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Water permeability of the polymer screen with a system of slits of hydraulic structures

Водопроницаемость полимерного экрана с системой щелей гидротехнических сооружений

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Abstract. The calculation of water permeability through a system of defects (long narrow slits in the junctions) in the screen is considered based on the filtration model. Its structural layout and description of the model main elements are given. The solution is carried out for the plane formulation of the problem by the methods of the filtration theory using the method of conformal mappings and the velocity hodograph method. A distinctive feature is the study of free filtration through a system of screen slits (the isolated standard fragment with a single defect), characterized by a pressure movement of the filtration flow in the protective layer and pressure/pressure-free movement at the bottom of the screen, taking into account the mutual influence of the slits on each other. We obtained the calculated dependences for the determination of the specific filtration flow through the screen slit, as well as the total flow through the system of slits and the averaged screen filtration factor, the results of the calculations are compared to the known formulas for the defect system. For practical use, a table of averaged screen filtration factors and a graph of the change in the reduced flow rate through the screen, considering and not considering the influence of the underlying base are made.

Аннотация. Рассматривается расчет водопроницаемости через систему дефектов (узких протяженных щелей в местах стыков) в экране на основе фильтрационной модели. Приводится ее структурная схема и описание основных элементов модели. Решение осуществляется для плоской постановки задачи методами теории фильтрации с использованием метода конформных отображений и способа годографа скорости. Отличительной особенностью является изучение свободной фильтрации через систему щелей экрана (выделенный стандартный фрагмент с единичным дефектом), характеризующиеся напорным движением фильтрационного потока в защитном слое и напорно-безнапорным движением в основании экрана с учетом взаимовлияния щелей друг на друга. Получены расчетные зависимости для определения удельного фильтрационного расхода через щель экрана, а также суммарного расхода через систему щелей и осредненного коэффициента фильтрации экрана, результаты расчетов сопоставлены с известными формулами для системы дефектов. Для практического использования составлена таблица осредненных коэффициентов фильтрации экрана и график изменения приведенного расхода через экран с учетом влияния подстилающего основания и без его учета.

1. Introduction

In hydraulic engineering construction, impervious screens of polymeric materials, geomembranes, and geocomposites are becoming increasingly used as more efficient and durable [1]. Their averaged filtration factor is 10^{-8}–10^{-10} cm/s, which is two to four orders of magnitude lower than for film screens, and the predicted service life is 50–100 years, which is 2–3 times longer than one of film screens.

The relevance of the use of geosynthetic materials in hydraulic engineering is also confirmed by B. E. Vedeneev Institute of VNIIG [2–4], SpbSTU [5, 6] and other authors [7, 9].

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The normative document [10] regulates the use of polymeric materials for anti-filtration devices of hydraulic structures. According to paragraph 5.50 [10], they can be used for dams of III and IV classes, and with proper justification for I and II class dams up to 60 m high.

European experience in the use of geosynthetic materials for anti-filtration screens of waste storage facilities is considered in the work of A.V. Pryamitsky and Yu. Shlee [11]. Here, the requirements of the American Environmental Protection Agency, the European Directive and the German Geotechnical Society for the construction of waste storage facilities are analyzed.

According to the authoritative experts on polymer screens [12], polymeric (film and sheet) geomembrane coatings can not be considered as waterproof, because through various defects and damages, filtration leaks occur that are formed during construction and operation.

At first, the problems of water permeability of polymer screens in the hydromechanical installation were raised and solved by V.P. Nedriga [13], and then by Yu.M. Kosichenko and O.A. Baev [13–16], A.V. Ishchenko [17]. Similar problems were considered by P.Ya. Polubarinova-Kochina and N.E. Zhukovsky [18] for a single drain, by V.V. Vedernikov [18, 19] for a system of drains, by S.F. Averyanov [20] for a channel with a finite depth of occurrence of a watertight stratum. In [15, 16] the main special functions are presented, which are used in the theory of filtration. In [21] methods of calculating the theory of analytic functions are given, and in [22] the main special functions used in the theory of filtration are presented.

The authors of [13] give the well-known approximate formula of V.P. Nedrigi (1976), which he obtained as a result of the solution of the problem of permeability for sheet pile walls, and used in connection with film screens with continuous gaps. However, the formula obtained is approximate, since the solution of such a problem is obtained in a simplified formulation, where the filtration region is an infinite half-strip. It was shown in [13] that V.P. Nedrigi's formula gives a significant discrepancy (from 30 to 50 %) with more exact dependencies.

The authors of this article considered the solution of the problem of axisymmetric filtration through small holes [14–16], and work [13] analyzes the main methods for calculating water permeability of polymer impervious screens for the first time. At the same time, special attention is paid to theoretical methods based on rigorous analytical solutions by methods of the theory of functions of a complex variable, as giving more accurate computational formulas with physically correct results. A hydraulic model to calculate water permeability of large channel liners is presented in [16].

Among the close problems of the theory of filtration, we consider the solution V.V. Vedernikov [18, 19] obtained for the drain system and it can be applied to the problems of water permeability of screens. The solution of this problem is carried out in a pressure installation on the basis of the application of Schwarz-Christoffel integral and elliptic integrals. However, the calculated formulas are rather cumbersome and do not allow us to calculate the filtration features with small parameters. Therefore, for their application in calculating the water permeability of the screen slits, it is necessary to reduce such dependences to an approximate form with the possibility of calculation with small parameters.

Methods for calculating the permeability of geomembrane screens with defects are widespread abroad, see works by J.P. Giroud, R. Bonaparte, K. Badu-Tweneboah [22–28], they are based on experimental and theoretical dependences of degree type, using empirical factors based on field research data reflecting the quality of the contact between geomembrane and base soil. The works by R.K. Rowe, N. Touze-Foltz and C. Duquennoi [29–31] present rather cumbersome theoretical dependences for computations of heads and flow rates through round and rectangular defects in geomembranes using special Bessel functions, which significantly complicates their calculations.

As shown below, when comparing the calculation results for the various formulas in Table 1, it was found that the dependences of J.P. Giroud on filtration through defects in geomembrane are considerably overestimated. In the last works of authors [32, 33] the review of efficiency of geosynthetics for environmental protection and methods of numerical modeling of geosynthetic systems is presented. The works [34–38] is devoted to the evaluation of the performance of underground dams and self-healing cracks of the polymer screen, as well as to the study of other anti-filtration devices.

As the main types of damage in the geomembrane, defects in joints in the form of narrow, long slits and round holes in the form of punctures are usually considered [8, 14, 23].

In present practice, we have to see not only individual defects in polymeric impervious screens, but also several damages in the form of long narrow slits that are formed at the joints of the cloths due to their poor connection. Such damage, located at an equal distance from each other, is called a system of slits.

They are especially common where the jointing of geomembranes or films is performed by welding with extruders. Due to poor quality welding, defects such as low current welding or burned welding of joints with the formation of the through slits can be observed.

The aim of the study is to develop a filtration model of water permeability for a polymer screen of broken continuity with a system of targets.

Objectives of the study:

- to obtain an analytical solution of a polymer screen water permeability with a system of slits by the methods of filtration theory, based on the methods of the theory of functions of a complex variable;
- to find the calculated dependencies to determine the main features of water permeability;
- to compare the obtained dependencies with ones of other authors and evaluate their applicability for practical calculations;
- for the purpose of practical use of the results of calculations based on the developed filtration model, to build a graph of the change in the reduced filtration flow through the slit of a polymer screen and make a table of the average screen filtration factor, depending on the parameters of the slits system.

Thus, the subject of the study is a filtration model of water permeability of a polymer screen of external continuity with a system of slits.

2. Methods

2.1. The filtration model description

To study the water permeability of a polymer screen with a slit system, an analytical method was chosen on the filtration model, which is most often used in the theory of filtration in the steady motion of the filtration flow [18]. In cases of unsteady, as well as spatial (three-dimensional and axisymmetric) motion of the filtration flow, it is advisable to use numerical calculation models, as, for example, in [33, 39].

Figure 1 shows a block diagram of the filtration model of the water permeability of a polymer screen with broken continuity, developed by the authors of this paper, in the form of a system of extended slits.

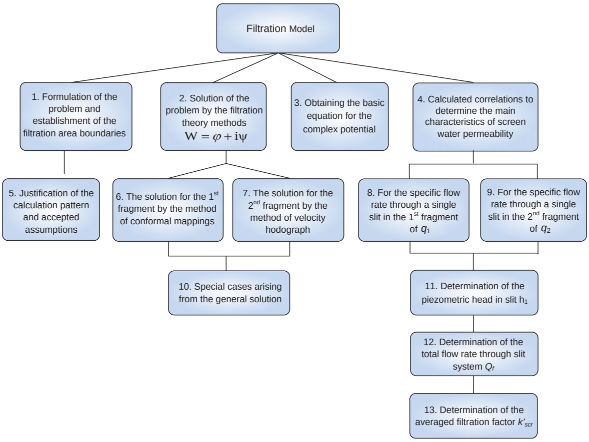

Figure 1. Structural diagram of the filtration model of a polymer screen water permeability through a system of slits

This model includes the following main structural blocks: block 1 – formulation of the problem of water permeability through the system of screen slits and the establishment of boundary conditions in the physical filtering area, which consists of sub-block 5 to justify the calculation pattern and accepted assumptions; block 2 – the solution of the problem with the methods of the filtration theory, which is divided into sub-blocks 6 and 7 in accordance with the solution of the general problem for I and II fragments with the method of conformal mappings and velocity hodograph; block 3 – obtaining the basic equation for the complex potential; block 4 – calculated dependences for determining the main characteristics of water permeability of the screen slits, consisting of sub-blocks 8, 9, 11–13, where specific flow rates are first determined per a single gap in I and II fragment (q_1) and (q_2), and then the piezometric head in the slit (h_1) is determined, the total flow through the slits system (Q_f) and the averaged filtration factor (k'_{scr}). Piezometric head (h_1) can be calculated by the iteration method (successive approximation) or can be determined directly with the formulas. In sub-block 10 there are particular cases arising from the general solution. Thus, for example, when the underlying base is composed of more permeable soils at $k_2 > 10k_1$, then it is neglected in the calculations.

More details of building the filtration model are considered further.

2.2. Research Methodology

The methodology for studying the water permeability of polymer screen slits system in the filtration model is based on the hydromechanical approach to the solution of the problem widely used in the theory of filtration [12, 13]. The method of conformal mappings and the velocity hodograph method are used.

The calculation pattern of the filtration model (Figure 2) includes a system of screen slits of small dimensions located at a distance of (l), protective layer with a thickness of (δ_0) and the underlying base of unlimited depth, in which sets of filtering areas in the protective layer are formed – when the filtration flow moves from the channel or a reservoir to each slit, in the underlying base - when the filtration flow moves from each slit to the ground base to infinity with their interaction.

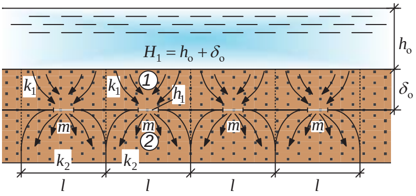

Figure 2. Calculation scheme of the polymer screen slit system

A feature of water permeability here is filtration through each slit with the formation of an independent filtration flow. Therefore, to solve this problem, it is advisable to consider a separate fragment of the filtration region, with its conventional release along the boundary line of the current in the protective layer on the right and on the left (fragment 1), and in the underlying base (where the flow movement will be forced/unforced) considering the interaction with the adjacent area in the general line of current (fragment 2), the velocity of the filtration flow is assumed to be infinite.

In solving this problem, we will make the following assumptions:

- the movement of the filtration flow through the slit is considered to be steady and according to the Darcy's law (with laminar conditions);
- the soil of the protective layer and the base of the screen is assumed to be homogeneous with the filtration factors, respectively, k_1 and k_2;
- the thickness of the polymer screen is neglected in view of its smallness in comparison with the thickness of the protective layer;
- the screen has a certain flexibility, in connection with which it fits snugly to the underlying base;
- capillary flow spreading in the base is accounted by the height of the soil capillary vacuum (H_c, m).

To solve the task in hydromechanical formulation, we shall separately consider the 1st fragment in the protective layer of the soil, and then the 2nd fragment in the underlying soil layer, while establishing a connection between the fragments at the slit boundary with the help of an unknown parameter (h_1), representing the piezometric head in the slit (Figure 1).

Figure 3 shows the pattern of conformal mappings for the 1st fragment of the filtration area.

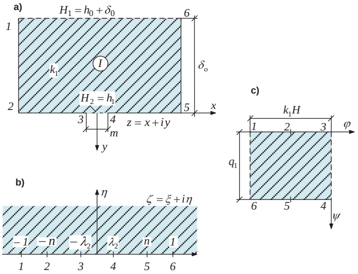

Figure 3. A pattern of conformal mappings for the 1st fragment of the filtration area: a) the filtration area *z*; b) an auxiliary semiplane ζ ; c) the region of the complex potential *W*

The physical area (Figure 3, a) is a rectangle with permeable boundaries 1-6 and 3-4, where the heads, respectively, are set as $H_1 = h_0 + \delta_0$ and $H_2 = h_1$. The impenetrable boundaries are lines 1-2-3 and 6-5-4, which are boundary lines of the current. Within the filtration range of 1-2-3-4-5-6, the flow will be purely forced, which occurs under the action of pressure $H = H_1 - H_2 = h_0 + \delta_0 - h_1$, with h_0 being water depth, m; δ_0 – the thickness of the protective layer, m; h_1 – piezometric head in the screen slit, m.

2.3. A solution by the method of conformal mappings

Now we will consider the solution for fragment I by means of conformal mappings [18, 19, 21].

We will map the polygon of the real filtration region to the upper semiplane (Figure 3, a, b) using the Schwartz-Christoffel integral:

$$z = A\int_0^{\zeta}(\zeta^2 - 1)^{-1/2} \cdot (\zeta^2 - n^2)^{-1/2} \cdot d\zeta + B = A\int_0^{\zeta}\frac{d\zeta}{\sqrt{(\zeta^2 - 1)\cdot(\zeta^2 - n^2)}} + B =$$
$$= A\lambda\int_0^{\zeta}\frac{d\zeta}{\sqrt{(1-\zeta^2)\cdot(1-\lambda^2\zeta^2)}} + B, \tag{1}$$

where $\lambda = \dfrac{1}{n} > 1$.

The resulting formula (1) is an elliptic integral of the first kind with λ modulus. Since in equation (1) $\lambda_1 > 1$, we make a change of variables, assuming:

$$\zeta = \frac{t}{\lambda},\ d\zeta = \frac{1}{\lambda}dt.$$

Then equation (1) can be written in the following form:

$$z = A\int_0^{t}\frac{dt}{\sqrt{\left(1 - \dfrac{t^2}{\lambda^2}\right)\cdot\left(1 - t^2\right)}} + B = AF\left(t, \lambda_1\right) + B, \tag{2}$$

where $\lambda_1 = \dfrac{1}{\lambda} < 1$ – being the elliptic integral modulus; $F\left(t, \lambda_1\right)$ is an elliptic integral of the first kind with λ_1 modulus.

From the correspondence of points 2 and 5 in the equation (2), we will find the constants A and B:

$$A = \frac{l}{2K(\lambda_1)};\ B = 0.$$

From the correspondence of point 1 we will have:

$$A = \frac{\delta_o}{K'(\lambda_1)} = \frac{l}{2K(\lambda_1)},$$

or:

$$\frac{l}{\delta_o} = \frac{K(\lambda_1)}{K'(\lambda_1)}, \tag{3}$$

where $K(\lambda_1)$, $K'(\lambda_1)$ are full elliptic integral of the fist kind, respectively, with modulus λ_1 and additional modulus $\lambda_1' \sqrt{1-\lambda_1^2}$.

The elliptic integral modulus in equation (2) is determined from correlation (3). Putting the identified constants (*A* and *B*) to equation (2), we will have:

$$z = \frac{l}{2K(\lambda_1)} F(t, \lambda_1), \tag{4}$$

or, replacing $t = \lambda\zeta$:

$$z = \frac{l}{2K(\lambda_1)} F(\lambda\zeta, \lambda_1). \tag{5}$$

Hence, from (5) the inverse function is expressed through elliptic sine:

$$z = \lambda_1 sn\left(\frac{mK(\lambda_1)}{l}; \lambda_1\right). \tag{6}$$

From the correlation of point 4 ($z = m/2$, $\zeta = \lambda_2$) and formula (6) we will determine the unknown parameter (λ_2):

$$\lambda_2 = \lambda_1 sn\left(\frac{mK(\lambda_1)}{l}; \lambda_1\right). \tag{7}$$

Mapping semiplane ζ to the region of complex potential W (see Figure 2.3, b, c) we will find:

$$W = C\int_0^\zeta (\zeta^2 - 1)^{-1/2} \cdot (\zeta^2 - \lambda_2^2)^{-1/2} d\zeta + D = \frac{C}{\lambda_2}\int_0^\zeta \frac{d\zeta}{\sqrt{(1-\zeta^2)\cdot(1-\lambda_3^2\zeta^2)}} + D, \tag{8}$$

where $\lambda_3 = \frac{1}{\lambda_2} > 1$.

As $\lambda_3 > 1$ we will perform the replacement $\zeta = \frac{\tau}{\lambda_3} = \tau\lambda_2$, $d\zeta = \lambda_2 d\tau$, then equation (8) will look as follows:

$$W = C\int_0^\tau \frac{d\tau}{\sqrt{(1-\tau^2)\cdot(1-\lambda_2^2\tau^2)}} + D = CF(\tau, \lambda_2) + D = CF\left(\frac{1}{\lambda_2}\tau, \lambda_2\right) + D. \tag{9}$$

From the correlation of points 4, 6 and equation (9) we will find the constants of Schwarz-Christoffel integral:

$$C = -i\frac{k_1 H}{K'(\lambda_2)};\ D = i\frac{k_1 H}{K'(\lambda_2)} K(\lambda_2) - k_1 H . \tag{10}$$

Putting correlations (2.10) into equation (2.9), we will obtain a complex potential that looks as follows:

$$W=-i\frac{k_1H}{K'(\lambda_2)}\cdot F\left(\tau,\lambda_2\right)+i\frac{k_1H}{K'(\lambda_2)}K(\lambda_2)-k_1H\,. \tag{11}$$

From formula (11) we will find specific filtration rate at section 1–6:

$$W=-k_1H+i\psi\,,\ \psi=q_{1-6}\,,\ \zeta=\xi\,,\ z=x\,.$$

Then:

$$q_{1-6}=\frac{k_1H}{K'(\lambda_2)}\left\{K(\lambda_2)-F\left(\tau,\lambda_2\right)\right\}, \tag{12}$$

where $\tau=\frac{1}{\lambda_2}\xi$,. $\zeta=\lambda_1 sn(U_1,\ \lambda_1)$.

We will determine the specific rate through a screen slit by putting $\xi=\lambda_2$, $q=q_1$, $F\left(\tau,\lambda_2\right)=F\left(-1,\lambda_2\right)=-K\left(\lambda_2\right)$ into formula (12):

$$q_1=\frac{2k_1H}{K'(\lambda_2)}K(\lambda_2), \tag{13}$$

or:

$$q_1=\frac{2k_1H}{K(\lambda_2')}K(\lambda_2), \tag{14}$$

where $\lambda_2=\lambda_1 sn\left(\frac{mK\left(\lambda_1\right)}{l},\lambda_1\right)$; $\frac{K\left(\lambda_1\right)}{K'\left(\lambda_1\right)}=\frac{l}{2\delta_o}\to\lambda_1$ [13]; $H=h_o+\delta_o-h_1$ – being head at the screen; $\lambda_2'=\sqrt{1-\lambda_2^2}$ – being a complementary elliptic integral modulus.

At a small value of the screen slit width (m) from a geomembrane, close to zero $(m\to 0)$, which is often the case in practice, the formula for the specific flow rate through the screen slit (14) (considering approximations [22, 40] $K\left(\lambda_2\right)\approx\frac{\pi}{2}$, $K\left(\lambda_2'\right)\approx\ln\frac{4}{\lambda_2}$) will look as follows:

$$q_1=\frac{\pi k_1\left(h_o+\delta_o-h_1\right)}{\ln\left(4/\lambda_2\right)}, \tag{15}$$

where $\lambda_2=\lambda_1K\left(\lambda_1\right)\frac{m}{l}$.

With $\delta_0/l<1$, when the elliptic integral modulus $\lambda_1\to 1.0$, the equation can be written in the following way: $K\left(\lambda_2\right)\approx\ln\left(4/\lambda_1'\right)$, $K\left(\lambda_1'\right)\approx\frac{\pi}{2}$.

Then the dependence for modulus λ_2 (7), considering the correlations (3) and the degeneracy of elliptic functions into hyperbolic functions [40], takes the following form:

$$\lambda_2=th\left(\pi m/4\delta_o\right). \tag{16}$$

Substituting (16) in (15), we will find the approximate formula for flow rate through the screen slit:

$$q_1=\frac{\pi k_1\left(h_o+\delta_o-h_1\right)}{\ln[4cth\left(\pi m/4\delta_o\right)]}. \tag{17}$$

This formula is applicable at $m/l\le 0.01$.

Under condition that $m/\delta_0 < 0.25$ formula (17) is simplified and expressed in terms of elementary logarithmic functions:

$$q_1 = \frac{\pi k_1 \left(h_\text{o} + \delta_\text{o} - h_1\right)}{\ln\left(16\delta_\text{o}/\pi m\right)}. \tag{18}$$

In the extreme event, when $m = l$, from equations (3) and (7) we have:

$$\lambda_2 = \lambda_1 \lim_{m \to l} sn\left\{\frac{mK(\lambda_1)}{l}, \lambda_1\right\} = \lambda_1 sn\{K(\lambda_1), \lambda_1\} = \lambda_1.$$

Then the specific filtration flow rate through the protective layer of soil (with no screen) from (2.14) can be written as:

$$q_1 = 2K_1 H \frac{K(\lambda_1)}{K(\lambda_1')}.$$

From this, according to (3), we will find $q_1 = k_1 l \dfrac{H}{\delta_\text{o}}$, which completely corresponds to Darcy's formula [12], if we take (l) as flow area, and $I = H/\delta_\text{o}$ as pressure gradient.

Thus, we can assume that the resulting formula of the specific filtration rate through the screen slit (14) gives correct results, and therefore it has a physically correct structure, which is confirmed by the classical Darcy's law in the limiting case, which is fundamental for filtration problems.

With $\delta_0 / l > 1.0$, when the elliptic integral modulus $\lambda_1 \to 0$:

$$K(\lambda_1) \approx \frac{\pi}{2};\ K(\lambda_1) \approx \ln\left(4/\lambda_1\right).$$

Taking into account the last correlations and the degeneracy of the elliptic sine into the trigonometric sine (with $\lambda_1 = 0$), formula (7) gives us this:

$$\lambda_2 = \lambda_1 \sin\left(\pi m/2l\right). \tag{19}$$

Modulus (λ_1) from (2.3) can also be expressed by an exponential function:

$$\lambda_1 = 4\exp\left(-\pi\delta_\text{o}/l\right). \tag{20}$$

Then the screen slit filtration flow formula from (15) looks like:

$$q_1 = \frac{\pi k_1 \left(h_\text{o} + \delta_\text{o} - h_1\right)}{\ln\left(\exp\dfrac{\pi\delta_\text{o}}{l} \Big/ \sin\dfrac{\pi m}{2l}\right)}. \tag{21}$$

Using the reciprocal representations of the exponential and logarithmic functions for large values of the arguments [22, 40], we will write the correspondence (21) as follows:

$$q_1 = \frac{\pi k_1 \left(h_\text{o} + \delta_\text{o} - h_1\right)}{Arsh\left(ch\dfrac{\pi\delta_\text{o}}{l} \Big/ \sin\dfrac{\pi m}{2l}\right)}. \tag{22}$$

Now let us find the main indicator of the screen water permeability – the averaged filtration factor with a protective layer, using the formula [13]:

$$k'_{scr} = \frac{Q_\text{f} \cdot \delta_\text{o}}{\left(h_\text{o} + \delta_\text{o}\right) \cdot F_\text{o}}, \tag{23}$$

where $Q_f = \sum_{i=1}^{n} (q_1 \cdot \bar{l}_s)$ – being total flow through a sealed screen with a slits system in the area F_o.

Putting formula (14) in (23), we obtain the most general and exact correspondence to determine the averaged screen filtration factor with a slits system:

$$k'_{\text{scr}} = \frac{2k_1 \cdot \delta_{\text{o}} \cdot K(\lambda_2) \cdot \bar{l}_{\text{s}} \cdot n}{F_{\text{o}} \cdot K(\lambda'_2)}, \tag{24}$$

where k_1 – being the filtration factor of the screen protective layer, m/day; $\bar{l}_s$ – the average statistical length of the screen slit, m; δ_{o} – protective layer thickness, m; n – the number of geomembrane screen slits, pcs.; F_{o} – screen area, m^2.

From correlation (24) and taking into account the approximated formula (18), we find the averaged screen filtration factor with a slits system without taking into account the underlying base:

$$k'_{\text{scr}} = \pi k_1 \frac{\delta_{\text{o}} \cdot \bar{l}_{\text{s}} \cdot n}{F_{\text{o}} \cdot \ln(16\delta_{\text{o}} / \pi m)}. \tag{25}$$

The obtained dependence (25) to determine screen water permeability index (k'_{scr}) is a simplified and approximated one, which can be used under condition $m/\delta_0 < 0.25$.

The averaged filtration screen factor, taking into account the influence of the underlying substrate permeability will be found from (22) as follows:

$$k'_{\text{scr}} = \frac{\pi k_1 \delta_0 (h_0 + \delta_0 - h_1) \cdot \bar{l}_s \cdot n}{F_{\text{o}} Arsh(ch \frac{\pi \delta_0}{l} / \sin \frac{\pi m}{2l} \cdot (h_0 + \delta_0)}. \tag{26}$$

2.4. The solution by the velocity hodograph method

Now let us consider the solution for fragment II in the screen underlying base, where forced/unforced filtration is observed, and when the slits are located closely to one another their mutual influence will occur (Figure 2).

Figure 4 shows the calculated pattern of half the filtration region fragment (a) and the sequence of conformal mappings onto the region of the complex potential (b-e). The surface curve (Figure 4, a) is characterized by a point of inflection (a).

The solution of this problem is carried out by the velocity hodograph method and the conformal mapping method [18]. The presence of a point of inflection on the surface curve is reflected in the velocity hodograph region by a double pass of a part of the circumference (Figure 4, b). Accordingly, the inversion of the complex velocity hodograph has the form shown in Figure 4, d.

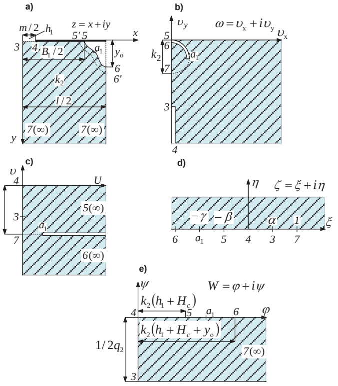

Figure 4. A pattern of conformal mappings for the 2nd fragment of the filtration area:
a) a half of the filtration area *z*; b) velocity hodograph ω; c) velocity inversion region $\bar{\omega}$;
d) an auxiliary semi-plane ζ; e) the region of the complex potential *W*

The mapping of the inversion region of velocity hodograph ($\overline{\omega}$) on a semi-plane (Figure 4, c, d) is performed by Schwarz-Christoffel formula [18, 21, 22]:

$$\overline{\omega} = C\int_0^{\zeta} \frac{\zeta + \gamma}{(\zeta + \beta)\cdot\sqrt{\zeta}} \cdot d\zeta + D. \tag{27}$$

From there, after *C* and *D* constants are determined from equation (27) in section 4-3-7(∞) we obtain this:

$$\overline{\omega} = -\frac{2i}{\pi k_2}\cdot\left[\frac{\sqrt{\zeta\beta}}{\gamma - \beta} + arctg\sqrt{\frac{\zeta}{\beta}}\right]. \tag{28}$$

Mapping the region of the complex potential *W* on the semi-plane ζ (Figure 4, d, e), we will have:

- in section 4-5-6:

$$W = \frac{q_2}{2\pi} Arch\frac{\alpha - \zeta(2-\alpha)}{\alpha(1-\zeta)}; \tag{29}$$

- in section 3-4:

$$W = -\frac{q_2}{2\pi} i \arccos\frac{\alpha - \zeta(2-\alpha)}{\alpha(1-\zeta)}; \tag{30}$$

where q_2 – being specific filtration rate through fragment 2 slit.

Applying the obtained correlations (29) and (30) to formula (31):

$$\overline{\omega} = \frac{dz}{dW} = \frac{1}{\upsilon_x - i\upsilon_y}, \tag{31}$$

we will find:

- in section 4-5:

$$z = \frac{iq_2}{\pi^2 k_2}\int_0^{\zeta}\left[\frac{\sqrt{-\beta\zeta}}{\gamma - \beta} + Arth\sqrt{\frac{-\zeta}{\beta}}\right]\cdot\frac{(1-\alpha)d\zeta}{\sqrt{\zeta(1-\alpha)\cdot(d-\zeta)}(1-\zeta)} + \frac{m}{2}; \tag{32}$$

- in section 5-6:

$$z = \frac{iq_2}{\pi^2 k_2}\int_{-\beta}^{\zeta}\left[\frac{\sqrt{\beta\zeta}}{\gamma - \beta} + Arcrc\sqrt{\frac{-\zeta}{\beta}} - \frac{\pi}{2}i\right]\cdot\frac{(1-\alpha)d\zeta}{\sqrt{\zeta(1-\alpha)\cdot(d-\zeta)}(1-\zeta)} + \frac{B_1}{2}, \tag{33}$$

where B_1 – being the width of the filtration flow rate spreading under the screen; k_2 – the base soil filtration factor.

From (32) and (33), omitting the intermediate transformations, we define the unknown coordinates of points 5 and 6:

$$B_1 = \frac{4q_2}{\pi^2 k_2}\left[\frac{\sqrt{\beta}}{\gamma - \beta}\left(arctg\sqrt{\frac{\alpha - \beta}{1-\alpha}} - arctg\sqrt{\frac{\alpha}{1-\alpha}}\right) + \frac{1}{\sqrt{1-\alpha}}\int_0^{\sqrt{\beta}}\frac{Arth\frac{t}{\sqrt{\beta}}dt}{(1+t^2)\sqrt{\beta + t^2}}\right] + m; \tag{34}$$

$$y_o = \frac{q_2\sqrt{1-\alpha}}{\pi k_2}\left[Arch\frac{1}{\sqrt{\alpha}} - Arch\sqrt{\frac{\beta}{\alpha(1-\beta)}}\right]. \tag{35}$$

The specific filtration rate through fragment 2 slit will be found, putting the coordinates of point 5 [$\zeta = -\beta; W = k_2(h_1 + H_c)$] into equation (29):

$$q_2 = \frac{2\pi k_2 (h_1 + H_c)}{Arch\left[\dfrac{\alpha + \beta(2-\alpha)}{\alpha(1+\beta)}\right]}, \tag{36}$$

where h_1 – being piezometric head in the slit; H_c – base soil capillary vacuum.

Parameters α and β are determined from the simultaneous equations:

$$\frac{\pi_2 m k_2}{4q_2} = arctg\sqrt{\beta}\arcsin\sqrt{\alpha} + \frac{1}{\sqrt{1-\alpha}}\int\limits_{arctg\sqrt{\beta}}^{\pi/2}\frac{arctg\left(\sqrt{\alpha/\beta}\sin\varphi\right)d\varphi}{1-\alpha^2\sin^2\varphi}; \tag{37}$$

$$\frac{\pi_2 (l-m) k_2}{4q_2} - arctg\sqrt{\beta}\arccos\sqrt{\alpha} + \frac{1}{\sqrt{1-\alpha}}\left[\int\limits_0^{\sqrt{\beta}}\frac{Arth\left(t/\sqrt{\beta}\right)dt}{(1+t^2)\sqrt{\alpha+t^2}} + \int\limits_{arctg\sqrt{\beta}}^{\pi/2}\frac{Arctg\left(tg\varphi/\sqrt{\beta}\right)d\varphi}{\sqrt{\alpha+tg^{2\varphi}}}\right]. \tag{38}$$

In a special case, when the forced/unforced flow mode of motion changes into forced flow and the point 5 coincides with the point 6, we will have $\beta \to \gamma \to \infty$. Putting coordinates of point 7 ($\bar{\varpi} = -i/k_2$; $\zeta = 1$), into equation (2.28) after simple transformations, we obtain $arctg\sqrt{\beta} = \pi/2$.

Then the simultaneous equations (37) and (38) will look as follows:

$$\frac{\pi_2 m k_2}{4q_2} = \frac{\pi}{2}\arcsin\sqrt{\alpha}; \tag{39}$$

$$\frac{\pi_2 (l-m) k_2}{4q_2} = \frac{\pi}{2}\arccos\sqrt{\alpha}. \tag{40}$$

Solving equations (39) and (40) jointly, we will find the filtration flow rate from the screen slit to infinity:

$$q_2 = k_2 l, \tag{41}$$

where l – being the width of the considered fragment of the filtration area.

A similar expression (41) is given by P.Ya. Polubarinova-Kochina in her work [18] for filtration from a channel of shallow depth to infinity, which confirms the validity of the theoretical solution obtained by the authors.

Thus, the authors obtained exact solutions of the problems of water permeability through a slits system for one of the filtration area fragments, including the problem of water permeability through a slit in the screen with a protective layer of soil and the problem of water permeability through a slit of the screen with underlying soil base.

To determine the piezometric head in the slit of the polymer screen, from the condition of continuity of the flow, we equate the rates through fragments 1 and 2 ($q_1 = q_2$) according to the approximate formulas (17) and (36), from which we will obtain:

$$h_1 = \frac{\sigma(h_o + \delta_o) Arch\left[\dfrac{\alpha+\beta(2-\alpha)}{\alpha(1+\beta)}\right] - 2H_c \cdot \ln\left[4cth\left(\pi m/4\delta_o\right)\right]}{\sigma Arch\left[\dfrac{\alpha+\beta(2-\alpha)}{\alpha(1+\beta)}\right] + 2\ln\left[4cth\left(\pi m/4\delta_o\right)\right]}, \tag{42}$$

where $\sigma = k_1/k_2$, α and β – being parameters determined from simultaneous equations (37) and (38).

To calculate the piezometric head in the slit of a polymer screen with a protective layer of soil (h_1), a correlation similar to that obtained earlier in [14] can also be used:

$$h_1 = \frac{\sigma(h_0+\delta_0)Arch(1/\sqrt{\alpha_1 - 1}) - H_c \ln(16\delta_0/\pi m)}{\sigma Arch(1/\sqrt{\alpha_1 - 1}) + \ln(16\delta_0/\pi m)}, \tag{43}$$

where α_1 – being a calculated parameter, defined in tabulated form by function $F_1(\alpha_1)$ [14].

On the basis of the above theoretical conclusions, the following simplified formulas can be used for practical application to determine the total flow rate through a polymer screen with a slit system:

- with the use of correlation (17):

$$Q_\mathrm{f} = \frac{\pi k_1(h_\mathrm{o} + \delta_\mathrm{o} - h_1)\cdot \bar{l}_s \cdot n}{\ln\left(4cth\left(\pi m/4\delta_\mathrm{o}\right)\right)}; \tag{44}$$

- with the use of correlation (18):

$$Q_\mathrm{f} = \frac{\pi k_1(h_\mathrm{o} + \delta_\mathrm{o} - h_1)\cdot \bar{l}_\mathrm{s} \cdot n}{\ln\left(16\delta_\mathrm{o}/\pi m\right)}; \tag{45}$$

- with the use of correlation (23):

$$Q_\mathrm{f} = k'_{scr}\frac{h_\mathrm{o} + \delta_\mathrm{o} - h_1}{\delta_\mathrm{o}}\cdot F_\mathrm{o}. \tag{46}$$

where h_1 – being piezometric head in the slit of the polymer screen, determined by correlation (42) or (43);

k'_scr – being the averaged screen filtration factor, calculated by formulas (24) or (25);

$\bar{l}_s$ – being the average statistical length of the screen slit;

n – the number of geomembrane screen slits;

F_o being the screen area.

3. Results and Discussion

For the purpose of the results reconciliation, we perform calculations of the water permeability of a polymer screen of a broken continuity with a slit system using the known formulas by V.P. Nedriga and V.V. Vedernikov, also obtained in the first case for a slits system in a film screen, and in the second case – for a drain system on watertight stratum as well as J.P. Giroud – for geomembranes with defects in the form of extended slits [17].

In the calculation, we use solutions for the general case, when we take into account the influence of the underlying base with slit head h_1 at $k_2 < k_1$, and for a special case without considering the influence of the base at $h_1 = 0$ and $k_2 \le 10k_1$.

Inputs for calculation: $h_1 = 3.0$ m, $\delta_0 = 0.5$ m, $k_1 = 1.0$ m/day, $k_2 = 0.3$ m/day, $l = 3.0$ m, $H_c = 0.5\cdot h_0 = 0.5\cdot 1.0 = 0.5$ m, $m = 0.001 - 0.1$ m, $\bar{l}_s = 1.0$ m, $n = 10$, $h_1 = 0$, $C_{q_0} = 0.21$, $C_{q_\infty} = 0.52$ (with C_{q0} and $C_{q\infty}$ – being the factors of quality of a contact between the geomembrane and the base soil with a good contact by J.P. Giroud) [23].

The total filtration rate through the slit system according to the authors' formula is calculated from the correlations (44) and (45), and the piezometric head in the slit h_1 – by formula (43).

Analysis of calculation results shows that in case of considering the permeability of the underlying base, the total filtration rate according to the formulas by the authors of the work is reduced 2.7–3.3 times.

Table 1. The comparison of the results of calculating the total filtration flow rate through the system of screen slits according to the known formulas

m, m	*h₁*, m	By the authors' formulas						
		Authors (43), (44)	V.P. Nedriga		V.V. Vedernikov		J.P. Giroud	
		Q_f, m^3/ day	Q_f, m^3/ day	ε , %	Q_f, m^3/ day	ε , %	$Q_{f\,(min)}$, m^3/ day	ε , %
0.001	2.21	5.17/14.02	4.53/12.28	-12.4	5.87/15.90	13.4	8.66	67.5
0.005	2.24	6.35/17.64	5.89/17.01	-7.2	7.02/19.85	10.5	12.10	90.5
0.01	2.28	6.92/19.84	6.57/18.84	-5.0	7.59/21.83	10.0	14.13	106.9
0.05	2.35	9.19/27.96	8.54/27.03	-7.1	11.35/36.12	23.5	20.00	117.6
0.1	2.43	10.37/33.92	9.53/31.13	-8.0	13.24/43.26	27.5	23.73	128.8

Note: The numerator shows the values of the total filtration rates through the slits system considering the head in the slit h_1, and in the denominator – without considering h_1 ($h_1 = 0$).

The reconciliation of the obtained flow values (Q_f) by the authors' formulas with ones by V.P. Nedriga, gives a variance from 5.0 to 12.4 %, and by V.V. Vedernikov's – from 10.0 to 27.5 %, which can be considered a satisfactory match. However, when comparing the results with the formula by J.P. Giroud, the discrepancy with the minimum flow rate at factor $C_{q_0} = 0.21$, $Q_{f\,\min}$ reaches from 67.5 % to 128.8 % in the whole range of possible defects (slits) in practice within $m = 0.001 \div 0.1$ m. At the same time, with the maximum value of factor $C_{q_0} = 0.15$ the rate will already increase 5 times. Such significantly overestimated values of rates by J.P. Giroud are explained by the fact that his formula differs fundamentally from other authors' ones in structure that can be considered experimental and theoretical, taking into account empirical factors C_{q_0} and $C_{q\infty}$ according to field study data and various degrees at the main design parameters m , h_1, and k_2. In this regard, its dependence does not comply with Darcy's law, which is fundamental in filtration problems. In addition, different degrees in the main variables indicate a violation of the principle of compliance with the dimension.

In contrast, in the three dependencies by the following authors: V.P. Nedriga and V.V. Vedernikov, who give close values of rates, which are based on strict filtration theory using the Darcy's law, the results are unquestionable and confirm each other.

Nevertheless, we believe that the assumed calculated correlations by J.P. Giroud also have their value and can be used for they are based on a large number of field and in-situ experiments and apparently have already been undergone extensive testing at US facilities.

Apparently, these formulas require their specification for Russian conditions according to field research data.

To calculate the piezometric head in the slit of a polymer screen with a protective layer of soil, the dependence obtained earlier in [14] can also be tested:

$$h_1 = \frac{\sigma(h_0 + \delta_0) Arsh(1/\sqrt{\alpha_1 - 1}) - H_c \ln(16\delta_0 / \pi m)}{\sigma \cdot Arsh(1/\sqrt{\alpha_1 - 1}) + \ln(16\delta_0 / \pi m)}, \qquad (47)$$

where d_1 – being calculated parameter, defined in tabulated form by function $F_1(d_1)$ [14].

For practical application, a graph was built to show the dependence of the reduced filtration flow through the slit of the polymer screen on its width and on the thickness of the protective layer $q_r = f(m, \delta_0)$ (Figure 5), where the reduced flow rate is understood as the correlation of the real filtration flow, the base soil filtration factor and the screen head factor: $q_r = q/(k \cdot H)$, with q_r – being the reduced filtration rate; k – soil filtration factor of protective layer or protective cover; H – the on-screen head.

The analysis of the obtained graphical dependencies shows that in the case of considering the permeability of the underlying base, the resulted rates q_r are reduced in comparison with the curves, without consideration of the base, both in the thickness of the protective layer at $\delta_0 = 0.5$ m and at $\delta_0 = 1.0$ m approximately by 12.5 %.

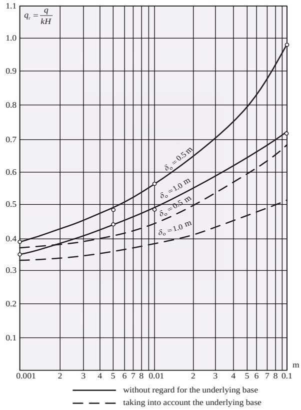

Figure 5. The graph of the dependence of the reduced filtration flow through the slit of the polymer screen on its width and the thickness of the protective layer

The averaged filtration factor of the polymer screen has also been calculated by formulas (25) and (26), with its values over a wide range of slit widths $m = 0.001 \div 0.1$ m and their numbers $n = 1.0 \div 20.0$ presented in Table 2.

Table 2. The values of the averaged filtration factor of a polymer screen with a slits system k_{scr}, m/s

Number of slits, *n*	The width of the slit in the screen, *m*						
	0.001	0.003	0.005	0.01	0.03	0.05	0.1
At $k_2 \geq 10k_1$ (without considering head h_1)							
1	5.8×10^{-10}	6.7×10^{-10}	7.3×10^{-10}	8.2×10^{-10}	1.0×10^{-9}	1.2×10^{-9}	1.4×10^{-9}
5	2.9×10^{-9}	3.3×10^{-9}	3.6×10^{-9}	4.1×10^{-9}	5.0×10^{-9}	6.0×10^{-9}	7.0×10^{-9}
10	5.8×10^{-9}	6.7×10^{-9}	7.3×10^{-9}	8.2×10^{-9}	1.0×10^{-8}	1.2×10^{-8}	1.4×10^{-8}
15	8.7×10^{-9}	1.0×10^{-8}	1.1×10^{-8}	1.2×10^{-8}	1.5×10^{-8}	1.8×10^{-8}	2.1×10^{-8}
20	1.2×10^{-8}	1.3×10^{-8}	1.5×10^{-8}	1.6×10^{-8}	2.0×10^{-8}	2.4×10^{-8}	2.8×10^{-8}
At $k_2 \leq 10k_1$ (considering head h_1)							
1	2.0×10^{-10}	2.3×10^{-10}	2.4×10^{-10}	2.6×10^{-10}	3.1×10^{-10}	3.3×10^{-10}	3.7×10^{-10}
5	1.0×10^{-9}	1.1×10^{-9}	1.2×10^{-9}	1.3×10^{-9}	1.5×10^{-9}	1.6×10^{-9}	1.8×10^{-9}
10	2.0×10^{-9}	2.3×10^{-9}	2.4×10^{-9}	2.6×10^{-9}	3.1×10^{-9}	3.3×10^{-9}	3.7×10^{-9}
15	3.0×10^{-9}	3.4×10^{-9}	3.6×10^{-9}	3.9×10^{-9}	4.6×10^{-9}	4.9×10^{-9}	5.5×10^{-9}
20	4.0×10^{-9}	4.6×10^{-9}	4.8×10^{-9}	5.2×10^{-9}	6.2×10^{-9}	6.6×10^{-9}	7.4×10^{-9}

The following data were taken into account: $\delta_0 = 0.5$ m, $h_0 = 3.0$ m, $k_1 = 1.0$ m/day, $k_2 = 0.3$ m/day, $\bar{l}_s = 1.0$ m, $F_0 = 400$ м², $l = 3.0$ m.

The comparison of the values of factor k_{scr} at $k_2 \geq 10k_1$ (without considering head h_1) and $k_2 \leq 10k_1$ (considering head h_1) shows that in the latter case the filtration factors are almost 3 times reduced. Obviously, this is due to the lower permeability of the underlying soil base compared to the protective layer ($k_2 < k_1$; 0.3 m/day < 1.0 m/day).

The reconciliation of calculated values k_{scr} for a slits system by J.P. Giroud [23] for round holes in the geomembrane at the same depths $h = 3.0$ m indicates a close order of the numbers, although here we also see that their values are overestimated as compared to our data, especially for large holes sizes.

4. Conclusions

1. A filtration model of water permeability for a polymer screen through a slits system has been developed, it includes an analytical solution of the problem by isolating standard fragments of the local filtration area including head filtration area to the screen slit with a soil protective layer and an unforced filtration area from the screen slit at its base.

2. As a result of the solution, the calculated correlations were obtained to determine the specific filtration flow rate through a system of slits with their small dimensions and averaged screen filter factors. The calculation results were compared to the formulas by V.P. Nedriga, V.V. Vedernikov, and J.P. Giroud.

3. For practical use, a graph of the change in the reduced filtration flow rate was made in its dependence of $q_r = f(m, \delta_0)$ as well as a table of changing the values of the averaged filtration factor.

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References

1. Kosichenko, Yu.M., Baev, O.A. Vysokonadezhnyye konstruktsii protivofil'tratsionnykh pokrytiy kanalov i vodoyomov, kriterii ikh effektivnosti i nadezhnosti [Highly reliable structures of anti-filtration coatings of canals and reservoirs, criteria of their efficiency and reliability]. Power technology and engineering. 2014. No. 8. Pp. 18–25. (rus)
2. Glagovskiy, V.B., Solskiy, S.V., Lopatina, M.G., Dubrovskaya, N.V., Orlova, N.L. Geosinteticheskiye materialy v gidrotekhnicheskom stroitelstve [Geosynthetics in hydraulic engineering]. Gidrotekhnicheskoye stroitelstvo. 2014. No. 9. Pp. 23–27. (rus)
3. Solskiy, S.V., Orlova, N.L. Perspektivy i problemy primeneniya v gruntovykh gidrotekhnicheskikh sooruzheniyakh sovremennykh geosinteticheskikh materialov [Prospects and problems of application in groundwater hydraulic structures of modern geosynthetic materials]. Izvestiya VNIIG im. B.Ye. Vedeneyeva. 2010. Vol. 260. Pp. 61–68. (rus)
4. Solskiy, S.V., Lopatina, M.G., Orlova, N.L. Laboratornyye issledovaniy geosinteticheskikh materialov dlya obosnovaniya ikh primeneniya v konstruktsiyakh gruntovykh gidrotekhnicheskikh sooruzheniy [Laboratory studies of geosynthetic materials to justify their use in the construction of groundwater structures]. Izvestiya VNIIG. im. B.Ye. Vedeneyeva. 2015. Vol. 276. Pp. 84–91. (rus)
5. Lupachev O.Yu., Teleshev V.I. The watertight geomembrane elements. Use experience in the hydraulic engineering. Magazine of Civil Engineering. 2009. No. 6. Pp. 35–43. (rus)
6. Lupachev, O.Yu., Teleshev, V.I. Primeneniye geosinteticheskikh materialov v gidrotekhnicheskom stroitelstve v kachestve protivofiltratsionnykh elementov plotin i damb [Application of geosynthetic materials in hydraulic engineering construction as anti-filtration elements of dams and dams]. Gidrotekhnika. 2009. No. 1. Pp. 71–75. (rus)
7. Koerner, R.M., Hsuan, Y.G., Koerner, G.R. Lifetime predictions of eposed geotextiles and geomembranes. Geosynthetics International. 2017. No. 24(2). Pp. 198–212.
8. Tian, K., Benson, C.H., Tinjum, Y.M., Edil, T.B. Antioxidant depletion and service life prediction for HDPE geomembranes exposed to low-level radioactive waste leachate. Journal of Geotechnical and Geoenvironmental Engineering, 2017. No. 143(6). Pp. 22–29.
9. Gallagher, E.M., Tonks, D.M., Shevelan, J., Belton, A.R., Blackmore, R.E. Investigations of geomembrane integrity within a 25-2017 old landfill capping. Geotextiles and Geomembranes. 2016. No. 4(5). Pp. 770–780.
10. Russian Construction Norms SP 39.13330.2012. (rus)

Литература

1. Косиченко Ю.М., Баев О.А. Высоконадежные конструкции противофильтрационных покрытий каналов и водоёмов, критерии их эффективности и надежности // Гидротехническое строительство. 2014. № 8. С. 18–25.
2. Глаговский В.Б., Сольский С.В., Лопатина М.Г., Дубровская Н.В., Орлова Н.Л. Геосинтетические материалы в гидротехническом строительстве // Гидротехническое строительство. 2014. № 9. С. 23–27.
3. Сольский С.В., Орлова Н.Л. Перспективы и проблемы применения в грунтовых гидротехнических сооружениях современных геосинтетических материалов // Известия ВНИИГ им. Б.Е. Веденеева. 2010. Т. 260. С. 61–68.
4. Сольский С.В., Лопатина М.Г., Орлова Н.Л. Лабораторные исследований геосинтетических материалов для обоснования их применения в конструкциях грунтовых гидротехнических сооружений // Известия ВНИИГ. им. Б.Е. Веденеева. 2015. Т. 276. С. 84–91.
5. Лупачев О.Ю., Телешев В.И. Противофильтрационные элементы из геомембран. Опыт применения в гидротехническом строительстве // Инженерно-строительный журнал. 2009. № 6. С. 35–43.
6. Лупачев О.Ю., Телешев В.И. Применение геосинтетических материалов в гидротехническом строительстве в качестве противофильтрационных элементов плотин и дамб // Гидротехника. 2009. № 1. С. 71–75.
7. Koerner R.M., Hsuan Y.G., Koerner G.R. Lifetime predictions of eposed geotextiles and geomembranes // Geosynthetics International. 2017. № 24(2). Pp. 198–212.
8. Tian K., Benson C.H., Tinjum Y.M., Edil T.B. Antioxidant depletion and service life prediction for HDPE geomembranes exposed to low-level radioactive waste leachate // Journal of Geotechnical and Geoenvironmental Engineering. 2017. № 143(6). Pp. 22–29.
9. Gallagher E.M., Tonks D.M., Shevelan J., Belton A.R., Blackmore R.E. Investigations of geomembrane integrity within a 25-2017 old landfill capping // Geotextiles and Geomembranes. 2016. № 4(5). Pp. 770–780.
10. СП 39.13330.2012. Актуализированная редакция СНиП 2.06.05-84. Плотины из грунтовых материалов.
11. Прямицкий А.В., Шлее Ю. Европейский опыт использования геосинтетических материалов в строительстве хранилищ отходов // Гидротехника XXI век. 2010. № 1(21). С. 52–56.
12. Чумаганов А.П. Сирота Ю.Я. За порогом надежности эксплуатации гидротехнических сооружений, входящих в промышленные комплексы различных

11. Pryamitskiy, A.V., Shleye, Yu. Yevropeyskiy opyt ispolzovaniya geosinteticheskikh materialov v stroitelstve khranilishch otkhodov [European experience of using geosynthetic materials in the construction of waste storages]. Gidrotekhnika XXI vek. 2010. No. 1(21). Pp. 52–56. (rus)
12. Chumaganov, A.P. Sirota, Yu.Ya. Za porogom nadezhnosti ekspluatatsii gidrotekhnicheskikh sooruzheniy, vkhodyashchikh v promyshlennyye kompleksy razlichnykh predpriyatiy [Beyond the threshold of reliability of operation of hydraulic structures within the industrial complexes of various enterprises]. Gidrotekhnika XXI vek. 2010. No. 1. Pp. 46–51. (rus)
13. Kosichenko, Yu.M., Bayev, O.A. Metody rascheta vodopronitsayemosti polimernykh protivofiltratsionnykh ekranov gidrotekhnicheskikh sooruzheniy [Methods for calculating the permeability of polymer anti-filtration screens of hydraulic structures]. Izvestiya VNIIG im. B.Ye. Vedeneyeva. 2017. Vol. 286. Pp. 10–21. (rus)
14. Kosichenko, Yu.M., Bayev, O.A. Teoreticheskaya otsenka vodopronitsayemosti protivofiltratsionnoy oblitsovki narushennoy sploshnosti [Theoretical assessment of the permeability of the impervious lining of the broken continuity]. Izvestiya vysshikh uchebnykh zavedeniy. Severo-Kavkazskiy region. Seriya: Tekhnicheskiye nauki. 2014. No. 3(178). Pp. 68–74. (rus)
15. Kosichenko, Yu.M., Bayev, O.A. Matematicheskoye i fizicheskoye modelirovaniye filtratsii cherez malyye povrezhdeniya protivofiltratsionnykh ustroystv iz polimernykh geomembran [Mathematical and physical modeling of filtering through small damages of impervious devices from polymeric geomembranes]. Izvestiya VNIIG im. B.Ye. Vedeneyeva. 2014. Vol. 274. Pp. 60–74. (rus)
16. Kosichenko, Yu.M., Baev, O.A. Design of impervious coatings with enhanced reliability made from innovative materials. Procedia Engineering. 2016. Vol. 150. Pp. 1503–1509.
17. Ishchenko, A.V. Gidravlicheskaya model vodopronitsayemosti i effektivnosti protivofiltratsionnykh oblitsovok krupnykh kanalov [Hydraulic model of permeability and efficiency of anti-filtration facing of large canals]. Izvestiya VNIIG im. B.Ye. Vedeneyeva. 2010. Vol. 258. Pp. 51–64. (rus)
18. Polubarinova-Kochina, P.Ya. Teoriya dvizheniya gruntovykh vod [Theory of groundwater movement]. Moscow: Nauka. 1977. 664 p. (rus)
19. Lavrik, V.I., Filchakova, S.P., Yashin, A.A. Konformnyye otobrazheniya fiziko-topologicheskikh modeley [Conformal mappings of physico-topological models]. Kiyev: Naukova Dumka. 1990. 376 p. (rus)
20. Averyanov, S.F. Filtratsiya iz kanalov i yeye vliyaniye na rezhim gruntovykh vod [Filtration from canals and its effect on groundwater regime]. Moscow: «Kolos». 1982. 237 p. (rus)
21. Hurwitz, A., Courant, R. Theory of functions. Springer-varlag. Berlin. 1964. 648 p.
22. Handbook of mathematical functions, with formulas, graphs and mathematical tables. Edited by Abramowitz M., Stegun A. National bureau standards applied mathematics series 55. 1964. 230 p.
23. Giroud, J.P. Equations for calculating the rate of liquid migration through composite lines due to geomembrane defects. Geosynthetics International. 1997. Vol. 4. No. 3-4. Pp. 335–348.
24. Bonaparte, R., Giroud, J.P., Gross, B.A. Rates of Leakage through Landfill Liners. Proceedings of Geosynthetics. 1989. Vol. 1. Pp. 18–29.
25. Giroud, J.P., Khatami, A., Badu-Tweneboah, K. Evaluation of the Rate of Leakage through Composite Liners. Geotextiles and Geomembranes. 1989. Vol. 8. No. 4. Pp. 337–340.
26. Giroud, J.P., Badu-Tweneboah, K., Bonaparte, R. Rate of Leakage Through a Composite Liner due to Geomembrane Defects. Geotextiles and Geomembranes. 1992. Vol. 11. No. 1. Pp. 1–28.

предприятий // Гидротехника XXI век. 2010. № 1. С. 46–51.

13. Косиченко Ю.М., Баев О.А. Методы расчета водопроницаемости полимерных противофильтрационных экранов гидротехнических сооружений // Известия ВНИИГ им. Б.Е. Веденеева. 2017. Т. 286. С. 10–21.
14. Косиченко Ю.М., Баев О.А. Теоретическая оценка водопроницаемости противофильтрационной облицовки нарушенной сплошности // Известия высших учебных заведений. Северо-Кавказский регион. Серия: Технические науки. 2014. № 3(178). С. 68–74.
15. Косиченко Ю.М., Баев О.А. Математическое и физическое моделирование фильтрации через малые повреждения противофильтрационных устройств из полимерных геомембран // Известия ВНИИГ им. Б.Е. Веденеева. 2014. Т. 274. С. 60–74.
16. Kosichenko Yu.M., Baev O.A. Design of Impervious Coatings with Enhanced Reliability Made from Innovative Materials // Procedia Engineering. 2016. Vol. 150. Pp. 1503–1509.
17. Ищенко А.В. Гидравлическая модель водопроницаемости и эффективности противофильтрационных облицовок крупных каналов // Известия ВНИИГ им. Б.Е. Веденеева. 2010. Т. 258. С. 51–64.
18. Полубаринова-Кочина П.Я. Теория движения грунтовых вод. М.: «Наука». 1977. 664 с.
19. Лаврик В.И., Фильчакова С.П., Яшин А.А. Конформные отображения физико-топологических моделей / Акад. наук АН УССР. Киев: Наукова Думка. 1990. 376 с.
20. Аверьянов С.Ф. Фильтрация из каналов и ее влияние на режим грунтовых вод. М.: «Колос». 1982. 237 с.
21. Hurwitz A., Courant R. Theory of functions. Springer-varlag. Berlin. 1964. 648 p.
22. Handbook of mathematical functions, with formulas, graphs and mathematical tables. Edited by Abramowitz M., Stegun A. National bureau standards applied mathematics series 55. 1964. 230 p.
23. Giroud J.P. Equations for calculating the rate of liquid migration through composite lines due to geomembrane defects // Geosynthetics International. 1997. Vol. 4. № 3-4. Pp. 335–348.
24. Bonaparte R., Giroud J.P., Gross B.A. Rates of Leakage through Landfill Liners. Proceedings of Geosynthetics. 1989. Vol. 1. Pp. 18–29.
25. Giroud J.P., Khatami A., Badu-Tweneboah K. Evaluation of the Rate of Leakage through Composite Liners // Geotextiles and Geomembranes. 1989. Vol. 8. № 4. Pp. 337–340.
26. Giroud J.P., Badu-Tweneboah K., Bonaparte R. Rate of Leakage Through a Composite Liner due to Geomembrane Defects // Geotextiles and Geomembranes. 1992. Vol. 11. № 1. Pp. 1–28.
27. Giroud J.P., Badu-Tweneboah K., Soderman K.L. Evaluation of Landfill Lines. Proceedings of the Filth International Conference on Geotextiles, Geomembranes and Related Products. Singapore, September 1994. Vol. 3. Pp. 981–986.
28. Giroud J.P., Touze-Foltz N. Equations for calculating the rate of liquid flow through geomembrane defects of uniform width and finite or infinite length // Geosynthetics International. 2005. Vol. 12. Pp. 191–204.
29. Touze-Foltz N., Rowe R.K., Duquennoi, C. Liquid flow through composite liners due to geomembrane defects: analytical solutions for axi-symmetric and two-dimensional problems // Geosynthetics International. 2000. Vol. 6. № 6. Pp. 455–479.
30. Rowe R.K. Geosynthetics and the Minimization of Contaminant Migration Through Barrier Systems Beneath Solid Waste. Keynote paper, Proceedings of the Sixth International Conference on Geosynthetics. Vol. 1. Atlanta, Georgia, USA, March 1998. Pp. 27–103.

27. Giroud, J.P., Badu-Tweneboah, K., Soderman, K.L. Evaluation of Landfill Lines. Proceedings of the Filth International Conference on Geotextiles, Geomembranes and Related Products. Singapore, September 1994. Vol. 3. Pp. 981–986.
28. Giroud, J.P., Touze-Foltz, N. Equations for calculating the rate of liquid flow through geomembrane defects of uniform width and finite or infinite length. Geosynthetics International. 2005. Vol. 12. Pp. 191–204.
29. Touze-Foltz, N., Rowe, R.K., Duquennoi, C. Liquid Flow Through Composite Liners due to Geomembrane Defects: Analytical Solutions for Axi-Symmetric and Two-Dimensional Problems. Geosynthetics International. 2000. Vol. 6. No. 6. Pp. 455–479.
30. Rowe, R.K. Geosynthetics and the Minimization of Contaminant Migration Through Barrier Systems Beneath Solid Waste. Keynote paper, Proceedings of the Sixth International Conference on Geosynthetics. Atlanta, Georgia, USA, March 1998. Vol. 1. Pp. 27–103.
31. Touze-Foltz, N. Large scale tests for the evaluation of composite liners hydraulic performance: a preliminary study. Proceedings of the Seventh International Waste Management and Landfill Symposium. S. Margherita di Pula, Italy. October 1999. Vol. 3. Pp. 157–164.
32. Touze-Foltz, N., Bannour, H., Barral, C., Stoltz, G. A review of the performance of geosynthetics for environmental protection. Geotextiles and Geomembrane. 2016. Vol. 44(5). Pp. 656–672.
33. Tano, B.F.G., Stoltz, G., Touze-Foltz, N., Dias, D., Olivier, F. A numerical modelling technique for geosynthetics validated on a cavity model test. Geotextiles and Geomembranes. 2017. Vol. 45. Pp 339–349.
34. Sainov, M.P., Zverev, A.O. Workability of high rockfill dam with a polymer face. Magazine of Civil Engineering. 2017. 75(7). Pp. 76–83.
35. Solsky, S.V., Orlova, N.L., Velichko, A.S. Crack self-healing in clay-sement concrete diaphragm of embankment dam. Magazine of Civil Engineering. 2018. 77(1). Pp. 3–12.
36. Sainov, M.P. Analysis of normal operation of a rockfill dam with combination of seepage-control elements: reinforced concrete face and a clay-cement-concrete wall. Magazine of Civil Engineering. 2016. 64(4). Pp. 3–9.
37. Anakhaev, K.N., Aniskin, N.A., Amshokov, B.Kh., Anakhaeva, Kh.K. Calculation of filtration in earth-fill dams with impervious diaphragms. Power technology and Engineering. 51(5). 2018. Pp.42–47.
38. Rasskazov, L.N., Yadgorov, E.Kh., Nikolaev, V.B. Fild observations of soil settlements, displacements, and pore pressure in dams. Power technology and Engineering. No. 51(2). Pp. 1–10.
39. Golovanov, A.I., Sukharev, Yu.I., Solskiy, S.V. Metod rascheta filtratsionnykh kupolov pod dnishchem basseynov sutochnogo regulirovaniya GES [The method of calculating the filtration domes under the bottom of the basins for daily regulation of hydropower]. Prirodoobustroystvo. 2014. Vol. 5. Pp. 37–42. (rus)
40. Yanke, Ye., Emde, F., Lesh, F. Spetsialnyye funktsii [Special Functions]. Moscow: «Nauka». 1977. 342 p. (rus)

31. Touze-Foltz N. Large Scale Tests for the Evaluation of Composite Liners Hydraulic Performance: a Preliminary Study. Proceedings of the Seventh International Waste Management and Landfill Symposium. S. Margherita di Pula, Cagliari, Sardinia, Italy. October 1999. Vol. 3. Pp. 157–164.
32. Touze-Foltz N., Bannour H., Barral C., Stoltz G. A review of the performance of geosynthetics for environmental protection // Geotextiles and Geomembrane. 2016. Vol. 44(5). Pp. 656–672.
33. Tano B.F.G., Stoltz G., Touze-Foltz N., Dias D., Olivier F. A numerical modelling technique for geosynthetics validated on a cavity model test // Geotextiles and Geomembranes. 2017. Vol. 45. Pp. 339–349.
34. Саинов М.П., Зверев А.О. Работоспособность высокой каменно-набросной плотины с полимерным экраном // Инженерно-строительный журнал. 2017. № 7(75). С. 76–83.
35. Сольский С.В., Орлова Н.Л., Величко А.С. Самозалечивания трещин в глиноцементобетонной диафрагме грунтовой плотины // Инженерно-строительный журнал. 2018. № 1(77). С. 3–12.
36. Саинов М.П. Анализ работоспособности каменной плотины с комбинацией противофильтрационных элементов – железобетонного экрана и глиноцементобетонной стены // Инженерно-строительный журнал. 2016. №4 (64). С. 3–9.
37. Anakhaev K.N., Aniskin N.A., Amshokov B.Kh., Anakhaeva Kh.K. Calculation of filtration in earth-fill dams with impervious diaphragms // Power technology and Engineering. № 51(5). 2018. Pp.42–47.
38. Rasskazov L.N., Yadgorov E.Kh., Nikolaev V.B. Fild observations of soil settlements, displacements, and pore pressure in dams // Power technology and Engineering. № 51(2). Pp.1–10.
39. Голованов А.И., Сухарев Ю.И., Сольский С.В. Метод расчета фильтрационных куполов под днищем бассейнов суточного регулирования ГЭС // Природообустройство. 2014. № 5. С. 37–42.
40. Янке Е., Эмде Ф., Леш Ф. Специальные функции. М.: «Наука». 1977. 342 с.

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