doi: 10.18720/MCE.76.23

# Dynamic stability of the lattice truss of the bridge taking into account local oscillations

# Динамическая устойчивость решетчатой фермы моста с учетом местных колебаний

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**Key words:** bar element; building structure; kinematic perturbation; parametric resonance; decomposition model; excitation coefficient; influence line

**Ключевые слова:** стержневой элемент; строительная конструкция; кинематическое возмущение; параметрический резонанс; декомпозиционная модель; коэффициент возбуждения; линия влияния

**Abstract.** The carrying capacity of the railway and the service life of artificial structures primarily depend on the operational category of the structure and the dynamic state: dynamic stability, the condition that dangerous vibrations do not appear, and the dangerous resonance of the amplitude of the oscillations. Studies on the dynamics of railway bridges have gained relevance in connection with the new construction and reconstruction of bridges of high-speed and high-speed railroads. When choosing the restoration measures for the reconstruction of existing railway lines or when designing and building new structures, taking into account the current high operational requirements, a thorough evaluation of the efficiency and reliability of the span structures is necessary, taking into account the type of construction and analysis of the dynamic impact. In the article the analysis of factors is produced influencing on the possible loss of dynamic stability of bars of the latticed truss under act of kinematics indignations of ends of bar at the general vibrations of flight structure caused by dynamic factors accompanying moving of the temporal loading on a bridge. A novelty is made by the account of mutually influencing general and local vibrations of flight structure at the estimation of dynamic stability of the cored latticed truss. The spectrum of parametric vibrations of bars of the latticed truss is investigational in the conditions of remoteness from the areas of dynamic instability. The method of decomposition of

Индейкин А.В., Чижов С.В., Шестакова Е.Б., Антонюк А.А., Евтюков С.А., Кулагин Н.И., Карпов В.В., Голицынский Д.М. Динамическая устойчивость решетчатой фермы моста с учетом местных колебаний // Инженерно-строительный журнал. 2017. № 8(76). С. 266–278.

decision of differential equalizations of vibrations is applied on the Bessel function with a whole icon. Practical limitation of spectrum of frequencies is got near-by the value of bearing frequency to equal frequency of free vibrations taking into account influence of central forces and also relatively small influence of parametric vibrations in areas remote from living parametric resonance. Taking into account the dynamic stability presented by the authors, it is possible to expand the possibilities of using the existing norms and update them for dynamic calculations of railway metal bridges with lattice trusses, as well as to take into account the main factors that influence the occurrence of additional dynamic influences.

**Аннотация.** Пропускная способность железной дороги и срок службы искусственных сооружений прежде всего зависят от эксплуатационной категории сооружения и динамического состояния: динамической устойчивости, условию не появления опасных вибраций и опасного резонанса амплитуды колебаний. Исследования по динамике железнодорожных мостов приобрели актуальность в связи с новым строительством и реконструкцией мостов скоростных и высокоскоростных железнодорожных магистралей. При выборе восстановительных мероприятий по реконструкции существующих железнодорожных линий или при проектировании и строительстве новых конструкций с учетом актуальных повышенных требований по эксплуатации необходима тщательная оценка работоспособности и надежности пролетных строений с учетом типа конструкции и анализе динамического воздействия. В статье производится анализ факторов, влияющих на возможную потерю динамической устойчивости стержней решетчатых ферм под воздействием кинематических возмущений концов стержня при общих вибрациях пролетного строения, вызванных динамическими факторами, сопровождающими перемещение временной нагрузки по мосту. Новизну составляет учет взаимовлияющих общих и местных вибраций пролетного строения при оценке динамической устойчивости стержневых решетчатых ферм. Исследован спектр параметрических колебаний стержней решетчатых ферм в условиях удаленности от областей динамической неустойчивости. Применен метод разложения решения дифференциальных уравнений колебаний по функциям Бесселя с целым значком. Получено практическое ограничение спектра частот вблизи значения несущей частоты, равной частоте свободных колебаний с учетом влияния продольных сил, а также относительно малое влияние параметрических колебаний в областях, удаленных от живого параметрического резонанса. Учет динамической устойчивости, представленный авторами, позволяет расширить возможности использования действующих норм и актуализировать их для динамических расчетов железнодорожных металлических мостов с решётчатыми фермами, а также учитывать основные факторы, влияющих на возникновения дополнительных динамических воздействий.

## *Introduction*

To ensure reliable and safe operation of the bridge structure throughout the life cycle, it is necessary to analyze and take into account many important factors, including dynamic stability.

In a historical aspect, it should be noted that the first work devoted to solving problems related to the dynamic stability of rods and rod systems subjected to longitudinal harmonic force is the work of N. Belyaev [9]. Since that time, the problem of studying the stability of elastic systems and related mathematical methods has attracted universal attention of scientists. Of the large number of scientists who worked and still work, it should be noted the work of A.V. Indeikin [1, 4], V.V. Bolotin [3], Ya.G. Panovko [2, 7], N.N. Moiseev [5], N.A. Alfutov [6], G. Ziegler [8], V.N. Chelomey [10] and many other authors.

In the literature, we mainly consider the power excitation of parametric oscillations of rods outside the connection with the general vibrations of the structure.

The interest in dynamic behaviour of railway different existing types including new ones bridges has increased in recent years, due to the introduction of high speed trains [11–17].

The main attention is paid to the complexes of measures to reduce the level of vibration of steel bridges, which subsequently ensures a reduction in costs for repair activities [18–22].

Under the loads of high speed, the bridges are subjected to large dynamic effects. Therefore, the demands on railway bridge structures are increased. The dynamic aspects have often shown to be the governing factor in the structural design. Generally, for all railway bridges induced by train speeds over 200 km/h, dynamic analysis is required. Correct understanding of Railway Bridge dynamic is essential, since a realistic prediction of the structural response contributes to an economic design of new bridges and to a rational exploitation of bridges in service. In railway bridge design, the dynamic effects are often considered by introducing dynamic amplification factors, specified in bridge design codes. Actually, the response of a railway bridge due to moving loads depends on span length, structure mass, stiffness and damping, train axle loads and speed. The dynamic factors are usually a function of the natural frequency or span length of the bridge, and states how many times the static effects have to be magnified in order to cover the additional dynamic loads. Another issue related to the dynamic of railway bridges, is the behaviour variations along the bridges, variations in the overall conditions, and in the materials. There exist a large number of studies, dealing with the dynamic moving load problem, by considering different bridge and vehicle models under different conditions. A more detailed list of previous investigations is given in works of professor Karoumi [23–28].

The dynamic effects for railway bridges are considered in Eurocode 1: Actions on structures – Part 2: Traffic loads on bridges, Dynamic effects (including resonance). For simple dynamic problems, only static analysis is required. The static analysis shall be carried out with the load models defined in Vertical loads – Characteristic values (static effects) and eccentricity and distribution of loading, and considered the load model LM71 and where required the load models SW/0 and SW/2. The results of the static analysis shall be multiplied by the dynamic factor Φ considered later on, and if required multiplied by a factor α in accordance with the load model LM71 [29, 30].

In this article kinematics excitation of vibrations of bars is examined as a result of the dynamic moving of nodes in composition the lattice truss. The method of decouplig is thus used at that the form of vibrations of the bar elements of the lattice truss is taken into account.

## *Methods*

### *Research of dynamic stability of elements at influence of the wave parametric load*

Structural design of the bar element a complementary model and conclusion of differential equalizations of vibration processes are presented on the Figure 1.

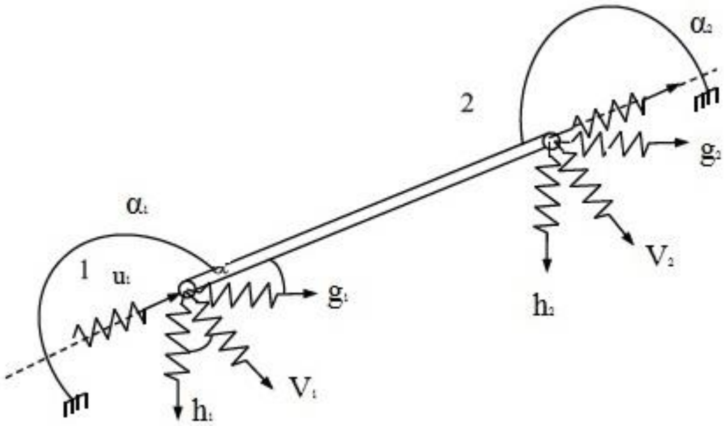


**Figure 1. Structural design [1]: $V_1$; $V_2$ – transverse components of the displacement of nodes 1 and 2, $g_1$; $g_2$ – horizontal components of the displacement of nodes 1 and 2, $u_1$; $u_2$ – longitudinal components of the displacement of nodes 1 and 2; $h_1$; $h_2$ – vertical components of the displacement of nodes 1 and 2, $\alpha_1$; $\alpha_2$ – spring stiffness characteristic**

In the case of a rod of a trellis truss loaded with a longitudinal force *S*(*t*) consisting of the static component $S_0$ and the dynamic component St, whose nodes make the transverse motion $v_1(t)$ and $v_2(t)$ in the first approximation, a differential equation describing the dynamic processes is obtained:

$$\ddot{q} + \Omega^2 \left[1 \pm 2\mu(t)\right] q = -\frac{2}{\pi}\left(\ddot{v}_1 + \ddot{v}_2\right), \tag{1}$$

where $\Omega^2 = \dfrac{EI\,\pi^4}{m\ell^4}\left(1 \pm \dfrac{S_0}{S_{\text{кр}}}\right)$, $2\mu(t) = \dfrac{S_t}{S_{\text{cr}} \pm S_0}$, a sign "+" behaves to the case of the stretched bar, sign of "-" to the case of the compressed bar;

$S_{cr} = \frac{\pi^2 EI}{\ell^2}$ – Euler's critical force;

$\ell$ – bar length;

*q* – generalized coordinate;

*m* – linear mass of bar.

Homogeneous equalization corresponding to equalization (1) on condition of $\mu(t) = \mu \cos\omega \cdot t$ is Mathieu equation and describes the parametric vibrations of the bar element of construction:

$$\ddot{q} + \Omega^2 [1 - 2\mu \cos\omega t] q = 0. \quad (2)$$

It is known [2] that at the value of frequency of excitation:

$$\omega_* = 2\Omega \sqrt{1 \pm \mu}, \quad (3)$$

there can be main parametric resonance.

Taking into account this condition amplitude of parametrik vibrations increases in time on an exponential law:

$$A = A_0 e^{\nu t}. \quad (4)$$

In the absence of resistance, the maximum value of the exponent is: $\nu_{max} = \frac{\mu \Omega}{2}$.

Taking into account all the above equations, it follows that:

$$A = A_0 e^{\frac{\mu \Omega t}{2}}. \quad (5)$$

If to take into account influence of viscid resistance Mathieu differential equation assumes a next form:

$$\ddot{q} + 2n\dot{q} + \Omega^2 [1 - 2\mu \cos\omega t] q = 0. \quad (6)$$

The values of the critical frequencies corresponding to the boundaries of the first (main) region of dynamic instability at parametric resonance:

$$\omega_* = 2\Omega \sqrt{1 \pm \sqrt{\mu^2 - \left(\frac{\Delta}{\pi}\right)^2}}, \quad (7)$$

where $\Delta$ – logarithmic decrement of local free vibrations of bar.

Value of exponential index of growth of amplitude in this case:

$$\nu_{max} = \frac{\mu\Omega}{2} - n = \frac{\mu\Omega}{2} - \frac{\Delta}{T} = \frac{\mu\Omega}{2} - \frac{\Delta\Omega}{2\pi} = \frac{\Omega}{2}\left(\mu - \frac{\Delta}{\pi}\right) = \frac{\Omega}{2}(\mu - \mu_*), \quad (8)$$

where $\mu_* = \frac{\Delta}{\pi}$ – the critical value of the excitation coefficient, when it exceeds the phenomenon of dynamic instability.

In case of periodic character of coefficient $\mu(t)$ presented by Fourier's series $\mu(t)=\sum_{k=1}^{\infty}\mu_k \cos k\omega t$ the critical frequencies corresponding to parametric resonances are given by [3]:

$$\omega_* = \frac{2\Omega}{k}\sqrt{1 \pm \mu_k}\ . \tag{9}$$

In case of polyharmonic excitation:

$$\mu(t)=\sum_{k=1}^{s}\mu_k \cos \omega_k t\ , \tag{10}$$

at that the values of $\omega_k$ are not multiple $k$.

$$\omega_{*k} = 2\Omega\sqrt{1 \pm \mu_k}\ . \tag{11}$$

In last In the latter case, the influence of higher forms of oscillations and combination parametric resonances is neglected.

At the stationary applying of the harmonic loading in the nodes of truss bar stress of oscillation of that examined on a decouple drawing determined by expression:

$$S_j(t)=\sum_{k=1}^{n}\alpha_{jk}\, P_k(\omega t)\, , \tag{12}$$

where $\alpha_{jk}$ – coefficients of influence taking into account influences in k-node of truss;

$S_j$ – force in the bar element;

$P_k$ – force impact profile.

Taking into account character of form of vibrations of truss (Figure 2) of value $\alpha_{jk}$ quasistatic is determined.

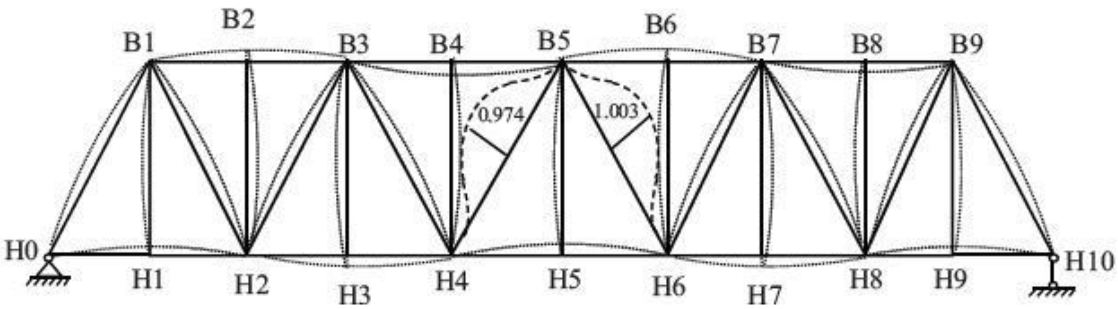


**Figure 2. Higher form (mode) of vibrations of truss [1]**

It is like possible to expect value $S_j(t)$ in case of the periodic key loading of general view presented through the Fourier series.

If node loads (forces) are applied at different nodes of the truss varying according to a harmonic law with different frequencies $\omega_k$, then the force in the j-th bar of the truss is described by a polyharmonic process:

$$S_j(t) = \sum_{k=1}^{n} \alpha_{jk} \, P_k(\omega_k t). \tag{13}$$

At excitation of parametric vibrations stationary forces attached in the nodes of truss and operating during the indefinite interval of time the only method of protection against vibrations there is an exception of possibility of origin of parametric vibrations by the increase of parameters of damping (coefficient of fading of n) with that inequality was provided:

$$\mu_* < \mu_k. \tag{14}$$

From a vibration can such methods of protecting become, for example, application of paint coat for the bars materials possessing the high degree of absorption of energy of vibrations in a superficial layer and also perfection of constructions of nodes – connection of the truss bars in the nodes on high-strength bolts instead of riveted joints.

In the case of the action of the mobile load, the force at the nodes of the truss is determined by the expression:

$$S(t) = P\,\Psi(t)\,\eta(vt), \tag{15}$$

where $\eta(vt)$ – ordinate of the force influence line in the truss bar from the amplitude value of the moving variable force *P*(*t*).

We expand the equation of the line of influence $\eta(vt)$ in a Fourier series with respect to $\sin npt$ continuing the function in an odd manner:

$$\eta(vt) = \sum_{n=1}^{\infty} h_n \sin npt, \tag{16}$$

where, $p = \frac{\pi v}{\ell}$;

$\ell$ – truss span;

$v = const$ – load speed.

The values of the Fourier coefficients:

- for a single-valued influence line

$$h_n = \frac{2\eta}{n^2 \pi^2 \alpha_0 (1-\alpha_0)} \sin \alpha_0 \, n\pi; \tag{17}$$

- for a two-digit influence line

$$h_n = \frac{2}{n^2 \pi^2 (\beta_0 - \alpha_0)} \left[ \left( \eta_1 \frac{\beta_0}{\alpha_0} + \eta_2 \right) \sin \alpha_0 \, n\pi - \left( \eta_1 + \eta_2 \frac{1-\alpha_0}{1-\beta_0} \right) \sin \beta_0 \, n\pi \right] \tag{18}$$

In equals (17) and (18): $\eta$, $\eta_1$, $\eta_2$ – absolute values of the ordinates of the vertices of the influence line, $\alpha_0 = \frac{\xi_1}{\ell}$, $\beta_0 = \frac{\xi_2}{\ell}$, $\xi_1$ and $\xi_2$ – abscissas of corresponding vertices.

When moving along the chord of force $P(t) = P(t)\cos\omega t$ the variable force in the truss bar is given by:

$$S(t) = P\cos\omega t \sum_{n=1}^{\infty} h_n \sin npt = \frac{P}{2}\sum_{n=1}^{\infty} h_n\left(\sin\omega_{1n}t - \sin\omega_{2n}t\right), \qquad (19)$$

where $\omega_{1n} = \omega + np = \omega + \frac{n\pi v}{\ell}$;

$\omega_{2n} = \omega - np = \omega - \frac{n\pi v}{\ell}$.

In this case the value of the coefficient $\mu(t)$ in equation (6):

$$\mu(t) = \frac{P}{4\left(S_{\kappa p} \pm S_0\right)}\sum_{n=1}^{\infty} h_n\left(\sin\omega_{1n}t - \sin\omega_{2n}t\right) = \sum_{n=1}^{\infty} \mu_n\left(\sin\omega_{1n}t - \sin\omega_{2n}t\right), \qquad (20)$$

where, $\mu_n = \frac{P}{4\left(S_{cr} \pm S_0\right)}$.

Frequencies $\omega_{1n}$ and $\omega_{2n}$ are modulated at carrier frequency $\omega$.

In the first approximation, the critical frequencies corresponding to single-frequency parametric resonances are given by:

$$\omega_* = 2\Omega\sqrt{1 \pm \mu_n} \mp \frac{n\pi v}{\ell}. \qquad (21)$$

The carrier frequencies are proportional to the speed of the load only if the source of the disturbance is the inertia forces of the unbalanced rotating masses associated with the object moving by the lattice truss (train).

I.e. $\omega = \frac{v}{R}$, where $R$ – radius of wheels.

In this case:

$$\omega_{(1,2)n} = \omega\left(1 \pm \frac{n\pi R}{\ell}\right), \qquad (22)$$

and

$$\omega_* = \frac{2\Omega}{1 \pm \frac{n\pi R}{\ell}}\sqrt{1 \pm \mu_n}. \qquad (23)$$

Usually $\frac{\pi R}{\ell} << 1$. For example, when the train is moving ($R$=0.525 m), the estimated span of the trusses $\ell$ =33…110 m value $\frac{\pi R}{\ell}$ is 0.005…0.015.

In this case, the regions of dynamic instability are concentrated around the region $\omega_* = 2\Omega\sqrt{1 \pm \mu_1}$ , width of these regions decreases with increasing n as the value of $\omega_* = 2\Omega$ increases, since the Fourier coefficients $h_n$ decrease substantially.

With the help of expression (23), it is possible to determine the critical speeds of load movement along the truss:

$$v_* = \omega_* R = \frac{2\Omega R\sqrt{1 \pm \mu_n}}{1 \pm \frac{n\pi R}{\ell}} . \tag{24}$$

Since the oscillations of the bar of the higher trusses are high-frequency, the critical speeds of the load motion along the railway bridge are realized only in the high-speed mode [4].

The increase in the time of the amplitudes of the resonance parametric vibrations occurs, without allowance for the resistance, according to the exponential law with the exponent $\nu = \frac{\mu\Omega}{2}$:

$$A = A_0 e^{\nu t} = A_0 e^{\frac{\mu_n \Omega t}{2}} . \tag{25}$$

Time of movement of load on the span of the truss is limited:

$$t = \frac{\ell}{v_{cr}} = \frac{\ell\left(1 \pm \frac{n\pi R}{\ell}\right)}{2\Omega R\sqrt{1 \pm \mu_n}} . \tag{26}$$

Consequently, the value of the exponent in this case:

$$\nu = \frac{\mu_n \ell}{4R} \frac{1 \pm \frac{n\pi R}{\ell}}{\sqrt{1 \pm \mu_n}} \approx \frac{\mu_n \ell}{4R} . \tag{27}$$

For a span structure with lattice trusses of a large railway bridge with parameter values $\mu_n$ =0.03, $\ell$=110 m, $R$=0.525 m values of the parameters $\nu$ is 1.7 and $\frac{A}{A_0}$ =4.806.

In this case, there is a significant increase in the vibration amplitudes even with a relatively small excitation coefficient $\mu_n$ .

It should be noted that the parametric resonances of the bar elements of trusses can be realized only in higher forms of vibrations (oscillations), since they are high-frequency.

In this case, there is no significant superposition of parametric and forced oscillations of the bars, since the critical frequencies are twice the vibration frequencies of the bar at which ordinary resonance can take place $(\omega = \Omega)$. Forced oscillations in this case occur in the supercritical region where the dynamic coefficient is less than unity (in the considered case it is 0.33).

When studying forced oscillations, one can neglect the effect of the variable frequency of free oscillations and use equation (1) with the value $\mu$=0. The values of the kinematic perturbations of the bar ends can be determined from an analysis of the general vibrations of the truss for the investigation of which it is necessary to apply known methods of structural dynamics or to use the corresponding computational complexes (for example, COSMOS/M).

In the domains of stable solutions we obtain the following asymptotic solution of the homogeneous equation (1) [5]:

$$q \approx \frac{1}{\sqrt[4]{1-2\mu(t)}} \left\{ A\cos\Omega \int_0^t \sqrt{1-2\mu(t)}\ dt + B\sin\Omega \int_0^t \sqrt{1-2\mu(t)}\ dt \right\}, \tag{28}$$

### *Investigation of oscillations of bars in regions of dynamic stability*

In the general case, the integrals in the right-hand side of equation (28) cannot be expressed in terms of elementary functions.

Taking this into account, we represent the equation $\sqrt{1-2\mu(t)}$ in the form of a uniformly convergent series:

$$\sqrt{1-2\mu(t)} = \sqrt{1-2\mu_0\ \Phi(t)} = 1-\mu_0\ \Phi(t) - \frac{\mu_0}{2}\ \Phi^2(t) - \ldots\ , \tag{29}$$

where, $\mu_0 = \frac{p}{2(S_{cr}-S_0)} << 1$ – small parameter.

Substituting equation (29) into (28), we obtain:

$$q \approx \frac{1}{\sqrt[4]{1-2\mu_0\ \Phi(t)}} \left\{ A\cos\Omega \left[ t - \mu_0 \int_0^t \Phi(t)\ dt - \frac{\mu_0^2}{2} \int_0^t \Phi^2(t)\ dt - \ldots \right] + \right.$$
$$\left. + B\sin\Omega \left[ t - \mu_0 \int_0^t \Phi(t)\ dt - \frac{\mu_0^2}{2} \int_0^t \Phi^2(t)\ dt - \ldots \right] \right\} \tag{30}$$

Integrals of the form $\int_0^t \Phi^n(t)\ dt$ in practically important cases are expressed in terms of elementary functions.

In the case where the function $\Phi(t)$ can be represented as a polyharmonic process $\Phi(t) = \sum_{n=1}^{\infty} b_k \sin\omega_k t$, then the solution of equation (30) will be represented in the complex form of writing.

$$q \approx \frac{u}{\sqrt[4]{1-2\mu_0\ \Phi(t)}} \exp\left\{ i \left[ \Omega t + \sum_{k=1}^{n} \xi_k \cos\omega_k t + \gamma \right] \right\}, \tag{31}$$

where $\xi_k = \frac{\mu_0\ \Omega\ b_k}{\omega_k}$.

Using relation $e^{i z \sin\theta} = \sum_{r=-\infty}^{\infty} J_r(z)\ e^{in\theta}$, where $J_r(z)$ Bessel functions with a whole icon we obtain the following equation:

$$q \approx \frac{u}{\sqrt[4]{1-2\mu_0\,\Phi(t)}} \sum_{r_1, r_2 \ldots = -\infty}^{\infty} J_{r_1}(\xi_1)\, J_{r_2}(\xi_2) \ldots J_{r_n}(\xi_n) \times \times \exp\left\{ i\left[ (\Omega - r_1\omega_1 - r_2\omega_2 - \ldots - r_n\omega_n)t + \gamma \right] \right\}. \quad (32)$$

The analysis of expression (31) indicates the presence in the total vibration of harmonic components with frequencies of the $\Omega + r_1\omega_1 + r_2\omega_2 + \ldots + r_n\omega_n$ and amplitudes proportional to the product of Bessel functions $J_{r_1}(\xi_1)\, J_{r_2}(\xi_2) \ldots J_{r_n}(\xi_n)$.

The oscillation spectrum is practically limited due to the properties of Bessel functions and because of negligible values provided that their argument is much smaller than the index. For example, under the $|\xi| << 1$ condition for a fixed index *r*, we get:

$$J_r(\xi) \approx \frac{1}{\Gamma(r+1)} \left( \frac{\xi}{2} \right)^r \approx \frac{1}{r!} \left( \frac{\xi}{2} \right)^2, \quad (33)$$

where $\Gamma(r+1)$ – gamma function of an integer argument with parameter values $\xi = 0.1$, $J_1(\xi) = 0.05$, $J_2(\xi) = 0.00125$.

At the same time, the value of the $J_0(\xi) = 1 - \left( \frac{\xi}{2} \right)^2$ function in this case is 0.9975, i.e. very close to unity.

Consequently, for small values of $\xi_k$ the frequency spectrum of the oscillations essentially consists of the fundamental frequency $\Omega$ and the frequencies $\Omega \pm \omega_k$ (in the case of polyharmonic excitation) and $\Omega \pm k\omega$ (in the case of periodic excitation of vibrations of the Fourier series).

The increase in the amplitudes of free oscillations of bar in modes far from parametric resonance is estimated using equation:

$$\frac{A}{A_0} = \frac{1}{\sqrt[4]{1-2\mu(t)}} = \frac{1}{\sqrt[4]{1-2\mu_0\,\Phi(t)}}. \quad (34)$$

With the value $2\mu_0\,\Phi(t) = 0.1$ this ratio is $\frac{A}{A_0} = 1.026$.

## *Results and Discussion*

Bridge structures for strength, stability and reliability must satisfy the conditions of uninterrupted and safe passage of trains with the maximum permissible axle loads and speeds depending on the class of tracks. The carrying capacity of the railway and the service life of artificial structures primarily depend on the operational category of the structure and the dynamic state (from dynamic stability, to the condition that dangerous vibrations do not appear and dangerous resonance of the amplitude of the oscillations).

The design of the span is made according to the conditions of strength, rigidity, dynamic stability with optimization of the design solution for the minimum cost of the entire life cycle. The tasks of optimizing the costs of maintaining the railway infrastructure require new approaches to managing reliability, risks, and the cost of the life cycle using the methodology for ensuring reliability, availability, maintainability and safety. The account of the dynamic stability of bridge structures is especially important at the initial stage of design development in the design, calculation and design, when the cost of making changes is minimal. This will make it possible to reduce the cost of the entire life cycle of bridge facilities, taking into account the repair work, while ensuring high reliability and the required level of safety for uninterrupted traffic.

There is a relationship between the degree of damage (wear, loss of bearing capacity) of structures and the dynamic state (deviation of the vibration indices from the normative values). Based on the application of the decomposition method for studying the vibrations of bar elements of latticed trusses of bridges, the possibility of a theoretical estimation of the growth indices of their amplitudes under the action of the dynamic load in the form of concentrated forces is investigated. These indices turn out to be high, even taking into account the small values of the excitation coefficients $\mu$ and the limited time for finding the force load on the structure $t = \frac{\ell}{v_{cr}}$. The nature of the interaction of forced and parametric oscillations of bar elements in resonance modes is also estimated. This factor has no significant effect on the system.

## Conclusions

The authors of the article have taken into account the mutual influencing general and local vibrations of the span structure in assessing the dynamic stability of lattice trusses. The spectrum of parametric oscillations of lattice truss rods under conditions of remoteness from the regions of dynamic instability is investigated. Practical limitation of the frequency spectrum near the value of the carrier frequency equal to the frequency of free oscillations taking into account the effect of longitudinal forces is obtained, as well as the relatively small influence of parametric oscillations in regions remote from living parametric resonance.

Conclusions and further research prospects:

1. The author has clearly demonstrated the effectiveness of the application of the decomposition method for solving similar problems.

2. A methodology for the theoretical estimation of indicators characterizing the increase in the amplitudes of parametric oscillations and the rod elements of lattice trusses of bridges under the action of a dynamic load in the form of concentrated forces is developed.

3. It was proved that there is no explicit interaction of forced and parametric oscillations of the bar elements of lattice truss bridges in resonance modes with each other.

4. Taking into account the dynamic stability analysis presented by the authors, it is recommended to update the current standards for railway metal bridges with latticed trusses.

5. Development of recommendations and technical solutions to increase the dynamic stability and service life of the bridge structures of railway bridges at high speeds of railway transport.

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Индейкин А.В., Чижов С.В., Шестакова Е.Б., Антонюк А.А., Евтюков С.А., Кулагин Н.И., Карпов В.В., Голицынский Д.М. Динамическая устойчивость решетчатой фермы моста с учетом местных колебаний // Инженерно-строительный журнал. 2017. № 8(76). С. 266–278.