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The nonlinear stress-strain state of the concrete-filled steel tube structures

Нелинейное напряженно-деформированное состояние трубобетонных конструкций

V.A. Snigireva*,
G.L. Gorynin,
Surgut State University, Surgut, Russia

Преподаватель В.А. Снигирева*,
д-р физ.-мат. наук, заведующий кафедрой Г.Л. Горынин,
Сургутский государственный университет, Сургут, Россия

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Ключевые слова: трубобетонные конструкции; нелинейная диаграмма деформирования бетона; бетонная стойка в стальной обойме

Abstract. The article is devoted to the mathematical modeling of the nonlinear stress-strain state of the concrete core in the concrete-filled steel tube structures. The paper uses a nonlinear diagram of concrete deformation under compression, which consists of two straight sections. In addition, the increase in the Poisson's ratio with increasing longitudinal deformations is taken into account. The article discusses two types of concrete-filled steel tube structures: the traditional concrete-filled steel tube column and the concrete column in steel tube cage. It is established that when the traditional concrete-filled steel tube column is loaded, the steel tube break contact with concrete core. This is the latent defect in such structures. The concrete column in steel tube cage does not have this defect. Concrete is acted to the three-dimensional compression here. It is concluded that taking into account the nonlinear behavior of concrete leads to the increasing of the calculated load-bearing capacity of the concrete column in the steel tube cage. In addition, change in the Poisson's ratio of concrete leads to the increasing of the load-bearing capacity too.

Аннотация. Статья посвящена математическому моделированию нелинейного напряженно-деформированного состояния бетонного ядра трубобетонных конструкций. В работе используется нелинейная диаграмма деформирования бетона при сжатии, которая состоит из двух прямолинейных участков. Кроме того, учитывается эффект возрастания величины коэффициента Пуассона при росте продольных деформаций. В статье рассматриваются два типа конструкций трубобетонных стоек: традиционная трубобетонная стойка и бетонная стойка в стальной обойме. Установлено, что при загружении традиционной стойки происходит отрыв стальной трубы от бетонного ядра, что является скрытым дефектом таких конструкций. Бетонная стойка в стальной обойме лишена этого недостатка, бетон в составе таких конструкций находится в состоянии трехосного сжатия. Сделан вывод, что учет нелинейного поведения бетона приводит к существенному увеличению расчетной несущей способности бетонной стойки в стальной обойме. Кроме того, учет эффекта изменения коэффициента Пуассона также приводит к дополнительному увеличению несущей способности стойки.

1. Introduction

The concrete-filled steel tube structures are composite structures, consisting of tube (often steel) and concrete filling, they are widely used as piles, columns or elements of trusses [1]. The stress-strain state of concrete-filled steel tube structures is complicated, since concrete and tube interact with each other in all directions. One of the variants of the cross section of concrete-filled steel tube structures is rectangular section [2], but the most rational form of the cross-section of such structures is the circle [3]. The object of our research is the stress-strain state of the concrete-filled steel tube column with circular cross-section.

According to many authors, the main advantage of concrete-filled steel tube structures is an increase of the load-bearing capacity of concrete. This fact is due to the reactive lateral pressure from the concrete to the tube side [4, 5]. The active use of concrete-filled steel tube structures is restricted due to the lack of a universal methodology for their calculation. Analytical calculation of such structures in accordance with the existing European standard EN1994-1-1 is based on empirical determination of the coefficients, these recommendations and formulas have a limited scope and can not be taken into account for the whole variety of building materials and structures [6]. The problem of calculating concrete-filled steel tube structures with a metal tube has been widely discussed since the first half of the 20th century. There are another point of view concerning the stress-strain state of concrete-filled steel tube columns under load presented by N. Skvortsov in 1953 [7]. In this work N. Skvortsov questions the availability of concrete compression by the steel tube. His argument is that the Poisson's ratio of concrete can never be greater than the Poisson's ratio of steel, and therefore the transverse deformation of concrete is always smaller than the transverse deformations of the steel tube, so concrete does not get compression by the steel tube. N. Skvortsov explains that the carrying capacity of compressible element, as observed from experiments, is increased due to the influence of frictional forces arising over the contact surface of the support plates of testing machines and the ends of the test specimens. Indeed, according to experimental studies of long samples, the steel tube is separated from the concrete core. The same effect was observed with the exploitation of the concrete-filled steel tube bridge across the Iset river [8]. The arguments advanced by N. Skvortsov were not supported by the majority of authors and in 1991, the book of L. Storozhenko "Calculation of concrete-filled steel tube structures" was published. The book describes an attempt to consider the problem of compression of a concrete-filled steel tube column as a spatial problem in the theory of elasticity [9]. However, in our opinion, the author did not take into account the fact that the normal longitudinal stresses in the concrete-filled steel tube element are negative (on page 49). As a result of that he made an incorrect conclusion that the difference in the Poisson's coefficients of concrete and steel "irrespective of the sign increases the rigidity of the composite bar brings the concrete core closer to the conditions of all-round compression". Many of the leading researchers refer to the works of L. Storozhenko [8, 10], but due to the presence of the error this work does not help clarify the question of the spatial work of the concrete-filled steel tube structures.

Thus, the topic of our research is actual, since the existing methods do not take into account the thickness of the tube wall and the forces of interaction between concrete and steel.

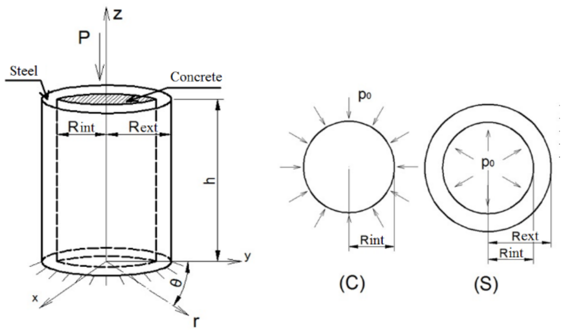
In connection with relevance of the above-described problem, the purpose of our research is to create the calculating method for the spatial stress-strain state of concrete-filled steel tube structures. To achieve the research purpose, it is necessary to fulfill the objectives: to derive formulas for simulating the spatial stress-strain state of concrete, taking into account the nonlinearity of the deformation; to analyze the interaction of concrete and steel tube in the structure; to suggest an improvement in concrete-filled steel tube structures; to derive formulas for the simulating the spatial stress-strain state of the proposed structure and analyze it.

Our research motivation is the desire to contribute to the effective application of the hidden advantages of concrete-filled steel tube structures, especially as they are widely used in construction.

2. Methods

2.1. Modelling of the stress-strain state of concrete in the nonlinear case

Modelling of the stress-strain state of concrete in the general case is a complex and unsolved problem because concrete has non-linearity of deformation [11]. In addition, the complex spatial stress-strain state is characteristic for concrete, which is part of concrete-filled steel tube structures due to interaction with the steel tube. We suppose that the concrete-filled steel tube column is under the action of the axial compressive force P. It is assumed that the stress-strain state of the structure has axial symmetry and the longitudinal displacements *W* depend only on the coordinate *z*, and the radial displacements *U* - on *r*, where "*z, θ, r*" is the cylindrical coordinate system (Figure 1). In this case, normal stresses $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}$, arise, and tangential stresses do not arise.



A) B) C)

Figure 1. The design model; A) a concrete-filled steel tube column; B) the concrete core cross section of the column; C) the steel tube cross section of the column

To take into account the non-linearity of concrete deformation as state diagram determining the relationship between stresses and deformations, we use the three-line diagram (Figure 2) describing the uniaxial stress-strain state, according to the Russian Construction Norms and Regulations 63.13330.2012 “Concrete and reinforced concrete structures”. There are no recommendations for taking into account the spatial stress-strain state of concrete in the Construction Norms and Regulations. We consider the proposed three-line diagram of uniaxial compression of concrete (Figure 2): when $\sigma < \sigma^* = 0.6R_b : \sigma = E\varepsilon$ (section 0-1); when $0.6R_b \le \sigma < R_b : \sigma = \sigma^* + \Delta\sigma$ (section 1-2), $\Delta\sigma$ – stress difference, R_b – design resistance of concrete under uniaxial compression. Many authors suggest a design model that takes into account the non-linearity of concrete deformation but the methods vary, because they are based on empirical dependencies [12].

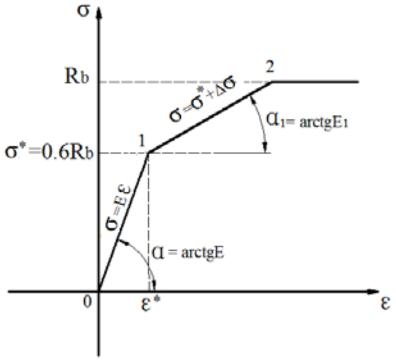


Figure 2. Three-line diagram of concrete deformation in uniaxial compression (Stresses and deformations are used without taking into account the sign)

Let us make natural assumption that the stresses in the radial and tangential directions of the concrete core of the concrete-filled steel tube column are much less than the axial stresses. Therefore, uniaxial stress-strain states, when radial and tangential stresses act separately, always correspond to section 0-1 of the diagram $(\sigma < 0.6R_b)$ (Figure2). But in the direction of the z-axis, two cases of the uniaxial stress-strain state are possible. Case 1: $\sigma_{zz} < 0.6R_b$ (section 0-1); or case 2: $0.6R_b \le \sigma_{zz} < R_b$ (section 1-2). At the initial stage (case 1), all components of the stress tensor $\sigma_{rr}, \sigma_{\theta\theta}$ σ_{zz} linearly depend on deformations, i.e. Hooke's law is valid (Figure 2):

$$\begin{cases} \sigma_{rr} = \dfrac{E}{(1-2\nu)(1+\nu)}\left((1-\nu)\cdot\varepsilon_{rr} + \nu\varepsilon_{zz} + \nu\varepsilon_{\theta\theta}\right); \\ \sigma_{zz} = \dfrac{E}{(1-2\nu)(1+\nu)}\left((1-\nu)\cdot\varepsilon_{zz} + \nu\varepsilon_{rr} + \nu\varepsilon_{\theta\theta}\right); \\ \sigma_{\theta\theta} = \dfrac{E}{(1-2\nu)(1+\nu)}\left((1-\nu)\cdot\varepsilon_{\theta\theta} + \nu\varepsilon_{rr} + \nu\varepsilon_{zz}\right), \end{cases} \tag{1}$$

where E, ν – the Young's modulus and the Poisson's ratio of concrete.

Let us consider case 2. In this state, the stresses and deformations in concrete are determined by the dependences:

$$\begin{gathered} \sigma_{rr} = \sigma_{rr}^* + \Delta\sigma_{rr};\ \sigma_{\theta\theta} = \sigma_{\theta\theta}^* + \Delta\sigma_{\theta\theta};\ \sigma_{zz} = \sigma_{zz}^* + \Delta\sigma_{zz} \\ \varepsilon_{rr} = \varepsilon_{rr}^* + \Delta\varepsilon_{rr};\ \varepsilon_{\theta\theta} = \varepsilon_{\theta\theta}^* + \Delta\varepsilon_{\theta\theta};\ \varepsilon_{zz} = \varepsilon_{zz}^* + \Delta\varepsilon_{zz}, \end{gathered} \tag{2}$$

where $\sigma_{rr}^*, \sigma_{\theta\theta}^*, \sigma_{zz}^*, \varepsilon_{rr}^*, \varepsilon_{\theta\theta}^*, \varepsilon_{zz}^*$ – values of stresses and deformations, when $\sigma_{zz} = 0.6R_b$; $\Delta\sigma_{rr}, \Delta\sigma_{\theta\theta}, \Delta\sigma_{zz}, \Delta\varepsilon_{rr}, \Delta\varepsilon_{\theta\theta}, \Delta\varepsilon_{zz}$ – increments of stresses and deformations respectively.

We set that the deformation increments caused by the simultaneous action of radial, tangential and axial stress increments are the sum of the deformation increments caused by the action of these stress increments separately. Thus the generalized relations of the deformation increments have the form:

$$\begin{cases} \Delta\varepsilon_{rr} = \dfrac{\Delta\sigma_{rr}}{E} - \nu\dfrac{\Delta\sigma_{\theta\theta}}{E} - \nu_1\dfrac{\Delta\sigma_{zz}}{E_1}; \\ \Delta\varepsilon_{\theta\theta} = -\nu\dfrac{\Delta\sigma_{rr}}{E} + \dfrac{\Delta\sigma_{\theta\theta}}{E} - \nu_1\dfrac{\Delta\sigma_{zz}}{E_1}; \\ \Delta\varepsilon_{zz} = -\nu_2\dfrac{\Delta\sigma_{rr}}{E} - \nu_2\dfrac{\Delta\sigma_{\theta\theta}}{E} + \dfrac{\Delta\sigma_{zz}}{E_1}, \end{cases} \quad (3)$$

where E,ν and E_1,ν_1 – the Young's modulus and the Poisson's ratio of concrete, when the stress values correspond to sections 0-1 or 0-2 of the diagram, respectively.

Relations (3) are analogous to the Hooke's law for an orthotropic elastic body, the symmetry of the matrices of elastic constants implies the fulfillment of the relation :

$$\frac{\nu_1}{E_1} = \frac{\nu_2}{E} \quad (4)$$

Express the stress increments through the deformation increments from formulas (3), (4). Then equalities (2) and (5) are the spatial law of the stress-strain state of the concrete core of the concrete-filled steel tube column when $\sigma_{zz} \ge 0.6R_b$ (case 2).

$$\begin{cases} \Delta\sigma_{rr} = \Delta\varepsilon_{rr}\left(\dfrac{E}{1-\nu^2} + \dfrac{E^2\nu_1^2}{(1-\nu)(E_1(1-\nu)-2\nu_1^2 E)}\right) + \Delta\varepsilon_{\theta\theta}\left(\dfrac{E\nu}{1-\nu^2} + \dfrac{E^2\nu_1^2}{(1-\nu)(E_1(1-\nu)-2\nu_1^2 E)}\right) + \Delta\varepsilon_{zz}\dfrac{EE_1\nu_1}{E_1(1-\nu)-2\nu_1^2 E}; \\ \Delta\sigma_{\theta\theta} = \Delta\varepsilon_{\theta\theta}\left(\dfrac{E}{1-\nu^2} + \dfrac{E^2\nu_1^2}{(1-\nu)(E_1(1-\nu)-2\nu_1^2 E)}\right) + \Delta\varepsilon_{rr}\left(\dfrac{E\nu}{1-\nu^2} + \dfrac{E^2\nu_1^2}{(1-\nu)(E_1(1-\nu)-2\nu_1^2 E)}\right) + \Delta\varepsilon_{zz}\dfrac{EE_1\nu_1}{E_1(1-\nu)-2\nu_1^2 E}; \\ \Delta\sigma_{zz} = \dfrac{E_1^2(1-\nu)}{E_1(1-\nu)-2\nu_1^2 E}\left(\Delta\varepsilon_{zz} + \dfrac{E\nu_1}{E_1(1-\nu)}\left(\Delta\varepsilon_{rr} + \Delta\varepsilon_{\theta\theta}\right)\right) \end{cases} \quad (5)$$

2.2. The traditional concrete-filled steel tube structures

We represent the solution of the problem of compressing the traditional concrete-filled steel tube structures.

Because of the axisymmetry of the problem, one of the equilibrium equations holds identically, while the remaining two simplify and have the form:

$$\frac{\partial\sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0, \quad \frac{\partial\sigma_{zz}}{\partial z} = 0. \quad (6)$$

In the future to indicate the physical characteristics related to the steel tube, we will use the superscript "S", to the concrete - the superscript "C".

The load P is acting on the entire cross-section, but the nature of load distribution on the steel shell the concrete core is not known. There are no longitudinal displacements of the points on the lower end of the column, this is due to an anchorage of the end (Figure 1), so the boundary conditions at the ends of the column have the form:

$W = 0$ *(when z=0);* $\int_F \sigma_{zz} dF = -P$ *(when z=h).* (7)

We assume that there is no lateral pressure on the outer cylindrical surface of the steel tube. Then the boundary condition on the lateral surface is valid (Figure 1):

$\sigma_{rr}^S = 0$ *(when r=R_{ext})* (8)

To ensure the joint operation of the steel tube-shell and the concrete cylinder, it is necessary to fulfill the conditions of layer interfacing (Figure1):

$\sigma_{rr}^C = \sigma_{rr}^S$ *(when r=R_{int}) (Figure1),* $U^C = U^S, W^C = W^S$. (9)

We introduce the following notation:

p_0 – radial pressure at the contact of the layers, then $\sigma^C_{rr} = \sigma^S_{rr} = -p_0$ *(when r=R$_{int}$), p^C, p^S* – longitudinal (along the z-axis) compressive pressures to the concrete core and steel tube, respectively.

The stress-strain state of the tube material corresponds to the generalized the Hooke's law (1).

Let us consider the operation of the structure when the stress-strain state of concrete corresponds to case 1. The problem (1), (6) – (9) is the problem of the spatial theory of elasticity in the formulation of Saint-Venant, the solution of problems of this type is considered in the paper [12]. The solutions of the problems (1), (6) – (8) for the steel tube and the concrete core, separately taking into account the first equation for the stresses (9) and without taking into account the second and the third equalities has the form:

$$\begin{cases} \sigma^S_{rr} = \dfrac{p_0 \cdot R^2_{\text{int}} \cdot (r^2 - R^2_{ext})}{(R^2_{ext} - R^2_{\text{int}}) \cdot r^2}; \\ \sigma^S_{zz} = -p^S; \\ \sigma^S_{\theta\theta} = \dfrac{p_0 \cdot R^2_{\text{int}} \cdot (r^2 + R^2_{ext})}{(R^2_{ext} - R^2_{\text{int}}) \cdot r^2}. \end{cases} \quad \begin{cases} \sigma^C_{rr} = -p_0; \\ \sigma^C_{zz} = -p^C; \\ \sigma^C_{\theta\theta} = -p_0. \end{cases} \tag{10}$$

Taking into account the second and the third equalities of the conditions of layer interfacing (9) and the equalities (7), we obtain the solution of problem (1), (6) – (9):

$$p^C = p_0 \cdot \left(\frac{(1-2\nu^C)\cdot(1+\nu^C)}{(\nu^C - \nu^S)} + 2\nu^C + \frac{E^C}{E^S} \cdot \frac{R^2_{\text{int}}(1-2\nu^S)\cdot(1+\nu^S) + R^2_{ext}(1+\nu^S)}{(\nu^C-\nu^S)\cdot(R^2_{ext} - R^2_{\text{int}})} \right),$$

$$p^S = p_0 \cdot \left(\frac{E^S}{E^C} \frac{(1-2\nu^C)\cdot(1+\nu^C)}{(\nu^C-\nu^S)} + \frac{R^2_{\text{int}}(1-\nu^S) + R^2_{ext}(1+\nu^S) - 2R^2_{\text{int}}\nu^S\nu^C}{(\nu^C-\nu^S)\cdot(R^2_{ext} - R^2_{\text{int}})} \right), \tag{11}$$

$$p_0 = P \cdot \frac{E^C E^S (\nu^C - \nu^S)(R^2_{ext} - R^2_{\text{int}})}{\alpha},$$

where

$$\alpha = \pi R^2_{\text{int}} E^C \left(E^S (1 - \nu^C - 2\nu^C\nu^S)(R^2_{ext} - R^2_{\text{int}}) + E^C (1+\nu^S)(R^2_{\text{int}}(1-2\nu^S) + R^2_{ext}) \right)$$
$$+ \pi (R^2_{ext} - R^2_{\text{int}}) E^S \left(E^S (1-2\nu^C)(1+\nu^C)(R^2_{ext} - R^2_{\text{int}}) + E^C \left(R^2_{\text{int}}(1-\nu^S - 2\nu^S\nu^C) + R^2_{ext}(1+\nu^S) \right) \right).$$

Let us analyze the obtained dependences and draw the main conclusions:

According to formula (11), the compression pressure sign p_0 is completely determined by the difference in Poisson's coefficients of concrete and steel $(\nu^C - \nu^S)$.

The Poisson's ratio of concrete is $\nu^C = 0.2$, the Poisson's ratio of steel is $\nu^S = 0.3$ (according to the Construction Norms and Regulations 63.13330.2012 “Concrete and reinforced concrete structures” and Construction Norms and Regulations 16.13330.2011 "Steel Structures"), therefore, the compression pressure p_0,<0 and the radial stresses in the concrete core are tensile (10), which leads to break of contact between the concrete core and the steel tube, and therefore, the joint work of concrete and steel is not realized. This is the latent defect of traditional the concrete-filled steel tube structures. This fact is also confirmed when calculating the traditional structures taking into account the nonlinear deformation of concrete.

2.3. The concrete column in steel tube cage

Since the traditional concrete-filled steel tube structures have the latent defect, as mentioned above, in [13] the authors proposed different concrete-filled steel tube structure - the concrete column in steel tube cage. When using this structure, the external load is applied only to the concrete core, and the tube is used as a cage, while the joint work of the concrete core and the steel tube in the longitudinal direction is excluded.

Let us consider the operation of the structure when the stressed state of concrete corresponds to case 1. The equations of equilibrium (6) and the Hooke's law (1) for concrete and steel and the boundary condition on the lateral surface (8) remain in effect in the formulation of this problem, but the boundary conditions at the ends of (7) vary and have the form:

$$W = 0 \text{ (when } z=0); \int_{F^C} \sigma_{zz} dF = -P \text{ (when } z=h). \tag{12}$$

In this design, there is no joint work of the concrete core and the steel tube in the longitudinal direction, and the interaction in the transverse direction is performed according to the following conditions of layer interfacing:

$$\sigma_{rr}^C = \sigma_{rr}^S,\ U^C = U^S. \text{ (when } r=R_{int}) \tag{13}$$

The boundary problem (1), (6), (8), (12), (13) is the problem of the spatial theory of elasticity in the formulation of Saint-Venant. We obtain the solution of problems (1), (6), (8), (12) for a steel tube and a concrete core separately, taking into account the fact that the longitudinal pressure acts only on concrete, and there is no joint work of the layers (13):

$$\begin{cases} \sigma_{rr}^S = \dfrac{p_0 \cdot R_{\text{int}}^2 \cdot (r^2 - R_{ext}^2)}{(R_{ext}^2 - R_{\text{int}}^2) \cdot r^2}; \\ \sigma_{zz}^S = -p^S = 0; \\ \sigma_{\theta\theta}^S = \dfrac{p_0 \cdot R_{\text{int}}^2 \cdot (r^2 + R_{ext}^2)}{(R_{ext}^2 - R_{\text{int}}^2) \cdot r^2}. \end{cases} \quad \begin{cases} \sigma_{rr}^C = -p_0; \\ \sigma_{zz}^C = -p^C = -\dfrac{P}{F^C}. \\ \sigma_{\theta\theta}^C = -p_0. \end{cases} \tag{14}$$

The equations (14) are obtained only with the use of equilibrium equations and do not depend on the linearity or nonlinearity of the concrete deformation law. The magnitude of the interaction pressure of the layers p_0 depends on the nature of the material deformation.

Taking into account the conditions of layer interfacing (13) and the equalities (12), we obtain the solution of the problem (1), (6), (8), (12), (13), in which the stress-strain state of concrete corresponds to the law (1) (case 1):

$$p_0 = \frac{P}{\pi R_{\text{int}}^2} \cdot \frac{E^S \nu^C \left(R_{ext}^2 - R_{\text{int}}^2\right)}{E^C \left(R_{\text{int}}^2 \left(1-\nu^S\right) + R_{ext}^2 \left(1+\nu^S\right)\right) + E^S \left(R_{ext}^2 - R_{\text{int}}^2\right)\left(1-\nu^C\right)}. \tag{15}$$

It follows from formula (15) that the compression pressure p_0 is always a positive value, hence the radial stresses in the concrete core are compressive, the concrete is in a state of triaxial compression (according to formula (14)). This fact has a positive effect on increasing the strength of the concrete core, as shown by the experiments of the researchers [15, 16].

We consider the operation of the structure when the stress state of concrete corresponds to case 2, that is, taking into account the nonlinearity of the concrete deformation diagram.

In the future, to denote the values related to the solution of the problem, taking into account the nonlinearity of the concrete deformation diagram, we will use the superscript "n", and when considering the deformation of concrete according to Hooke's linear law, the superscript "l".

Taking into account the conditions of layer interfacing (13) and the solutions obtained for the independently operating steel tube cage and the concrete cylinder (14), we obtain the solution of the non-linear problem (1), (2), (5), (6), (8), (12), (13), in which the stress-strain state of concrete obeys the law (2), (5) (case 2):

$$p^n{}_0 = \left(\frac{P}{\pi R_{\text{int}}^2} \cdot \frac{\nu_1^C}{E_1^C} + \frac{E^C \nu_1^C - E_1^C \nu^C}{E_1^C \left(1-2\nu^C\right)\left(1+\nu^C\right)} \cdot \left(\nu^C \varepsilon_{rr}^* + \nu^C \varepsilon_{\theta\theta}^* + \left(1-\nu^C\right)\varepsilon_{zz}^*\right) \right) \cdot \\ \cdot \frac{E^C E^S \left(R_{ext}^2 - R_{\text{int}}^2\right)}{E^S \left(1-\nu^C\right)\left(R_{ext}^2 - R_{\text{int}}^2\right) + E^C \left(\left(1-\nu^S\right) R_{вн}^2 R_{\text{int}}^2 + \left(1+\nu^S\right) R_{ext}^2\right)} \tag{16}$$

3. Results and Discussion

3.1. The increasing the load-bearing capacity of concrete under comprehensive compression

Let us analyze the increase in the load-bearing capacity of the concrete column in steel tube cage due to strengthening of the concrete core. The most common form of presenting the strength of concrete under a three-axis contraction is the formula proposed by F. Richard, A. Brown and A. Brandraeg [15]:

$$R_{b,3} = R_b + K \cdot p_0 \ ,$$

where $R_{b,3}$ – the design resistance of concrete under three-axial compression, K – the concrete strengthening coefficient $K \approx 4$.

Thus the formula for determining the strength of concrete under triaxial compression has the form:

$$R_{b,3} = R_b + 4 \cdot p_0$$

In order to further increase the load-carrying capacity of concrete-filled steel tube structures, in cases where vibration is impossible it is more preferable to apply self-compacting concrete mixtures [17].

3.2. An example

Let us consider an example of calculation of the concrete-filled steel tube column. We compare two different approaches: the nonlinear model of the concrete deformation diagram (case 2) and the linear model (case 1). We take different thickness of the tube wall and R_{int} = 0.210 m. We assume the deformation-strength characteristics of materials: E^C = 30000 MPa, $E_1{}^C$ = 8571.4 MPa, ν^C = 0.2, ν_1^C = 0.2, R_b = 14.5 MPa (concrete B 25) and E_S = 206000 MPa, ν^S = 0.3. The results are presented in Table 1, where Δ is the increment of pressure. For example, when the wall thickness is 4 mm, we have the maximum value P_3^n = 2.56 MN, P_3^i = 2.2 MN, and the values of the design resistance of concrete under three-axial $R_{b,3}^n$ = 18.56 MPa, $R_{b,3}^l$ = 15.95 MPa(Figure 3).

Table 1. Results of calculations when ν^C = 0.2, ν_1^C = 0.2

Tube wall thickness, mm	$\frac{\Delta p_0^l}{\Delta p^C}$	$\frac{\Delta p_0^n}{\Delta p^C}$	$\frac{\max p_0^n}{\max p_0^l}$	$\frac{R_{b,3}^l}{R_b}$	$\frac{R_{b,3}^n}{R_b}$	$\frac{R_{b,3}^n}{R_{b,3}^l}$
4	0.02	0.08	2.69	1.10	1.28	1.16
4.5	0.03	0.09	2.81	1.12	1.33	1.19
5	0.03	0.10	2.96	1.13	1.38	1.22
10	0.05	0.18	5.41	1.25	2.36	1.89

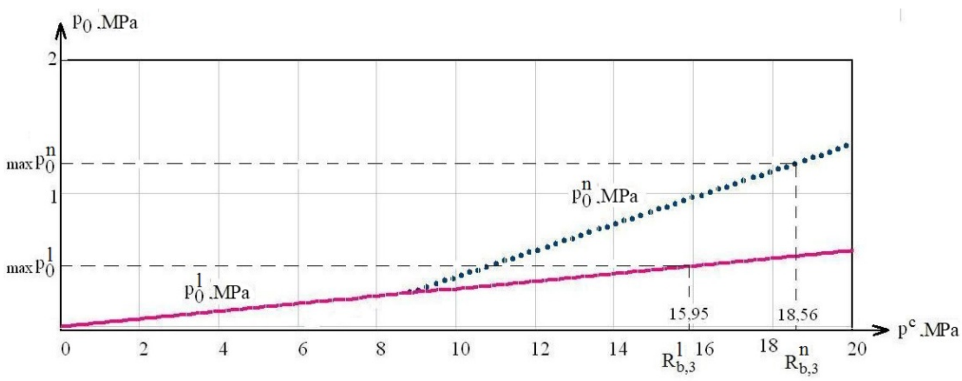


Figure 3. Dependence of the radial compression pressure on the axial compressive pressure on concrete when ν^C = 0.2, ν_1^C = 0.2, and the wall thickness of the pipe is 4 mm.

According to the results (Table 2) and the presented graph (Figure 3), the calculated compression pressure of concrete is underestimated. Thus the design load-bearing capacity of the structure is also underestimated as the result of calculation by the linear model.

It is known that the Poisson's ratio of concrete is the variable that increases with increasing of load on concrete [18]. We perform calculations with the initial data presented above but the Poisson's ratio of concrete is different ν_1^C = 0.25. The results are presented in Table 2.

Table 2. Results of calculations when ν^C = 0.2, ν_1^C = 0.25

Tube wall thickness, mm	$\frac{\Delta p_0^l}{\Delta p^C}$	$\frac{\Delta p_0^n}{\Delta p^C}$	$\frac{\max p_0^n}{\max p_0^l}$	$\frac{R_{b,3}^l}{R_b}$	$\frac{R_{b,3}^n}{R_b}$	$\frac{R_{b,3}^n}{R_{b,3}^l}$
4	0.02	0.10	3.60	1.10	1.37	1.24
4.5	0.03	0.11	3.86	1.12	1.45	1.30
5	0.03	0.12	4.15	1.13	1.53	1.36
10	0.05	0.22	15.67	1.25	4.95	3.95

Comparison of the results presented in Tables 1 and 2 shows that in the case when we taking into account the increase in the Poisson's ratio of concrete, the value of the calculated bearing capacity of the concrete core is higher than this value when the calculation is made with an unchanged coefficient.

3.3. Discussion

According to the calculation of the formulas obtained by us (10-11), when loading traditional concrete-filled steel tube structures, contact between the concrete core and the steel tube breaks, these results confirm the position of N. Skvortsov [7].However, a universal calculation model for concrete-filled steel tube structures was not developed by him.

Different approaches to calculate concrete-filled steel tube structures are mainly based on the calculation of the uniaxial stress-strain state. In this case the load-bearing capacity of the centrally compressed concrete-filled steel tube element is determined by the formula [3, 6, 19]:

$$N = (c \cdot R_b + d) \cdot F_b + \alpha \cdot F_s \cdot R_s, \tag{17}$$

where R_b and R_s – the design resistances to the axial compression of concrete and steel respectavly; F_b and F_s - the cross-sectional area of the concrete core and the steel tube; c, d, α – the constant coefficients determined experimentally.

The limits of the coefficients *c, d* and *α* for various geometric and mechanical characteristics of steel and concrete have been determined as a result of numerous experiments and thus it possible to formulate practical recommendations on their calculation. However, since this calculation method (17) is based on empirical determination of the coefficients, these recommendations and formulas have limited scope and can not be taken into account for the whole variety of building materials and structures [19–21]. The refinement of the empirical coefficients requires new extensive and time-consuming experiments. This is due to the fact that the empirical dependence (17) does not take into account the spatial work of the concrete-filled steel tube structures. In this article, we presented formulas for three-dimensional calculating the concrete-filled steel tube structures taking into account the non-linearity of spatial deformation of concrete, the interaction of steel tube and concrete, the characteristics of materials, and the structure dimensions.

4. Conclusions

1. The model for three-dimensional the stress-strain state of concrete taking into account the non-linear concrete behavior (2)-(3) are presented.

2. We present the derivation of analytical solution for the three-dimensional model of the concrete column in steel tube cage taking into account the non-linear model.

3. Taking into account the nonlinear behavior of concrete leads to the increasing of the calculated load-bearing capacity of the concrete column in the steel tube cage by 16–89 %, depending on the tube wall thickness.

4. Change in the Poisson's ratio of concrete leads to the increasing of the load-bearing capacity of the concrete column in the steel tube cage by 7–109 % too, depending on the tube wall thickness.

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Vera Snigireva,*
+7(982)416-44-14; snow-vera@mail.ru

Gleb Gorynin,
+7(904)872-84-76; ggorynin@list.ru

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Вера Алексеевна Снигирева,*
+7(982)416-44-14;
эл. почта: snow-vera@mail.ru

Глеб Леонидович Горынин,
+7(904)872-84-76; эл. почта: ggorynin@list.ru

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