Flat rod systems: optimization with overall stability control

Плоские стержневые системы: оптимизация с контролем общей устойчивости

I.N. Serpik,

Bryansk State University of Engineering and Technology, Bryansk, Russia **A.V. Alekseytsev,** National Research Moscow State Civil Engineering University, Moscow, Russia **P.Yu. Balabin, N.S. Kurchenko,** Bryansk State University of Engineering and Technology, Bryansk, Russia

Д-р техн. наук, заведующий кафедрой И.Н. Серпик,

Брянский государственный инженернотехнологический университет, г. Брянск, Россия

канд. техн. наук, доцент А.В. Алексейцев, Национальный исследовательский Московский государственный строительный университет, г. Москва, Россия без степени, аспирант П.Ю. Балабин, канд. техн. наук, доцент Н.С. Курченко, Брянский государственный инженернотехнологический университет, г. Брянск, Россия

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Abstract. An algorithm for discrete optimization of steel flat rod systems was developed on the basis of an evolutionary search. The task is to minimize the weight of the bars via taking into account constraints on stresses, displacements, and overall stability. The cross-sectional dimensions of the bars and the coordinates of their node connections were varied. Buckling is taken into account when stability is lost both in the object plane and out of the plane. Analysis of deformations of the considered structure variants was performed via the displacement-based finite element method. An iterative procedure for solving the task was formulated by using an auxiliary elite population, combined approaches to selection and mutation, and single-point crossover. The primary feature of the proposed computing scheme is simplified structure stability verification by determining stress-strain conditions with a tangent stiffness matrix and the additional self-balanced system of small fictitious forces. Assessment as to how constraint on stability was met was performed based on the results of the considered convergence of the internal iteration cycle used for analyzing load-carrying system behavior by taking into account the influence of longitudinal forces on the bars while bending. It was calculated that it is sufficient to perform only 3–5 iterations of this procedure to verify structure stability. Efficiency of the proposed algorithm is illustrated via the example of optimization of bar system with two supports and a frame with a girder truss.

Аннотация. Разработан алгоритм дискретной оптимизации изготовленных из стали плоских стержневых систем на основе эволюционного поиска. Ставится задача минимизации веса стержней с учетом ограничений по напряжениям, перемещениям и общей устойчивости. Варьируются размеры поперечных сечений стержней и координаты узлов их соединения. Учитывается выпучивание при потере устойчивости как в плоскости, так и из плоскости объекта. Анализ деформаций рассматриваемых вариантов конструкции выполняется методом конечных элементов в форме метода перемещений. Сформулирована итерационная процедура решения поставленной задачи с использованием вспомогательной элитной популяции, комбинированных подходов к селекции и мутации, одноточечного кроссинговера. Основной особенностью предлагаемой вычислительной схемы является упрощенная проверка устойчивости конструкций путем расчета их напряженно-деформированного состояния с использованием касательной матрицы жесткости и дополнительной самоуравновешенной системы малых фиктивных сил. Оценка удовлетворения ограничения по устойчивости выполняется по результатам рассмотрения сходимости внутреннего итерационного цикла, реализующего анализ поведения несущей системы при учете влияния для стержней продольных сил на изгиб. Расчетным путем установлено, что для проверки устойчивости конструкции достаточно выполнить только 3-5 итерации этой процедуры.

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Работоспособность предлагаемого алгоритма иллюстрируется на примере оптимизации двухопорной стержневой системы и рамы с ферменным ригелем.

Introduction

For newly designed building systems, the percentage of buildings and constructions with steel bearing structures is increasing. These being the case, flat rod systems are in wide usage. The process of optimally designing such objects often includes the choice of bar profile dimensions and the arrangement of their connecting nodes, taking into account constraints on stresses, displacements, and stability. To verify the provision of strength and stiffness of flat rod systems included in steel framework systems, one should usually implement two-dimensional computational models. At the same time, a stability analysis of such objects, in many cases, is required to be performed while taking into account the possibility of buckling out of the structure plane.

The issue of optimally designing bar systems is given great attention in scientific literature. The most universal approaches to solving these problems have been obtained via the use of meta-heuristic methods [1-10], which allow performing an effective search using discrete sets of variable parameters taking into account a set of standard requirements as the conditions for a building structure design. In particular, the use of genetic algorithms in such issues was considered [1, 4]. At the same time, metaheuristic procedures for the optimal synthesis of frameworks are often related to the need for performing a significant number of working capacity checks of structure solution variants taken into consideration. A significant volume of work hours for such checks to be spent, particularly for taking into account the necessity of ensuring conditions of stability, to a certain extent restrains the use of such algorithms for practical purposes. In a number of papers that represent optimization methods of various types, stability constraints were considered only for separate bars [11-15], which greatly simplifies the task, but reduces the possibilities of application in real design practice. In [16, 17], compliance with constraints on a structure's overall stability was verified on the basis of the classical solution of the eigenvalue problem within the framework of the Euler approach. In [18, 19], the condition of ensuring stability at size optimization of the flat rod systems unfolded from displacements from the plane of the structure was approximately taken into account by considering the effect of longitudinal forces on bending.

It should be noted that during the optimization process, it is usually not necessary to determine the critical load values and buckling shapes, but only to check structure stability. To do so, it is sufficient to confirm that the determinant of the tangent stiffness matrix of the system is positive [20]. Such a criterion was taken into account for optimization of bar structures in [21, 22]. In this paper, to optimize flat rod systems, even a simpler assessment of in-plane and out-of-plane stability is realized, including analysis of convergence of the iterative process of the structure calculation via the finite element method using a tangent stiffness matrix and a self-balanced system of small auxiliary forces. In this case, the stability test is combined with the analysis of the stress-strain condition of the bar system.

Minimization of the rods' weight is performed with a set of strength, stiffness, and overall stability constraints by varying rod profiles and coordinates of nodal points on discrete sets of permissible variants. Constructive and technological requirements are all taken into account when assigning sets of permissible values of variable parameters. The structure of the optimization scheme is carried out by using the general provisions for genetic algorithm of work [23].

The purpose of this paper is to develop an evolutionary algorithm for optimization of flat rod systems considering with acceptable computational complexity the overall stability as one of the active constraints. To this effect the problem of creating a simplified iteration scheme of overall stability evaluation of rod structures without defining the critical load level for intermediate alternatives of a deformable object is solved.

Methods

Statement of the optimization problem

We believe that the plane steel structure is made of rectilinear bars with constant cross-sections along their lengths. The axes of bars and one of the main axes of bar cross-sections are located in plane XY of the Overall Cartesian coordinate system XYZ (Fig. 1). We set the task of minimizing the weight W of all the bars of the structure:

$$W(H_1, H_2, ..., H_N, R_1, R_2, ..., R_M) \to \min$$
, (1)

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where H_n (n = 1, 2, ..., N) – set of permissible combinations of sizes of independently varied crosssections of the bar n, N – total number of such sections, R_m (m = 1, 2, ..., M) – set of permissible values for the independently varied nodal coordinate m, M – the total number of such coordinates.

We take into account that the bars with variable cross-sectional dimensions can be combined into groups in which such parameters are assumed to be the same. Accordingly, variable coordinates can be combined into groups, the values of which are required to be the same.

We assume that the structure is subject to loading in its plane. The stress-strain condition of the object is calculated using the finite element method within the framework of the displacement method. This being the case, the possibility of structure spatial deformation should be taken into account, considering the need to assess compliance with stability requirements both in the framework plane and out of the plane. To ensure the possibility of stability testing of separate bars, each of them is divided into not less than 5–6 finite elements.



Figure 1. Example of the bar system with separation into bars and finite elements: 1-3 are bar numbers, G – bar connection nodes, U – nodes of the finite element model

A set Ω of all coordinates of the finite-element model nodes can be represented as follows

$$\Omega = \{\Omega_X, \ \Omega_Y\},\tag{2}$$

where $\Omega_X = \{X_1, X_2, ..., X_I\}$, $\Omega_Y = \{Y_1, Y_2, ..., Y_I\}$ - sets of values of the node coordinates along axis X and Y, respectively, I - total number of nodes.

We expand the set Ω into nonoverlapping subsets:

$$\Omega = \Omega_A + \Omega_B + \Omega_D, \qquad (3)$$

where Ω_A – set of independently changeable coordinates, Ω_B – set of changeable coordinates being linear functions of the coordinates belonging to the set Ω_A , Ω_D – set of unchangeable coordinates.

Each of sets R_m (m = 1, 2, ..., M) is given for one coordinate of set Ω_A . In this case, if the position of the rectilinear structural element varies in the plane, each coordinate of nodes G_1 , G_2 of its marginal sections (Fig. 2) may belong only to sets Ω_A or Ω_D only. Then, coordinates X_k , Y_k of a certain internal node k of such a structural element for the current configuration of the structure can be determined using equations

$$X_{k} = X_{1} + x_{k} (X_{2} - X_{1}) / l, \ Y_{k} = X_{2} + x_{k} (X_{2} - X_{1}) / l,$$
(4)

where X_1 , Y_1 , X_2 , Y_2 - coordinates for nodes G_1 and G_2 in the current configuration, respectively, x_k and l - value of coordinate x of node k and the length of the structural element for the basic configuration, respectively.

We believe that, in the general case, the following constraints can be taken into account:

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A) Strength condition:

$$\sigma_m \le R_y , \tag{5}$$

where σ_m – Mises stress; R_v – design steel resistance assigned with regard to the yield strength [24].

B) Stiffness requirements. For each node i, of the discretizable structure, inequations should be met

$$\left|u_{j}\right| \leq \left[u\right]_{j}, \left|v_{j}\right| \leq \left[v\right]_{j} \quad , \tag{6}$$

where u_j , v_j – projections of *j* node displacement on *X* and *Y* axis, respectively, $[u]_j$, $[v]_j$ – permissible values for such displacements.

C) Overall stability of bar system structure, including stability of individual bars.

D) Stability of side flanges and walls of profiles.

E) Stability of plane bending of bars.

F) Provision of local structural strength.

G) Unification regarding topologies and parameters.

H) Design constraints (possibility of determining layout of nodal connections, support conditions, etc.).



Figure 2. Straight-line structural element in plane *XY* : 1, 2 – basic and current configurations

Principles of constraints accounting

Constraints A, B, and C are considered active and are directly taken into account during the optimization process. Other constraints are taken into account when choosing the initial prerequisites for optimal design and are controlled after the optimal search has been performed. In general, it is understood that the result of optimization, a variant of the design solution, in any case, should be checked for compliance with all the constraints imposed, by applying certain adjusted schemes. If necessary, the initial prerequisites for optimization can be adjusted and the optimization process performed again.

We assume that the structure material operates under conditions of linear elasticity. In general, we take into account that bars are subject to deformations caused by tension/compression, bending in both main planes, and pure torsion. The finite element model of the bar system is formed in accordance with known provisions [25].

We will check compliance with active constraints based on an analysis of the stress-strain condition of the design variants using the method of a step-by-step approach. We consider the system deformations under conditions of small displacements, while taking into account longitudinal-transverse bending of the bars. For each standard combination of loads, during the first iteration, we perform calculations by applying conventional stiffness matrices of the finite element method and the actual load, and, in iteration $r \ge 2$, we solve the following system of linear algebraic equations:

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$$\left[K\right]_{\tau}\left\{\delta\right\}^{\left(r\right)} = \left\{Q\right\} + \left\{\Delta\right\},\tag{7}$$

where $[K]_r$ – tangent of the stiffness matrix [20] of the spatial finite element model, $\{\delta\}^{(r)}$ – vector of nodal displacements calculated in iteration r, $\{Q\}$ – vector of nodal forces that takes into account the standard impact, $\{\Delta\}$ – randomly generated vector of a self-balanced system of small nodal forces that can take non-zero values for any nodal degree of freedom.

The tangent matrix of stiffness can be represented as [20]

$$\begin{bmatrix} K \end{bmatrix}_{\tau} = \begin{bmatrix} K \end{bmatrix} + \begin{bmatrix} K_G \left(\{N\}^{(r-1)} \right) \end{bmatrix}, \tag{7}$$

where [K] – stiffness matrix of the finite element model, $[K_G(\{N\}^{(r-1)})]$ – geometric matrix of the finite element system [25] expressed through longitudinal bar forces found in iteration r-1 that are combined into vector $\{N\}^{(r-1)}$.

Vector $\{\Delta\}$ should not have any significant influence on the task solution if the load does not approach the Euler critical level. In this case, additional fictitious displacements associated with this vector should allow creating the conditions for the manifestation of stability loss by a planar or spatial scheme. Then, according to the theory of stability [26], as the load approaches the critical level, the energy of the object deformation obtained through the effect of longitudinal forces on the bars bending will tend to infinity. We introduce an estimate of structure stability based on the premise that when implementing the iterative process on a computer in accordance with equation (7), the absence of the convergence of solutions indicates non-fulfillment of constraint C. We control the approach to the condition of instability by verifying the following condition for the preset iteration number $r_0 \ge 3$:

$$\left|1 - \frac{U^{(r_o)}}{U^{(r_o - 1)}}\right| \le \alpha ,$$
(9)

where $U^{(r_o)}$, $U^{(r_o-1)}$ – discretized object deformation energies obtained in iterations r_o and $r_o - 1$, respectively, α – fixed small positive number.

According to calculations, a sufficiently effective verification of this type is ensured if $r_o = 5$, $\alpha = 0.001$. If all the standard requirements are met for the bar system, then practically 3–5 iterations are required for the iterative process to be implemented.

Management of the optimal search process on the basis of genetic algorithm

In our case, an individual is the variant of the structure obtained by choosing values of parameters from predefined discrete sets. Each of such sets shall be arranged in descending order: by area of cross-sections for sets H_n and the values of the coordinates for sets R_m .

We believe that during each iteration of the genetic algorithm, the following two populations (sets) of individuals shall be considered:

1. Current population Φ_1 . It has the fixed size N_1 and is used for individuals that can be processed by genetic operators.

2. Elite population Φ_2 . It is introduced to save the best variants of solutions in the search process. The genetic material of this population can be used in the modification of population Φ_1 . It is envisaged that the number of individuals in population Φ_2 should not exceed preset value N_2 . Initially, population Φ_1 is formed. N_1 identical projects based on the first elements of sets H_n (n = 1, 2, ..., N), R_m (m = 1, 2, ..., M) are introduced. Next, an iterative procedure is performed, each iteration of which includes the following steps:

1. Check of the individuals' fitness for work. The stress-strain condition is calculated for each variant of the structure related to population Φ_1 . This population is divided into groups: Φ'_1 and Φ''_1 . If for an individual belonging to group Φ'_1 , any of the active constraints is not met, then such individual is replaced by the best individual from the elite population Φ_2 not available in population Φ_1 . If there are no such individuals in population Φ_2 , then, for this purpose, random information is generated about parameters for a new variant of the structure not considered in population Φ_1 . For an individual from group Φ''_1 , constraint (9) is taken into account in the same way. At the same time, if only constraints A and B are violated for individuals belonging to this group, a significant penalty shall be introduced by multiplying the value of the objective function by a factor

$$k_p = (1 + \xi_\sigma \chi(\alpha_\sigma))(1 + \xi_\delta \chi(\alpha_\delta)), \tag{10}$$

where $\xi_{\sigma}, \xi_{\delta}$ - set positive numbers, $\chi(x)$ - heaviside function of some argument ω ($\chi(\omega) = 0$, if $\omega < 0$ and $\chi(\omega) = 1$, if $\omega \ge 0$),

$$\alpha_{\sigma} = \max_{t} \left(\max_{i} \left(\frac{\sigma_{m}^{(i \max)}}{R_{yi}} - 1 \right) \right), \alpha_{\delta} = \max_{t} \left(\max_{j} \left(\max_{j} \left(\frac{|u_{j}|}{[u]_{j}} - 1 \right), \max_{j} \left(\frac{|v_{j}|}{[v]_{j}} - 1 \right) \right) \right), \quad (11)$$

t – number of load combination variants, $\sigma_m^{(i \max)}$ – maximum value σ_m for *i* bar, R_{yi} – value R_y for this bar material.

2. Modification of the elite population. For each individual φ_{1l} of population Φ_1 , the objective function value $W(\varphi_{1l})$ is calculated and the following condition shall be verified:

$$\left(\neg \left(\left(\varphi_{1l} = \varphi_{2j} \right) \lor \left(\varphi_{2j} \in \Phi_2 \right) \right) \right) \lor \left(W(\varphi_{1l}) < W_{2\max} \right), \tag{12}$$

where $\varphi_{2j} - j$ individual from population Φ_2 , $W_{2 \max}$ – the maximum value of the objective function for this population individuals.

If the criterion (12) is met, the individual from population Φ_1 is copied into population Φ_2 . This being the case, if the elite population already contains individuals N_2 , then the individual with the largest value of the objective function is removed from it.

3. Check of conditions for the optimization completion. Calculations show that in the absence of changes in population Φ_2 , throughout 200-300 iterations, further continuation of the search, as a rule, does not lead to a change in the best value of the objective function. It was assumed that, providing that the elite population is stable for 300 iterations, the solution of the optimization problem is complete.

4. *Mutation.* For individuals of population Φ_1 , where in each generation there exists the possibility of random change in part of the parameters in several individuals. The following mixed scheme for changing the position of parameter j in the discrete set T_j of its permissible values is used. Suppose that this set has w_j elements. On the interval (0; 1), with the help of a random number generator, values m_a, m_b are selected by applying the uniform distribution law to be compared with the control numbers of mutation f, f_1, f_2, f_3 . If $m_a > f$, then the number of the parameter value in set T_i shall be chosen

randomly with equal probability. When $m_a \leq f$, the choice shall be made in accordance with Table 1,

where r_j is the number of the parameter value in set T_i before mutation, \tilde{r} is the value by which this number changes as a result of mutation.

	\widetilde{r}				
r_j	$m_b < f_1$	$f_1 \le m_b < f_2$	$f_2 \le m_b < f_3$	$m_b \ge f_3$	
1	0	0	1	2	
2	-1	-1	1	2	
3	-2	-1	1	2	
4	-2	-1	1	2	
w _{j-2}	-2	-1	1	2	
w _{j-1}	-2	-1	1	1	
w _j	-2	-1	0	0	

Table 1. Values of additions to the parameter number at $m_a \leq f$

5. Selection and crossover. The selection operator is applied to all individuals from population Φ_1 by applying the roulette method [27, 28] on the basis of the obtained objective function values in accordance with the mixed scheme used in [22]. Crossover was performed using the single-point operator [27, 28].

Results

The use of a developed computational scheme is considered for examples of a bar structure with two supports and a frame with a girder truss. It was assumed that the optimized objects were made of S235 steel [24]. The following was taken into account: modulus of elasticity $E = 2.06 \cdot 10^5 \,\text{MPa}$, weight density $\rho = 77 \,\text{kN/m}^3$. The following was set out: $N_1 = N_2 = 20$, $r_o = 5$, $\xi_\sigma = 10$, $\xi_\delta = 100$, f = 0.9, $f_1 = 0.5$, $f_2 = 0.75$, $f_3 = 0.9$. Compliance with constraints on strength, stiffness, and stability were confirmed for the obtained optimization task solutions by applying the Autodesk NEi Nastran software package (License FGBOU VO "BGITU" N PR-05918596) and with regard to the existing standards [24].

Example 1 considered the bar structure with two supports consisting of six bars 1–6 of equal length (Fig. 3a) with an H-shaped cross-section (Fig. 3b) at $z \parallel Z$. Dimensions of each bar 1–3 cross-section of the symmetric system were varied independently. Permissible combinations of bar cross-sections are indicated in Table 2. Coordinates of the central node *C* also varied, provided that straightness of segments $D_{\rm I}C$ \bowtie $D_{\rm II}C$. remain unchanged. The following set of permissible values was specified for this coordinate: {-20, -17.5, -15, -12.5, -10, -7.5, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 7.5, 10, 12.5, 15, 17.5, 20} (cm). The uniform finite element mesh was introduced (see Fig. 3a). Independently, three cases of optimization were implemented with the following loading conditions: $P_1 = 2.5$ MN, $P_2 = P_3 = P_4 = 0$; 2) $P_1 = 2.5$ MN, $P_2 = P_3 = P_4 = 0.5$ MN \bowtie 3) $P_1 = 2.5$ MN, $P_2 = P_3 = 1.25$ MN, $P_4 = 0$. It was assumed that $[u]_i = [v]_i = 8$ cm, $R_y = 230$ MPa. Optimization results are shown in Figure 4, where k_{st} – safety factor of stability by the Euler method obtained through the use of Autodesk NEi Nastran software. These design parameters were achieved in no more than 60 generations.



Figure 3. Bar system with two supports: a – design diagram applying the basic geometry of the object with subdivision on finite elements: 1–6 – bar numbers, b – bar cross-section in local system of axes

Table 2. Permissible combinations of cross-section variable parameters for Example 1

Combination number	b_f , cm	t_f ,cm	h_w , cm	t_w, cm
1	16	1	15	0.6
2	20	1.5	18	0.8
3	25	2	20	1
4	28	2	25	1.2
5	30	2.2	30	1.2
6	34	2.2	35	1.4
7	38	2.4	40	1.4
8	42	2.6	45	1.6
9	46	2.8	50	1.6
10	50	3	55	1.8



Figure 4. Results of an optimal search for a bar system with two supports with numbers of bar cross-section combinations (see Table 2)

Example 2 demonstrates the object of optimization represented by a symmetrical plane frame with a girder truss, span $L = 36 \,\mathrm{m}$ (Fig. 5). In the basic configuration of the object. $H_1 = 10 \,\mathrm{m}$, $H_2 = 3.5 \,\mathrm{m}$, $H_3 = 1.75 \,\mathrm{m}$. The frame has rigid supports A, B and is fixed in the nodes of the upper chord of the truss from displacement in the direction of axis Z. The operational load of the weight of the structure and snow in the form of the system of forces P_1 , P_2 , wind pressure represented by distributed loads $q_1 - q_4$, and bars weight adjusted during the process of optimization according to the varied parameters, was taken into account. Values of loads were determined considering the structure location in Bryansk (Russian Federation). The following was set out: $P_1 = 108 \,\mathrm{kN}$, $P_2 = 54 \,\mathrm{kN}$, $q_1 = 1.38 \,\mathrm{kN/m}$, $q_2 = 4.42 \,\mathrm{kN/m}$, $q_3 = 1.035 \,\mathrm{kN/m}$, $q_4 = 3.31 \,\mathrm{kN/m}$, $[u]_i = 9 \,\mathrm{cm}$, $[v]_i = 12 \,\mathrm{cm}$, $R_y = 234 \,\mathrm{MPa}$.

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Figure 5. Frame basic configuration

All in all, 22 variables were varied independently: 15 combinations of cross-section dimensions and 7 coordinates. This being the case, provisions were made to ensure structure symmetry. In Figure 5, for the left half of the object, $g_1 - g_{15}$ groups of bars are shown with independently varied combinations of cross-section dimensions and nodes $U_1 - U_7$. with independently variable coordinates Y. It was supposed that the bars have an H-beam cross-section (see Fig. 3b), provided that $z \parallel Z$. For each group $g_1 - g_{15}$ of bars, the search was performed with reference to the combinations of dimensions indicated in Table 3. For Y coordinates of $U_1 - U_4$ nodes, a set of permissible values {700; 800; 900; 1000; 1050 1100} (cm) was taken into account, and for $U_5 - U_7$ nodes – the set {1150; 1175; 1200; 1225; 1250} (cm).

Number of combination	b_f ,cm	t_f , cm	h_w, cm	t_w, cm
1	10	1	8	0.6
2	20	1.2	18	0.6
3	25	1.6	23	0.8
4	30	1.8	28	0.8
5	35	1.8	33	0.8
6	40	2	38	1
7	43	2	40	1
8	45	2.2	43	1
9	50	2.4	48	1.2
10	55	2.4	52	1.4
11	60	2.6	58	1.6
12	70	2.8	68	1.8

Table 3. Permissible combinations of bar cross-sections in Example 2

In performing the structure discretization, each bar was divided into five finite elements. As a result of the optimal search, the solution was obtained and shown in Fig. 6 with the value of the objective function $W = 88.50 \,\mathrm{kN}$, where digits indicate the number of combinations of bar cross-section dimensions (see Table 3). In this case, 300 iterations of the outer cycle were required with less than $6 \cdot 10^4$ structure variant analysis. The optimization process took 37 hours of computer time by using an Intel Core i7 Processor. For this project, coefficient $k_{st} = 1.19$. From Figure 6, it is clear that during the course of optimization, for the most important bar cross-sections, an arch configuration was reproduced approximately.



Figure 6. Result of frame optimization

Discussion

It should be noted that in the presented algorithm used for the performance of optimal search iterations, an object stability assessment is provided for, including the stability of individual bars according to a spatial pattern. The introduction of a random self-balanced system of small forces can be considered as a practical means to reduce complexity of calculations when there is an opportunity to approximately reproduce buckling corresponding to the Euler mode of stability loss in the case where a normative load does not allow it to be done. Such conditional additional loading cannot lead to rejection of an efficient structure variant. At the same time, it is presumed that, in any case, the solution of an optimization task must be analyzed in detail, including by applying widely-distributed software for finite element analysis.

In the considered examples, the proposed procedure provided for obtaining the resulting framework variants and stability conditions both via the Euler method and in accordance with standard requirements [24]. The values of the Euler stability coefficient for such project solutions did not exceed 1.19. Since the search was performed using discrete sets of parameters, the stability coefficients are quite acceptable for practical purposes. In this case, in assessing the structure variant stability combined with the calculation of stress and strain of the bar system. This approach does not require intermediate search stages of multiple consideration of the generalized problem of eigenvalues or the performance of calculating the stress-strain structure in a geometrically nonlinear setting, and allows solving, with an acceptable level of requirements to the complexity of the calculations, complex optimization problems of bar systems.

Conclusion

1. A computational scheme is proposed which allows optimization of flat rod systems made of steel, on discrete sets of bar cross-section dimensions and geometrical parameters defining the shape of a structure.

2. An optimal search is performed using the genetic algorithm procedures earlier proposed by the authors for effectively solving complex tasks with a large number of constraints.

3. In the optimization process, verification of the structure variants for stresses, displacements, and overall stability is provided for. Assessment of compliance with requirements for stability is performed both for buckling in the structure plane and out of plane by applying an approximate approach providing for the implementation of 5 iterations of the object deformation analysis via the finite element method based on the tangent of the stiffness matrix. In this case, no multiple generalized eigenvalue problems require solving.

4. Examples of steel load-bearing structures were used to make it possible, due to the proposed approach, to find solutions for flat rod systems optimization. Verification of compliance with stability constraints based on the adjusted schemes showed that factors of safety for the obtained constructive solutions were within the interval [1.12, 1.19].

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Igor Serpik, +7(4832)64-88-00; iserpik@online.debryansk.ru

Anatoly Alekseytsev, +7(960)564-33-58; aalexw@mail.ru

Pavel Balabin, 7(919)196-53-92; pavelbalabin90@yandex.ru

Natalia Kurchenko, +7(920)602-32-40; inserpik@gmail.com

- Серпик И.Н., Алексейцев А.В. Оптимизация системы стальной плоской рамы и столбчатых фундаментов // Инженерно-строительный журнал. 2016. № 1(61). С. 14–24.
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Игорь Нафтольевич Серпик, +7(4832)64-88-00; эл. почта: iserpik@online.debryansk.ru

Анатолий Викторович Алексейцев, +7(960)564-33-58; эл. почта: aalexw@mail.ru

Павел Юрьевич Балабин, +7(919)196-53-92; эл. почта: pavelbalabin90@yandex.ru

Наталья Сергеевна Курченко, +7(920)602-32-40; эл. почта: inserpik@gmail.com

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