Computational modelling of stiffness and strength properties of the contact seam

Численное моделирование жесткостных и прочностных свойств контактного шва

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Abstract. The problem of contact interaction of structures taking into account the deformation and strength properties of the contact seam material is considered. To discretize the contact layer, frame-rod contact finite elements (CFE) are used, by means of which the physical properties of the seam material (initial strength, line and nonlinear deformability) are modelled. By means of CFE are also modelled various contact conditions – separation, clutch, friction-sliding, etc. On the base of the proposed discrete contact model and the method of step-by-step analysis, a numerical algorithm for solving the contact problem has been developed taking into account the deformation and strength properties of the seam material. This approach allows in one step-by-step cycle to perform simultaneous account of the initial strength of the contact seam, as well as the conditions of unilateral deformable constraints and friction-sliding of its surfaces in areas where the strength of the contact seam is broken. The use of frame-rod contact elements allows the physical nonlinearity of the contact layer to reduce to the internal nonlinearity of only the system of contact elements, while the nonlinear properties of the seam are set by means of the nonlinear characteristics of the individual rods of contact elements. Iterative refinement of the nonlinear solution for the current level of loading is performed by the method of compensating loads. With the help of the proposed approach, the numerical solution of the problem of contact of the structure with the base under different conditions of the contact seam has been obtained and analyzed.

Аннотация. Рассматривается задача контактного взаимодействия конструкций с учетом деформативных и прочностных свойства материала контактного шва. Для дискретизации контактного слоя используются рамно-стержневые контактные конечные элементы (ККЭ), посредством которых моделируются физические свойства материала шва (начальная прочность, линейная и нелинейная податливость). С помощью ККЭ также моделируются различные условия контакта – отрыва, сцепления, трения-скольжения и т.п. На основе предложенной дискретной модели контакта и метода пошагового анализа разработан численный алгоритм решения контактной задачи с учетом деформативных и прочностных свойств материала шва. Данный подход позволяет в одном пошаговом цикле выполнить одновременный учет начальной прочности контактного шва, а также условия односторонних податливых связей и трения-скольжения его поверхностей на участках, где прочность контактного шва нарушена. Использование рамно-стержневых контактных элементов позволяет физическую нелинейность контактного слоя свести к внутренней нелинейности только системы контактных элементов, при этом нелинейные свойства шва задаются через нелинейные характеристики отдельных стержней контактных элементов. Итерационное уточнение нелинейного решения для текущего уровня нагружения выполняется посредством способа компенсирующих нагрузок. С помощью предложенного подхода получено и проанализировано численное решения задачи контакта сооружения с основанием при различных условиях работы контактного шва.

1. Introduction

The problems of contact interaction of structures and their parts have a wide range of applications in construction and other fields of engineering. For example, deformation and technological seams can be opened and closed, both with slippage, and with clutch of contacting surfaces, at various combinations of external loadings. The same can happen at interaction of the cracks coasts, on contact of a sole of the construction with the basis or on the supports allowing a separation or slippage of the construction leaning on them. Herewith, it is often the state of the contact zone can be determining when assessing the stress-strain state, strength and reliability of structures and construction [1–8]. In turn, the state of contact is also influenced by the deformation and strength properties of the material of the contact seam, the accounting of which approximates the design scheme to the real working conditions of construction.

The deformation of the contact seam can be conditioned by the presence of the locally deformable layer between the contact surfaces, the roughness of the boundary surfaces of interacting bodies and other reasons. Similar calculation schemes can be used for structures where an opening and closing of seams, friction and sliding of surfaces, formation of cracks, etc. The corresponding problems (taking into account the deformation of the contact layer, both in normal and tangential directions) were considered in the works [2, 9–15]. They used numerically-analytical or iterative methods for solving the contact problem, particularly, the method of iterations over the gaps [9, 12], the method of iterations on limit friction forces [2, 13–15].

In some cases it is necessary to consider nonlinear properties of the contact seam. Using the finite element method allows to take into account this kind of physical nonlinearity, reducing it to the internal nonlinearity of the system of contact elements discretizing the intermediate layer. The solution of such problems leads to the solution of a nonlinear system of equations at each step of loading or iterative approximation. Various iterative schemes of accounting of nonlinear effects on the contact were considered in the works [16–21]. The nonlinear relationship between the deformation of the contact layer and the contact stresses is given here by means of the corresponding nonlinear dependencies or diagrams.

The contact seam may have some initial strength as well, both in the normal and tangential direction. It should be emphasized that issues related directly to destruction of construction are not considered here. The strength of the seam is described by simple dependencies, which, however, may be useful in solving some practical problems. Account of the strength of unilateral constraints was discussed in several papers [22–25], where the rather complex algorithms of successive approximations were proposed. For example, in [23] a combined algorithm consisting of four nested iterative cycles is presented, for practical implementation of which it is recommended to apply a preliminary lowering of the order of discretization of the problem.

The numerical solution of contact problems is usually realized on the basis of different schemes of the finite element method (FEM). In this case, the continuum problems of contact of elastic bodies are reduced to finite-dimensional problems with discrete unilateral constraints [10–14, 17–26]. In this paper, for the modeling of the contact seam contact finite elements (CFE) of the frame-rod type have been used, that allows to calculate the forces and displacements in the contact zone with the sufficiently high accuracy. By means of the FEM both physical properties of the seam material (initial strength, line and nonlinear deformability), and a variety of contact state (separation, clutch, friction-sliding) are modeled.

The purpose of this work is to develop an algorithm for the numerical realization of the proposed finite element model, which allows simultaneous accounting of the strength and nonlinear deformability of the contact seam, as well as the conditions of unilateral constraints and friction--sliding in the contact areas. To achieve this purpose, numerical studies, as well as a comparison of the results with the solutions having been obtained by alternative methods, have been carried out.

2. Methods

Let us consider the case of contact between the boundary surfaces S_g^+ and S_g^- of the elastic bodies V^+ and V^-. It is believed that the surface data are connected by contact seam having thickness ζ^0, possessing deformability ρ_n and ρ_τ (in the normal and tangent direction respectively) and the initial tensile strength (break) $R_n > 0$ and shift $R_\tau > 0$ ($\alpha_R = R_\tau / R_n$). Under the assumption that the destruction of the material of the contact seam occurs according to the fragile scenario, we use the strength criterion of the Coulomb-Mohr:

$$\sigma_n + \frac{1}{\alpha_R}\left|\sigma_\tau\right| \le R_n; \quad \left|\sigma_\tau\right| \le R_\tau - \alpha_R\,\sigma_n. \tag{1}$$

The property of the contact problem with seams having an initial strength is that the process of breaking bonds is an irreversible phenomenon. The solution of such problems depends on the sequence of application of external loads, and the form of boundary conditions having been implemented for contact surfaces at the moment depends on whether the destruction of the initial clutch of constraints from the beginning of loading happened or not. If there was no destruction, then the surfaces at this point remain of clutched, and if there was, then the usual conditions for unilateral constraints with friction (without taking into account their strength) are fulfilled.

The contact finite elements proposed by the author – in the form of a flat or spatial frame are used to model the contact interaction in the seam zone. The CFE data interacts with the usual finite elements of a discrete calculation scheme and thus providing a connection between the mesh nodes located on the boundary surfaces of the contacting bodies (Figure 1).
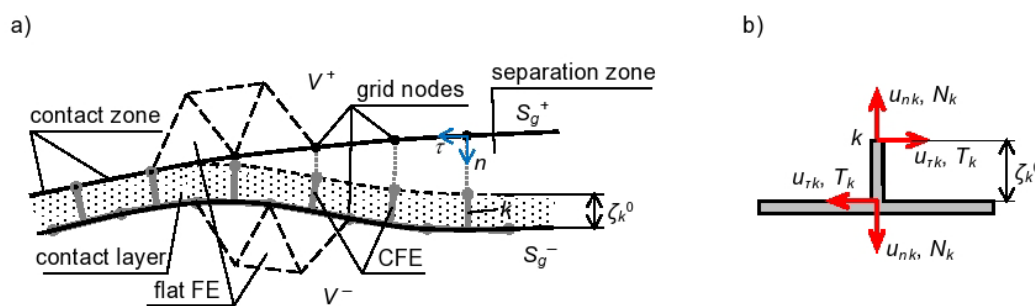
Figure 1. Modeling of contact by means of frame-rod CFE:
a – CFE in the contact zone; b – displacement and forces in the CFE

When using the contact elements data, there is no need to match the coordinates of the nodes of the contacting surfaces, i.e. inconsistent grids can be used. Various properties of the contact seam, such as deformability, initial strength, physical nonlinearity, etc., can be taken into account with the help of CFE. Modeling of various contract conditions is carried out by changing the physical properties of the contact layer, which, in turn, are expressed through the stiffness and strength characteristics of individual rods of frame CFE [26, 27].

Considering the k contact element as the k discrete unilateral deformable constraint (in normal and tangential directions) between interacting bodies, we express the conditions (1) through contact forces in k CFE ($k \in S_g$, $S_g = S_g^+ \cup S_g^-$)

$$N_k + \frac{1}{\alpha_R}\left|T_k\right| \le N_{Rk}; \quad \left|T_k\right| \le T_{Rk} - \alpha_R\,N_k. \tag{2}$$

Let us write down the conditions on the contact in terms of forces and displacements for the k contact element:

$$u_{nk} + u_{nk}^c \le 0; \quad N_k + \frac{\left|T_k\right|}{\alpha_R} \le N_{Rk}; \quad (u_{nk} + u_{nk}^c)(N_k + \frac{\left|T_k\right|}{\alpha_R} - N_{Rk}) = 0; \tag{3}$$

$$\left|T_k\right| \le T_{Rk} - \alpha_R\,N_k; \quad T_k\,(u_{\tau k} + u_{\tau k}^c) \ge 0; \quad (T_k - T_{Rk} + \alpha_R\,N_k)\,(u_{\tau k} + u_{\tau k}^c) = 0; \tag{4}$$

$$u_{nk}^c = N_k / C_{nk}; \quad u_{\tau k}^c = T_k / C_{\tau k}. \tag{5}$$

Here u_{nk}, $u_{\tau k}$ is the mutual displacement of the opposite nodes at S_g^+ and S_g^- in the normal and tangential direction; u_{nk}^c, $u_{\tau k}^c$ is the longitudinal and transverse deformation in k CFE; N_k, T_k is the

forces in the support rod of k CFE; $T_{Uk} = -f_k N_k$ is the ultimate «Coulomb» friction force; $f_k \geq 0$ is the coefficient of friction in k contact; $N_{Rk} = R_n \omega_k$, $T_{Rk} = R_\tau \omega_k$ are the ultimate tensile and shear contact forces for k discrete unilateral constraint (i.e. CFE). An additional group of conditions Eq. (5) characterize the deformability of the contact seam: C_{nk}, $C_{\tau k}$ is the normal and tangential stiffness; ω_k is the contact area related to k CFE. At linear deformability of the seam material $C_{nk} = \omega_k / \rho_{nk}$, $C_{\tau k} = \omega_k / \rho_{\tau k}$.

The conditions in the form (3) – (4) are valid only until the initial strength of unilateral constraints on the k contact is reached, and after the destruction the conditions (6) – (7), describing unilateral constraints with additional deformability without taking into account the strength, are valid [27]:

$$u_{nk} + u_{nk}^c \leq 0; \quad N_k \leq 0; \quad (u_{nk} + u_{nk}^c) N_k = 0; \tag{6}$$

$$|T_k| \leq |T_{Uk}|; \quad T_k (u_{\tau k} + u_{\tau k}^c) \geq 0; \quad (|T_k| - T_{Uk})(u_{\tau k} + u_{\tau k}^c) = 0. \tag{7}$$

Thus, in one step cycle simultaneous account of the strength of the seam, as well as the disclosure or friction of its surfaces in areas where the strength of the seam is broken are provided.

Modeling of physical and strength properties, as well as contact states, is carried out by assigning of the corresponding stiffness characteristics of frame-rod CFE. The stiffness of the contact seam in the normal and tangential directions to the boundary surfaces is expressed by the longitudinal and bending stiffness of the support rod CFE. In the state of contact with the clutching, the stiffness values of this rod in k CFE must correspond to the corresponding stiffness of the contact layer:

$$EA_k = C_{nk}\zeta_k^0 = \omega_k \zeta_k^0 / \rho_n; \quad EI_k = C_{\tau k}(\zeta_k^0)^3 / 3 = \omega_k (\zeta_k^0)^3 / 3\rho_\tau. \tag{8}$$

With the disengagement of the contact surfaces or slippage, respectively assigned a «zero» hardness.

The computational implementation of the conditions (3) – (5) is performed using a step-by-step analysis of changes in the state of the contact in the process of sequential application of a given load. In this case, the friction conditions can be satisfied to the best extent, since the solution of the friction problem depends on the history of loading of the structure. The moment of transition from one state to another is, respectively, the event of destruction of unilateral constraints, separation or contact, slippage or clutching. The method of step-by-step analysis is the most effective for the considered class of contact problems, besides, it is possible to monitor the current state of the contact seam in the process of loading the structure.

The algorithms of step-by-step analysis, including the sequence of actions at each step, both under static and dynamic loading, are described in sufficient detail in [27–31]. In the foundation of these algorithms is the representation of constructively nonlinear contact problem in the form of a sequence of finite number of linear problems with a sequential change of the working schemes of the structure. The analysis of the working scheme behavior determines the time of the next event on the contact. As a result of the next step, the working scheme is changed and the new state of the contact is set, while the method of compensating loads is used to fulfill the contact friction-slip conditions [27, 29].

Below are some expressions of the step-by-step algorithm determining, in particular, the moments of the occurrence of the closest event on the contact (change in the state of contact), taking into account the deformability and strength of the contact seam.

The moment of destruction for the bond (in the direction of normal and tangential respectively), which has been earlier in the state of initial strength

$$\left. \begin{aligned} \Delta\lambda_k^{s+1} &= \Delta\tilde{\lambda}^{s+1}\left(\frac{N_{Rk} - N_k^s - |T_k^s|/\alpha_R}{\Delta\tilde{N}_k^{s+1}}\right) \\ \Delta\lambda_k^{s+1} &= \Delta\tilde{\lambda}^{s+1}\left(\frac{T_{Rk} - T_k^s - \alpha_R N_k^s}{\Delta\tilde{T}_k^{s+1}}\right) \end{aligned} \right\}, \quad k \in S_{1g}. \tag{9}$$

The slippage torque for connection in the state of the clutching

$$\Delta\lambda_k^{s+1} = \Delta\tilde{\lambda}^{s+1}\left(\frac{T_{Uk}^s - T_k^s}{\Delta\tilde{T}_k^{s+1} - \Delta\tilde{T}_{Uk}^{s+1}}\right), \quad k \in S_{2g}. \tag{10}$$

The separation torque for constraint, which has been earlier in the contact

$$\Delta\lambda_k^{s+1} = \Delta\tilde{\lambda}^{s+1}\left(\frac{-N_k^s}{\Delta\tilde{N}_k^{s+1}}\right), \quad k \in S_{2g}, S_{3g}. \tag{11}$$

The moment of contact for constraint, which has been earlier in of the separation

$$\Delta\lambda_k^{s+1} = \Delta\tilde{\lambda}^{s+1}\left(\frac{-(\Delta u_{nk} + \Delta u_{nk}^c)^s}{(\Delta\tilde{u}_{nk} + \Delta\tilde{u}_{nk}^c)^{s+1}}\right), \quad k \in S_{4g}. \tag{12}$$

Here the sign "tilda" denotes the values corresponding to the trial step of loading $\Delta\tilde{\lambda}^{s+1}$, the force values T_{Uk}^s, T_k^s and N_k^s correspond to the s level of loading. $k \in S_{1g}$ are the connections in the state of initial strength of the contact seam; $k \in S_{2g}$ are the constraints in the state of pre-ultimate friction (clutching); $k \in S_{3g}$ – in conditions of ultimate friction (sliding); $k \in S_{4g}$ – in the state of separation.

The estimated loading step corresponds to the minimum of the obtained values, i.e., the one at which the nearest event occurs on the contact:

$$\Delta\lambda_{S_g}^{s+1} = \min(\Delta\lambda_k^{s+1}), \quad k \in S_{1g}, S_{2g}, S_{3g}, S_{4g}. \tag{13}$$

In other respects, the computational algorithm for solving the contact problem, including the implementation of limiting friction conditions by means of compensating loads, corresponds to the algorithms used in [26–29].

Under the nonlinear law of deformation of the contact layer of stiffness in the direction of the normal and tangential there will be functions from the values respectively compression and shear of the contact seam: $C_n = C_n(u_n^c)$; $C_\tau = C_\tau(u_\tau^c)$. Using a frame-rod CFE allows the physical nonlinearity of the contact layer to reduce the internal nonlinearity of the system, only the contact elements, herewith the nonlinear properties of the weld are determined through the nonlinear characteristics of the single members of the CFE.

When the step-by-step solution of the contact problem (based on the discrete model of FEM) for each (s+1) level of loading, the following matrix equilibrium equation is valid:

$$\left[K_{lin} + K_{nel}(u^{s+1})\right]\Delta u^{s+1} = P^{s+1} - \left[K_{lin} + K_{nel}(u^{s+1})\right]u^s. \tag{14}$$

Here u^{s+1} and P^{s+1} are respectively the vectors of nodal displacements and external loads at the end of step (s+1); Δu^{s+1} is the increment of displacement at step (s+1); K_{lin}, $K_{nel}(u^{s+1})$ are linear and non-linear components of the stiffness matrix of the system of finite elements.

By moving the nonlinear component of the equation $K_{nel}(u^{s+1})\Delta u^{s+1}$ to the right side, it is possible to write the following recurrent equation to determine the increments of displacements Δu^{s+1} at step (s+1):

$$K_{lin}\Delta u_i^{s+1} = P_{lin}^{s+1} - P_{i-1}^{s+1}(u_{i-1}^{s+1}), \tag{15}$$

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where Δu_i^{s+1} are the values of increments of displacements on the current iteration i; $P_{lin}^{s+1} = \left(P^{s+1} - K_{lin}\, u^s \right)$ is the constant (linear) part of the vector of right parts; $P_{i-1}^{s+1}(u_{i-1}^{s+1}) = K_{nel}(u_{i-1}^{s+1}) \left(u^s + \Delta u_{i-1}^{s+1} \right)$ is the changeable part of the vector of right parts.

Let us present a nonlinear matrix, $K_{nel}(u)$ corresponding to a discrete layer, as the sum of the nonlinear components of the CFE stiffness matrices

$$K_{nel}(u) = \sum_k \left[K_{n\,nel}^{(k)}(u_n^c) + K_{\tau\,nel}^{(k)}(u_\tau^c) \right]. \tag{16}$$

In turn, the nonlinear component of the stiffness matrix of k CFE, for example, $K_{n\,nel}^{(k)}(u_n^c)$, can be written as follows:

$$K_{n\,nel}^{(k)}(u_n^c) = C_n(u_n^c) \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}. \tag{17}$$

Here, the contribution of the separate CFE to the changeable (nonlinear) part of the right-hand side vector $K_{nel}(u_{i-1}^{s+1}) \left(u^s + \Delta u_{i-1}^{s+1} \right)$ will be as follows:

$$C_n(u_n^c) \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} (u_n^+)^s + (\Delta u_n^+)_{i-1}^{s+1} \\ (u_n^-)^s + (\Delta u_n^-)_{i-1}^{s+1} \end{Bmatrix} = C_n(u_n^c) \begin{Bmatrix} -(u_n^c)^s - (\Delta u_n^c)_{i-1}^{s+1} \\ (u_n^c)^s + (\Delta u_n^c)_{i-1}^{s+1} \end{Bmatrix}, \tag{18}$$

where $(u_n^+)^s$, $(u_n^-)^s$ are moving opposite points to S_g^+, S_g^- at the end of the step s; $(\Delta u_n^\pm)_{i-1}^{s+1} = (u_n^\pm)_{i-1}^{s+1} - (u_n^\pm)^s$ are respectively the increments of displacements of opposite points in (i–1) approximation for step ($s+1$); $(u_n^c)^s$ is the layer compression between boundary surfaces for s loading level; $(\Delta u_n^c)_{i-1}^{s+1} = (u_n^c)_{i-1}^{s+1} - (u_n^c)^s$ is the increment of layer compression in (i–1) approximation for step ($s+1$).

Taking into account the nonlinear deformation of the contact layer, both in the normal and tangential direction, according to (16), the iterative expression (15) takes the following form:

$$K_{lin}\, \Delta u_i^{s+1} = P_{lin}^{s+1} - \left(\sum_{k\,\in\,S_{2g},\,S_{3g}} \begin{Bmatrix} F_{nk}^+ \\ F_{nk}^- \end{Bmatrix} + \sum_{k\,\in\,S_{2g}} \begin{Bmatrix} F_{\tau k}^+ \\ F_{\tau k}^- \end{Bmatrix} \right)_{i-1}^{s+1}. \tag{19}$$

Here F_{nk}^+, F_{nk}^-, $F_{\tau k}^+$, $F_{\tau k}^-$ are the forces applied on the iteration i (in the process of iterative refinement of the value Δu^{s+1} for step ($s+1$) of loading) to the opposite nodes of contact surfaces and, respectively, normal and tangential.

The left part of equations (19) for the given working scheme of contact here is not changed that makes it able to hold the factorization of the stiffness matrix once, and then adjusting the value Δu_i^{s+1} until the difference between two subsequent iterations does not satisfy the given accuracy of calculation. The values of the correcting forces F_{nk}^+, F_{nk}^-, $F_{\tau k}^+$, $F_{\tau k}^-$ are calculated based on the results of the previous iteration (i–1):

$$\begin{Bmatrix} F_{nk}^+ \\ F_{nk}^- \end{Bmatrix} = C_n(u_n^c) \begin{Bmatrix} -(u_n^c)^s - (\Delta u_n^c)_{i-1}^{s+1} \\ (u_n^c)^s + (\Delta u_n^c)_{i-1}^{s+1} \end{Bmatrix}; \quad \begin{Bmatrix} F_{\tau k}^+ \\ F_{\tau k}^- \end{Bmatrix} = C_\tau(u_\tau^c) \begin{Bmatrix} -(u_\tau^c)^s - (\Delta u_\tau^c)_{i-1}^{s+1} \\ (u_\tau^c)^s + (\Delta u_\tau^c)_{i-1}^{s+1} \end{Bmatrix}. \tag{20}$$

Thus, the corresponding expressions to clarify the moment of occurrence of the next event on the contact (switching off, switching of constraint, slipping or clutching) will now be put down in the form of iterative formulas. So, expression (10), (11) will take respectively the following form:

$$(\Delta\lambda_k^{s+1})_i = (\Delta\lambda_k^{s+1})_{i-1}\left(\frac{T_{Uk}^s - T_k^s}{(\Delta T_k^{s+1} - \Delta T_{Uk}^{s+1})_{i-1}}\right), \quad k \in S_{2g}; \tag{21}$$

$$(\Delta\lambda_k^{s+1})_i = (\Delta\lambda_k^{s+1})_{i-1}\left(\frac{-N_k^s}{(\Delta N_k^{s+1})_{i-1}}\right), \quad k \in S_{2g}, S_{3g}. \tag{22}$$

Here T_k^s, N_k^s are the values of contact forces for the "s" level of loading; $(\Delta T_k^{s+1})_{i-1}$, $(\Delta N_k^{s+1})_{i-1}$ are the increment of forces on the iteration (i–1) to the step (s+1); $(\Delta\lambda_k^{s+1})_i$ is the iterative refinement of the step value (s+1). In other respects step-by-step algorithm defining expressions also take the iterative form. The end of the iterative refinement for the loading step is to achieve the specified accuracy of calculations. In other respects, the sequence of computational solution of the contact problem with a nonlinearly deformable layer corresponds to the algorithms described in [26–29].

In fact the nonlinear deformation law of the contact layer can be much more complicated than that recorded in the form of Eq. (17). First of all, this applies to tangential forces, which should take into account not only the shear deformation, but also the compression of the contact layer. In this case, using the method of step-by-step analysis allows us to establish the dependence of the forces on the deformation at each individual step of loading, then iterative refinement of the solution for the current level of loading.

3. Results and Discussion

Using the stated algorithm, numerical solutions for the model problem of the contact interaction of the structure with the base are obtained. Conditions of Coulomb friction-sliding (friction coefficient f = 0.2), as well as separation of boundary surfaces from each other, are possible at the contact areas. The representation by finite elements is shown in Figure 2a (to the right of the axis of symmetry). Contact seam was simulated by nine frame-rod CFE. The aim of the calculations was to assess the various conditions of contact interaction on the stress state of the soles of the structure.

The comparative analysis of the results of calculations obtained under the following conditions of the contact seam was conducted:

1. with zero deformability of the contact seam (hard unilateral contact taking into account the Coulomb friction);

2. with linear deformability of the contact seam in normal and tangential directions;

3. with nonlinear deformability of the contact seam in the direction of normal to the contact surfaces (at constant deformability in the tangential direction);

4. simultaneously with deformability the initial tensile strength and shear strength of the contact seam were taken into account.

The results of calculations - mutual displacements and stresses on the contact are shown in Figure 2b–d. The solid line corresponds to the calculation of the first option, dashed – on the second, dash-dotted – on the third, dotted – on the fourth.

As it can be seen from the results of the calculation, the area of separation of contact surfaces has the largest dimensions in the first variant – with the hard contact of the structure with the base. When taking into deformability of the seam (thickness is ζ^0 = 2 mm, deformability is ρ_0 = 1.25·10⁻⁷ m³/MN, modulus of elasticity of material of structure is 3.06·10³ MPa) the area of separation is slightly reduced. At the same time, due to shear deformations of the intermediate layer, the mutual displacement of the contact surfaces horizontally increases by 20%. The intensity of contact stresses here is slightly less than in the case of hard contact. It should be noted that the calculation results for the first and second variants fully correspond to the solutions of the problem under consideration, obtained in a number of works by iterative methods [12–14, 23].
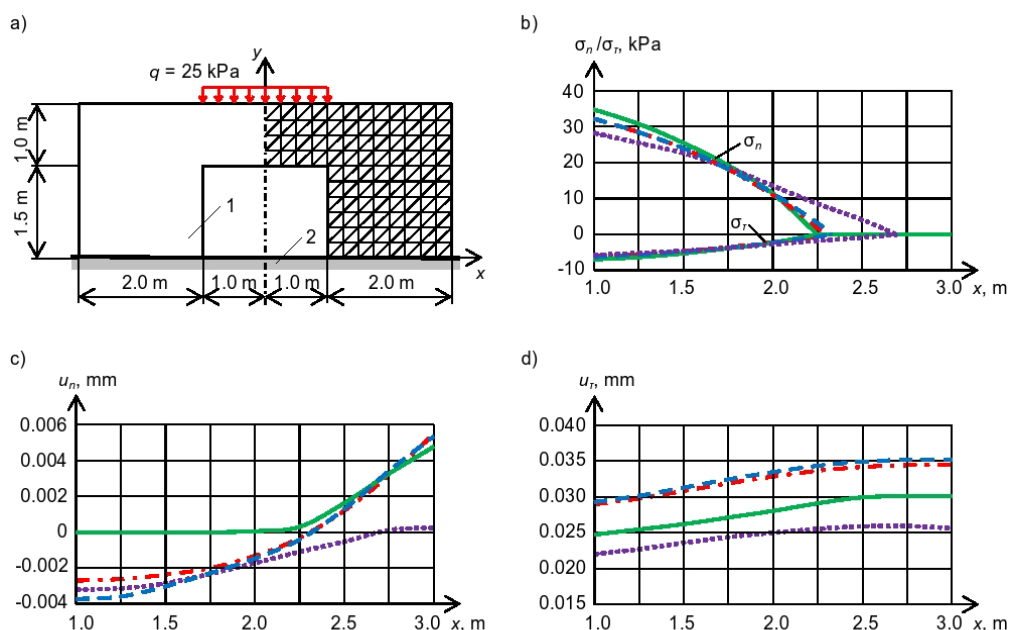
Figure 2. The problem of the interaction of a structure (1) with a hard base (2):
a – design scheme; b – contact stresses; c – displacement in the normal direction;
d – displacement in the tangential direction

The nonlinear law of deformation of the seam material in the direction of the normal to the boundary surfaces was conditionally set by the following function [17]:

$$C_{nk}(u_{nk}^c) = \omega_k \left/ \left[\rho_0 \left(1 - \sqrt[3]{|u_{nk}^c| / \zeta^0} \right) \right] \right. . \tag{21}$$

In the tangential direction the stiffness of the contact seam was taken constant $C_{\tau k} = \omega_k / \rho_0$. The comparison of the calculation results shows that taking into account the nonlinear deformation of the contact seam leads to some redistribution of deformations when it is compressed. If the initial tensile strength of the contact seam R_n = 10 kPa and shear strength R_τ = 5 kPa are taken into account, the opening of the seam surfaces is significantly reduced: the separation zone is reduced by 3 times, the maximum separation value – by 20 times. Due to the redistribution of contact stresses their intensity also decreases.

The given calculations confirm that taking into account the malleability and strength of the contact seam (in particular, between the structure and the base) is important in assessing the stress-strain state and, therefore, for the normal operation of the structure. Alongside this, the strength characteristics of the contact layer are insufficiently defined factors, first of all, due to the irregularity of the properties of the actual contact of the structure with the base. For its evaluation the integral criteria having been derived from experiments are generally used, local strength criteria are not developed sufficiently. Under these conditions, it is quite acceptable – as another approximation to the solution of this problem – to use the model of the initial strength of the contact layer as described here.

4. Conclusion

1. A finite element model of contact interaction of structures and facilities has been proposed, taking into account the physical properties of the contact seam, including initial strength, linear and nonlinear deformability of the seam material. For the modeling of unilateral constraints with the relevant physical properties has been used to contact finite elements of the frame-rod type that allows to calculate the forces and displacement in the contact zone with the same accuracy, apply an incoordinate finite element mesh, to consider the various types and conditions of contact.

2. The numerical algorithms developed on the basis of the proposed discrete contact model and of the stepping method provide the possibility of step-by-step analysis of the contact interaction and have

advantages in cases where the solution of the problem depends on the loading history, in particular, taking into account the initial strength and friction of the contact seam surfaces.

3. The proposed approach allows in one step-by-step cycle to perform simultaneous accounting of the initial strength of the contact seam, as well as the conditions of unilateral deformable constraints and friction-sliding of its surfaces in areas where the strength of the contact seam is broken. It should be noted that the results of the model problem calculations performed using the constructed algorithms are in good agreement with the numerical solutions having been obtained by alternative methods.

4. In the case of nonlinear deformability of the contact layer, the proposed numerical model allows a physically nonlinear problem to reduce to the internal nonlinearity of only the system of frame-rod contact elements, while the nonlinear properties of the seam material will be set through the nonlinear characteristics of the individual rods of the contact elements. Iterative refinement of the nonlinear solution at each step of loading for the system of contact elements is performed by means of the method of compensating loads.

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