doi: 10.18720/MCE.78.13

The temperature waves motion in hollow thick-walled cylinder

Распространение температурных волн в пустотелом толстостенном цилиндре

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Key words: hollow cylinder; thermal conductivity equation; finite-difference scheme; temperature wave; temperature profile; cylindrical symmetry

Ключевые слова: полый цилиндр; уравнение теплопроводности; конечно-разностная схема; температурная волна; профиль температуры; цилиндрическая симметрия

Abstract. A hollow cylinder with thick walls is one of the most complex objects to calculate the unsteady temperature field, so this field is the least studied. However, such objects are found in many modern constructions of systems of generation and distribution of heat. In the proposed article it deals with the study of propagation of temperature waves in the wall of the hollow cylinder after a sudden temperature change of the internal environment, fuss-causes upon termination of the movement or circulation of the heated stream. The algorithm of calculation of temperature fields numerically is shown using an explicit finite-difference scheme of high accuracy in conditions of cylindrical symmetry with boundary conditions of the first kind. The results of calculations of the penetration depth of the temperature wave according to the considered algorithm, depending on the time since the start of heat exposure and their comparison with the existing data for one-dimensional case are given for the implementation of the identification obtained mathematical model. Calculated radial profiles of relative temperature in the cylinder wall within the temperature waves in dimensionless coordinates and the analytical approximation relations for the description of these profiles are presented. The results are compared with the existing analytical solution for an unlimited array in rectangular coordinates and it is marked that the common results are found regardless of the material and geometry of the cylinder, as well as of temperatures of inner and outer environment. Presented dependences are invited to apply for the analytical evaluation of the minimum temperature on the inner surface of the heated cylindrical structures that will allow the use of engineering methods to verify compliance with industrial safety requirements.

Аннотация. Полый цилиндр с толстыми стенками является одним из наиболее сложных объектов для расчета нестационарного температурного поля, поэтому такое поле является наименее изученным. Вместе с тем подобные объекты встречаются во многих современных конструкциях систем генерации и распределения теплоты. В предлагаемой работе рассматривается исследование распространения температурной волны в стенке полого цилиндра при скачкообразном изменении температуры внутренней среды, возникающем при прекращении движения или циркуляции нагретого потока. Показан алгоритм расчета температурного поля численным методом с помощью явной конечно-разностной схемы повышенной точности в условиях цилиндрической симметрии при граничных условиях первого рода. Приведены результаты вычисления глубины проникновения температурной волны по рассмотренному алгоритму в зависимости от времени с момента начала теплового воздействия и их сопоставление с существующими данными для одномерного случая для осуществления идентификации получаемой математической модели. Представлены рассчитанные радиальные профили относительной температуры в стенке цилиндра в пределах температурной волны в безразмерных координатах и предложены аналитические аппроксимационные зависимости для описания данных профилей. Полученные результаты сопоставлены с имеющимся аналитическим решением для неограниченного массива в прямоугольных координатах и отмечена общность найденных результатов независимо от материала и геометрии цилиндра, а также температур внутренней и наружной среды. Представленные зависимости предложено применять для аналитической оценки минимальной температуры на внутренней поверхности цилиндрических

Самарин О.Д. Распространение температурных волн в пустотелом толстостенном цилиндре // Инженерно-строительный журнал. 2018. № 2(78). С. 161–168.

нагреваемых конструкций, что позволит использовать инженерные методы проверки выполнения требований промышленной безопасности.

1. Introduction

The proposed work has unsteady state temperature field of hollow thick-wall cylinder at a single heat exposure as target of research.

The task of research of unsteady state conduction in bodies of various geometric forms has been investigated for quite a long time. Most such investigations come from a solution of differential equation of heat transfer in solids, known as Fourier [1–2]. For the last time, due to development of computing tools, the center stage is taken by numerical methods of its solution obtaining, if required, approximation analytical dependences [2–3]. In most cases, though, they belong to one-dimensional case, or the consideration is performed in rectangular coordinates. This, though, corresponds to prevailing part of actually met tasks, and also not only in unsteady, but also in steady state [3]. In condition of cylindrical symmetry the most developed are the issues of heating and cooling of solid or thin-wall cylinders [1–2]. At the same time in some applications the calculation of unsteady state temperature field of hollow cylindrical structures with thick walls is of interest. But even in fundamental monograph [1] corresponding solution is not provided. For the last time there arise a set of publications, where such issues are considered both analytically and numerically. But the results obtained by the authors, as a rule, are extremely complicated for application in engineering practice [4, 5, 10, 15, 17, 18] or, otherwise, are too rough [12]. Others relate to specific options of structures, applied in limited fields and functioning in specific conditions, for example, under supercritical modes [9], in nuclear power industry [14] or composite materials production [6–8, 11, 13], and in the presence of phase transitions [16, 19], or fuel combustion processes [20].

Thus, the relevance of the proposed research lies in the need of finding sufficiently accurate and physically based, but at the same time acceptable for engineering use variations of the temperature of the heated cylindrical structures in emergency conditions. For example, it can be useful when resolving the issue about beginning of condensate formation at internal surfaces of smoke stacks when boiler unit is stopped or at outer surface of heat insulation of heat pipelines when heat supply is switched off. One can in general note that these tasks mainly bear a relation to safety, both industrial and connected with human livelihood. The obtained results can be applied in this case for quite wide range of power facilities of such structure.

The target of the work is the calculation of unsteady temperature field at discontinuous variation of environment temperature at internal surface of cylinder. The tasks of the research will be:

- development of algorithm realizing a finite difference scheme of resolving an equation of heat conduction in cylinder wall;
- obtaining of analytical dependences for a depth of penetration of temperature wave and temperature at internal surface of cylinder on the results of approximation of program generation results.

2. Methods

Figure 1 shows the scheme of cylinder under consideration and some conventional symbols. Main of them are R and r_0 – correspondingly outer and inner radius, m.

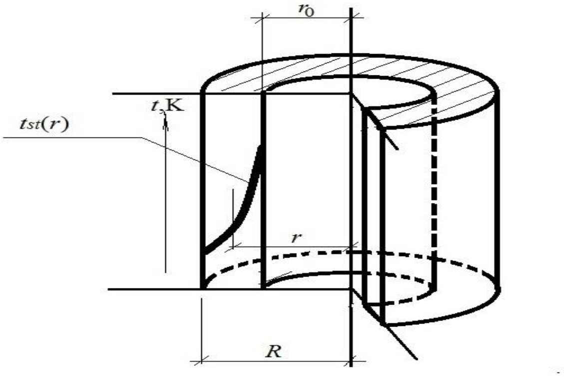

Figure 1. Scheme of hollow cylinder and steady state temperature field in its wall

In this case in cylinder wall a steady state temperature field $t_{st}(r)$ of logarithmic type is set. It can be calculated in a usual way [1, 2]. Its typical character is also shown at Figure 1. For convenience of further narration and to achieve maximum collectivity of the obtained results it is convenient to pass to a relative non-dimensional temperature θ. It can be defined according to the equation:

$$\theta = \frac{t - t_{ex}}{t_{in} - t_{ex}}, \tag{1}$$

where t_{ex} and t_{in} – correspondingly environment temperatures, K, from outer side of cylinder and inside its chamber. Then the differential equation of unsteady state conduction for cases of cylindrical symmetry can be written in the following way [1–2]:

$$\frac{\partial\theta}{\partial\tau} = a\left[\frac{\partial\theta}{r\partial r} + \frac{\partial^2\theta}{\partial r^2}\right], \tag{2}$$

where $a = \frac{\lambda}{c\rho}$ – coefficient of chamber wall material temperature conductivity, m^2/s; λ – its heat conductivity W/(m·K); c and ρ – specific heat capacity, J/(kg·K), and density, kg/m^3; t, s – time moment for which θ value is calculated; r – radial coordinate (see figure 1). Solution in this case will show current deviation of temperature field from initial steady state.

This equation in the simplest case can be tried to investigate at boundary conditions of first order, i.e. consider that θ = 1 at $r = r_0$ and θ = 0 at $r = R$. This just corresponds to the mode of non-continuous variation of heat current at switching off or, vice versa, switching on of heat supply. Then at a first approximation the actual value θ at internal surface of chamber can be calculated, if we input an additional cylindrical layer. Its thickness can be defined from conditions of internal heat exchanging, on account of equality of thermal resistance of such layer to actual resistance of heat exchange:

$$\delta_0^* = r_0\left(\exp\left(\frac{\lambda}{\alpha_0 r_0}\right) - 1\right) = r_0\left(\exp\left(\frac{1}{\mathrm{Bi}_0}\right) - 1\right), \tag{3}$$

where α_0 – coefficient of convective heat exchange to internal surface, W/(m^2·K);

$\mathrm{Bi}_0 = \frac{\alpha_0 r_0}{\lambda}$ – non-dimensional criterion Bio for this surface. It is easy to see that at big values Bi_0 thickness of additional layer with sufficient preciseness can be determined as λ/α, and for flat wall too. Then actual radius r_0, for which θ = 1 is fixed, is obtained from actual by reduction by value δ_0^*. The same is actual radius R of outer surface, where θ = 0 is maintained, will be increased by thickness δ^*. It can be calculated according to (3), but instead r one shall use R, and as α_0 a corresponding coefficient of outer heat exchange will be used.

To resolve the equation (2) one can use numerical methods. The main importance in this case is the preciseness of the obtained result, and volume of the used memory and amount of performed operations is not critical due to high computational resources of modern computing devices. So the explicit finite-difference scheme appears to be advantageous due to its simplicity for programming. Then temperature value in i-th mesh point in j+1-st time moment can be calculated according to equation:

$$t_{i,j+1} = Fo_\Delta\left(\frac{2i-1}{2i-2}t_{i+1,j} + \left(\frac{1}{Fo_\Delta} - 2\right)t_{i,j} + \frac{2i-3}{2i-2}t_{i-1,j}\right). \tag{4}$$

Here $t_{i,j}$, $t_{i-1,j}$ and $t_{i+1,j}$ – value of temperature in j-th moment of time in i-th mesh and neighboring on the left and on the right (mesh numeration from chamber axle in direction of outer surface);

$Fo_\Delta = \frac{a\Delta\tau}{(\Delta r)^2}$ – non-dimensional local criterion of Fourier, where Δτ, s, and Δr, m – correspondingly steps on time and coordinate, representing parameters of finite-differentiate scheme. As it is known [1],

[2], the adopted scheme is concurred at $Fo_\Delta \le 1/2$. So in calculations they selected the value $Fo_\Delta = 1/6$, providing improved accuracy of approximation – of 4-th order on space and 2-nd – on time coordinate.

3. Results and Discussion

For analysis and interpretation of results of calculations it is required first of all to note that from general theoretical considerations, including dimensional analysis method, it arises that the depth of penetration of temperature wave into the depth of material Δ, m, shall be increased with the flow of time in general as $\Delta = k\sqrt{a\tau}$ [1]. Of course, within the frames of the applied phenomenological model, providing the basis of derivation of differential equation of hat conductivity, specific level of coefficient k will always be an issue of some reasonable agreement. Really, as a result of supposition about infinitely big speed of spreading heat in a substance, the values θ appear to be different from zero at any r for each moment $t > 0$. That is why it is required to stipulate, what the calculated value θ will be equal to at the boundary of the area of penetration of temperature wave. If we accept for this case $\theta = 0.01$, series of sources gives value $k = 3.6$. In this case the approximation of results of numerical calculations allows with good accuracy to consider $k = 3.75$. Some deviation from known data can be explained, apparently, with the fact that in this case we investigate the case of cylindrical symmetry. Figure 2 shows the results of calculations according to scheme (4) for correlation $r_0/R = 0.7$ with the help of the program developed by author for computing device at algorithm language *Fortran*.

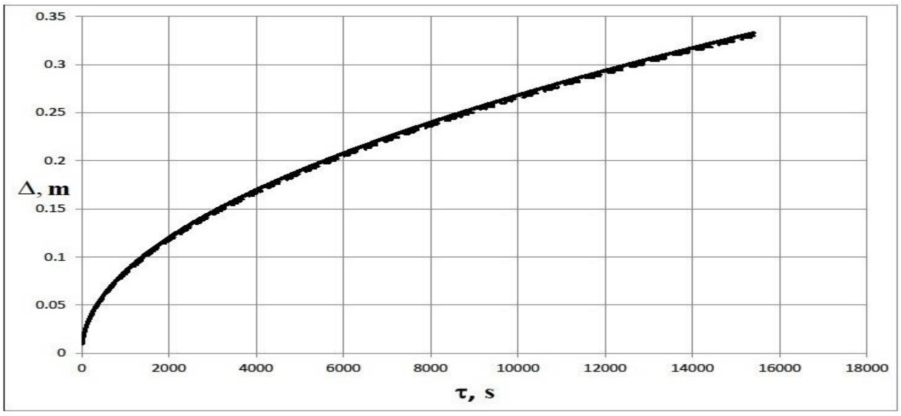

Figure 2. Dependence Δ from τ according to data of numerical calculation (dot line) and its approximation in the form of $\Delta = 3.75\sqrt{a\tau}$ for $r_0/R = 0.7$

The calculated non-dimensional profile of temperature on chamber profile web within the limits of the layer with thickness Δ at the value of Fourier criterion, related to the outer chamber radius $Fo_R = \frac{a\Delta\tau}{R^2} = 5\cdot 10^{-4}$, is shown at Figure 3 with dots. Here parameter δ is a difference $(r - r_0)$, i.e. distance along chamber radius from its internal surface. At other Fo_R of the same order the results differ only in the within the limits of thickness of approximating lines.

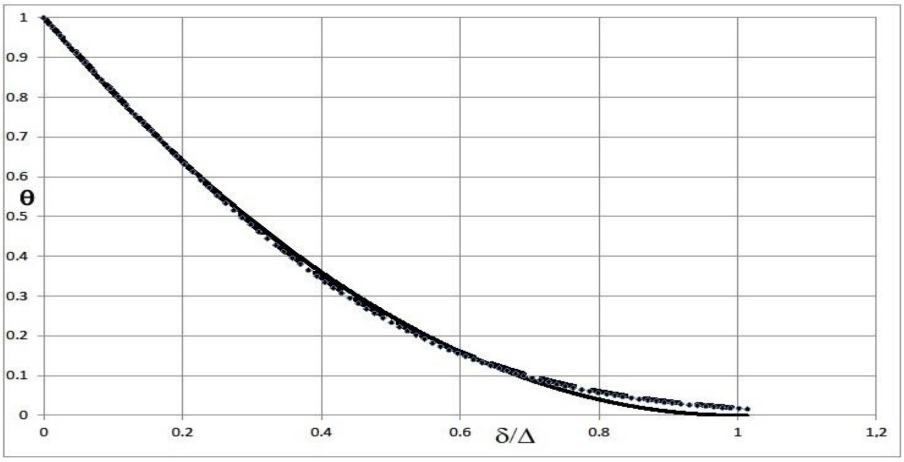

Figure 3. Non-dimensional temperature field in chamber wall at small Fo_R

The calculated curve is well approximated with the following dependency:

$$\theta=\left(1-\frac{\delta}{\Delta}\right)^2=\left(1-\frac{\delta}{k\sqrt{a\tau}}\right)^2=\left(1-\frac{1}{k\sqrt{Fo_\delta}}\right)^2, \tag{5}$$

Here $Fo_\delta=\frac{a\tau}{\delta^2}$ – Fourier criterion, related to the current distance δ. The plot of the obtained expression is shown at figure 3 with full line. It is easy to see that taking into account the easiness of this formulae and meaning an inadvertent error, which is entered when passing to a finite-differentiate scheme, this solution can be acknowledged as quite successful. The condition of its application is, naturally, performance of inequation $r_0+\delta<R$. It can be compared with theoretical solution for spreading of temperature wave in non-limited single-dimension array [1]:

$$\theta=1-\mathrm{erf}\left(\frac{\delta}{n\sqrt{a\tau}}\right)=1-\mathrm{erf}\left(\frac{1}{n\sqrt{Fo_\delta}}\right), \tag{6}$$

where erf – special function of errors, and numerical coefficient $n=2$. Calculations show that for approximation of results of program calculations the dependency (6) is also useful on condition that parameter n will be taken in the amount of $2.27=1+4/\pi$. Corresponding plot is provided at figure 3 with dot line. This coincidence additionally confirms the fairness of obtained solution. Like in the case of coefficient k when determining Δ, the deviation on n can be explained by cylindrical symmetry. Correlations (5)-(6) are written in non-dimensional form. So they are general, regardless actual values t_{ex} and t_{in}, of definite material of chamber walls and correlation r_0/R, at least, at $r_0/R>0.5$, for which numerical calculations were performed. These results are in principle agreed also with data which were previously obtained by author for another structure geometry [21–23], and, as it was previously noted, with theoretical considerations, provided in [1–2]. Besides, general view of the calculated temperature fields corresponds to results of some other authors, for example [16], and their found analytical description finds out analogies in theoretical solutions from other sources, in particular [14, 17].

To illustrate the practical use of the ratio (5) and its comparison with other solutions, we calculate the cooling of the brick chimney with the parameters $r_0=0.6$ m, $R=1$ m at an initial flue gas temperature $t_{in}=120$°C and the outside air temperature $t_{ex}=-25$ °C. In this case, the $r_0/R>0.5$ condition is met. For brickwork $\lambda=0.81$ W/(m·K), $c=880$ J/(kg·K), $\rho=1800$ kg/m^3, so $a=5.11\cdot10^{-5}$ m^2/s. Since we are primarily interested in the temperature on the inner surface of the chimney t_s in terms of estimating the time of condensation of water vapor, for the actual value of r_0, which is fixed $t_{in}=120$ °C, we accept, as it was mentioned above, the value of $\delta_0^*=r_0-\lambda/\alpha$. The factor α at a given radius of the tube and the speed of movement of gases about 12 m/s will be near 21 W/(m^2·K), then $\delta_0^*=0.039$ m, $r_0=0.561$ m and the t_s is still defined at $r=0.6$ m. In the calculations it was taken into account that in the considered mode the value θ, as it was already noted, shows the deviation of the current dimensionless temperature from the initial stationary distribution $t_{st}(r)$, therefore it was initially calculated the value of t_{st} on the inner surface of the chimney. In this case it is equal to 104 °C. In Figure 4 the dependence of the t_s from τ, constructed according to the formula (5), is shown by a solid line. The dotted line in figure 4 shows also the results of calculations using the analytical solution in the form of a series of data [24], and the dash – also on the analytical solution in the form of a series of [17]. As an example, we can cite a series of [24] with some changes corresponding to the peculiarities of the problem under consideration and the symbols accepted in this paper:

$$\theta=\frac{1}{4\ln(R/r_0)}\left(-0.5772+\ln\left(\frac{4a\tau}{r^2}\right)+\frac{r^2}{4a\tau}-\frac{r^4}{64a^2\tau^2}+\ldots\right), \tag{7}$$

where –0.5772 is the Euler's constant.

It can be noted that the coincidence of the curves in figure 4 is close enough both in quality and quantity. In all cases, the decrease of value t_s over time occurs practically by the same law, which in the considered time interval can be approximated by hyperbolic (5) or logarithmic [17, 24] dependence from τ. We can notice that, in general, the decrease of t_s looks somewhat sharper when calculating by (5) and smoother when calculating by analytical solutions in the form of series. However the difference lies within the accuracy of engineering calculation and the deviation lies within the accuracy of the engineering calculation, but the expression (5) is much easier in structure and is available for use in engineering practice.

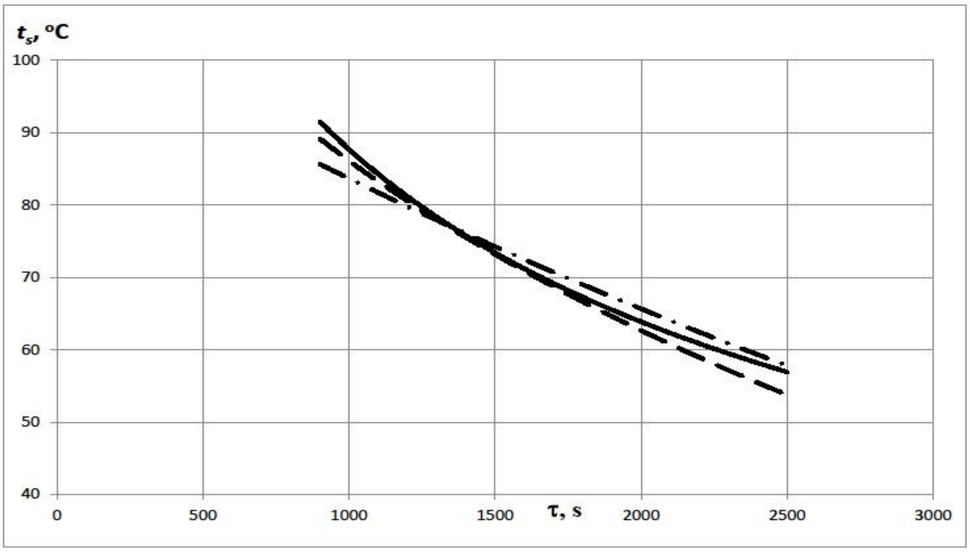

Figure 4. Dependence of the temperature on the inner surface of the cooling chimney from the value τ

4. Conclusion

1. It is noted that the speed of spreading of temperature wave in cylindrical wall complies to the same conformities that is in one-dimensional case, but with another proportional coefficient.

2. It was found that correlation of inner and outer radius of hollow chamber does not influence at the character of temperature field in the area of temperature wave, at least, at $r_0/R > 0.5$.

3. It is proved that the radial temperature profile in the wall of the hollow cylinder when $r_0/R > 0.5$ within the temperature wave propagation is described with good accuracy by quadratic dependence or with the use of an error function similar to a flat wall.

4. It is shown that the rate of temperature change over time at a fixed point in the cylinder wall within the temperature wave is sufficiently well approximated by hyperbolic dependence into which the expression for θ passes at r = const.

5. It is proved that the error in the use of formulas obtained in the paper in comparison with the results of numerical calculations and existing analytical solutions in the form of series lies within the accuracy of engineering calculations.

6. It is proposed to apply correlations obtained in the research for analytical estimation of minimal temperature at inner surface of cylindrical heated structures, primarily when resolving the issue about beginning of condensate formation at internal surfaces of smoke stacks when boiler unit is stopped or at outer surface of heat insulation of heat pipelines when heat supply is switched off, which will allow applying not only program, but also engineering methods of checking execution of industrial safety requirements.

7. It is noted that the ratio r_0/R, typical for modern designs of chimneys and thermal insulation of pipelines, allows to use the found formulas for θ when assessing the cooling time of their surfaces.

8. It is shown that the cooling rate of the inner surface of the smoke stack of a typical design, calculated in accordance with the found dependencies, before condensation of water vapor from flue gases of a standard composition is limited to tens of minutes which determine the available time for repair.

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