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Deformations of the periodic truss with diagonal lattice

Деформации периодической фермы с раскосной решеткой

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Abstract. A scheme for statically determinate flat lattice-type truss is investigated. The truss has two supports and uniformly loaded at the upper zone. Analytical dependencies of the deflection of the structure and displacement of the movable support from its size, load and number of panels are obtained. It is shown that if the number of panels in multiples of three, the truss cinematically altered, as is clear from the zero determinant of the system of equilibrium equations. The relevant diagram of the possible speeds of the nodes was defined. The system of computer mathematics Maple with induction method was used for output results. This approach was previously proposed and developed by the author in some problems related to the pivotal plane and spatial trusses. It was discovered intermittent character of the dependence of deflection on the number of panels. Asymptotic properties of the solution were found. The features of the solution allow us to optimize the size of the structure.

Аннотация. Рассмотрена схема плоской статически определимой фермы решетчатого типа. Ферма имеет две опоры и загружена равномерной нагрузкой по нижнему или верхнему поясу. Найдены аналитические зависимости прогиба конструкции и смещения подвижной опоры от ее размеров, нагрузки и числа панелей. Показано, что если число панелей кратно трем, то ферма кинематически изменяема, что явствует из равенства нулю определителя системы уравнений равновесия. Найдена соответствующая схема возможных скоростей узлов. Для вывода результатов использована система компьютерной математики Maple и метод индукции, ранее предложенный и развитый автором в ряде задач о стержневых плоских и пространственных фермах. Обнаружен скачкообразный характер зависимости прогиба от числа панелей. Найдены асимптотические свойства решения. Выявленные особенности решения позволяют оптимизировать размеры конструкции.

Introduction

Regular (or structurally scalable) trusses represent a large class of rod systems, to which analytical methods of calculation based on the method of induction may be applicable. Flat trusses tend to have one natural parameter scale — the number of panels. In some problems, for example, in cantilever truss, there is a second parameter scaling (number of panels on the console). The problem of finding and calculating the new statically determinate rod structures of the regular type (periodic) studied in [1, 2]. Some questions of numerical methods for calculation, optimization, and fracture of beam structures considered in [3-8]. Examples of analytical and numerically-analytical solutions of problems of elasticity feature in the system of computer mathematics Maple [9] are given in [10, 11]. The exact solution for the deflection of the lattice trusses by the method of induction received and analyzed in [12–19]. Overview of analytical solutions for flat trusses, obtained in the system Maple can be found in [20, 21].

Design scheme

The truss contains the upper horizontal zone of rods, vertical struts and the inclined struts (Fig. 1). The truss with n panel contains $m = 4n + 12$ rods, including three support bar, and $s = 2n + 9$ joints with three joints (hinges) at the ends of the support rods. Given that for each node in the method of

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cutting of knots one can write two equations of equilibrium in projections, the system of equilibrium equations is closed, the design is statically determinate.

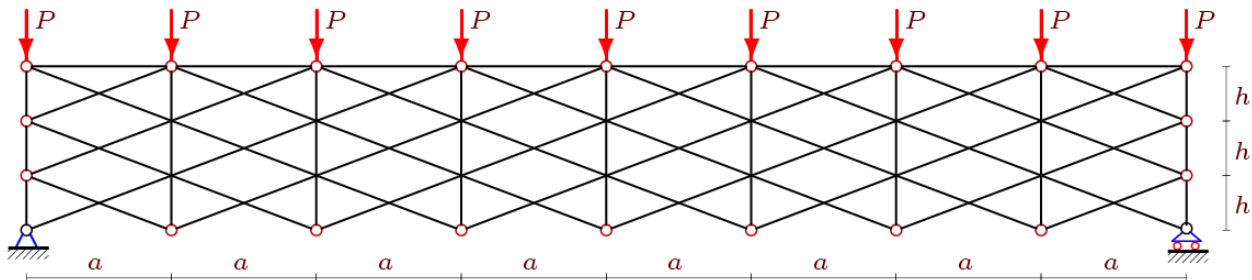


Figure 1. Truss, $n = 8$

Let us introduce a coordinate system with the origin at the left fixed support to specify the geometry of the truss. For making the equilibrium equations of the nodes and finding the guides of the cosines of the effort, it will take the coordinates of hinges:

$$x_i = x_{i+n+1} = (i-1)a, \quad y_i = 0, \quad y_{i+n+1} = 3h, \quad i = 1, \dots, n+1,$$

$$x_{i+2+2n} = 0, \quad x_{i+2+4n} = an, \quad y_{i+2+2n} = y_{i+4+2n} = (3-i)h, \quad i = 1, 2.$$

The fixed bearing is modeled by two rigid bars that are attached to node 1: $x_{s-2} = 0, y_{s-2} = -h, x_{s-1} = -h, y_{s-1} = 0$, a movable rod with one end pivot at the point $x_s = x_{n+1}, y_s = -h$. While the length of the support rods is taken arbitrarily since according to the problem, they are rigid and their length does not influence the solution. The lattice structure of the truss is set the same way as is specified in discrete math graph conditional vectors $\bar{q}_i, i = 1, \dots, m$. The vectors of the diagonals of the lattice ($i = 1, \dots, n-2$), for example, have the following form: $\bar{q}_i = [i, i+n+4]$, $\bar{q}_{i+n-2} = [i+3, i+n+1]$. The rack is coded by the vectors: $\bar{q}_{i+3n+10} = [i+n+2, i+1], i = 1, \dots, n-1$. Left and right hinge correspond to the vectors $\bar{q}_{m-2} = [1, s-2], \bar{q}_{m-1} = [1, s-1], \bar{q}_m = [n+1, s]$.

Solution

To determine the stresses in the bars program [12–16] based on the method of cutting of knots is used. The equilibrium equations of all nodes are organized in the general system of equilibrium equations:

$$G\bar{S} = \bar{T} \quad (1)$$

with the matrix G of the guides of the cosines of effort calculated at the given coordinates. Here \bar{S} is a vector of stresses in the bars, \bar{T} a vector of given loads. For loads uniformly distributed on the upper zone, this vector has the form: $T_{2i} = -P, i = n+2, \dots, 2n+2$. Other components of the vector are equal to zero. A solution of a system of linear equations (1) finds in the symbolic form using Maple: $\bar{S} = G^{-1}\bar{T}$ where G^{-1} is the inverse matrix. The results of the program are analytical expressions for the stresses in the bars farm. For the computation of deflection use the formula of Maxwell-Mohr:

$$\Delta = \sum_{j=1}^{m-3} \frac{S_j N_j l_j}{EF}, \quad (2)$$

where E is the modulus of elasticity of cores F – the cross-sectional area of the rods (the same as for the whole structure), l_j and S_j the length of the j -th core and force in it from the action of a given load;

N_j — efforts from a single vertical force applied at mid-span. The summation is conducted on all rods of the truss, except hinges rods, which are assumed to be rigid. For the derivation of deflection for an arbitrary number of panels required to first obtain a sequence of solutions for a truss with different numbers of panels, and then by induction to generalize the solution to the arbitrary case. However, in the process of counting it was found that the determinant of the matrix G of the system (1) becomes zero if

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the number of panels in multiples of three. Obviously, this corresponds to a kinematic degeneration of the structure. Indeed, kinematic analysis for a truss with $n=3$ gave the following consistent characteristic of lattice schemes [14–16], the picture of possible velocity joints:

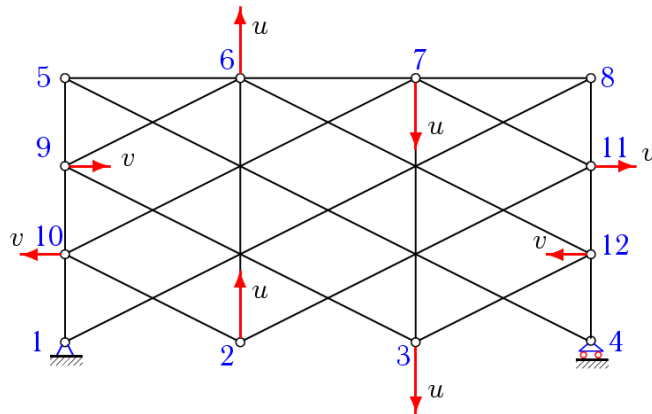


Figure 2. Scheme of possible velocity of joints

Rods with hinges 2–10, 3–12 commit instantaneous rotation around the poles 1 and 4, rods 6–9, 7–10 and 7–11, 6–12 – around corner points 5 and 8. Terminals 9–10, 11–12, 6–7 rotate around their mid-points. Rods 2–6 and 7–3 move forward with instantaneous speed u , the terminals 1–8 and 5–4 are fixed. Between the speeds there is the ratio $u/a = v/h$, following from a consideration of the provisions of the instant centers of velocity.

The same distribution of speeds obtained for other trusses in which the number of panels in multiples of three. In [16] discovered the kinematic degeneration and a diagram of possible speeds for stud truss with an odd number of panels. There are kinematically modified lattice in the statically indeterminate structures [15] also.

To obtain a sequence of solutions of trusses with the number of panels not multiple of three, we distinguish two cases. In the first case, the number of panels will be set by formula $n = 1 + 3k$, $k = 1, 2, \dots$, in the second place $n = 2 + 3k$, $k = 1, 2, \dots$. Induction for these cases gives the general formula:

$$\Delta = P \frac{A_k a^3 + H_k h^3 + C_k c^3}{2h^2 EF}, \quad (3)$$

where $c = \sqrt{a^2 + h^2}$. Two solutions differ only in the expression for the coefficients. In the first case the coefficients have the form:

$$A_k = \left(30k^4 + 40k^3 - 2((-1)^k + 15)k^2 + (20(-1)^k - 36)k - 7(-1)^k + 7 \right) / 32,$$

$$H_k = \left(10k^4 + 8(23(-1)^k - 6)k^3 + 6(7(-1)^k - 11)k^2 + 4(29(-1)^{k+1} + 37)k + 79(-1)^k + 113 \right) / 32,$$

$$C_k = \left(10k^4 + 8(23(-1)^k - 6)k^3 + 2(45(-1)^k - 17)k^2 + 4(11(-1)^{k+1} + 3)k + 9(-1)^{k+1} + 9 \right) / 64.$$

In the second ($n = 2 + 3k$, $k = 1, 2, \dots$):

$$A_k = \left(30k^4 + 40k^3 + 2(3(-1)^k - 19)k^2 + 4((-1)^k + 5)k - 15(-1)^k + 15 \right) / 32,$$

$$H_k = \left(10k^4 + 136k^3 + 6(9(-1)^{k+1} + 5)k^2 + 4((-1)^k + 7)k + 47(-1)^k + 145 \right) / 32,$$

$$C_k = \left(10k^4 + 136k^3 + 2(27(-1)^{k+1} + 55)k^2 + 4(3(-1)^{k+1} + 11)k + 9(-1)^{k+1} + 9 \right) / 64.$$

The coefficient in the first case received in Maple system is a generalization of a sequence 0, 20, 97, 302, 712, 1446, 2625, 4412, 6976, ..., 105372. Therefore, it is necessary to calculate a sequence of

16 trusses to get the pattern. For finding the general term of the sequence, the special operator `rgf_findrecur` from the Maple package `genfunc` is used and returns a recurrence equation:

$$A_k = 2A_{k-1} + 2A_{k-2} - 6A_{k-3} + 6A_{k-5} - 2A_{k-6} - 2A_{k-7} + A_{k-8}. \quad (4)$$

A feature of the operator `rgf_findrecur` is that it only works with an even number sequence. The solution of equation (3) for the eight initial data 0, 20, 97, 302, 712, 1446, 2625, 4412 found operator `rsolve`. Verification of the obtained solution can be performed in the numerical solution that does not have significant limitations to run time. Similarly, but somewhat more complicated, for 18 trusses and with the equation of higher order other factors decision are defined.

To determine the *horizontal* displacement of the movable support is necessary to obtain analytical expressions of the forces in the bars from the action of a single force applied to this pole [22]. The solution according to the formula (2) takes the form

$$\Delta_h = P \frac{A_k a^3 + H_k h^3 + C_k c^3}{ahEF}.$$

If $n = 1 + 3k$, $k = 1, 2, \dots$ we have the following coefficients

$$A_k = k(3k^2 + 3k - 2) / 2, H_k = k^3 + 9k^2 + 5k + 2, C_k = k(k^2 + 9k + 2) / 2.$$

If $n = 2 + 3k$, $k = 1, 2, \dots$ we have

$$A_k = (3k^3 + 6k^2 + k - 2) / 2, H_k = k^3 - 6k^2 - 6k + 1, C_k = (k^3 - 6k^2 - 13k - 6) / 2.$$

Note that the problem of horizontal offset of the support the degree of the polynomials that specify the coefficients is smaller than for the case of mid span deflection.

Analysis

Graphically, the dependence of the dimensionless relative deflection $\Delta' = \Delta EF / (PL)$ where $L = na$ from the number of panels k for a given panel length and height is displayed by points in figure 3. In this structure, this dependency is by definition discrete and non-monotonic, due to the "flashing" of the component species $(-1)^k$ in the coefficients of the solution. By the methods of Maple (operator, limit), you one can find out the extent of this growth:

$$\lim_{k \rightarrow \infty} \Delta' / k^3 = \frac{5(6a^3 + 2h^3 + c^3)}{192ah^2}. \quad (5)$$

Noticeable jumps in the values of the deflection is due in part to the fact that the point at which deflection is determined, is, or exactly in the middle of the span for even values of n , or near the geometric center of the span. When the number of panels is small, the deflection may even have a different sign for trusses with different from each other at the one panel. With the increasing number of panels, this effect disappears, however, the difference in deflection similar in a set of trusses can achieve up to four times. The last conclusion shows the possibility of optimizing the stiffness of the truss by the rational choice of a number of panels.

The results of computing the relative displacement of the support $\Delta'_h = \Delta_h EF / (PL)$ also show an abrupt change in the result of which depends mainly on the parity of the number of panels. The growth rate in this case is less and determined by the limit

$$\lim_{k \rightarrow \infty} \Delta'_h / k^2 = \frac{3a^3 + 2h^3 + c^3}{6a^2h}.$$

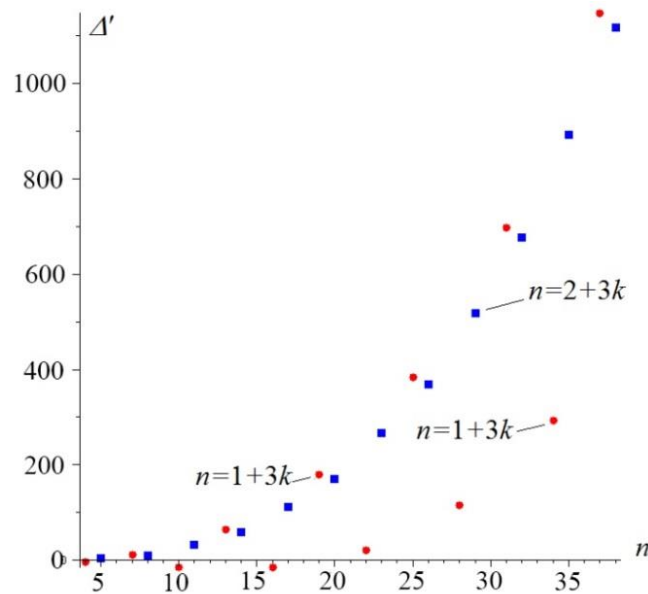


Figure 3. The Deflection, $a = 3\text{m}, h = 2\text{m}$

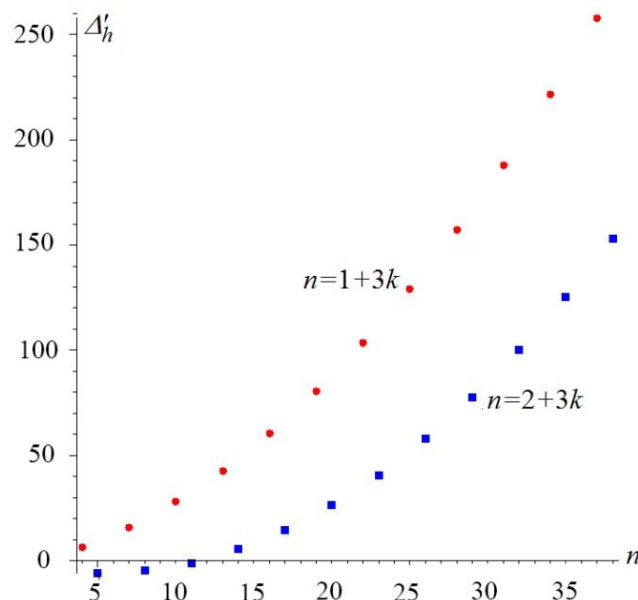


Figure 4. Offset of the support, $a = 3\text{m}, h = 2\text{m}$

The solution to (3) when any load is easily generalized to the case when the stiffness of the rods of the zones and grids are different. If to express the stiffness of the groups of rods of length a , h and c respectively using some given rigidity $EF_j = EF_0 / \mu_j$, $j = 1, \dots, 3$, the result would be

$$\Delta = P \frac{A_k \mu_1 a^3 + H_k \mu_2 h^3 + C_k \mu_3 c^3}{2h^2 EF_0}.$$

Results and Discussion

The formulas for calculation of the deflection and displacement of the support trusses with an arbitrary number of panels are obtained. The main advantage and at the same time the main difficulty here is the dependence on the number of panels obtained by the method of induction. Induction method proposed and developed by the authors [12–15]. It should be noted that in the courses of structural mechanics the same formulas for estimating the deflection in the domestic and foreign literature are not

found. The only well-known formula V.K. Kachurina [23, c. 310] for the optimal height of the truss selected for rigidity, is very approximate, intended for almost all of the girders and not take into account any particular structure of the lattice or the number of panels. Another alternative discussed in the article the approach is a fundamental theory of periodic lattices L.S. Rybakov [24], applicable to a wide class of structures. However, this theory analytically implements the finite element method, can't give a simple formula, closed form solutions, suitable for analysis and implementation. The method of induction, together with a system of symbolic mathematics allows not only to obtain a compact closed solution but also to identify in some cases, the threat of its features. In this calculation, it has been shown the kinematic system degeneration when the number of panels in multiples of three. Similar features were found in [14–16]. The analytical form of the solution allowed to reveal the asymptotic behavior of it. Comparative asymptotics of analytic solutions is dedicated in [20]. Special note is the object of study. This truss with complex diagonal bars not previously been investigated. The scheme of this statically determinate truss was developed by the authors.

Conclusions

The proposed scheme of the lattice girder is not quite normal. Bottom chord is not straightforward. It is more correct to say that it is not in traditional form. One of the purposes of introducing this scheme was to add architectural expression to the truss, in order to break the habitual way of the trusses with triangular lattice and rectilinear lines. By the way, it was discovered one important constructive feature of the design. It turned out that when the number of panels is three-fold, truss becomes instantaneously changeable mechanism. It is confirmed by the diagram of possible speeds. However, despite this feature, the purpose – output compact and convenient for practice of the formula, is achieved. A sequence of numbers is generated in which numbers of multiples of three are eliminated and formulas for deflection are derived for it and asymptotic estimates are obtained, which are necessary mainly for the comparative characterization of various lattice schemes.

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