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Min-cut algorithm for network schedule by merging the vertices

Алгоритм поиска минимального разреза сетевой модели слиянием вершин

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Ключевые слова: календарное планирование и управление проектом; метод критического пути; оптимизация календарных графиков по срокам; минимальный разрез критической сети

Abstract. The task of reducing the total duration of the project ("compression", "crashing") occurs when developing and adjusting schedules. The activities requiring crashing are determined by the minimum of added resources at the unit reduction of the schedule. The minimum cut in a critical network determines such activities. The essence of searching the minimum cut by merging vertices (edge tightening) of the critical network is presented. The procedure of selecting vertices consists of the following. Possible merge options (tentative steps) are evaluated by equivalent cuts. The minimum cut will define priority vertex of a critical network. After this merging algorithm re-examines all the possible options of joining of adjacent vertices (on next step). The general applicability of the algorithm is demonstrated. Conditions of effective application of a method of the merge of vertexes are established. Search results of the minimum cuts of the critical network by means of this method are presented. Calculations have shown that in 14 of 15 examples the algorithm has established the global minimum cuts. Implementation of the proposed approaches will allow determining the activities to be optimized, calculate the size of reduction and the number of resources involved. The suggested methodology can be recommended for use by construction project managers.

Аннотация. Задача сокращения общей продолжительности проекта («сжатие») возникает при разработке и корректировке календарных графиков. Работы, требующие «сжатия», определяются минимумом дополнительно привлекаемых ресурсов при их единичном сокращении. Минимальный разрез критической сети определяет такие работы. Представлена сущность поиска минимального разреза путем слияния вершин (стягивания ребер) критической сети. Порядок выбора вершин состоит в следующем. Возможные варианты слияния (предварительные шаги) оцениваются эквивалентными разрезами. Минимальный разрез определяет приоритетную для слияния вершину критической сети. После этого алгоритм повторно исследует все возможные варианты присоединения смежных вершин (на следующем шаге). Показана общая применимость алгоритма. Установлены условия эффективного применения метода слияния вершин. Представлены результаты поиска минимальных разрезов критической сети с помощью данного метода. Расчеты показали, что в 14 из 15 примеров алгоритм установил глобальные минимальные разрезы. Реализация предложенных подходов позволит определить работы графика, подлежащие оптимизации, рассчитать размер сокращения и количество привлекаемых ресурсов. Предложенная методика может быть рекомендована для использования руководителями строительных проектов.

1. Introduction

A construction project is a complex process, which includes a large number of different tasks performed by different crews and displayed by calendar charts. When forming the schedules in case of

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exceeding the planned duration over deadlines requires a reduction in the total duration. In addition, with the operational management of the progress of work, it is also necessary to periodically adjust the schedule by dates.

Reduces the planned time schedule and decision-making (compression) involves primarily the intensification of activities critical path. It is obvious that critical activity that requires a minimum number of attracted resources for such compression must be reduced in the first place [1–6].

After the compression of the critical path by the minimum value of the total float [6], the schedule is recalculated with the repeated determination of all critical activities. It is obvious that in this case the chain of operations of the only critical path is transformed into a critical network, for optimization (compression) which can be used appropriate approaches and methods [1, 3, 6, 7].

The Ford-Fulkerson algorithm for maximum flow (minimum cut) and its variety [8, 9] is most often used to find such critical actions of the network model. The use of flow algorithms implies integer values of flows and multistage iterative procedures [10–19].

A simple and quite effective method of finding the minimum cut in critical networks is the methods based on the use of dual graph [20–23] and the method of branch-and-bound [24–26].

The dual graph is constructed as follows: on each face of the primary graph (a network model consisting of critical activities the vertex of the dual is placed. The vertices of the dual graph lying on the neighboring faces of the primary are connected so that each arc of the primary graph corresponds to one arc of the dual graph. The shortest path on the network of the dual graph defines the minimum cut of the primary graph (critical network) [23].

This approach is applicable only to planar networks. Although the vast majority of network schedules are planar, there are cases when network schedules that is not planar.

An interesting way to find the minimum cut in an undirected graph is the Stoer-Wagner algorithm and related approaches [27, 28].

The idea of the method is as follows. The most "connected" vertices are added sequentially to the randomly selected vertex of the graph, forming the first set. Then the local minimum cut separates either the last two vertices, or these vertices are in the second set, and the cut separates the two sets. After merging the last two vertices, the loop repeats until there are two vertices. The minimum among all the local sections and will be the desired global minimum cut.

It seems appropriate to consider the possibility of using a similar approach to find the minimum cut in a network model, which is a directed graph with one start and one end vertices.

The aim of the present paper is to substantiate the algorithm for finding the minimum cut in a directed graph by merging vertices.

Objectives of the study are:

1. Justification of general approach;
2. Substantiate the procedure for selection of vertices;
3. Determining the scope of effective application of the vertex merge method.

2. Methods

2.1. General approach

The general approach to finding the minimum cut of the network model by merging vertices (edge tightening) is quite simple [29]. Consider a critical network (a fragment of the network schedule in which all activities are critical) (Figure 1). Over arcs are represented the values of the resources involved to reduce the activity per unit time.

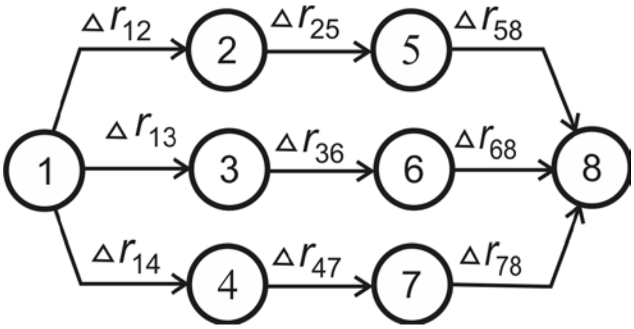

Figure 1. Fragment of a critical network

It is obvious that for any cut, the start and end vertices must be in different sets. First, a cut is defined between vertex 1 and adjacent vertices 2, 3, 4 (Figure 2), (1).

$$S_0 = \sum \Delta r_{1j}. \tag{1}$$

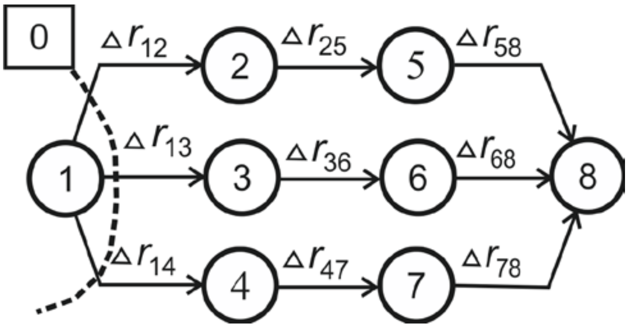

Figure 2. Initial cut

Then there is a sequential merge (Union) of vertex 1 with these neighboring vertices. The first step merges vertices 1 and 2 (Figure 3).

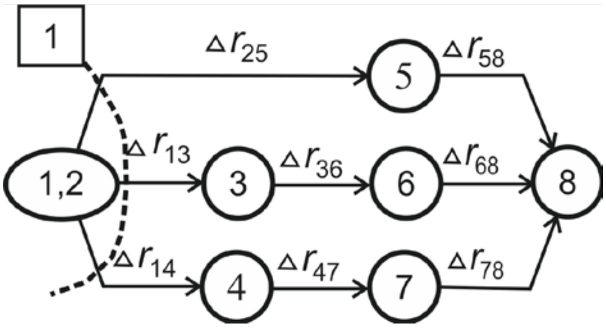

Figure 3. First step of the merger

In the second step, vertex 3 is added to the combined vertices 1 and 2 (combined vertex 1, 2), (Figure 4).

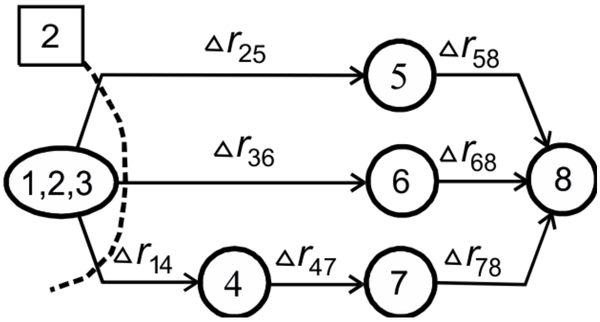

Figure 4. Second step of the merger

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At each m -th step is determined by the value of the corresponding cut (2).

$$S_m = \sum \Delta r_{ij}, \tag{2}$$

i – is the number of the event entering the merged vertex;

j – is the number of the event adjacent to the merged vertex.

In the third step, vertex 4 is added to the combined vertices 1, 2 and 3 (combined vertex 1, 2, 3), (Figure 5).

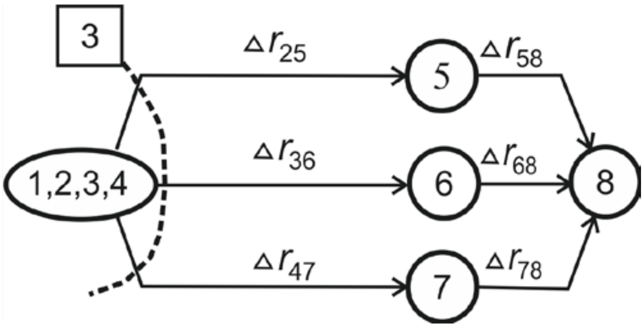

Figure 5. Third step of the merger

After a number of similar procedures, the network acquires the following form at step 6 (Figure 6).

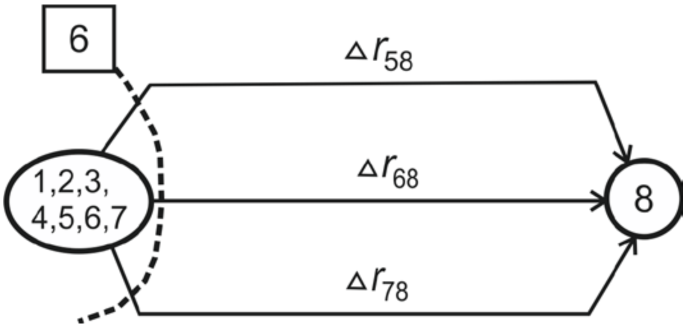

Figure 6. Sixth step of the merger

The minimum of all cuts at each step determines the desired activities for "compression".

$$S = \min S_m. \tag{3}$$

The calculations [29] showed that the correctness of determining the global minimum cut significantly depends on the selected sequence of merging vertices. Under these conditions, it is necessary to justify the order of selection of vertices to merge, allowing you to determine the global (or close to it local) minimum cut.

2.2. Procedure of selecting vertices

The procedure for selecting promising to join (merge) neighboring vertices of the network at each step is as follows. The author examines the possible merging of the vertices (tentative steps).

For each variant (tentative step), the corresponding cuts separating the combined vertex from the rest of the network are calculated. The minimum value of the corresponding cut determines the priority for merging vertex at this step.

After this merge (definite step) again considers all possible options of merging the neighboring vertices (the next step).

As an illustration of this procedure, the first step considers the following possible options for merging vertices (tentative steps) – (1,2); (1,3); (1,4), (Figures 7, 8, 9).

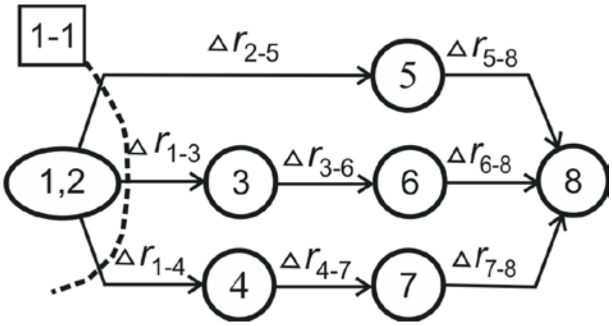

Figure 7. The variant of merging vertices 1 and 2 (step 1)

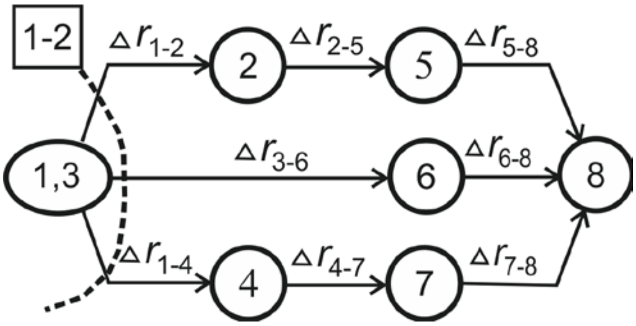

Figure 8. The variant of merging vertices 1 and 3 (step 1)

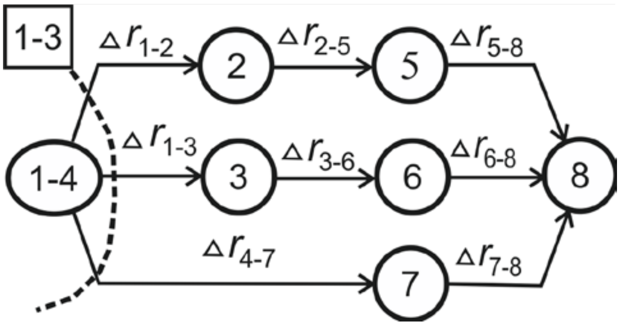

Figure 9. The variant of merging vertices 1 and 4 (step 1)

Let the minimum cut in the first step corresponding to the combined vertex (1,4), (Figure 9). This variant is a priority definite step. The value of the corresponding cut is remembered.

The second step sequentially considers the variants of merging the merged vertex (1-4) and of vertices 2, 3, 7.

The process repeats as long as is not merged by all the vertices except the end vertex (Figure 10).

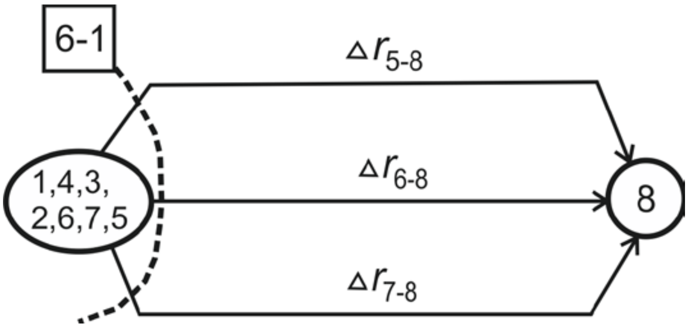

Figure 10. The variant of merging the vertices 1,4,3,2,6,7,5 (step 6)

Therefore, each step defined a local minimum cut. In total, taking into account the initial cut (at near the vertex 1), eight local minimum cuts were established (with the total number of considered cuts – 15). The minimum value of these local cuts will determine the required global minimum cut.

Table 1 presents the results of the implementation of the algorithm for finding minimal network sections (Figure 1) according to [29]. The parameters of the activities (the resource values involved to reduce activity by one unit time) are random numbers from 1 to 9.

Table 1. Results of determination cuts of the network when using a rule of the choice preferable neighbor vertex (the minimum cut of tentative step)

##	The parameters of the activities									Local minimum cuts	Global minimum
	Δr_{12}	Δr_{13}	Δr_{14}	Δr_{25}	Δr_{36}	Δr_{47}	Δr_{58}	Δr_{68}	Δr_{78}		
1	2	6	1	3	4	9	4	5	8	12-36-14 S=7	12-36-14 S=7
2	5	8	9	1	9	4	9	3	6	25-68-47 S=8	25-68-47 S=8
3	4	3	8	6	4	7	5	5	9	12-13-47 S=14	12-13-47 S=14
4	2	1	9	6	2	3	3	9	7	12-13-47 S=6	12-13-47 S=6
5	9	1	1	8	4	3	7	6	6	58-13-14 S=9	58-13-14 S=9
6	2	6	9	5	5	1	3	3	4	12-68-47 S=6	12-68-47 S=6
7	9	9	2	4	4	3	2	1	4	58-68-14 S=5	58-68-14 S=5
8	3	8	4	5	4	3	6	5	1	12-36-78 S=8	12-36-78 S=8
9	5	1	5	6	8	2	2	6	4	58-13-47 S=5	58-13-47 S=5
10	6	4	1	7	2	3	5	8	4	58-36-14 S=8	58-36-14 S=8
11	1	8	9	9	4	2	8	1	5	12-68-47 S=4	12-68-47 S=4
12	9	1	3	1	8	4	6	7	9	25-13-14 S=5	25-13-14 S=5
13	7	5	4	3	7	3	2	3	5	58-68-47 S=8	58-68-47 S=8
14	4	9	2	1	6	3	3	8	7	25-36-14 S=9	25-36-14 S=9
15	7	7	1	9	5	8	4	6	3	58-68-14 S=11	58-36-14 S=10

An analysis of the data presented in table 2 shows that, in 14 of 15 cases, global lows have been identified. In the last example, the algorithm set only the local minimum.

The presented example does not guarantee that the stated algorithm determines the global minimum cut. In this regard, it is necessary to justify the conditions for the effective application of the method of merging vertices (conditions under which the minimum cut is found to be global).

2.3. The scopes of effective application of the vertex merge method

All activities of any critical network can be divided as follows.

Activities of stage 1, for which the merged vertex is the initial event: 1-2; 1-3; 1-4 (A_1, B_1, C_1), (Figure 11).

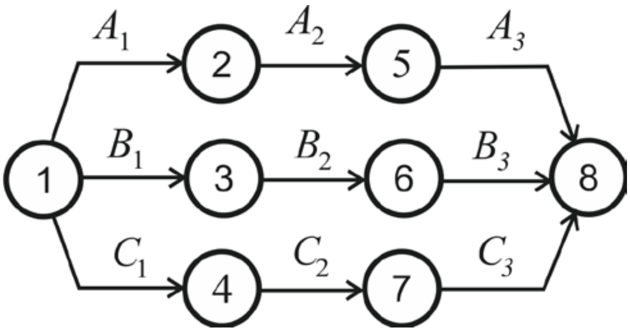

Figure 11. Fragment of a critical network

Activities of the 2 stage-initial events are the final events of the first stage: 2-5; 3-6; 4-7 (A_2, B_2, C_2).

Activities of the 3 stage-initial events are the final events of the second stage: 5-8; 6-8; 7-8 (A_3, B_3, C_3).

Activities of the i-th stage, for which the initial events are the final work events (i – 1) of the stage.

The proposed algorithm is guaranteed to determine the global minimum cut, passing through the activities of one or two adjacent stages.

If the global minimum cut passes through three stages of a critical network, the following variants are possible (Figure 12).

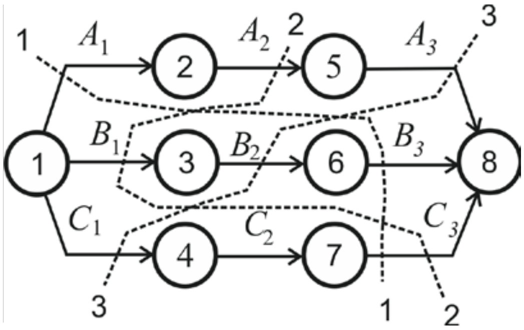

Figure 12. Variants of cuts through the activities of three stages

When two-way merging vertices (left and right), the algorithm detects all global minimum cuts of type 1-1, most of the cuts of type 2-2 and 3-3.

Studies have shown that the probability of finding a global minimum cut in the presented critical network (Fig. 11) is greater than 0.95. This is consistent with the data of table 1.

The practice of optimization of schedules shows that the compressible activities is not substantially spaced in time and are typically located in the progress line area. Therefore, the presented algorithm, which determines the global minimum cut, passing through 1-2-3 adjacent stages, is quite effective.

3. Results and Discussion

The method presented was used to optimize the network graph (Figure 13), [12, 23].

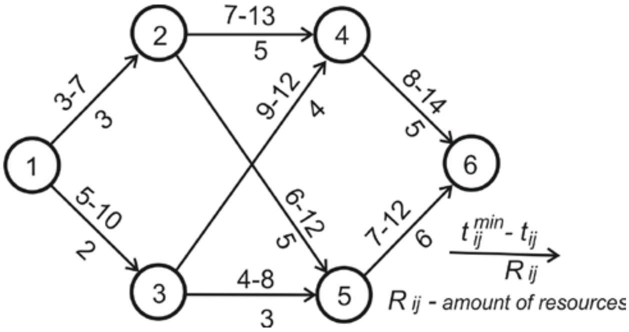

Figure 13. The Initial network schedule

Compression of the critical path from 36 to 22 days was carried out for 14 iterations.

After 13 iterations, the network graph was as follows (Figure 14). The total duration was 23 days. All paths of this graph are critical. Activities 3-4 and 4-6 are reduced to a minimum. Further, their compression is impossible.

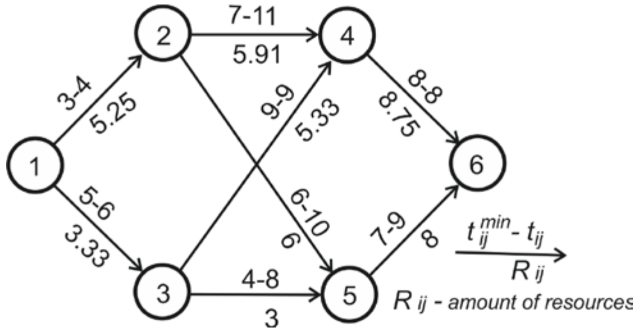

Figure 14. The network schedule after 13th iteration

The next iteration defines the activities that allow you to compress the schedule for another day with a minimum increase in resources (find the minimum cut off the critical network).

The critical network on 14th iteration looked as follows (Figure 15). The values of increase of resources for reduction of the corresponding work for one day are presented over the activities of the critical network.

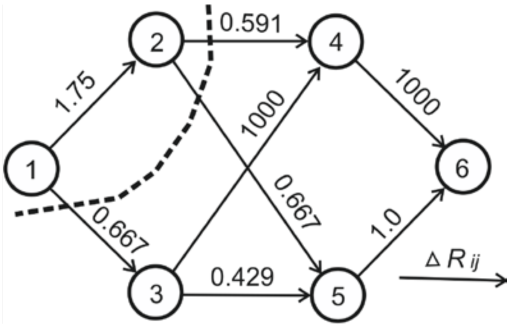

Figure 15. The critical network on the 14th iteration

After compression of activities 2-4, 2-5, 1-3 on the unit (one day) the optimum schedule was received (Figure 16).

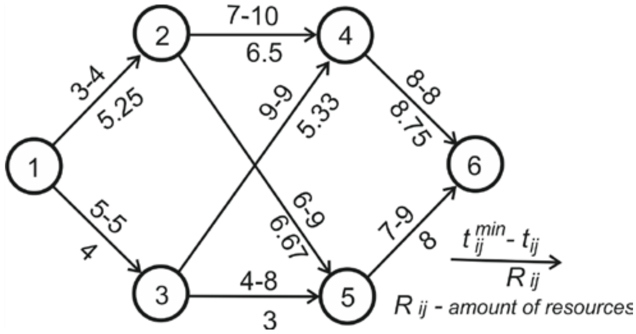

Figure 16. The optimal network schedule

The schedule provides a required duration (22 days). At the same time, the number of additional attracted resources is minimal (14.5).

The method presented in this study used Microsoft Excel [30].

4. Conclusions

1. When forming the schedules in case of exceeding the planned duration over deadlines requires a reduction in the total duration. Reduces the planned time schedule and decision-making (compression) involves primarily the intensification of activities critical path (critical network). Critical activities that require a minimum number of attracted resources for such compression must be reduced in the first place. These critical network operations are determined by the minimum cut.

There are a number of methods for finding the minimum cut in the network.

2. The essence of searching the minimum cut by merging vertices (edge tightening) of the critical network is presented.

The procedure of selecting vertices consists in the following. Possible merge options (tentative steps) are evaluated by equivalent cuts. The minimum cut will define priority vertex of a critical network. After this merge (definite step) again considers all possible options of merging the neighboring vertices (the next step).

Results of determination minimum cuts of the 15 networks when using a rule of the choice preferable neighbor vertex (the minimum cut of tentative step) are presented. They show that, in 14 of 15 cases, global lows have been identified.

3. Conditions of effective application of a method of the merge of vertexes are established. In networks activities of 1, 2, 3 stage is revealed. The proposed algorithm is guaranteed to determine the global minimum cut, passing through the activities of one or two adjacent stages. The probability of finding a global minimum cut in any critical networks is very high.

4. The method presented was used to optimize the network schedule. Compression of the critical path from 36 to 22 days was carried out for 14 iterations. As a result, the optimal schedule was obtained. At the same time, the number of additional attracted resources is minimal.

The method presented in this study used Microsoft Excel.

The suggested method can be recommended for use by construction project managers in order to prevent a potential failure of project completion deadlines.

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