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Non-uniform torsion of thin-walled open-section multi-span beams

Стесненное кручение многопролетных тонкостенных балок открытого профиля

*A.D. Pavlenko,
V.A. Rybakov,
A.V. Pikh,т,
E.S. Mikhailov,
Peter the Great St. Petersburg Polytechnic
University, St. Petersburg, Russia*

*Студент А.Д. Павленко,
канд. техн. наук, доцент В.А. Рыбаков,
студент А.В. Пихт,
студент Е.С. Михайлов,
Санкт-Петербургский политехнический
университет Петра Великого, г. Санкт-
Петербург, Россия*

Key words: structural mechanics; light steel thin-walled structures; bending moment; bimoment; multi-span continuous thin-walled beams; frame system; beam deformation; normal stress; coefficient of proportionality; method by B.P.E. Clapeyron

Ключевые слова: энергоэффективность; коэффициент теплотехнической однородности; геометрический параметр; сопротивление теплопередаче; ограждающая конструкция здания; температурное поле; теплота; термограмма; здание; строительство

Abstract. In the article a calculation of the bending moment and bimoment for different occasions single- and multi-span continuous thin-walled channel section beams using B.P.E. Clapeyron and N.L. Kuzmin methods have been made. Article considers the cases of uniformly distributed loading with the eccentricity relative to the center line of the bend sections of one-, two-, three- and five-span beams with spans 2.5 meters long and 5 meters. Ratio between the bending moment and bimoment in feature points and proportionality coefficients between the bending moment and bimoment has been calculated experimentally. Article suggests a formula that allows to determine bimoment, if only the bending moment and the eccentricity of applied load are given.

Аннотация. В статье выполнен расчет изгибающего момента и бимоента для разных случаев одно- и многопролетных неразрезных тонкостенных балок швеллерового сечения с использованием методов Б.П.Э. Клапейрона и Н.Л. Кузьмина. Рассмотрены случаи загрузки равномерно распределенной нагрузкой с эксцентриситетом относительно линии центров изгиба сечений одно-, двух-, трех- и пятипролетной балок с пролетами: длиной 2,5 метров и 5 метров. Посчитаны отношения величин изгибающего момента и бимоента в характерных точках пролетов и определены коэффициенты пропорциональности между изгибающим моментом и бимоментом. Предложена формула, позволяющая определить бимомент, зная лишь изгибающий момент и эксцентриситет приложения нагрузки.

Introduction

At the turn of XXI century in all over the earth metallurgy industry is quite actively developing. Today there is widespread application of metal structures in civil engineering. The main advantages of steel structures are universal, strength and, at the same time, lightness. Moreover, these constructions are pre-fabricated, that is very important in construction.

The most profitable and effective way to construction of prefabricated structures is to use the system of light steel thin-walled structures (LSTS below), and also thermal insulation, facing and vapor sealing.

However, such a significant feature as lightness, can effect on the ability of constriction's bearing strength. So, testing and analysis of bearing strength of the product are necessary (especially for open sections, such as I-profiles and Z-profiles, angle and channel profiles).

Павленко А.Д., Рыбаков В.А., Пихт А.В., Михайлов Е.С. Стесненное кручение многопролетных тонкостенных балок открытого профиля // Инженерно-строительный журнал. 2016. № 7(67). С. 55–69.

LSTS can be used as an economic variant in low-rise housing construction.

Despite the widespread application of LSTS, there are significant weaknesses in the standard and methodological bases of LSTS calculation.

To solve these problems, it is necessary to refer to the fundamental science – structural mechanics.

Cold-rolled thermoprofile and light thin galvanized sheet beams are the main elements of LSTS. Beams are used to build the frame of the building, and for the installation of individual elements.

The special shape of the profile ensures high durability of the product and makes on the thermal profile walls slots reduce heat loss and avoids the emergence of "cold bridges". [1]

In construction practice bending is the most common type of strain, which is more typical for beam structures. In most cases, share force occurs with the bending moment in the beams and that bending is called cross.

Normal stresses dominate in cross bend of thin-walled beams. It determines the bearing capacity of the beam.

The normal stress is a summation of longitudinal force acting, bending moments of two planes and bimoment acting.

Bimoment – additional force characteristic. It is self-balanced factor and it can not be found from equilibrium conditions of clipping part. Bimoment characterizes the changes, introduced in linear zones stress distribution of distortion of cross-section. It is a pair of bending moments applied in opposite directions, or the four forces, the action direction of two is not the same as the rest.

V.Z. Vlasov made a huge technical contribution to the thin beams theory development. He is considered as the founder of the theory. He suggested the formula of bimoment calculation for thin-walled metal-roll. His theory was developed and continued throughout the twentieth century by dozens of Russian scientists. S.P. Tymoshenko made a big contribution to the development of the thin-walled beams stability theory. Engineer E.N. Popova engaged the problem of accounting embossments in thin-walled profiles. [1]

A.R. Tusnin and M. Prokich examined the combined effect of bending moment and bimoment on I-profiles. They have given recommendations for determination their load-bearing capacity with the development of plastic deformations. [2]

N.L. Kuzmin, P.A. Lucash, I.E. Mileikovskii engaged the methods of bimoment and stresses calculation for multi-span beams, but they did not define the link between bimoment and bending moment. [3]

D.A. Trubina, L.A. Kononov, A.A. Kaur, E.D. Pichugin, D.A. Abdulaev in their article [5] examined various methods of thin-walled structures behaviour, and also determined the dependence of local stability of the cross-section of the various parameters and found the degree of influence on the intermediate amplifiers LPA wall.

V. Rybakov and O.S. Gamayunova in their article [6] provide an overview of the characteristics and problems of the stress-strain state (SSS) analysis of the thin-walled frame structures elements of open and closed profiles, and also some specific effects of their work, such as warping of the cross-section and the "bimoment" normal stress distribution over the cross section.

M.K. Bronzova, N.I. Vatin, M.R. Garifullin in their article [7] describe a new design of frame buildings using the skeleton of LSTS monolithic foam concrete.

M.R. Garifullin, N.I. Vatin considered domestic and foreign publications relating to cold-formed thin-walled profile, which works on a bend. [8]

T.V. Nazmeeva in her article [9] provides numerical and experimental study of bearing capacity of racks of various lengths from cold-formed C-profile solid and perforated section in order to obtain reliable methodology for their calculation.

A.S. Sinelnikov made a numerical analysis of the expanded stretching profile based on the finite element method, and conducted experimental and analytical studies. [10, 11]

In their work [13] V.V. Lalin, V.A. Rybakov have developed a numerical method for the calculation of thin-walled rod systems semi-shearing and unshearing theories calculation. [13]

Pavlenko A.D., Rybakov V.A., Pikht A.V., Mikhailov E.S. Non-uniform torsion of thin-walled open-section multi-span beams. *Magazine of Civil Engineering*. 2016. No. 7. Pp. 55–69. doi: 10.5862/MCE.67.6

According to the researches [13–16] in thin-walled bar structures, which are subjected by bending torsion, component of normal stresses from bimoment can exceed component from bending moment and tangent strain effects on stress strain behavior small by the side of normal stresses. [4]

V.V. Lalin, V.A. Rybakov, S. Morozov in their publication reviewed test problems of non-uniform torsion of thin-walled beam with different boundary conditions. In the terms of the search static force factor in a non-uniform torsion were examined: bimoment, sectorial torque and moment of pure torsion. [17]

The article [18] considers the technology of multi-storey buildings construction using LSTS, and also offers the possibility of constructive solutions on the basis of existing projects. Article offers a comparison of LSTS with other materials used as cladding elements in buildings with high altitude. The main advantages and disadvantages of the application of light steel thin-walled structures in high-rise construction were showed in the issue.

The article [20] of A.R. Tusnin shows the features of calculation and design of thin-walled structures, which are widely used in construction. The numerical method of calculation of bar beam systems is considered.

In [21] free vibration analysis of thin-walled channel section beam including warping effects has been carried out using Finite element method and compared with experimentally measured results.

The article [22] discusses the dynamics of a composite thin-walled box-beam built into a rigid hub and performing combined motion of rotation and in-plane translation.

The article [23] shows that for the calculation of thin-walled structures considering restrained torsion can be successfully applied finite element method.

In [24] the crushing behavior of circular aluminum thin-walled tubes with functionally graded thickness (FGT) is investigated experimentally.

The lateral stability of bending non-prismatic thin walled beams is calculated using orthogonal Chebyshev series in the work [25].

This paper [26] investigates the effects of fatigue material data and finite element types on accuracy of residual life assessments under high cycle fatigue conditions.

The article [27] presents an experimental study on the behaviors and modes of failure of square thin-walled steel tubed RC columns subjected to concentrically axial load applied directly to the RC core.

The purpose of this paper [28] is to resolve aforesaid problems and find out a kind of new technics substitute for present forming method.

According to available sources, bimoment is calculated by different formulas, depending on the type of design schemes. The disadvantage is in cumbersome formulas of the application of a hyperbolic sine and cosine. [4]

The goal of this article is to obtain a design proportionality coefficient (maximum) value of the bending moment and bimoment for multi-span beams with equal span, loaded with a uniformly distributed load equipartition, which will facilitate the difficult calculation of bimoment.

The objectives of the article – a function definition (diagrams) of the bending moment in the statically indeterminate single- and multi-span beams, including the method of B.P.E. Klaperyon and N.L. Kuzmin.

Methods

Assume channel section of beam (Figure 1) with dimensions: 70 mm – width of a channel, 220 mm – height:

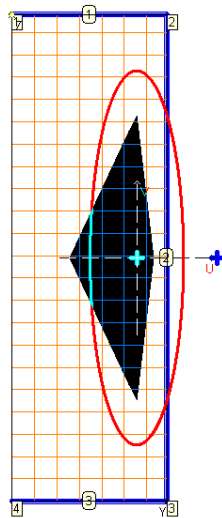


Figure 1. Cross-section of the beam under consideration in the program TONUS (SCAD Office)

Consider the calculation of bending moments and bimoment for the case of three-span beams (Figure 2), loaded with a uniformly distributed load $q = 20 \text{ kN/m}$, the length of each span $l = 2.5 \text{ m}$.

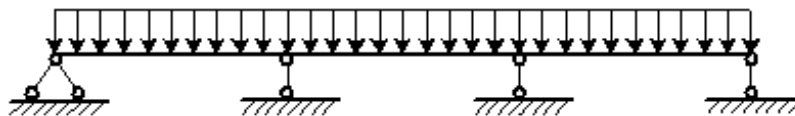


Figure 2. Design model

Define of values of bending moment, using the method of B.P.E. Clapeyron [4]:

The value of the bending moment in the middle supports will be found with the equation of three moments.

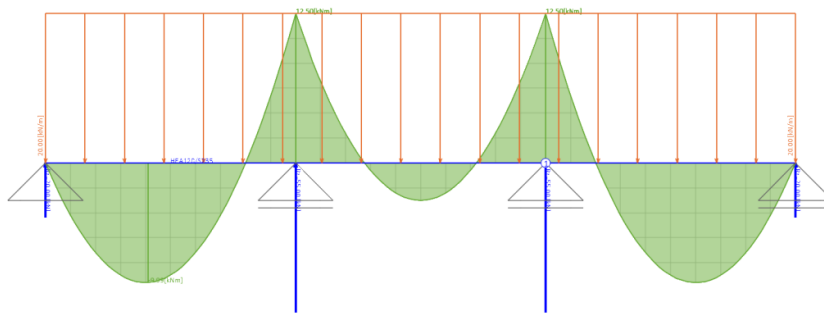


Figure 3. The bending moment (kN·m) for the three-span beam

Finding bimoment is divided into four steps.

Consider bimoment value for each span of the uniformly distributed load, using (1) according to [3]:

$$B_w = \frac{qe}{k^2} \left[1 - \frac{\text{chk}\left(\frac{l}{2} - z\right)}{\text{ch}\frac{kl_1}{2}} \right], \quad (1)$$

where z – horizontal coordinate of span varies from 0 to 2.5 m, the eccentricity e is calculated by the formula (2):

$$e = \frac{b}{3} + \alpha, \quad (2)$$

where $\frac{b}{3}$ – accepted position in the load center from the origin application center of wall channel gravity [1], b – the width of the channel, α – coordinate of the bending channel beams center found using a satellite program “TONUS” (SCAD Office). [19]

Flexural-torsional characteristics of response section k will be found with the formula (3):

$$k = \sqrt{\frac{GI_d}{EI_w}}, \quad (3)$$

where the shear modulus steel is $G = 79.3 \cdot 10^9$ Pa, elastic modulus steel is $E = 2 \cdot 10^{11}$ Pa; torsional inertia moment $I_d = 0.095 \text{ cm}^4$ and sectorial inertia moment $I_w = 2647.298 \text{ cm}^6$. Geometric characteristics are defined in the "TONUS" program [19].

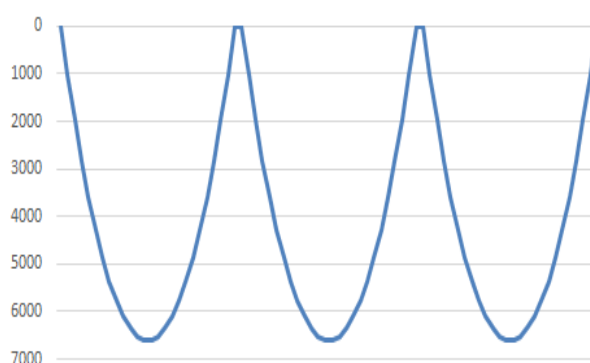


Figure 4. Diagram of bimoment ($\text{kN}\cdot\text{m}^2$) from uniformly distributed load with eccentricity in each span

Define values of bimoment B_1 on supports, with using formula of three bimoments (4) [3]:

$$B_{n-1}l_n\psi_n + 2B_n(l_n\varphi_n + l_{n+1}\varphi_{n+1}) + B_{n+1}l_{n+1}\psi_{n+1} = -6EJ_\omega(\bar{\theta}_{n+1}^{(\text{left})} - \bar{\theta}_{n+1}^{(\text{right})}), \quad (4)$$

where ψ_n and φ_n – values, determined with interpolation method from table [3, p. 64], n – number of supports, θ_{n+1} – rotation angles on anchorages, determined with formula (5):

$$\theta = -\frac{qe}{k^3EJ_\omega} \left(\frac{kl}{2} - \frac{\text{sh} \frac{kl}{2}}{\text{ch} \frac{kl}{2}} \right), \quad (5)$$

Define bimoments for each span from determined bimoment B_1 , applied in the middle supports by formula (6):

$$B_{\frac{1}{2}} = B_1 \frac{1}{2\text{ch} \frac{kl_1}{2}}, \quad (6)$$

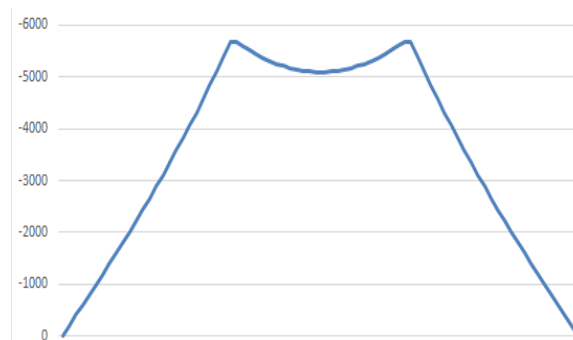


Figure 5. Diagram of bimoment (kN·m²) from concentrated bimoments on anchorages

Conclusive diagram of bimoments frame as algebraical sum of couple diagrams (Figures 4 and 5)

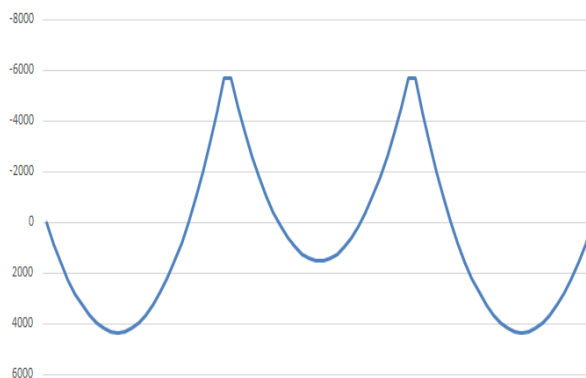


Figure 6. Conclusive diagram of bimoments (kN·m²)

Comparison of bending moment diagrams and bimoment diagrams

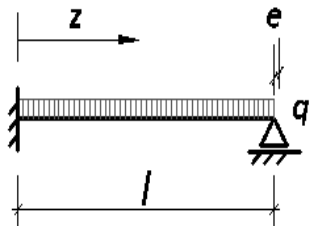
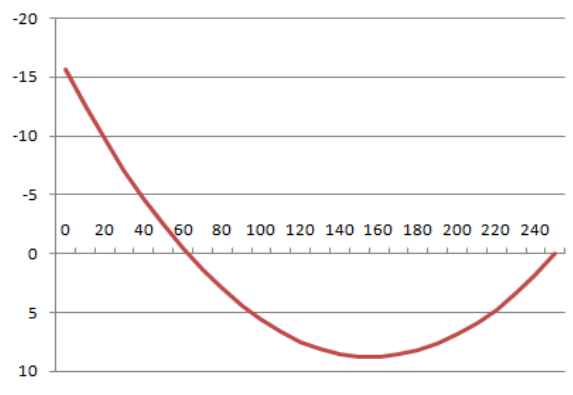
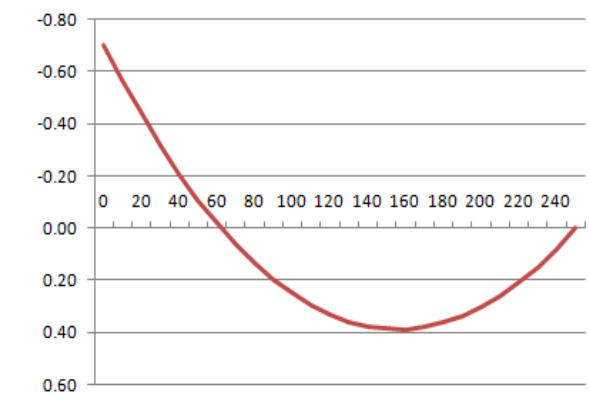
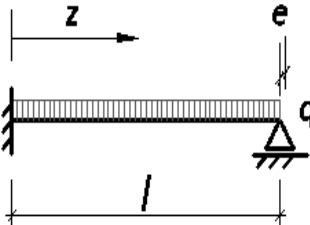
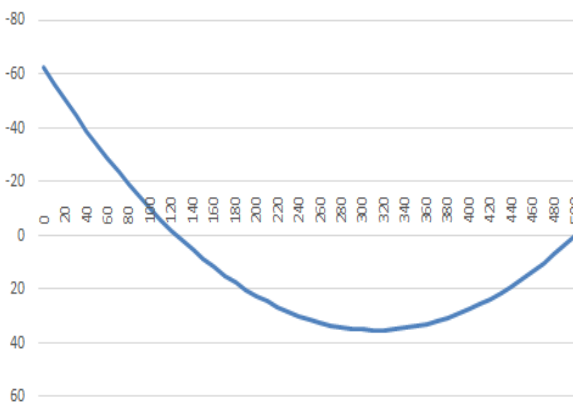
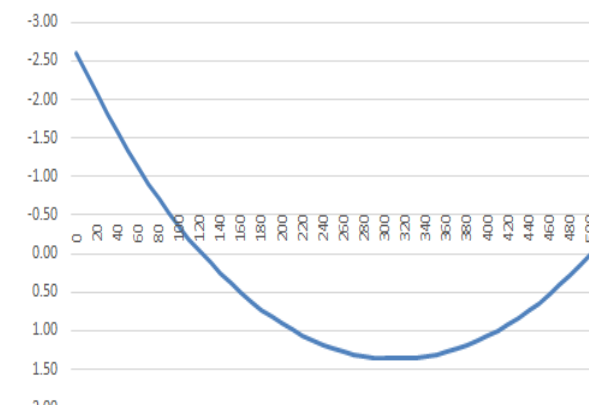
In the same way calculations were made for others modifications of analytical models and span's lengths. Analytical models and totals of calculations (diagram of bending moment and bimoment) were summarized in the Table 1.

As is seen from Figures 3–6, diagrams of bending moment and bimoment have similar configuration.

Suppose, that coversine of bimoment it is possible to express by coversine of bending moment using the proportionality coefficient.

To prove it, compare diagrams of bending moment and bimoment.

Table 1. Diagrams of bending moment and bimoment for different single-span analytical models

Analytical model  $l = 2.5 \text{ m}$ $e = 0.0463 \text{ m}$	
Diagram of bending moment 	Diagram of bimoment 
Analytical model  $l = 5 \text{ m}$ $e = 0.0463 \text{ m}$	
Diagram of bending moment 	Diagram of bimoment 

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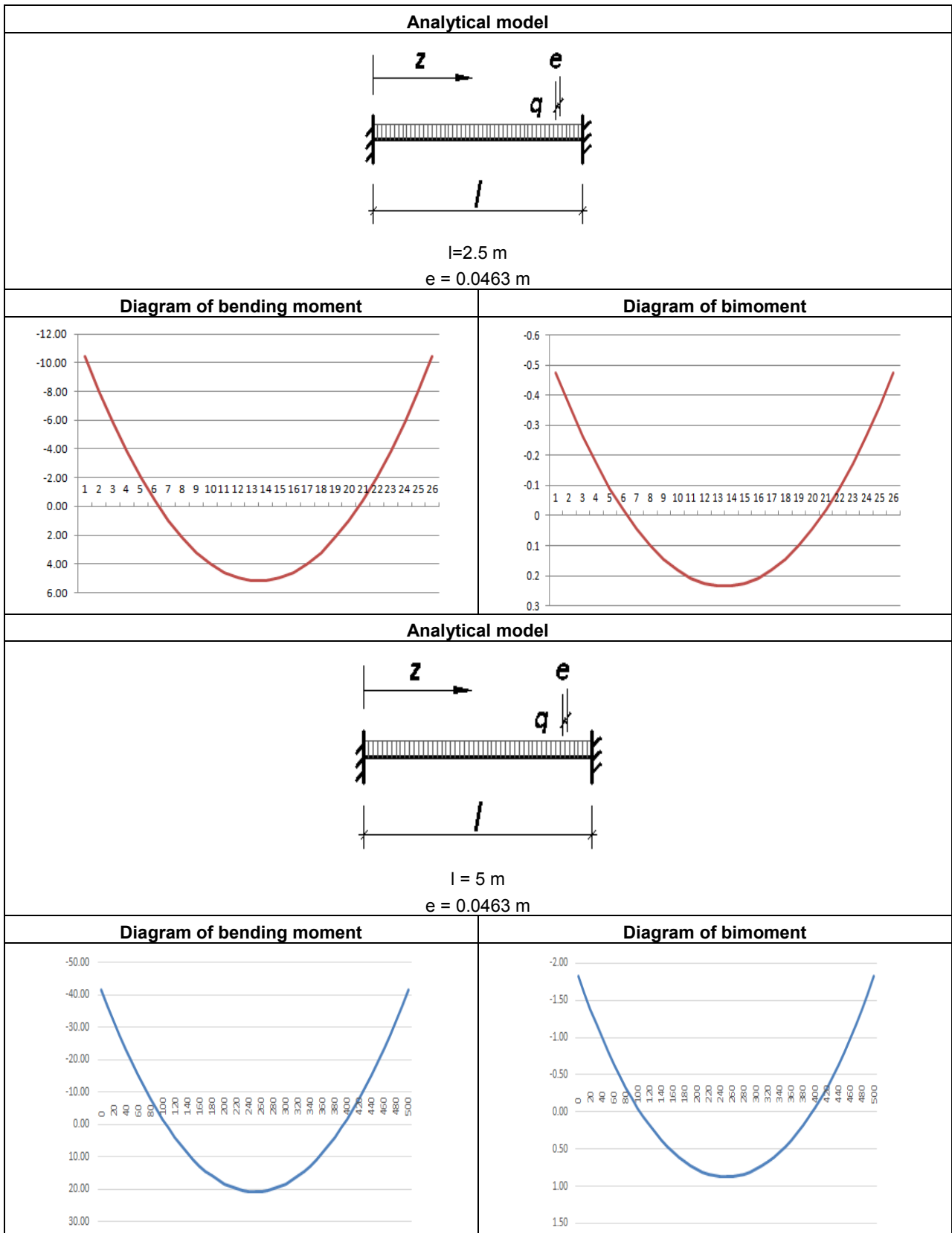
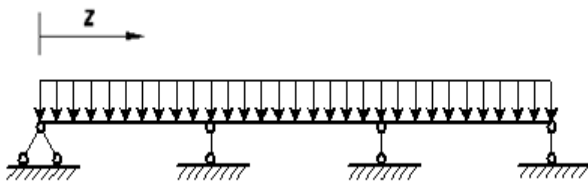
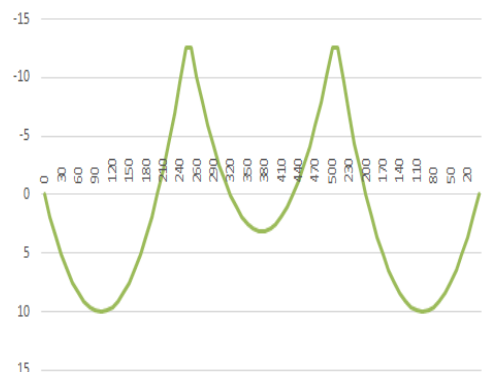
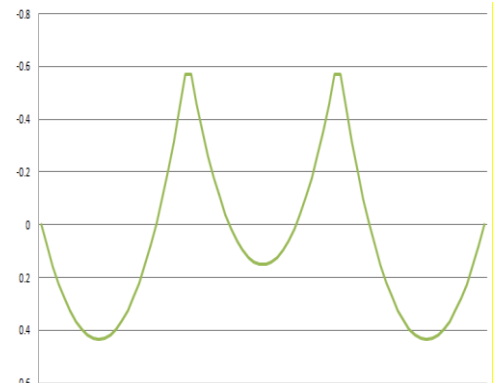
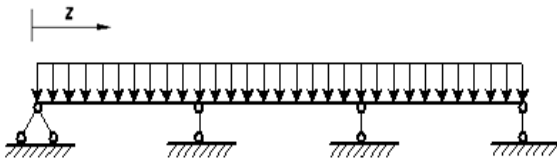
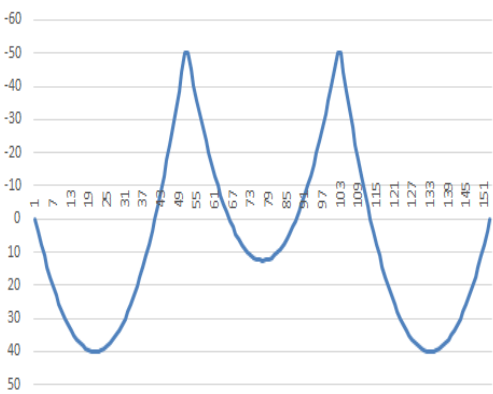
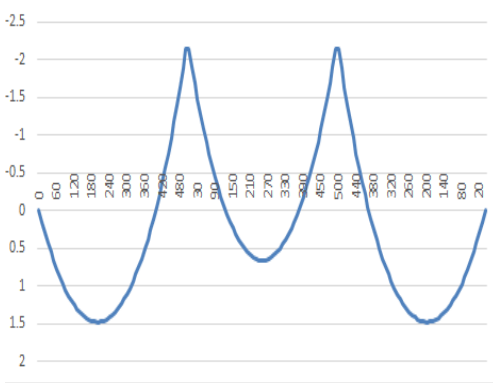
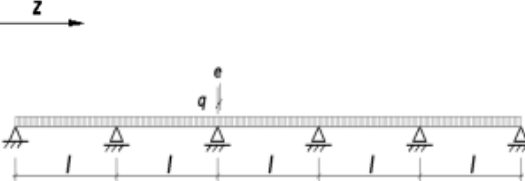
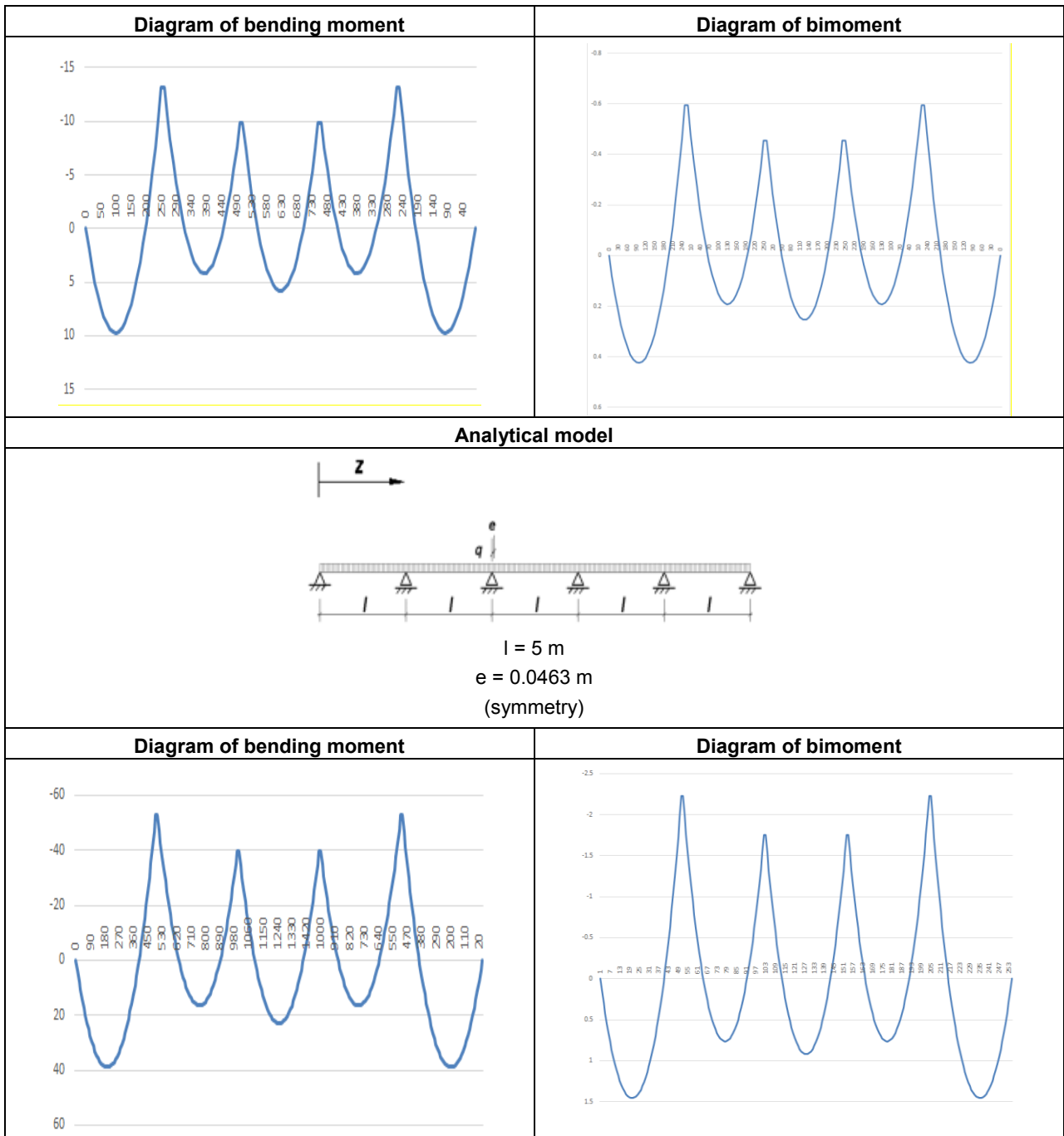


Table 2. Diagrams of bending moment and bimoment for multi-span beams

<p>Analytical model</p>  <p>$l = 2.5 \text{ m}$ $e = 0.0463 \text{ m}$</p>	
<p>Diagram of bending moment</p> 	<p>Diagram of bimoment</p> 
<p>Analytical model</p>  <p>$l = 5 \text{ m}$ $e = 0.0463 \text{ m}$</p>	
<p>Diagram of bending moment</p> 	<p>Diagram of bimoment</p> 
<p>Analytical model</p>  <p>$l = 2.5 \text{ m}$ $e = 0.0463 \text{ m}$ (symmetry)</p>	

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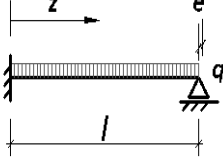
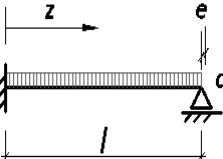
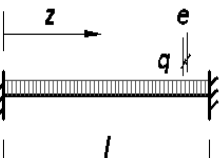
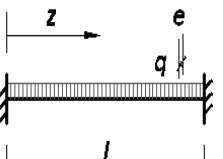


From Tables 1, 2 it appears, that diagrams of bending moment and bimoment really are alike both for single-span and for multi-span beams.

Result and Discussion

In Tables 3–5 bending moment and bimoment ratio present for beams with different number of spans (1, 3 and 5 spans), span lengths (2.5 and 5 metres) and fixing type (fixed and hinged).

Table 3. Values of bending moment and bimoment for single-span beams

Bending moment (bimoment) Analitical model	Moment value in fixed support (kN·m) Bimoment value in fixed support (kN·m ²) Moment and bimoment ratio (m ⁻¹)	Moment value in the middle of span (kN·m) Bimoment value in the middle of span (kN·m ²) Moment and bimoment ratio (m ⁻¹)	Moment value in point of local extremum (kN·m) Bimoment value in point of local extremum (kN·m ²) Moment and bimoment ratio (m ⁻¹)
 l = 2.5 m; e = 0.0463 m	-15.63	7.81	8.78 (z=1.6 m)
	-0.70	0.346	0.39 (z = 1.6 m)
	22.23	22.56	22.64 (z = 1.6 m)
Coefficient value k	0.967	0.957	0.959
 l = 5 m; e = 0.0463 m	-62.50	31.25	35.15 (z = 3.1 m)
	-2.60	1.23	1.36 (z = 3.1 m)
	24.05	25.45	25.77 (z = 3.1 m)
Coefficient value k	0.898	0.850	0.914
Bending moment (bimoment) Analitical model	Moment value in fixed support at the left (kN·m) Bimoment value in fixed support at the left (kN·m ²) Moment and bimoment ratio (m ⁻¹)	Moment value in the middle of span (kN·m) Bimoment value in the middle of span (kN·m ²) Moment and bimoment ratio (m ⁻¹)	Moment value in fixed support at the right (kN·m) Bimoment value in fixed support at the right (kN·m ²) Moment and bimoment ratio (m ⁻¹)
 l = 2.5 m; e = 0.0463 m	-10.42	5.21	-10.42
	-0.48	0.24	-0.48
	21.94	22.17	21.94
Coefficient value k	0.995	0.995	0.995
 l = 5 m; e = 0.0463 m	-41.67	20.83	-41.67
	-1.82	0.87	-1.82
	22.85	23.86	22.85
Coefficient value k	0.943	0.902	0.943

Analysis for similarity coversines bending moment and bimoment for multispan beams view values in feature points (maximum values; values in middle of spans and values on intermediate supports).

Table 4. Values of bending moment and bimoment for three-span beams

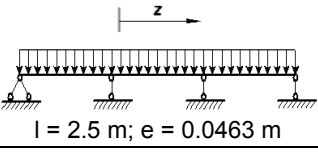
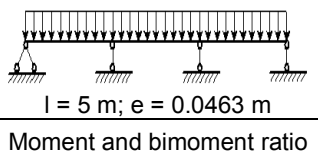
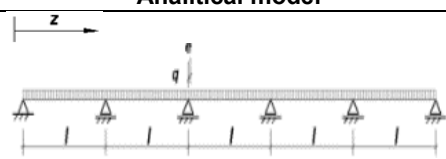
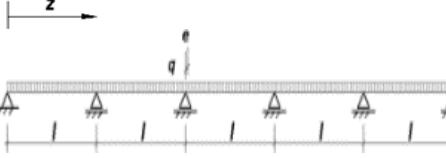
z	z = 1.0 m	z = 1.25 m	z = 2.5 m	z = 3.75 m	z = 5.0 m	z = 6.25 m	z = 6.5 m
 $l = 2.5 \text{ m}; e = 0.0463 \text{ m}$	10.00	9.38	-12.50	3.13	-12.50	9.38	10.00
	0.43	0.41	-0.57	0.15	-0.57	0.41	0.43
Moment and bimoment ratio	23.01	23.03	22.02	20.56	22.02	23.03	23.01
Coefficient value k	0.929	0.944	0.985	1.035	0.985	0.944	0.929
z	z = 2.0 m	z = 2.5 m	z = 5.0 m	z = 7.5 m	z = 10.0 m	z = 12.5 m	z = 13.0 m
 $l = 5 \text{ m}; e = 0.0463 \text{ m}$	40.00	37.50	-50.00	12.50	-50.00	37.50	40.00
	1.48	1.38	-2.14	0.66	-2.14	1.38	1.48
Moment and bimoment ratio	27.09	27.15	23.32	19.05	23.32	27.15	27.09
Coefficient value k	0.799	0.795	0.924	1.140	0.924	0.795	0.799

Table 5. Values of bending moment and bimoment for five-span beams

Analytical model						
 $l = 2.5 \text{ m}$ $e = 0.0463 \text{ m}$ (symmetry)						
z = 1.0 m	z = 1.25 m	z = 2.5 m	z = 3.75 m	z = 3.8 m	z = 5.0 m	z = 6.25 m
9.74	9.05	-13.16	4.11	4.17	-9.87	5.76
0.43	0.39	-0.59	0.19	0.19	-0.45	0.25
Moment and bimoment ratio						
22.91	22.91	22.13	21.51	21.63	21.74	22.65
Coefficient value k						
0.954	0.931	0.968	0.998	0.984	0.985	0.937
Analytical model						
 $l = 5 \text{ m};$ $e = 0.0463 \text{ m}$ (symmetry)						
z=2.0 m	z=2.25 m	z=5.0 m	z=7.5 m	z=7.6 m	z=10.0 m	z=12.5 m
29.47	36.18	-52.65	16.43	16.59	-39.50	23.00
1.12	1.35	-2.23	0.76	0.76	-1.75	0.92
Moment and bimoment ratio						
26.41	26.73	23.64	21.60	21.70	22.52	24.99
Coefficient value k						
0.821	0.806	0.915	0.999	0.989	0.957	0.864

Analogous research was made by A.M.Sergei in his master's thesis [4]. It was considered an approximation of the theoretical relationships between bimoment and bending moment in a general way. He offered number of coefficients and tables that allows framing a diagram of bimoment by using the diagrams of the bending moment. However, from the practical point of view, the proposed communication continues to be complex and time-consuming for routine engineering tasks.

Conclusions

1. Values of bending moments and bimoments were determined for typically using analytical models and span's lengths.
2. It was established, that bimoment value is directly-proportional of bending moments values for the same spans and cross-sections; proportionality factor is eccentricity e of load application.
3. It was established, that ratio values of bending moment and bimoment change from 0.929 to 1.035 – for span with length 2.5 metres, from 0.795 to 1.14 – for span with length 5 metres in case of single eccentricity for typically widespread spans (2.5 m...5 m).
4. It was recommended the dependence:

$$B_w = M \cdot e \cdot k, \quad (7)$$

where k – proportionality factor, determined according to the Tables 2–4.

The set dependence allows determining bimoment value quickly in feature points of the span, knowing bending moment, eccentricity of load application, and also coefficient k , given from tables.

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Anna Pavlenko,
+79819875221; *anna.pavlenko.1996@yandex.ru*

Vladimir Rybakov,
+7(964)3312915; *fishermanoff@mail.ru*

Artem Pikht,
+7(911)8442933; *tamas96@rambler.ru*

Egor Mikhailov,
+7(921)9834269; *egormi95@mail.ru*

Анна Дмитриевна Павленко,
+79819875221;
эл. почта: *anna.pavlenko.1996@yandex.ru*

Владимир Александрович Рыбаков,
+7(964)3312915;
эл. почта: *fishermanoff@mail.ru*

Артём Вячеславович Пихт,
+7(911)8442933;
эл. почта: *tamas96@rambler.ru*

Егор Сергеевич Михайлов,
+7(921)9834269; эл. почта: *egormi95@mail.ru*

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