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Ultimate state of the support zone of reinforced concrete beams

Предельное состояние приопорной зоны железобетонных балок

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Ключевые слова: бетон; железобетон; арматура; наклонное сечение; поперечная сила; прочность; численное моделирование

Abstract. A brief review of actual methods of calculation of the shear forces is performed; the calculated dependences are analyzed and their estimation is given. The shortcomings of the methods are noted, the main one is the empirical approach to the solution of the problem. The features of destruction of reinforced concrete elements along an inclined section are considered; the mechanism of destruction, on which the actual normative documents are based, is analyzed. A method of design of reinforced concrete elements under the action of shear forces is proposed. The method is developed on the basis of analysis of the stress-strain state of the support zone of a reinforced concrete beam of rectangular cross-section. In the design work, two stages are distinguished. This made it possible to avoid empirical coefficients in the equations. A numerical model was created. Stress distribution in the beam was obtained and analyzed. Conclusions about the actual stress distribution in the compressed zone of concrete above the top of the inclined crack and the actual compressive strength of concrete are made. The proposed design method made it possible to describe the process of destruction of the inclined section in strict accordance with the experimental data. The proposed approach allows to obtain accurate results not only for reinforced concrete elements of simple cross-section (rectangular, T-section), but also for other shapes of sections of a more complex configuration.

Аннотация. Выполнен краткий обзор современных методов расчета на действие поперечных сил; проанализированы расчетные зависимости, дана их оценка. Отмечены недостатки современных методов, главным из которых является эмпирический подход к решению задачи. Рассмотрены особенности разрушения железобетонных элементов по наклонному сечению; проанализирован механизм разрушения, заложенный в основу современных нормативных документов. Предложен метод расчета железобетонных элементов при действии поперечных сил. Метод разработан на основе анализа напряженно-деформированного состояния приопорной зоны железобетонной балки прямоугольного сечения. В работе конструкции было выделены два этапа, что позволило избежать эмпирических коэффициентов в расчетных зависимостях. Создана численная модель, получена и проанализирована картина распределения напряжений в балке. Сделаны выводы о фактическом распределении напряжений в сжатой зоне бетона над вершиной наклонной трещины и о фактической прочности бетона при сжатии. Предложенный метод расчёта позволил описать процесс разрушения по наклонному сечению в строгом соответствии с опытными данными. Предложенный подход позволяет получить точные результаты не только для железобетонных элементов простого поперечного сечения (прямоугольного, таврового), но и для других форм сечений, имеющих более сложную конфигурацию.

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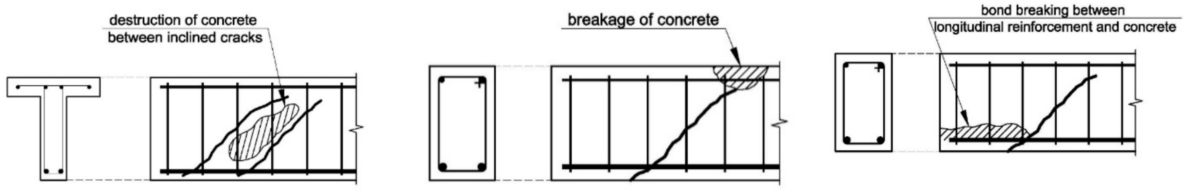
1. Introduction

The issue of calculation of structures under the action of shear forces is still actual, since an exact analytical solution of the problem is not found. The methodologies underlying the major domestic and foreign regulations have similar approaches to addressing this issue. However, the calculation introduces various factors, and this leads to different results. Moreover, all design formulas without exception are based on empirical and semi-empirical dependences [1], which allows to obtain the final result only with a certain degree of accuracy and with a number of limitations. The purpose of this study is to develop a method that does not have empirical dependencies at its core; the method should have an area of application not only for reinforced concrete elements of simple cross-section (rectangular, T- section), but also for other shapes of sections of a more complex configuration.

In the works of various authors, the issues of a general nature, such as the stress-strain state of reinforced concrete elements in the area of shear forces, are considered, as well as a narrower range of issues concerning one of the force factors acting in the inclined section [2, 3] or, for example, the loading conditions [4], type of shear reinforcement [5]. There are works in which analysis of cracking is made [6]. In connection with the development of technological processes in the manufacture, creation and research [7, 8, 9] of new types of concrete, many publications are devoted to the review of high-performance concretes (HPC and UHPC) behavior [10–13] and fiber concrete [11, 13–16] under the action of shear forces. A significant part of the publications includes experimental studies of the elements behavior along an inclined section [17–20]. In addition to these, we can note the work on the study of reinforced concrete elements in various environments, including under the influence of water [21]. There are publications in which biaxial stress state is considered [22]. Special attention is paid to the behavior of composite steel and concrete structures [10, 23]. Let us also mention the publications devoted to improving the methods of computer modeling when considering the features of the calculation of structures under the action of shear forces [24, 25]. Much of the work is aimed at developing theoretical bases of calculation under the action of shear forces [26–28], most publications note the shortcomings of the empirical approach and suggest new ways to resolve this issue [1, 29, 30].

2. Methods

In general, there are three possible variants of destruction of elements along an inclined section – along a concrete strip between inclined cracks, along a compressed zone of concrete above the top of an inclined crack, and along a stretched zone in case of violation of the anchoring or the yield strength in the longitudinal reinforcement is reached (Figure 1).



a) along a concrete strip between inclined cracks

b) along a compressed zone of concrete

c) along a stretched zone of concrete (bond breaking between longitudinal reinforcement and concrete)

Figure1. Schemes of a reinforced concrete element destruction along an inclined section

Let us consider in more detail the approach implemented in the Russian Standard SP 63.13330 “Concrete and won concrete construction. Design requirements” and consider the main design case – the calculation along the inclined section for the action of the shear force. It corresponds to the case of destruction in the compressed zone as a result of the concrete fragmentation over the crack (or its cutoff in the direction of the inclined crack).

The main difficulty in the design is caused by determination of the magnitude of the transverse force received by the concrete part of the cross section of the element. The main parameters, which determine the magnitude of the shear force received by the concrete in an inclined section in accordance with Russian Standard SP 63.13330, are the cross section width of the element b and the working section height h_0, as well as the concrete tensile strength R_{bt}:

$$Q_b = \frac{\varphi_{b2} R_{bt} b h_0^2}{c} \tag{1}$$

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Thus, in the design formula there is only the value of R_{bt}, and the concrete compression strength R_b is not used. This indicates that the inclined section reaches the limit state only due to the formation and opening of the inclined crack. In fact, even after the formation of inclined cracks, reinforced concrete element continues to receive the applied loads and does not immediately lose the load-bearing capacity. In formula (1) this is taken into account by introducing the coefficient φ_{b2}. As a result, the destruction occurs due to the destruction (fragmentation) of compressed concrete over the crack, where the deformation and stress of the material exceed its compression limits. That is, the physical essence of the phenomenon differs from the approach taken in regulatory documents.

It is also worth mentioning the significant similarity in the approaches to solving problems related to the action of shear forces and the punching. The difference is the coefficient equal to 1.5.

Let us mention that the calculation dependences in the problems on the action of shear forces in both domestic and foreign regulations are based on a similar principle – the behavior of a concrete tensile area equal to the projection of the inclined section on the longitudinal axis of the element is considered. The area of concrete that is entered into the design is represented as a product of the working height of the section by its width, with the former being equal to the projection of the inclined crack with an angle of its inclination 45º. In addition, the design formulas in Russian Standard and Eurocode 2 “Design of concrete structures” include empirical coefficients that “transform” the basis of the formula (the product $R_{bt}bh_0$ – for formula (1)) in the final result, which is confirmed by numerous experiments. Additionally, Standard SP 63.13330 takes into account the influence of the distance from the support to the top of the crack on the result of the calculation. In Eurocode 2, this effect is not taken into account, which makes the result less accurate.

In the formulas of Eurocode 2, the value of the concrete tensile strength does not appear explicitly. The value of the shear force received by the concrete section is determined by the formula:

$$V_{Rd,c} = \left[C_{Rd,c}k(100\rho_1 f_{ck})^{\frac{1}{3}} + k_1\sigma_{cp}\right]b_w d, \qquad (2)$$

where $C_{Rd,c}$, k, k_1 – coefficients;

ρ_1 – percentage of the cross-section reinforcement;

σ_{cp} – stresses in the cross section caused by prestressing;

b_w, d – the element cross-section dimensions;

f_{ck} – the cylindrical compression strength of concrete.

Cube root of the value f_{ck} means transition from concrete compression strength to tensile strength that is somewhat different from the table values. It looks close to the Feray formula. It can be directly verified that the expression $f_{ck}^{1/3}$ is an averaged transition function from the normalized cylindrical compression strength to the average tensile strength. For low-strength concretes, the tensile strength is somewhat overestimated, and for high-performance concrete it is underestimated, with an error of up to 12 %.

Let us mention that the formulas for design of limit shear force received by the concrete section and for determination of the load-bearing capacity in the punching calculation in Eurocode 2 coincide completely, unlike Russian Standard.

A significant advantage of the domestic dependencies for determining the limit shear force received by the concrete is the taking into account the value of the projection of the inclined section on the longitudinal axis of the element, which greatly affects the final result (1).

The reason for the simplifications and empirical approach in the calculation dependencies in the regulation documents is a consequence of the main problem in the calculation of the strength of the concrete elements with respect to the action of transverse forces – the number of equations is insufficient for determining all the unknowns. In the general case, the following forces act in the inclined section (Figure 2):

- in the concrete - the longitudinal N_b and the transverse Q_b components;
- in the shear reinforcement – Q_{sw};
- in the tension bars of longitudinal reinforcement – the longitudinal N_s and the transverse Q_s components;

- the forces of bonding on the sides of the inclined crack F_{crc} – the longitudinal N_{crc} and the transverse Q_{crc} components.

In addition, in order to determine the entire complex of forces, it is also necessary to know the height of the compressed section area.

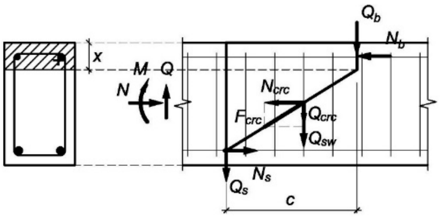


Figure 2. Forces acting in the inclined section

The modern approach in regulatory documents does not take into account the bonding forces along the sides of the inclined crack ($F_{crc}, Q_{crc}, N_{crc}$) and the transverse component in the longitudinal reinforcement (Q_s). In the proposed method, these values, in the strength margin, also will not be taken into account. Therefore, the following unknowns remain:

- in the concrete - the longitudinal N_b and the transverse Q_b components;
- in the shear reinforcement – Q_{sw};
- in the stretched bars of longitudinal reinforcement – the longitudinal component N_s;
- the reactive force at support Q.

In the general case, in order to determine all the unknowns, the design should be based on the joint solution of the system of three equilibrium equations – the sum of the projections on the longitudinal and transverse axes and the sum of the moments of all forces relative to the selected axis. Moreover, the equations will still include the height of the compressed zone as an unknown value. Even in this simplified form, the equilibrium equations alone are not sufficient to fully solve the problem.

3. Results and Discussion

Let us consider the most common case of destruction of the reinforced concrete element from the action of shear forces - as a result of exhaustion of the bearing capacity of the compressed concrete over the crack. We will consider an element that does not have shear reinforcement.

In order for the problem to have a mathematically correct solution, one more additional condition to the existing system of equilibrium equations is necessary. To obtain the missing condition, we single out two stages:

Stage 1. Inclined crack formation. Development and formation of cracks takes place. In the compressed zone of the concrete above the crack, the main force factor is the longitudinal force; the state of the compressed concrete can be considered as conditionally unidirectional and, accordingly, the strength characteristics of the material will accurately correspond to the standard strength values. At this stage, we determine the shear force at which the crack is initiated.

We introduce the assumption that the inclined crack is formed as a result of the separation of the lower part of the concrete (located below the crack) from the upper part of the concrete (located above the crack) – Figure 3.

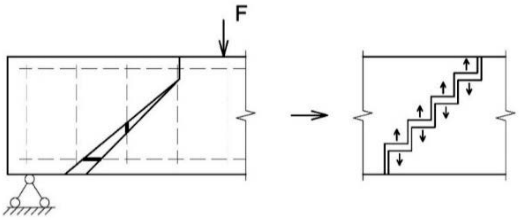


Figure 3. Scheme of the inclined crack development

The presented mechanism of destruction is similar to the destruction at punching. Therefore, the value of the shear force corresponding to the moment of crack development can be determined by a similar formula for design the limit force received by the concrete during punching.

$$Q_1 = R_{bt}A_b, \tag{3}$$

where A_b – is the calculated cross-sectional area defined as:

$$A_b = bc, \tag{4}$$

where c – is the length of the inclined section projection onto the longitudinal axis of the element,

b – is the width of the cross section.

Let us direct our attention to the following features of the structure behavior: after the initiation of an inclined crack and with a further increase in the load, its length varies slightly. In addition, the crack is not strictly inclined. Its upper part, following the trajectory of the main stresses (Figure 4), smoothly approaches the horizontal. This gives a ground to believe that the height of the compressed zone of the concrete after the initiation of an inclined crack remains constant until the loss of the bearing capacity of the element. Such assumption allows to determine the height of the compressed zone of the cross section from the equilibrium conditions at the initial stage of the crack development (in fact, considering the reinforced concrete element at the time before formation of the crack), when the strength characteristics of the concrete have not changed much due to the complex stress state arising in the future as a result of the biaxial compression over the top of the inclined crack.

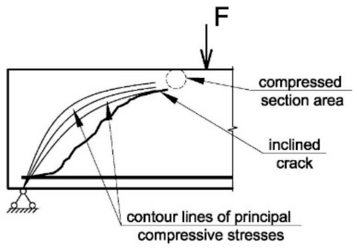


Figure 4. Schematics of the inclined crack propagation along the trajectory of the main compression stresses

The schematics of forces, when considering the equilibrium of the reinforced concrete beam section, are shown in Figure 5. From the equation of equilibrium of moments with respect to C (5), the height of the compressed zone x is determined.

$$\sum M_C = 0; \; -Qc + N_b(h_0 - 0.5x) = 0, \tag{5}$$

from which we obtain:

$$N_b = \frac{Qc}{h_0 - 0.5x} \tag{6}$$

On the other hand, the force in the compressed concrete is represented as

$$N_b = \sigma_b bx = R_b bx \tag{7}$$

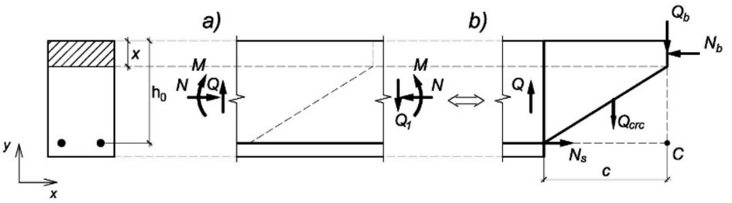
We equate expressions (6) and (7):

$$\frac{Qc}{h_0 - 0.5x} = R_b bx \tag{8}$$

$$0.5x^2 - h_0 x + \frac{Qc}{R_b b} = 0 \tag{9}$$

The value of *x* is determined from the solution of the square equation (9).

Knowing the value of the compressed section area, it is possible to determine from the available equilibrium equations the value of the shear force. When it is reached, the loss of the load-bearing capacity of the element occurs. But before that, we will analyze the stress-strain state of the concrete of the compressed zone above the top of the inclined crack.



a) the beginning of the development of the inclined crack
b) the inclined crack had some propagation
Figure 5. Equilibrium of a section of the reinforced concrete beam with an inclined crack

Stage 2. Inclined crack propagation. In connection with the opening of the inclined crack, the contribution of the bonding forces to the load-bearing capacity of the element decreases, the tangential and normal stresses in the concrete of the element are mainly concentrated within the compressed section zone. Here, a horizontal compression force is observed not only along the axis of the element, but also across it (from the bending plane) – due to the arising expansion of the material (characterized by the Poisson's ratio), which is resisted by the adjacent layers of concrete. Therefore, in the compressed section area above the crack, the stress state of the concrete can be characterized as three-axial (Figure 6).

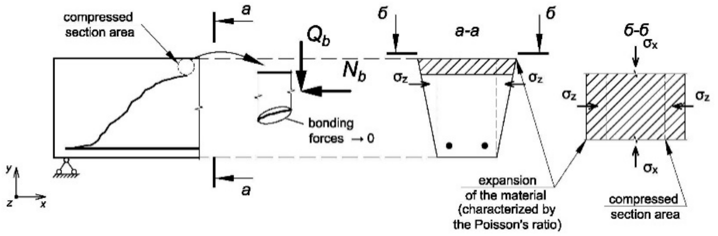
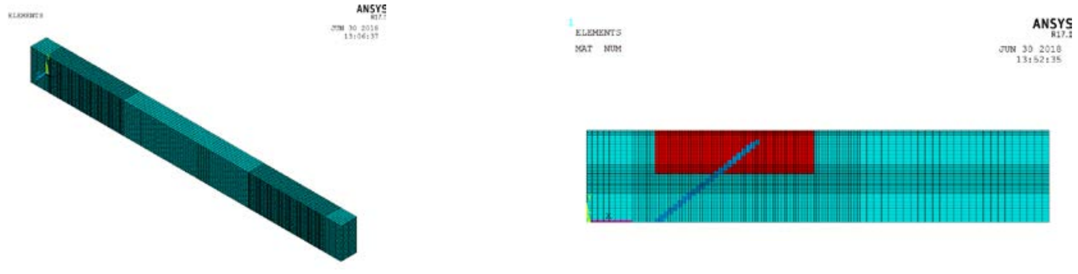


Figure 6. Forces in the compressed concrete area above the crack before the loss of the load-bearing ability of the element

Changing the stress-strain state of the material leads to a significant change in its strength and deformation characteristics. The strength of the concrete will depend on the ratio of the existing forces in the compressed zone (for different directions). To clarify this value, a numerical model is made in the ANSYS software package. The modeling of a reinforced concrete hinged beam with a crack in the support zone was performed. Solid 185-type volumetric finite elements were used in the creation of the model. The reinforced concrete beam of rectangular cross-section having only longitudinal reinforcement in the stretched zone is considered. A general view of the calculation scheme is shown in Figure 7. The inclined crack is modeled by FE (finite elements) of small rigidity. To prevent unintended destruction, nonlinear FE are introduced only in the study zone – around the top of the inclined crack. The nonlinear FE in Figure 8 are marked by red.



a) general view

b) color representation of different types of elements (turquoise – concrete with linear characteristics, red – concrete with non-linear characteristics)

Figure 7. Computational model of a reinforced concrete beam with an inclined crack

The results of calculations are given for the supporting beam fragment with nonlinear elements at the top of the critical crack in Figures 8-10.

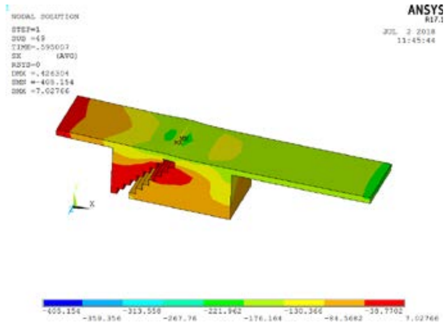


Figure 8. Longitudinal stresses σ_x, MPa×10^{-1}

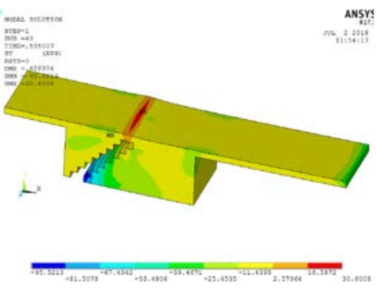


Figure 9. Vertical stresses σ_y, MPa×10^{-1}

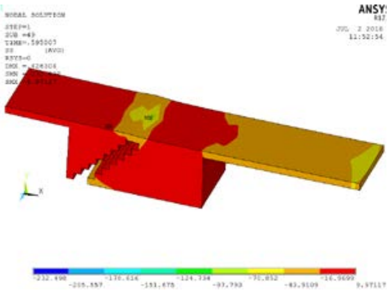


Figure 10. Horizontal stresses (from bending plane) σ_z, MPa×10^{-1}

Having obtained a stress distribution at triaxial stress state and referring to the data of [31], it is possible to determine the actual value of the concrete strength in case of compression from the surface limiting the area of strong resistance (Figure 11). A more detailed analysis of the stress-strain state shows that the vertical stresses concentrated within the height of the compressed zone above the crack quickly changes along the length of the beam and rapidly decreases along the height of the section. In addition, their value is much lower (up to ten times) compared to the horizontal longitudinal stress. At the same time, the horizontal stresses across the beam make up almost half of the longitudinal compression stresses. This allows us to neglect the vertical stresses and to classify the condition of the concrete above the crack as conditionally biaxial. This assumption simplifies the problem, allowing you to refer to the results of the study of concrete in a biaxial stress state. According to data of [31], the strength limit with obtained the ratio between principal stresses σ_1 and σ_2 rises up to 1.5 times (Figure12). This means that the maximum permissible transverse load will be 1.5 times higher than the one for the first stage of the element operation – the case of crack formation. These data correspond to experimental studies and materials of Russian Standard. Now, knowing the actual compression strength of the concrete and having the height of the compressed zone, calculated at the first stage, it is possible to determine from the equation of equilibrium of moments (5) the limiting shear force received by the concrete element.

$$Q_{b,ult} = \frac{N_b(h_0 - 0.5x)}{c}, \tag{10}$$

where

$$N_b = \sigma_b bx = 1.5R_b bx \tag{11}$$

The value *x* in formulas (10) and (11) should be taken equal to the value determined at the first stage of operation of the reinforced concrete element from equation (9).

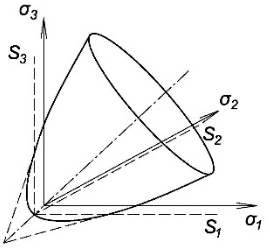


Figure 11. Surface limiting the area of strong resistance at triaxial stress state according to [31]

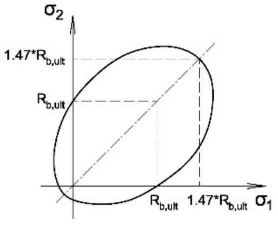


Figure 12. Curve limiting the area of strong resistance at biaxial stress state according to [31]

The proposed method of calculation allows to describe the process of destruction of the inclined section in strict accordance with the experimental data, in contrast to the methods adopted in the current regulatory documents. For rectangular sections, the proposed approach does not provide any significant advantages, except for the correct description of the physics of the phenomenon. But for cross-sections, different from rectangular ones (T-shaped, I-beam, round and generally arbitrary symmetric sections), the proposed approach allows to obtain much more accurate results compared to the known methods.

In contrast to the classical approach of the stress-strain state of concrete over the top of an inclined crack, realized in the calculated dependences in [1, 30], in this paper, a revision of the material RSS is made, which became a prerequisite for the implementation of this method. Of course, the proposed method needs some refinement and it is not the only alternative to modern normative documents. For a long time both semi-empirical methods, such as the modified theory of compression fields [29], and methods approaching the complete rejection of empirical dependences [1, 30] have been proposed. However, most of the proposed methods have some scope of applicability, or have excessively complex dependencies in relation to manual calculation. These methods are based on approaches different from those proposed in this paper.

4. Conclusions

1. The existing methods for design structures under the action of shear forces are analyzed. The shortcomings are noted, the main one being the approach based on empirical dependencies and the inability to use them to determine the ultimate shear force in concrete for an arbitrary cross section.

2. A method of design of reinforced concrete elements under the action of shear forces is proposed. It is based on the strict principles of mechanics. There are no empirical coefficients in the calculated dependencies. The method is applicable not only to reinforced concrete elements of simple cross-section (rectangular, T- section), but also for other shapes of sections of a more complex configuration.

3. A numerical model of a reinforced concrete element of a rectangular cross-section with an inclined crack under the action of shear forces is created. The analysis of the stress-strain state of the support zone of the beam is performed. A numerical substantiation of the accepted hypotheses and assumptions is given.

4. A good agreement between the results of numerical calculations and the results of regulatory documents for rectangular cross sections is obtained. The discrepancy is 5 %.

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