculation of reliable completion times. *Magazine of Civil Engineering*. 2016. No. 7. Pp. 70–80. doi: 10.5862/MCE.67.7

the mean duration is equal to 1.34 shifts; however, with a probability of 0.75 included, works will end after 1.42 shifts. The calculations lasted some seconds.

4. These results show the efficacy of the universal DCI method to calculate more realistic completion times. This method considers an increase in the actual completion times in comparison with the planned completion times (using the PERT method), especially since in reality, numerous possible critical paths may exist.

The suggested DCI method can be recommended for use by construction project managers in order to prevent a potential failure of project completion deadlines.

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- Инженерное оборудование зданий и сооружений
- Технология и контроль качества строительства
- Управление инвестиционно-строительными проектами. Выполнение функций технического заказчика и генерального подрядчика
- Управление строительной организацией
- Экономика и ценообразование в строительстве
- Управление строительной организацией
- Организация, управление и планирование в строительстве
- Автоматизация сметного дела в строительстве

**П-03 «Инженерные системы зданий и сооружений»**

Программа включает учебные разделы:

- Основы механики жидкости и газа
- Инженерное оборудование зданий и сооружений
- Проектирование, монтаж и эксплуатация систем вентиляции и кондиционирования
- Проектирование, монтаж и эксплуатация систем отопления и теплоснабжения
- Проектирование, монтаж и эксплуатация систем водоснабжения и водоотведения
- Автоматизация проектных работ с использованием AutoCAD
- Электроснабжение и электрооборудование объектов

**П-04 «Проектирование и конструирование зданий и сооружений»**

Программа включает учебные разделы:

- Основы сопротивления материалов и механики стержневых систем
- Проектирование и расчет оснований и фундаментов зданий и сооружений
- Проектирование и расчет железобетонных конструкций
- Проектирование и расчет металлических конструкций
- Проектирование зданий и сооружений с использованием AutoCAD
- Расчет строительных конструкций с использованием SCAD Office

**П-05 «Контроль качества строительства»**

Программа включает учебные разделы:

- Основы строительного дела
- Инженерное оборудование зданий и сооружений
- Технология и контроль качества строительства
- Проектирование и расчет железобетонных конструкций
- Проектирование и расчет металлических конструкций
- Обследование строительных конструкций зданий и сооружений
- Выполнение функций технического заказчика и генерального подрядчика

По окончании курса слушателю выдается диплом о профессиональной переподготовке  
установленного образца, дающий право на ведение профессиональной деятельности

