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Additional load on barrel vaults of architectural monuments

Дополнительная нагрузка на коробовый свод в памятнике архитектуры

A-Kh.B. Kaldar-ool*,
V.V. Babanov,
St. Petersburg State University of Architecture and Civil Engineering, St. Petersburg, Russia
B.M. Allahverdov,
Petersburg State Transport University, St. Petersburg, Russia
S.S. Saaya,
Tuvan State University, Kyzyl, Republic of Tuva, Russia

Соискатель А.Б. Калдар-оол*,
канд. техн. наук, доцент В.В. Бабанов,
Санкт-Петербургский государственный архитектурно-строительный университет, Санкт-Петербург, Россия
канд. техн. наук, доцент Б.М. Аллахвердов,
Петербургский государственный университет путей сообщения Императора Александра I, Санкт-Петербург, Россия
канд. техн. наук, доцент С.С. Саая,
Тувинский государственный университет, г. Кызыл, Республика Тыва, Россия

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Ключевые слова: коробовый свод; метод сил; прочность; напряжения; деформация; модуль упругости; нагрузка

Abstract. In old buildings, which are architectural monuments, the stone three-centered barrel vaults and arches have been often used. In case of reconstruction of old buildings, it becomes necessary to increase the load on these elements. Calculation of additional load is based on instrumental examination of the vaults. While creating an analytical model of a barrel vault, it is necessary to use the experimental results of determination of the vault geometric parameters and material strength characteristics. The vault bearing capacity determined on base of analytical estimation is confirmed by similar calculations done by means of software packages, as well as by the data obtained by means of real loading of the vault. The proposed procedure for determination of additional load onto barrel vault may be also used in designing of similar architectural monuments.

Аннотация. В старинных зданиях, являющиеся памятниками архитектуры, часто в виде несущих элементов использовались каменные трехцентровые коробовые своды и арки. При реконструкции сооружений появляется необходимость увеличения нагрузки на эти элементы. Определение величины дополнительной нагрузки основано на результатах инструментального обследования сводов. При создании расчетной схемы модели коробового свода необходимо использовать результаты экспериментальных исследований: определение геометрических параметров свода и прочностных характеристик материалов. Несущая способность свода, определенная по результатам аналитического расчета, подтверждается аналогичными расчетами с использованием программных комплексов, а также данными, полученными при нагружении реального свода. Предложенная методика определения возможной дополнительной нагрузки на коробовый свод может быть использована при расчете аналогичных памятников архитектуры.

1. Introduction

Barrel vaults were widely used for construction of unique architectural buildings in XVIII to XIX century. They are often found in large-span inter-floor ceilings and covers of civil and industrial buildings, in church architecture, in the ceilings above basements, passages, corridors of low-rise residential and public buildings, as well as in the structures of stairwells and as span structures of bridges.

Cross-section of barrel vault has a shape of curve formed on base of several centers. There are three-centered, five-centered and multicentral barrel vaults. In this article, the symmetrical three-centered barrel vaults consisting of three parts of circles of two different radii are described (Figure 1).

In contrast to elliptic vaults, the generatrix of barrel vaults is a curve consisting of several circular curve segments. These segments have common first-order derivative in their point of compound curvature; therefore, the centers of neighboring circles must be on the same line.

Until recently, axis plotting and approximate calculation of barrel vaults were carried out by various graphical methods.

The best-known works on calculation of arches and vaults of different shapes are [1–7] and experimental studies of stone vaults and arch structures are [8–20]. In these researches, methods of calculation of the stress-strain state for different shapes of stone arches and vaults have been developed, and behavior of vaults has been studied on base of numerical models.

Usually, methods of calculation have been worked out for parabolic vaults [21], but they are scantily suitable for designing of barrel vault axis configuration. These methods have just a historical value and have become the first attempt to solve the problem, but it has created a base for development of more perfect methods. In this case, the vault was represented as twice statically indeterminable single-hinged arch, which calculation is simplified due to symmetry properties [22].

Today the existing method of barrel vaults' calculation is usually based on graphic-analytical method for development of statically determined design model. No other methods were found in the cited literature.

In this regard, it is necessary to carry out more precise calculations, taking into account the specific geometry of barrel arches and the real extent of static indeterminacy of the vault analytical model.

It is obvious that development of methods permitting to estimate technical condition of the barrel vaults taking into account their construction technology and history of loading is of immediate interest.

The purpose of this article is to create a methodology for determination of the value of additional load on the vault during its further upkeeping.

To determine the value of the additional load it is necessary to solve several related tasks:

1. Creation of analytical expressions describing the barrel vault axis on base of the results of measuring the real vault geometry.

2. Static calculation of the vault taking into account all internal stresses.

3. Primary structure loading condition.

4. Obtaining the strength characteristics of the barrel vault brickwork on base of the results of experimental tests.

5. Determination of deformation characteristic (deformation modulus) of the vault brickwork taking into account changes in the material within the structure service period.

6. Determination of the additional load value. Experimental test.

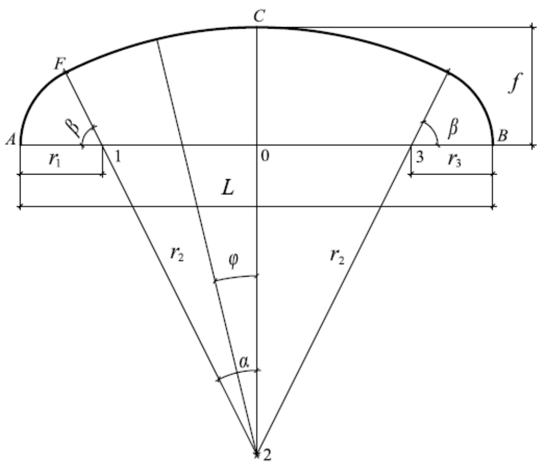

Figure 1. The axis of three-centered barrel vault, where L is the span of vault, f is the rise of vault, r_1 is small radius, r_2 is large radius.

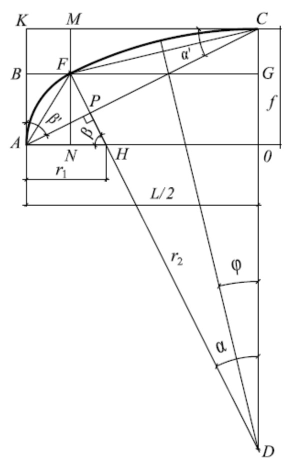

Figure 2. Method of constructing the three-centered barrel vault.

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2. Methods

1. Creation of equations describing the shape of vault (arch) axis on base of the results of measuring the real span L and rise of vault f (Figure 2).

To design the shape of the arch axis, it is necessary to determine the values of small and large radii r_1 and r_2. If $A0 = L/2$, $C0 = f$ the rectangle is build and angles are measured $\alpha' = \arctan\left(\frac{2f}{L}\right)$ and $\beta' = \arctan\left(\frac{L}{2f}\right)$. The triangles *KCA*, *GDF* and *CDP* are similar, as their sides are proportional and the angles between them are the same; in this case $\alpha = \alpha'$, $\beta = \beta'$ (Figure 2).

Let us make a system of equations:

$$\begin{cases} MF + FN = f = r_1 \cdot \sin\beta + (1-\cos\alpha)\cdot r_2 \\ BF + FG = L/2 = (1-\cos\beta)\cdot r_1 + \sin\alpha \cdot r_2. \end{cases} \tag{1}$$

Transformation matrix and output matrix are:

$$A = \begin{pmatrix} \sin\beta & 1-\cos\alpha \\ 1-\cos\beta & \sin\alpha \end{pmatrix}, \quad D = \begin{pmatrix} f \\ L/2 \end{pmatrix}. \tag{2}$$

Matrix equality of known radii is:

$$R = A^{-1}\cdot D = \begin{pmatrix} r_1 \\ r_2 \end{pmatrix}. \tag{3}$$

The equations of the vault axis as a function of current angle φ in the parametric form $y(\varphi) = f(x(\varphi))$ are:

$$x(\varphi) = \left| \begin{array}{l} r_2 \cdot \sin(-\alpha) + r_1 \cdot \cos\ \beta - r_1 \cdot \cos\left(\pi/2 + \varphi\right), if\ -\pi/2 < \varphi \le -\alpha \\ r_2 \cdot \sin\varphi, if\ \ \varphi \le |\alpha| \\ r_2 \cdot \sin\alpha - r_1 \cdot \cos\beta + r_1 \cdot \cos\left(\pi/2 - \varphi\right), if\ \ \alpha < \varphi \le \pi/2 \end{array} \right. \tag{4}$$

$$y(\varphi) = \left| \begin{array}{l} r_1 \cdot \sin\left(\varphi + \pi/2\right), if\ \ -\pi/2 \le \varphi < -\alpha \\ \left[\left(r_2 \cdot (\cos\varphi - 1)\right) + f\right], if\ \ -\alpha < \varphi \le \alpha \\ r_1 \cdot \sin\left(\pi/2 - \varphi\right), if\ \ \alpha < \varphi \le \pi/2 \end{array} \right. \tag{5}$$

Thus, for analytical designing of the axis of spacial curve of symmetric vault, it is sufficient to know the span L and the rise f of the vault, as well as the design requirement: the point of contact of two curves composing the axis of the spatial curve should be located at the intersection of the bisectrixes of the triangle formed by L and f values.

Each vault may be presented as a system of elementary arches forming the shape of the vault and bearing their load [24]. The vault analytic model may be approximately presented as a system of independent parallel arches. For these reasons, for the barrel vault calculation, we adopt a simplified calculation model in the form of no-hinged arch under the following boundary conditions:

$$\text{if } z=-\frac{L}{2} \text{ (left abutment) } y\left(-\frac{L}{2}\right)=0;\ \theta=y'\left(-\frac{L}{2}\right)=0;$$

$$\text{if } z=\frac{L}{2} \text{ (right abutment) } y\left(\frac{L}{2}\right)=0;\ \theta=y'\left(\frac{L}{2}\right)=0.$$

In the investigated three-centered arch on the segments with different radii there are gaps in the second derivative of the geometric axis of the arch. In this case, the internal forces (bending moment, etc.) have no breaks, since the moment is not associated with the second derivative of the geometry, but depends on the change in curvature.

First derivative:

$$y'(\phi)=\left|\begin{array}{l}(-\tan(\phi)), if \quad -\frac{\pi}{2}\le\phi<-\alpha \\ (-\tan(\phi)), if \quad -\alpha<\phi\le\alpha \\ (-\tan(\phi)), if \quad \alpha<\phi\le\frac{\pi}{2}\end{array}\right. \tag{6}$$

Second derivative:

$$y''(\phi)=\left|\begin{array}{l}-\dfrac{1}{r_1\cdot\cos(\phi)^3}, if \quad -\frac{\pi}{2}\le\phi<-\alpha \\ -\dfrac{1}{r_1\cdot\cos(\phi)^3}, if \quad -\alpha<\phi\le\alpha \\ -\dfrac{1}{r_1\cdot\cos(\phi)^3}, if \quad \alpha<\phi\le\frac{\pi}{2}\end{array}\right. \tag{7}$$

Since the moment diagram has no gaps and the stresses in the arch are directly proportional to the change in the axis curvature, the stresses practically do not change on the butt sections with different radii of curvature.

For flat segments, the moment of inertia of the cross section may be determined by the general formula for the straight rod:

$$J=\frac{b\cdot h^3}{12}. \tag{8}$$

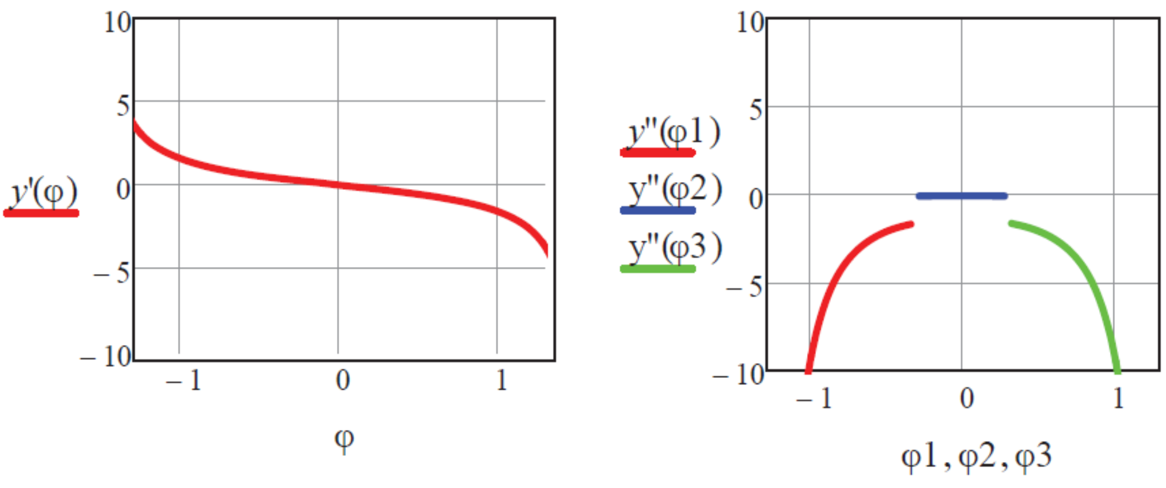

Figure 3. Diagrams of derivatives
a) diagram of the first derivative: function is continuous;
b) diagram of the second derivative: function has the gaps at -α and α.

In the steep part of the arches, where the neutral line is distinctly displaced, the moment of inertia of the section should be determined by the formula:

$$J' = \int_{-h/2}^{h/2} \frac{y^2 \cdot b}{1 + y/r} dy. \tag{9}$$

2. Static calculation.

The static calculation of triply statically indeterminate arch is performed by method of forces using Maxwell-Moore formula with a full set of internal forces for determining changes (bending moment, longitudinal and transverse forces) [24].

$$\begin{aligned} \delta_{ik} = \sum\int \frac{M_i(\varphi)\cdot M_k(\varphi)\cdot rd\varphi}{EA\cdot r^2} + \sum\int \frac{M_i(\varphi)\cdot M_k(\varphi)\cdot rd\varphi}{EJ_r} + \sum\int \frac{N_i(\varphi)\cdot N_k(\varphi)\cdot rd\varphi}{EA} + \\ + \mu\cdot\left[\sum \frac{Q_i(\varphi)\cdot Q_k(\varphi)\cdot rd\varphi}{GA}\right] + \sum\int \frac{M_k(\varphi)\cdot N_i(\varphi)\cdot r_1 d\varphi}{EA\cdot r_1} + \sum\int \frac{M_i(\varphi)\cdot N_k(\varphi) d\varphi}{EA}. \end{aligned} \tag{10}$$

$$\begin{aligned} \Delta_{iq} = \sum\int \frac{M_i(\varphi)\cdot M_q(\varphi)\cdot rd\varphi}{EA\cdot r^2} + \sum\int \frac{M_i(\varphi)\cdot M_q(\varphi)\cdot rd\varphi}{EJ_r} + \sum\int \frac{N_i(\varphi)\cdot N_q(\varphi)\cdot rd\varphi}{EA} + \\ + \mu\cdot\left[\sum\int \frac{Q_i(\varphi)\cdot Q_q(\varphi)\cdot rd\varphi}{GA}\right] + \sum\int \frac{N_i(\varphi)\cdot M_q(\varphi)\cdot r_1 d\varphi}{EA\cdot r_1} + \sum\int \frac{M_i(\varphi)\cdot N_q(\varphi) d\varphi}{EA}. \end{aligned} \tag{11}$$

3. General case of loading is described by four types of loads: self-weight of vault, brickwork back filling weight, evenly distributed load on top of the backfilling (weight of the floor), and different types of temporary loads F (weight of people and equipment) (Figure 4).

4. When examining the bearing capacity of the above-basement vaulted ceiling of Ethnographic Museum building in St. Petersburg, the question has arisen about the strength characteristics of the brickwork. More than 100 years have passed since its construction (the building was built in 1911). The vaulted ceiling has a three-centered arc shape.

Visual inspection of the above-basement vaulted ceiling has proved that the overall structure condition is satisfactory (usable) (Figure 5).

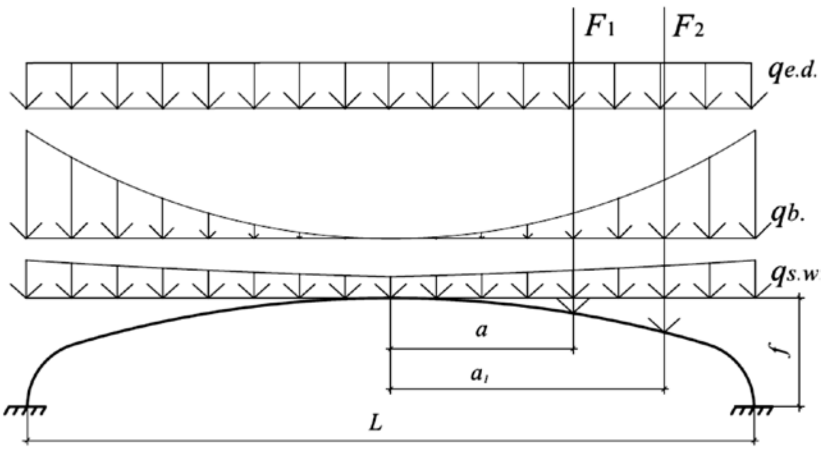

Figure 4. Actual loads on the vault: $q_{s.w.}$ is the self-weight of vault; $q_{b.}$ is the brickwork back filling weight; $q_{e.d.}$ is the load from structures on the top of vault; F – different types of temporary loads.

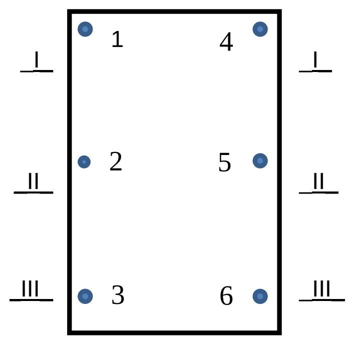

Figure 5. Location of vault materials testing points: UT test points on vault lower surface.

The arch bond strength was determined by the following methods:

- The strength of the brick and arch bond mortar on the upper surface of the vault according to Russian State Standards (GOST) 8462-85 [23] and 5802-86 [24] is determined by the destructive method. The selected samples have been tested in the laboratory of Saint Petersburg State University of Architecture and Civil Engineering.

- The strength of brickwork on the lower surface of the vault has been determined by means of ultrasonic method using UK-14P device.

Determination of the brickwork strength has been carried out by means of ultrasonic method using UK-14P device as follows:

1. brick strength limit (at the base on the surface of 100 mm)

$$R_1 = 7.5201 + 0.008\,\upsilon_1 \text{ MPa};$$

2. the limit strength of the arch bond mortar (at the base of 40 mm)

$$R_2 = 7.3975 - 0.0068\upsilon_2 \text{ MPa}.$$

The brickwork strength has been determined by the formula of Prof. L.I. Onishchik [25]:

$$R = 0.5R_u - 0.5AR_1(1 - a/(b + 0.5R_2/R_1))\gamma\,.$$

in this case:

$$A = (9.81 + R_1)/(9.81\,m + n\,R_1);$$

$$a = 0.2;\ b = 0.3;\ m = 1.25;\ n = 3.$$

The value of correction factor γ:

– according to the table: $R_{1,m} = 10.4$ MPa, $R_{2,1} = 0.04R_{1,m} = 0.4$ MPa $< R_{2,m} = 2.14$ MPa,

thus $\gamma = 1$.

- In accordance with impact resilience using Schmidt hammer, according to Russian State Standard (GOST) 22690-88 [26].

To determine the brickwork load-carrying ability, the strength properties of brickwork have been examined. As a result of 12 tests, $R_1,\ R_2, ...,\ R_{12}$ sample data of brick and mortar of which the brickwork is made have been obtained.

Statistical processing of test results.

To estimate the mathematical expectation of normal distribution the following function (arithmetic mean of observed values of random variable) is used:

$$R_m = \frac{1}{n}(R_1 + R_2 + ... + R_n)\,,$$

where R_m is the mean value of the limit strength according to test results;

n is the number of the test sample;

RMS deviation of the observed values of R_i factor from their mean value R_m has been determined by the formula:

$$s = \sqrt{\frac{\sum_{i=1}^{n}(R_i - R_m)^2}{n}}\,;$$

Using Student's distribution, the confidence interval of the brick and mortar strength has been determined:

$$R_{1,2} = R_m - t\frac{S_q}{\sqrt{n}};$$

where $t =$ 2.23 is Student's coefficient at $n =$ 12 and the confidence interval with a probability of 95 %.

5. Definition of the stiffness (deformation) characteristics of vault brickwork.

Before making calculations, it is necessary to determine the stiffness (deformation) characteristics of vault brickwork.

For this purpose, it is necessary to know the initial modulus of elasticity E_0, the change of the modulus of deformation in time, taking into account the material plasticity, which in its turn results in a change in the initial modulus of deformation.

To determine the initial deformation modulus of the material, it is necessary to use the results presented in the work of N.S. Khamidzhanov. As a result of the step-by-step regression analysis, he has got an analytical dependence that relates the initial modulus of elasticity E_0 to the brick and mortar grade, which match with a probability of 0.99 the experimental data received by the author by F Fisher criterion and t Student's criterion. In our calculations, it is necessary to follow the linear model proposed by N.S. Khamidzhanov:

$$E_0 = 187R_{brick} + 173.2R_{mortar} + 5766, \tag{12}$$

To determine the value of the initial modulus of elasticity it is also possible to use the generally accepted formula of Professor L.I. Onishchik that is in force in normative documents and in the Russian Set of Rules SP 15.13330.2012 [27, 28]:

$$E_0 = \alpha_1 \cdot R_u, \tag{13}$$

where α_1 is brickwork elastic characteristic determined experimentally by L.I. Onishchik. The value of α_1 elastic characteristics for non-reinforced brickwork with the strength of the brick and mortar is accepted by means of interpolation.

$R_u = R \cdot k$ is temporary brickwork resistance (average strength limit);

R is calculated brickwork resistance;

k is the material safety factor, for brickwork it is 2.0.

Within the linear creeping, it is possible to use the concept of modulus of long-term deformation, which is determined by formula proposed by A.M. Rozenblyumas. It allows for validity of Hooke law for linear time-dependent deformations [29], but with a modified value of the initial deformation modulus:

$$E_t = \frac{E_0}{1+\phi_t}, \tag{14}$$

where φ_t is the brickwork creep characteristic, which depends on time.

To determine the brickwork creep characteristic φ_t, we have adopted the exponential function in accordance with the data given in Eurocode 1992-1 [30]:

$$\varphi_t = \varphi_0 \cdot \left(\frac{T}{T+\beta_n}\right)^{0.3}, \tag{15}$$

where $\varphi_0 = 1.5$ is theoretical creep factor adopted by Eurocode 1992-1-1 (how many times the deformation increases over the infinite duration of the load action).

$\beta_n = 657.82$ is a coefficient depending on relative humidity and theoretical size of the element (adopted for materials given in documentation of "Lira-9.6" software complex).

T is the number of days (the age of material), after which it is necessary to take into account the effect of creep.

To account for possible creep according to the Russian Set of Rules SP 15.13330.2012, the adopted coefficient of creep is $\nu = 2.2$ [28]. The deformation modulus in this case is determined by the formula $E_t = \frac{E_0}{2.2}$ [27, 28].

In accordance with the Russian Set of Rules SP 15.13330.2012, when determining the deformation of masonry in case of stress calculation in statically indeterminate systems, the structures' stiffness characteristic is specified by the formula (accounting for material plastic properties) [27, 28]:

$$E = 0.8 \cdot E_t . \tag{16}$$

6. Determination of internal forces and stresses in barrel vault.

The barrel vault calculation was done using analytical method in "Mathcad 14" software system and for checking the finite element method was used in "Lira-Windows" software complex. In this case the vault surface deformations accumulated over the vault lifetime and found out by means of geodetic survey were taken into account.

There have been chosen the variants of elements that can provide triaxial calculation of the vault composed of 3D finite elements (Figure 6) and the shell (Figure 7), as well as the design in form of the arch composed of the rod-type finite elements (Figure 8). It is shown that the vault stressed state is described by the rod-type model without significant errors.

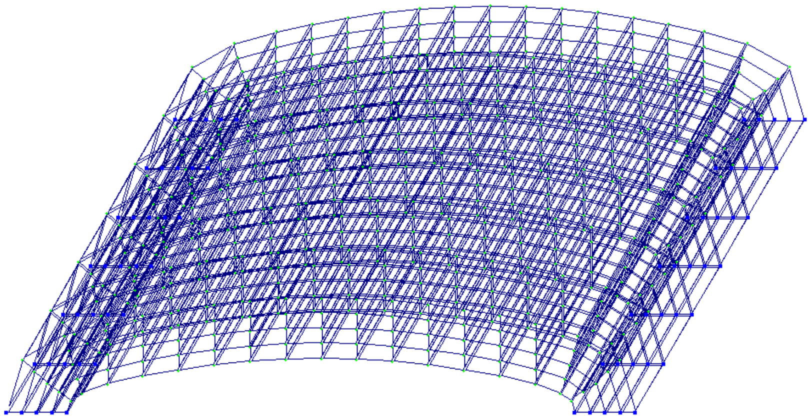

Figure 6. 3D Volumetric finite elements model of vault.

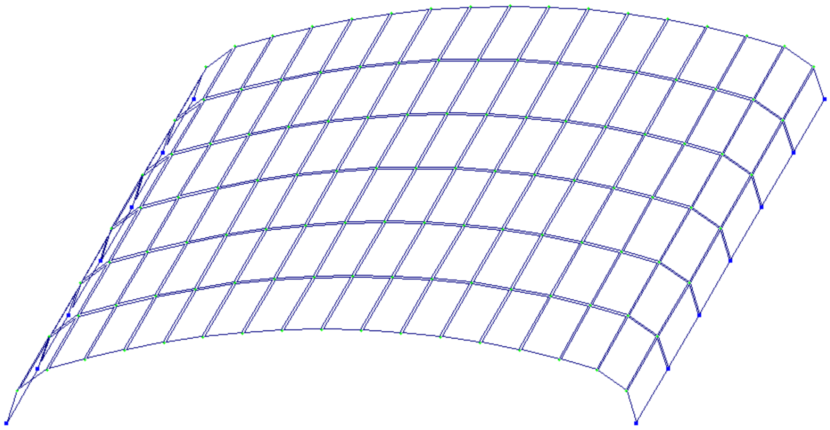

Figure 7. 3D Shell model of vault.

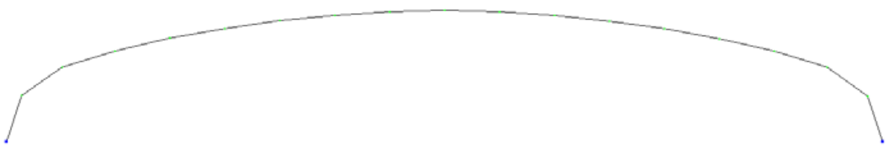

Figure 8. Flat arch design model of vault.

Determination of additional load onto vault is based on the results of calculations. Taking into account estimation of the value of the additional load onto the vault $(F_1 + F_2 = 10\ \text{kN/m})$, obtained by numerical calculation of the vault as an arch and spatial elements, it is possible to set this value $F_{additional\ load}$ for experimental research of the vault.

To identify the stressed state under load mechanical strain gauges with a base of 100 mm were used. Barrel vault vertical displacement was measured using deflectometer.

Loading was done using metal loads of 0.5 kN, placed on the vault to create a uniformly distributed load along a strip. For each stage of loading, the value of distributed load was taken to be equal to $\frac{1}{10} F^{\text{destroying}} \approx 2$ kN/m. After five steps the maximum load of 10 kN/m was achieved. Loading was begun along the longitudinal section of the vault from both sides to the centre. At each stage after loading instrument readings were taken.

3. Results and Discussion

Table 1. The results of brick and mortar strength determination by means of nondestructive ultrasonic control method.

Item No.	t_1, microsecond	ϑ_1, m/s	R_1, MPa	$t2$, microsecond	ϑ_1, m/s	R_2, MPa	R_i, MPa	$p_i = \frac{R_i}{R_{min}}$	p_iR_i	$S_i = R_i - R_{\text{medium}}$
1	32	3125	10	47	851	1.78	1.14	1.00	1.14	-0.053
2	31	3226	10.1	54	741	2.51	1.25	1.10	1.37	0.056
3	36	2778	9.74	51	784	2.22	1.19	1.04	1.24	-0.006
4	29	3448	10.3	48	833	1.90	1.17	1.03	1.21	-0.019
5	30	3333	10.2	53	755	2.42	1.24	1.09	1.35	0.049
6	34	2941	9.87	49	816	2.01	1.17	1.02	1.19	-0.028
Σ	6.28	7.50								

$$R_{med.} = \sum p_i R_i / \sum p_i = 1.19\ \text{MPa}.$$

$$\Delta R = \sqrt{\frac{\sum p_i s_i^2}{(n-1)\sum p_i}} = 0.12\ \text{MPa}.$$

The brickwork design strength is:

$$R = R_{medium} - \Delta R = 1.07\ \text{MPa or } 10.92\ \text{kg/cm}^2;$$

at medium values:

brick grade $M_{brick} = 102$; $R_{1,\,medium} = 10.03$ MPa;

mortar grade $M_{mortar} = 22$; $R_{2,\,mortar} = 2.14$ MPa.

Table 2. Results of tests of brick and mortar strength by means of rebound hammer method with the help of Schmidt hammer.

Sample number	1	2	3	4	5	6	7	8	9	10	11	12
Brick strength, MPa	9.8	1.0	9.9	9.7	1.01	9.6	9.9	1.03	9.9	1.0	1.08	1.07
Mortar strength, MPa	2.1	2.7	1.9	1.8	2.2	3.1	3.0	2.6	2.2	2.9	1.8	2.9

Brick breaking strength is $R_1 = 9.8$ MPa; mortar tensile strength is $R_2 = 2.1$ MPa.

Table 3 compares the moments of inertia taking into account the effect of *J* axis curvature (formula (8)) and *J'* neutral axis of the vault according to the formula (9).

Table 3. Comparison of values of moments of inertia.

Steep part of vault			Flat part of vault		
J_1	J'_1	%	J_2	J'_2	%
0.0629	0.0704	10	0.0486	0.0486	0

The difference between the results in the estimation of influence of the neutral axis shift in the flat part of the vault is 0 %, and in the steep part is 10 %. According to the results of the study it should be noted that the offset of neutral line may be neglected.

According to the obtained values of brick and mortar strength characteristics, the calculated brickwork resistance according to the Russian Set of Rules SP 15.13330.2012 is determined and compared with L.I. Onishchik formula (Table 4) [27, 28].

Table 4. Comparison of values of brickwork design strength.

Brickwork design strength	According to the Russian Set of Rules SP 15.13330.2012	According to L.I. Onishchik formula			%
		Rebound hammer method	%	Ultrasound control	
R (MPa)	1.126	0.915	18.7	1.07	4.97

Evaluation of the design resistance of the brickwork by an elastic rebound according to the formula of Professor L.I. Onishchik and the Russian Set of Rules SP 15.13330.2012 differ by 18.7 %, in the ultrasonic inspection at 4.97 %. According to the Set of rules the strength of brickwork gets highest properties.

Table 5. Comparison of values of initial deformation modulus.

Initial deformation modulus	According to the Russian Set of Rules SP 15.13330.2012	According to N.S. Hamidzhanov formula	%
E_0 (MPa)	2252	2883	21.88

The difference between the values of the initial modulus of elasticity according to N.S. Khamidzhanov formula and the Russian Set of Rules SP 15.13330.2012 is 21.88 % (Table 5). Taking into account that in the research [30] various correlations between brick and the mortar strength are close to those obtained in our tests, in further calculations we take into account the initial modulus of elasticity equal to E_0 = 2883 MPa.

Table 6. Estimation of values of modulus of long-term deformation.

Modulus of long-term deformation	According to the Russian Set of Rules SP 15.13330.2012	According to A.M. Rozenblyumas formula	%
E_t (MPa)	1023.63	1156	11.45

Estimation of the values of modulus of long-term deformation calculated according to A.M. Rozenblyumas formula and according to the Russian Set of Rules SP 15.13330.2012 gives a difference of 11.45 %. In our calculation the adopted value of the modulus is taken according to A.M. Rozenblyumas data.

In accordance with the Russian Set of Rules SP 15.13330.2012, when determining the brickwork deformation in case of determining the strains in statically indeterminate systems, the stiffness properties of structures are taken in accordance with the formula:

$$E = 0.8 \cdot E_t.$$

Finally the modulus of deformation is adopted to be equal to $E =$ 925.55 MPa.

According to the results of calculation, the normal stress and vertical displacement comparison was performed for E_0 initial modulus and E modulus of deformation in the Tables 7 and 8.

Table 7. Comparison of stresses at E_0 initial modulus of deformation and E modulus of deformation (coordinate x = 3.342 m).

Stresses	E_0 vault (Lira)	E vault (Lira)	%	E_0 arch (Lira)	E arch (Lira)	%	E_0 Mathcad	E Mathcad	%
σ, (MPa)	-0.4755	-0.4755	0	-0.5305	-0.5305	0	-0.53357	-0.52316	1.95

Table 7 shows that the change in the modulus of deformation has almost no effect on the stress values.

Table 8. Comparison of displacements at different moduli of deformation (coordinate x = 0 m).

Displace-ment	E_0 vault (Lira)	E vault (Lira)	%	E_0 arch (Lira)	E arch (Lira)	%	E_0 Mathcad	E Mathcad	%
f, (mm)	0.9147	2.8494	67.9	0.9674	3.0136	67.9	1.19	3.71	67.92

From the Table 8 it is clear, that the change of modulus of deformation significantly affect the magnitude of displacements.

So for more than 100 years of service life of barrel vault structure, which description is given below, due to creep and plasticity, the deflections have grown by more than half.

Distribution of normal stresses in Mathcad and Lira:

The results of analytical and numerical methods of calculation are generally consistent.

Taking into account the spatial service of the vault analyzed by means of finite element method with volumetric elements, as well as in form of plates (compared to an arch analytic design), it proves that the calculated stress values are almost independent of the analytical solution.

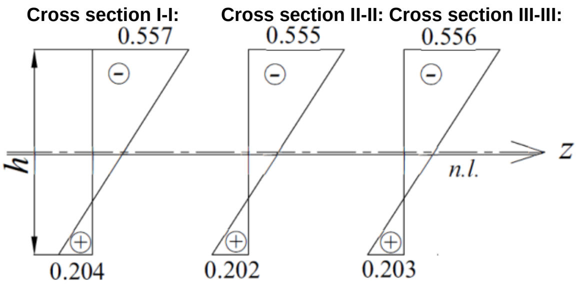

Figure 9. Distribution of normal stresses σ (MPa) in the cross-section near the vault skew-back (x = 3.342 m) under the action of constant loads in Mathcad.

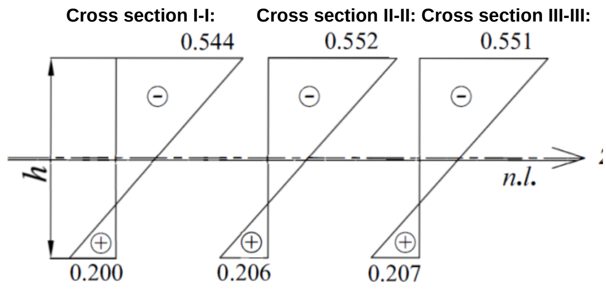

Figure 10. Distribution of normal stresses σ (MPa) in the cross-section near the vault skew-back (x = 3.342 m) under the action of constant loads in Lira.

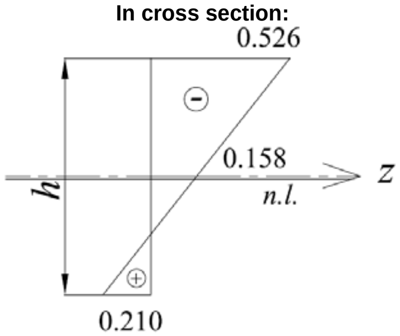

Figure 11. Distribution of normal stresses σ (MPa) in the cross-section near the vault skew-back (x=3.342 m) under the action of constant loads, three-dimensional analysis of vault in form of a shell in Lira.

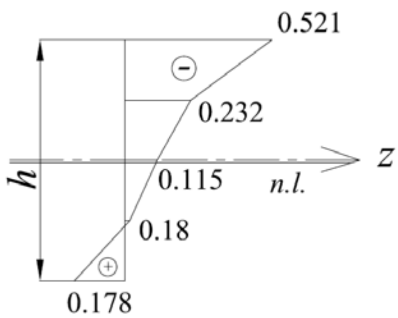

Figure 12. Distribution of normal stresses σ (MPa) in the cross-section near the vault skew-back (x=3.342 m) under the action of constant loads, three-dimensional analysis of vault according to the model with volumetric finite elements in Lira.

Based on the brickwork strength properties, the value of the additional load and the stressed state of the vault is determined. The analytical calculation, taking into account the displacement of the neutral layer of the vault, shows that the vault will withstand an additional load up to 10 kN/m under bending tensile stresses.

The dependence of displacements under specified load acting onto the test structure is shown in the Figure 13.

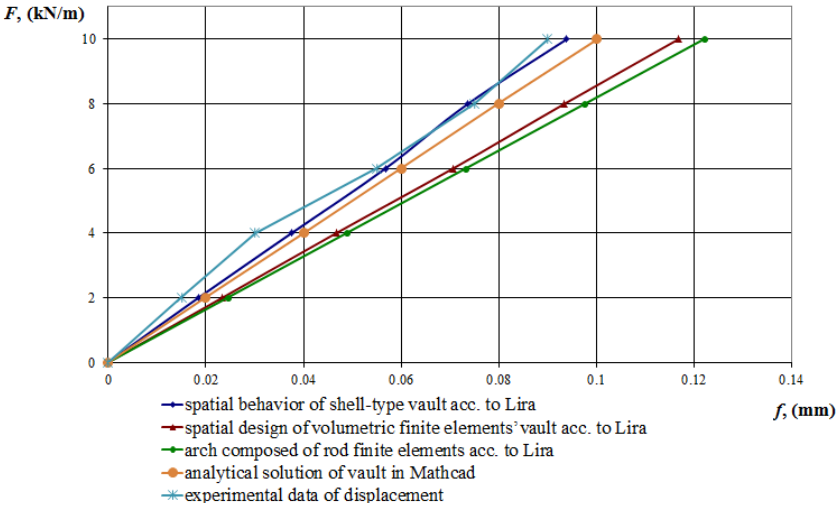

Figure 13. Dependence of vertical displacements in the crown of the vault.

Comparison of experimental and theoretical values of stresses and displacements of the structure shows that increase in stresses and displacements in case of load gradual increase is practically proportional to these loads. In this case, the modulus of deformation of object is approximately equal to the value taken in analytical calculations. It was the correct method of accounting for creep and plasticity in form of coefficients to the initial modulus of elasticity in the calculations.

To continue the comparison with the known works, this analytical solution is compared with the results of calculation of the arch outlined by the square parabola [21], which has been carried out for efficient selection of axis outline. Such arches are also often found in the ancient architectural components of the buildings.

The adopted outline of the arch axis is described by the expression:

$$y = f[\mu(x/d)^2 + (1-\mu)(x/d)^5],$$

where *f* is the rise of vault; $\mu = 0.966$.

Table 9. Comparison of stress values according to Kachurin and our data.

Stress	According to Kachurin	According to our data	%
σ, (MPa)	0.3997	0.2035	49

The difference between normal stresses in the results of design calculation of the arch defined by the parabolic law with the analytical solution for the arch as a three-centered compound curve is 49 %.

Thus, qualitatively, the experimental data completely confirm the derived analytical theory; but the calculation of the arch, described by the square parabola, has a noticeable discrepancy compared to the experiment results. It follows that calculation should be carried out with more accurate geometry and static uncertainty of the designed model of the vault.

4. Conclusions

1. The proposed herein a confirmatory method of calculation of barrel vaults of buildings constructed long ago (architectural monuments) allows to consider changes of physical and mechanical characteristics of the brickwork taking place in time. Modulus of initial deformation is taken from the works of N.S. Khamidzhanov, creep and plasticity are taken into account using A.M. Rozenblyumas formulas and adjustments to the Eurocode (material creep), as well as the nonlinearity of these characteristics (according to the Russian Set of Rules SP 15.13330.2012).

2. On the basis of analytical and numerical confirmatory calculations, the studies of the bearing capacity and the stressed state of the three-centered barrel vaults under the acting loads were carried out, which made

it possible to solve the task, i.e. to determine the amount of additional load that the considered barrel vault can withstand. The vault will be able to withstand the additional load up to 10 kN/m.

3. The experimental verification was carried out of the proposed method of studying the deformed state of the brickwork barrel vaults when loaded by short-term static load applied after a long period of time since the vault construction (more than 100 years).

4. The received test data and their comparison with the theoretical calculations of forces and displacements performed with the help of finite element method proved the proximity of the results.

5. The above calculations may be applied in the surveying and strength testing of barrel vaults used in architectural monuments.

References

1. Stefanou, I., Sab, K., Heck, J.V. Three dimensional homogenization of masonry structures with building blocks of finite strength: a closed form strength domain. International Journal of Solids and Structures. 2015. No. 54. Pp. 258–270.
2. Sab, K. Overall ultimate yield strength of a quasi-periodic masonry. Comptes Rendus Mecanique. 2009. No. 337. Pp. 603–609.
3. Skripchenko, I.V., Bespalov, V.V., Lukichev, S.Y., Zimin, S.S. Unconventional cases of the stone vaults. Construction of Unique Buildings and Structures. 2017. No. 2 (53). Pp. 87–95.
4. Basilio, I., Fedele, R., Lourenco, P.B., Milani, G. Assessment of curved FRP-reinforced masonry prisms: Experiments and modeling. Construction and Building Materials. 2014. No. 51. Pp. 492–505.
5. Milani, G. Upper bound sequential linear programming mesh adaptation scheme for collapse analysis of masonry vaults. Advances in Engineering Software. 2015. No. 79. Pp. 91–110.
6. Milani, G., Tralli, A. A simple meso-macro model based on SQP for the non-linear analysis of masonry double curvature structures. International Journal of Solids and Structures. 2012. No. 49. Pp. 808–834.
7. Kamal, O.A., Hamdy, G.A., El-Salakawy, T.S. Nonlinear analysis of historic and contemporary vaulted masonry assemblages. HBRC Journal. 2014. No. 10(3). Pp. 235–246.
8. Pottman, H., Eigensatz, M., Vaxman, A., Wallner, J. Architectural geometry. Computers & Graphics. 2015. No. 47. Pp. 145–164.
9. Cancelliere, I., Imbimbo, M., Sacco, E. Experimental tests and numerical modeling of reinforced masonry arches. Engineering Structures. 2010. No. 32. Pp. 776–792.
10. Ramaglia, G., Lignola, G.P., Prota, A. Collapse analysis of slender masonry barrel vaults. Engineering Structures. 2016. Vol. 117. Pp. 86–100.
11. Rizzi, E., Rusconia, F., Cocchettia, G. Analytical and numerical DDA analysis on the collapse mode of circular masonry arches. Engineering Structures. 2014. No. 60. Pp. 241–257.
12. Viola, E.L., Panzacci, F., Tornabene, F. General analysis and application to redundant arches under static loading. Construction and Building Materials. 2007. No. 21. Pp. 1129–1143.
13. De Santis, S., Tomor, A.K. Laboratory and field studies on the use of acoustic emission for masonry bridges. NDT & E International. 2013. No. 55. Pp. 64–74.
14. D'Ambrisi, A., Feo, L., Focacci, F. Masonry arches strengthened with composite unbunded tendons. Composite Structures. 2013. No. 98. Pp. 323–329.
15. Betti, M., Drosopoulos, G.A., Stavroulakis, G.E. Two non-linear finite element models developed for assessment of masonry arches. Comptes Rendus Mecanique. 2008. No. 336. Pp. 42–53.
16. Felice, G. Assessment of the load carrying capacity of multi span masonry arch bridges using fibre beam elements. Engineering Structures. 2009. No. 31. Pp. 1634–1647.

Литература

1. Stefanou I., Sab K., Heck J.V. Three dimensional homogenization of masonry structures with building blocks of finite strength: a closed form strength domain // International Journal of Solids and Structures. 2015. № 54. Pp. 258–270.
2. Sab K. Overall ultimate yield strength of a quasi-periodic masonry // Comptes Rendus Mecanique. 2009. № 337. Pp. 603–609.
3. Скрипченко И.В., Беспалов В.В., Лукичев С.Ю., Зимин С.С. Нетипичные случаи каменных сводов // Строительство уникальных зданий и сооружений. 2017. №2 (53). С. 87–95.
4. Basilio I., Fedele R., Lourenco P.B., Milani G. Assessment of curved FRP-reinforced masonry prisms: Experiments and modeling // Construction and Building Materials. 2014. № 51. Pp. 492–505.
5. Milani G. Upper bound sequential linear programming mesh adaptation scheme for collapse analysis of masonry vaults // Advances in Engineering Software. 2015. № 79. Pp. 91–110.
6. Milani G., Tralli A. A simple meso-macro model based on SQP for the non-linear analysis of masonry double curvature structures // International Journal of Solids and Structures. 2012. № 49. Pp. 808–834.
7. Kamal O.A., Hamdy G.A., El-Salakawy T.S. Nonlinear analysis of historic and contemporary vaulted masonry assemblages // HBRC Journal. 2014. № 10(3). Pp. 235–246.
8. Pottman H., Eigensatz M., Vaxman A., Wallner J. Architectural geometry // Computers & Graphics. 2015. № 47. Pp. 145–164.
9. Cancelliere I., Imbimbo M., Sacco E. Experimental tests and numerical modeling of reinforced masonry arches // Engineering Structures. 2010. № 32. Pp. 776–792.
10. Ramaglia G., Lignola G.P., Prota A. Collapse analysis of slender masonry barrel vaults // Engineering Structures. 2016. Vol. 117. Pp. 86–100.
11. Rizzi E., Rusconia F., Cocchettia G. Analytical and numerical DDA analysis on the collapse mode of circular masonry arches // Engineering Structures. 2014. № 60. Pp. 241–257.
12. Viola E.L., Panzacci F., Tornabene F. General analysis and application to redundant arches under static loading // Construction and Building Materials. 2007. № 21. Pp. 1129–1143.
13. De Santis S., Tomor A.K. Laboratory and field studies on the use of acoustic emission for masonry bridges // NDT & E International. 2013. № 55. Pp. 64–74.
14. D'Ambrisi A., Feo L., Focacci F. Masonry arches strengthened with composite unbunded tendons // Composite Structures. 2013. № 98. Pp. 323–329.
15. Betti M., Drosopoulos G.A., Stavroulakis G.E. Two non-linear finite element models developed for assessment of masonry arches // Comptes Rendus Mecanique. 2008. № 336. Pp. 42–53.
16. Felice G. Assessment of the load carrying capacity of multi span masonry arch bridges using fibre beam elements // Engineering Structures. 2009. № 31. Pp. 1634–1647.

17. Madani, K. A study of fiber deboning in circular composite arches. Comptes Rendus Mecanique. 2002. No. 330. Pp. 535–541.
18. Zhang, Y., Macorini, L., Izzuddin, B.A. Mesoscale partitioned analysis of brick-masonry arches. Engineering Structures. 2016. Vol. 124. Pp. 142–166.
19. Ramaglia, G., Lignola, G.P., Prota, A. Collapse analysis of slender masonry barrel vaults. Engineering Structures. 2016. Vol. 117. Pp. 86–100.
20. Felice, G. Assessment of the load-carrying capacity of multi span masonry arch bridges using fibre beam elements. Engineering Structures. 2009. No. 31. Pp. 1634–1647.
21. Zimin, S.S., Bespalov, V.V., Kazimirova, A.S. Raschetnaya model kamennoy arochnoy konstruktsii [The computational model stone arch] [online]. System requirements: Adobe Acrobat Reader. URL: http://donnasa.ru/publish_house/journals/vestnik/2015/vestnik_2015-3(113).pdf. (assessed: 01 August 2018).
22. Zimin, S.S., Kokotkova, O.D., Bespalov, V.V. Vault structures of historical buildings. Construction of Unique Buildings and Structures. 2015. No. 2 (29). Pp. 57–72. (rus)
23. Russian State Standard GOST 8462-85. Materialy stenovyye. Metody opredeleniya predelov prochnosti pri szhatii i izgibe [Wall materials. Methods for determination of ultimate compressive and bending strength]. Moscow. Gosstandart Soyuza SSR, 1985. 10 p. (rus)
24. Russian State Standard GOST 5802-86 Rastvori stroitelnie. Metodi ispitaniya [Mortars. Test methods]. Moscow. Gosstandart Soyuza SSR, 1986. 22 p. (rus)
25. Zubkov, S.V., Ulybin, A.V., Fedotov, S.D. Assessment of mechanical properties of brick masonry by flat-jack method. Magazine of Civil Engineering. 2015. No. 8. Pp. 20–29. (rus)
26. Russian State Standard GOST 22690-88 Betony. Opredeleniye prochnosti mehanicheskimi metodami nerazrushayuschego kontrolya [Concretes. Determination of strength by mechanical methods of nondestructive testing]. Moscow. Gosstandart Rossii, 1988. 26 p. (rus)
27. Russian Construction Norms and Rules SNiP II-22-81. Kamennyye i armokamennye konstruktsii [Stone and reinforced structures]. Moscow: FGUP CPP, 2004. 40 p. (rus)
28. Russian Set of Rules SP 15.13330.2012. Svod pravil. Kamennye i armokamennyye konstruktsii. Aktualizirovannaya redaktsiya SNiP II-22-81* [Masonry and reinforced masonry structures. The actual formulation of Construction Norms and Regulations II-22-81*]. Moscow: FAU "FCS", 2012. 74 p. (rus)
29. EN 1996-1-1 (English). Eurocode 6: Design of masonry structures. Part 1-1: General rules for reinforced and unreinforced masonry structures. Management Centre: rue de Stassart. Brussels: 2005. Pp. 123.
30. EN 1992-1-1 (English). Eurocode 2: Design of concrete structures. Part 1-1: General rules and rules for buildings. Management Centre: rue de Stassart. Brussels: 2004. Pp. 226.

17. Madani K. A study of fiber deboning in circular composite arches // Comptes Rendus Mecanique. 2002. № 330. Pp. 535–541.
18. Zhang Y., Macorini L., Izzuddin B.A. Mesoscale partitioned analysis of brick-masonry arches // Engineering Structures. 2016. Vol. 124. Pp. 142–166.
19. Ramaglia G., Lignola G.P., Prota A. Collapse analysis of slender masonry barrel vaults // Engineering Structures. 2016. Vol. 117. Pp. 86–100.
20. Felice G. Assessment of the load-carrying capacity of multi span masonry arch bridges using fibre beam elements // Engineering Structures. 2009. № 31. Pp. 1634–1647.
21. Зимин С.С., Беспалов В.В., Казимирова А.С. Расчетная модель каменной арочной конструкции [Электронный ресурс]. Систем требования: Adobe Acrobat Reader. URL: http://donnasa.ru/publish_house/journals/vestnik/2015/vestnik_2015-3(113).pdf (дата обращения: 01.08.2018).
22. Зимин С.С., Кокоткова О.Д., Беспалов В.В. Сводчатые конструкции исторических зданий // Строительство уникальных зданий и сооружений. 2015. № 2(29). С. 57–72.
23. ГОСТ 8462-85. Материалы стеновые. Методы определения пределов прочности при сжатии и изгибе. М.: Госстандарт Союза ССР, 1985. 10 с.
24. ГОСТ 5802-86. Растворы строительные. Методы испытания. М.: Госстандарт Союза ССР, 1986. 22 с.
25. Зубков С.В., Улыбин А.В., Федотов С.Д. Исследование механических свойств кирпичной кладки методом плоских домкратов // Инженерно-строительный журнал. 2015. № 8. С. 20–29.
26. ГОСТ 22690-88. Бетоны. Определение прочности механическими методами неразрушающего контроля. М.: Госстандарт России, 1988. 26 с.
27. Строительные нормы и правила. Каменные и армокаменные конструкции: СНиП II-22-81*: нормативно-технический материал. Госстрой России. Офиц. изд. Взамен СНиП II-В.2-71; Введ. с 1 янв. 1983. М.: ФГУП ЦПП, 2004. 40 с.
28. СП 15.13330.2012. Каменные и армокаменные конструкции. Актуализированная редакция СНиП II-22-81. М.: ФАУ "ФЦС", 2012. 74 с.
29. EN 1996-1-1 (English). Eurocode 6: Design of masonry structures. Part 1-1: General rules for reinforced and unreinforced masonry structures. Management Centre: rue de Stassart. Brussels: 2005. Pp. 123.
30. EN 1992-1-1 (English). Eurocode 2: Design of concrete structures. Part 1-1: General rules and rules for buildings. Management Centre: rue de Stassart. Brussels: 2004. Pp. 226.

Anay-Khaak Kaldar-ool,*
+7(981)801-26-87; oorzhaka-h@mail.ru

Vladimir Babanov,
+7(911)242-96-55; babanov_vladimir@mail.ru

Boris Allahverdov,
+7(921)743-59-66; aborism@mail.ru

Svetlana Saaya,
+7(923)549-82-78; sedip@mail.ru

Анай-Хаак Бугалдаевна Калдар-оол.*
+7(981)801-26-87; эл. почта: oorzhaka-h@mail.ru

Владимир Владимирович Бабанов,
+7(911)242-96-55;
эл. почта: babanov_vladimir@mail.ru

Борис Михайлович Аллахвердов,
+7(921)743-59-66; эл. почта: aborism@mail.ru

Светлана Сергеевна Саая,
+7(923)549-82-78; эл. почта: sedip@mail.ru