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The modeling method of discrete cracks in reinforced concrete under the torsion with bending

Метод моделирования дискретных трещин в железобетоне при кручении с изгибом

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Ключевые слова: железобетонные плосконапряженные составные конструкции; пространственные трещины; кручение с изгибом; двухконсольная схема; расстояние между трещинами; ширина раскрытия трещин

Abstract. The most famous of computer systems for the design of reinforced concrete structures in the world are taken into account only regular dispersed cracks. Completely different criteria should be used when analyzing the appearance and development of discrete cracks in reinforced concrete, the modeling methodology of which has not been developed to date. Therefore, the article provides working prerequisites, a methodology for simulating discrete cracks and calculating their rigidity. Several levels of cracking are considered. The development of spatial cracks is carried out on special bilinear surfaces. Then they fit in the approximating spatial finite elements that "expand", modeling a spatial crack, the opening of which is given in the form of a deformation effect with allowance for the discontinuity effect. When solving the inverse problem of determining the crack opening width, the deformation effect is not specified, and only the presence of a gap of the minimum possible width is modeled by means of an embossing; its opening, with appropriate loading, determines the width of the crack opening as the divergence of the shores of this gap. It is considered another variant of simulation of discrete spatial cracks in the article, in case of their implicit manifestation. Here pairs of finite elements adjacent to such a crack are distinguished from opposite sides. It is a special two-element design console model. These pairs are considered in two states: before their "expand" and after their "expand" taking into account the deformation effect and the discontinuity effect of concrete. It is introduced the classification of basic spatial cracks in spatial reinforced concrete composite constructions, cracks that develop to zones or from zones of geometric concentration; cracks that develop to zones or from zones of force and strain concentration of loading; cracks that develop in zones of inter-medial strain concentration. Their scheme is supplemented and based on these basic cracks by applying adjacent cracks, which are sought using the deformation criterion for their formation and the method of finding the extremum of a function of many variables using the Lagrange multipliers.

Аннотация. В большинстве известных в мире вычислительных комплексов при проектировании железобетонных конструкций учитываются лишь регулярные дисперсные трещины. Совершенно иные критерии необходимо использовать при анализе появления и развития дискретных трещин в железобетоне, методика моделирования которых на сегодняшний день не разработана. Поэтому в статье разработаны рабочие предпосылки, методика моделирования дискретных трещин и расчета их жесткости. При этом рассматривается несколько уровней трещинообразования. Развитие пространственных трещин осуществляется по специальным билинейным поверхностям. Затем в них вписываются аппроксимирующие пространственные конечные элементы, которые «расшиваются», моделируя пространственную трещину, раскрытие которой задается в виде деформационного воздействия с учетом эффекта нарушения сплошности. При решении обратной задачи определения ширины раскрытия трещин, деформационное воздействие не задается, а с помощью расшивки моделируется лишь наличие щели минимально возможной ширины, ее раскрытие при соответствующем нагружении и определяет ширину раскрытия трещины, как расхождение берегов этой щели. В статье рассмотрен и другой вариант моделирования дискретных пространственных трещин, при их неявном проявлении. Здесь выделяются пары конечных элементов, прилегающих к такой трещине с противоположных сторон,

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– специальная расчетная двухэлементная консольная модель. Эти пары рассматриваются в двух состояниях: до их «расшивки» и после их «расшивки» с учетом деформационного воздействия и эффекта нарушения сплошности бетона. Вводится классификация базовых пространственных трещин в пространственных железобетонных составных конструкциях, – трещины, которые развиваются к зонам или из зон геометрической концентрации; трещины, которые развиваются к зонам или из зон концентрации силового и деформационного нагружения; трещины, которые развиваются в зонах межсредовой концентрации деформаций. Опираясь на эти базовые трещины их схема дополняется путем нанесения смежных трещин, которые отыскиваются с привлечением деформационного критерия их образования и метода отыскания экстремума функции многих переменных с использованием множителей Лагранжа. Для отыскания уровневого расстояния между трещинами и ширины их раскрытия вырезается представительный объем из железобетонной конструкции, подверженной кручению с изгибом (расчетная модель второго уровня), и в итоге записывается дифференциальное уравнение, необходимое для определения искомых параметров.

1. Introduction

In connection with the recommendations of bionics, more and more unique reinforced concrete buildings are being introduced into construction practice in recent decades, which are experiencing complex resistance, necessitating the use of their spatial design scheme [1, 2]. Alongside with them, flat-stressed reinforced concrete (including composite) structures (FSRCCS, bearing walls occupy up to 40% of the total volume of reinforced concrete) are used in a very wide scale in construction. The problem of determining their rigidity in the presence of cracks remains practically unexplored [3], therefore its development is an urgent problem and the need, which is one of the most important problems of capital construction [4, 5].

The resulting cracks significantly reduce the rigidity of reinforced concrete structures [6]. In most computer systems known in the world [7], accounting for the formation of cracks in reinforced concrete structures is performed using criteria for achieving the main stresses or the main deformations of the elongation of concrete of its limit values [8]. However, such criteria reflect the appearance of only dispersed, regular cracks in reinforced concrete structures.

Absolutely different criteria should be used when analyzing the appearance and development of discrete cracks in reinforced concrete [9]. Here the main role is played by the concentration of deformations at the places of sharp changes in geometric dimensions, zones of concentration of force and deformation loading, inter-medial concentration [10], and the like. Nevertheless, to this day, the technique for modeling discrete cracks, including the use of computer systems known in the world, has not been developed.

In order to fill the existing gap in reinforced concrete under conditions of complex torsional resistance with bending [11–13], including under seismic influences [4, 5, 14], the essence of the proposed technique for simulating discrete cracks is described below, taking into account the effect of discontinuity and inconsistency of deformations of concrete and reinforcement. The need to develop this technique is also due to the fact that finding the width of the discrete crack opening with the help of the known computer systems in the world (direct and inverse problem) is absent.

The article is also devoted to the construction of a method for calculating the rigidity of reinforced concrete structures, buildings and structures in the presence of discrete cracks under conditions of complex resistance, torsion with bending arising in conditions of seismic influences (Figure 1).

The basis of this technique is the scheme of discrete cracks, which takes place during seismic actions in buildings and structures made of reinforced concrete, obtained as a result of an analysis of a number of earthquakes [4, 14, 15].

Analysis shows [4, 5, 14, 15] that, as a rule, the development of discrete cracks under seismic actions occurs according to the "envelope" scheme, which manifests itself in the resistance of flat-strained reinforced concrete constructions, and not only, as already known scheme of the "envelope", as applied to slabs with static vertical load. overlapping with a static vertical load.

In the future, based on these basic diagonal cracks (Figure 1, b, position 1), the scheme of discrete cracks ("envelope" scheme) can be refined by applying adjacent cracks (Figure 1, b, position 2), which are sought with the use of the deformation criterion for their formation and the method of finding the extremum of a function of several variables using the Lagrange multipliers. Definition of the scheme of discrete cracks will undoubtedly contribute to the classification of basic cracks for reinforced concrete plane-stressed composite structures, proposed by I.A. Iakovenko [16, 17]. This classification is based on the geometric, force (deformation) and interspersed concentration of the stress-strain state with the corresponding sources-concentrators.

The cracking scheme, based on the envelope scheme, is used as the main one if flat-stressed finite elements (panel buildings, pylons, etc.) are used in the design model of a building, structure or reinforced concrete construction, Figure 2, a, b.

If the modeling of buildings and volumetric reinforced concrete constructions is performed with the help of volumetric finite elements taking into account their complex resistance, it becomes necessary to develop a technique for calculating the rigidity in the presence of already spatial cracks.

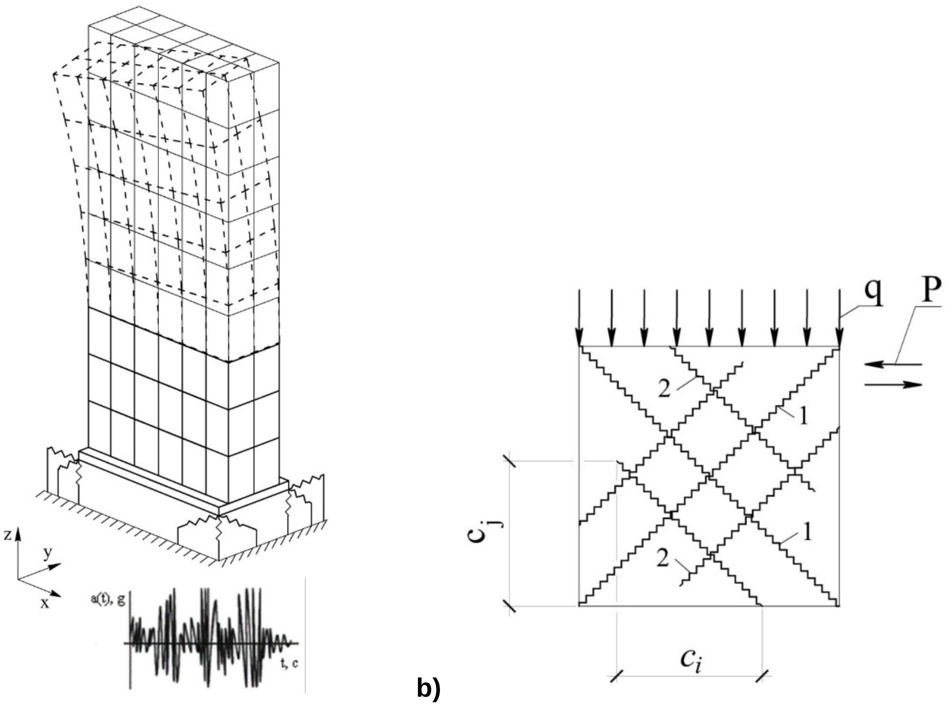
2. Methods

If the modeling of buildings and volumetric reinforced concrete structures is performed with the help of volumetric finite elements taking into account their complex resistance, it becomes necessary to develop a technique for calculating the rigidity in the presence of already spatial cracks. Considering that at present the development of the finite element model of reinforced concrete in the calculation of spatial and plane-stressed structures with allowance for nonlinear deformation has reached a rather high level, therefore it is advisable to focus on the use of the most sophisticated software complexes for the calculation of their stiffness and simulation of discrete cracks. In this case, it is necessary to adequately take into account the nature of the development and the opening of cracks in them.

The method of modeling discrete cracks and calculating the rigidity of spatial reinforced concrete structures, buildings and structures with their complex resistance, under the action torsion with bending, will be based on the following design positions:

1. The formation of a subsequent level of spatial cracks occurs after the concrete fibers stretched along the axes of the working reinforcement (longitudinal or transverse) of their ultimate deformations $\varepsilon_{bt,u}$ are reached. There may be several levels of cracking. The development of spatial cracks is carried out on special bilinear surfaces. For this purpose, knowing the equation of a bilinear surface in parametric form [19], the value of the corner points of the cross-section of the reinforced concrete **constructions –** points A, B, C, D is inscribed in it; the equation of a bilinear surface is concretized with respect to a given cross-section.

$$\begin{aligned}[x_k;\, y_k;\, z_k] = [x_A;\, y_A;\, z_A]\cdot(1-u_k)\cdot(1-w_k) + [x_B;\, y_B;\, z_B]\cdot(1-u_k)\cdot w_k\, + \\ + [x_C;\, y_C;\, z_C]\cdot u_k\cdot(1-w_k) + [x_D;\, y_D;\, z_D]\cdot u_k\cdot w_k\,.\end{aligned} \qquad (1)$$



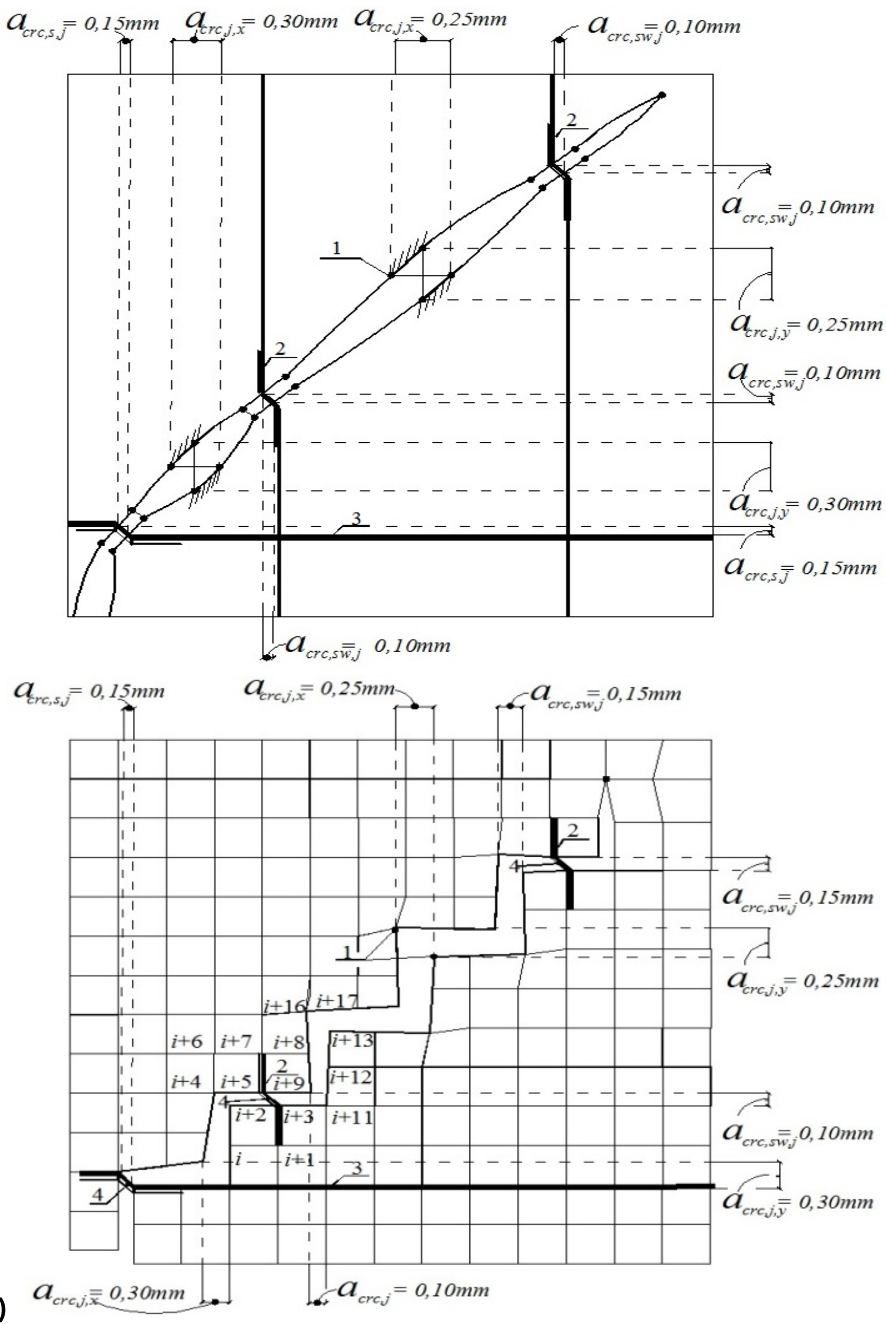
a) accelerogram b)

Figure 1. The calculation model of the aboveground and underground parts of the building under seismic action (*a*) and the scheme of basic and adjacent discrete cracks in RC wall panels (*b*)
1 – basic cracks; 2 – adjacent cracks

Then the spatial cracks are the surfaces which approximated by the parallelepipeds inscribed in them after modeling the reinforced concrete construction with the same spatial finite elements (Figure 2, c).

2. The classification of basic discrete spatial cracks is introduced. In spatial reinforced concrete composite structures [10, 16], such basic cracks can occur: 1) cracks that develop to zones or from zones of geometric concentration of stress-strain state, etc.); (in places where the cross-sectional dimensions change, in the incoming corners, in the areas of non-circular holes and the state (in places where cross-sectional dimensions change, in the incoming corners, in zones of non-circular holes, and 2) cracks that develop to zones or from zones of concentration of power and deformation loading (the location of the supporting reactions and concentrated, the place of the change in the loading intensity along the contour of the structure, the place of deformation loading from the subsidence, the type of loading is of particular importance-bending, shearing 3) longitudinal cracks that develop in zones of inter-medium stress concentration (in joints between concrete in flat-strained reinforced concrete composite constructions, along longitudinal reinforcement in anchoring zones, etc.).

In construction practice (for example, in complex engineering-geological conditions), the most often encountered are schemes of force and deformation loading, which, as a rule, cause the imposition of different cracks [17].



a)

b)

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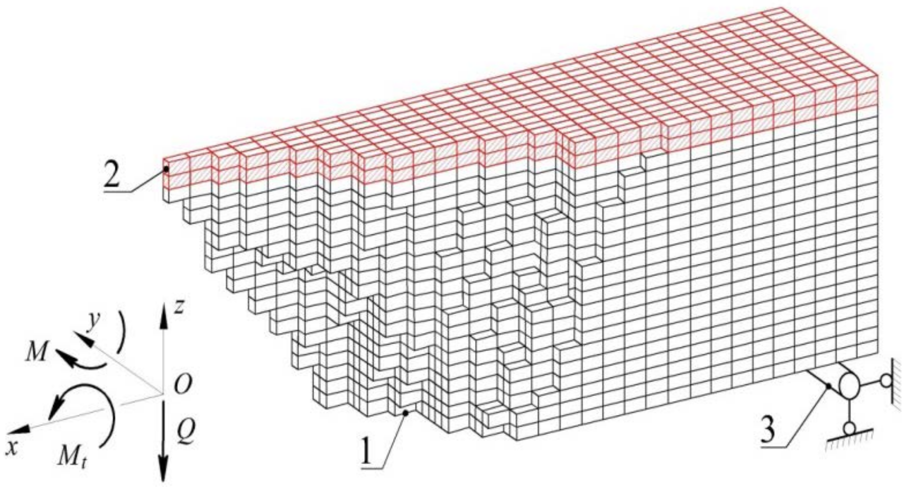


c)

Figure 2. The proposed model of cracks: a – real crack; b – modeled with the help of "disconnection" plane-stressed finite elements (FE) and deformation effects $\Delta = a_{crc,j}$; c – modeled with the help of "disconnection" spatial finite elements (FE) and deformation effects; $\Delta_1 = a_{crc}^l$; $\Delta_2 = a_{crc}^m$; $\Delta_3 = a_{crc}^n$ in a block design model with a spatial and normal section passing through the end of the spiral-shaped crack; 1 – crack; 2 – transverse reinforcement and its simulation with 201 FE, 3 – longitudinal reinforcement and its simulation with 201 FE; 4 – possible closure of the fracture and its simulation with the help of 255 FE

3. With a complex resistance, as already noted above, the building (Figure 1, a) can be modeled with the help of flat ones, or with the help of spatial finite elements.

The essence of the proposed model of cracks [20, 21] is that the actual crack (Figure 2, a for flat reinforced concrete structures and described by (2) formula for spatial reinforced concrete constructions) is replaced by a model in the form of a broken line corresponding to inscribed finite elements (Figure 2, b for planar and Figure 2, c for spatial, is considered on the example of the computer complex "Lira-CAD"), which "expand" by modeling a crack, and its expansion is given in the form of a deformation action $\Delta = a_{crc,j}$, directed perpendicular to the surface of the spatial cracks [19], described by the dependence (1). The effect of discontinuity [23, 24] is taken into account by means of introducing a variable crack width opening depending on its distance from the axis of the working (longitudinal or transverse) reinforcement.

The solving of inverse problem [21, 22] is determining the width of crack opening; the deformation effect is not specified, and only the presence of disconnection of the minimum possible width is modeled with the help of an embossing, and its opening, with the appropriate loading, determines the opening width of the crack, as the divergence of the shores of this gap.

4. There is another possible variant of simulation of discrete cracks [14, 21, 22]. It is used in the case when the renumbering of the nodes of the design scheme of the reinforced concrete construction (building or structure), connected with the necessity of "disconnection", considered in the first variant, is undesirable.

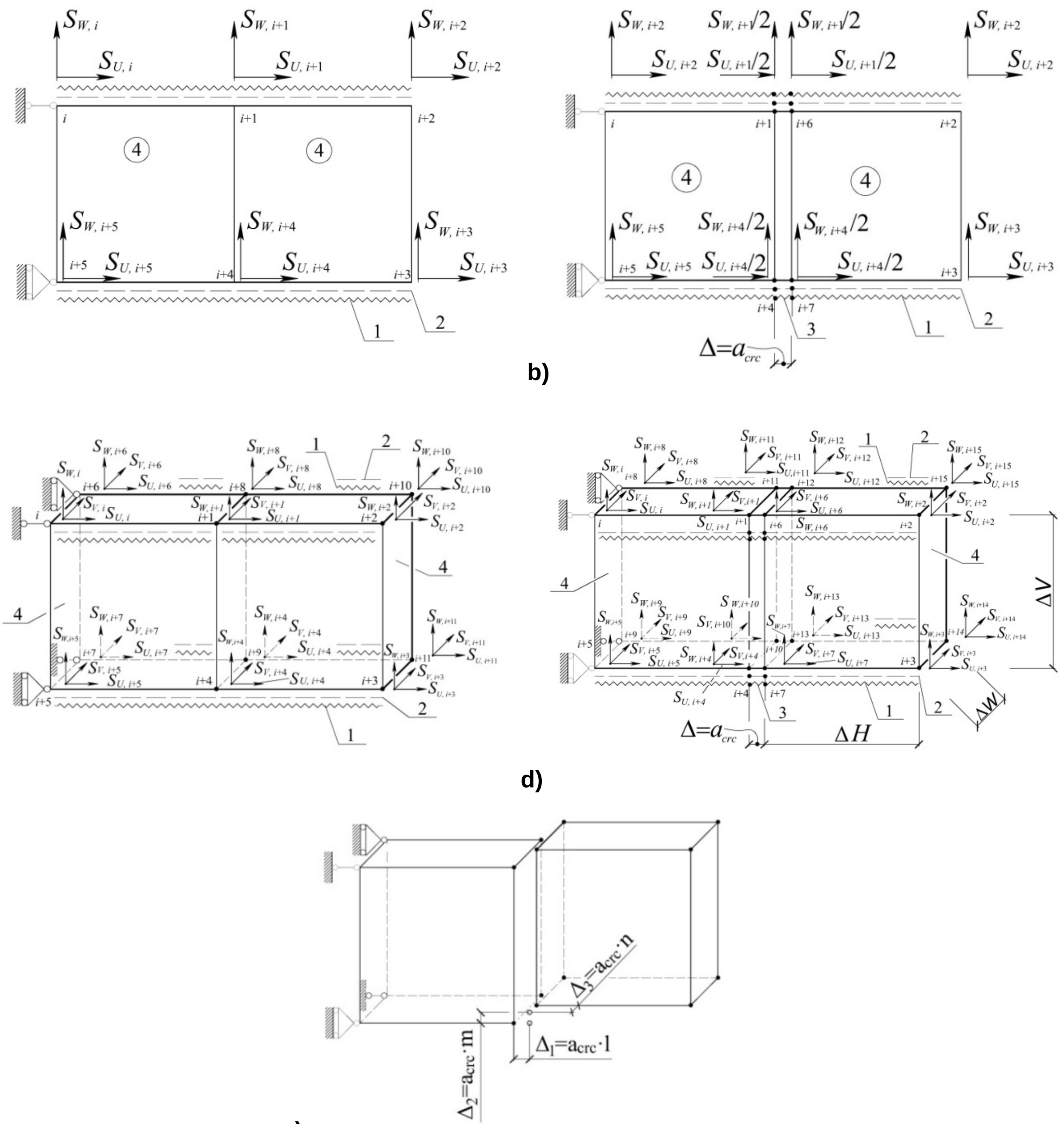
In this variant, the final elements do not "expand" along the entire crack, and in the first stage of discrete cracking simulation only imaginary discrete cracks are used, the development of which is predicted by the introduced classification of cracks as applied to a specific calculation.

At the second stage of modeling cracks along the trajectory of an imaginary crack, pairs of finite elements adjacent to such a crack from opposite sides are distinguished. These pairs are considered in two states: before their "barking" and after their "barking".

For this purpose, a special design two-element console model is involved in the calculation (in the first case, a flat one in accordance with Figure 3, a, b.

In the second case, the spatial crack in accordance with Figure 3, c–e, which is used to perform a sequential iterative analysis of the stress-strain state of the spatial console adjacent to the simulated spatial crack on opposite sides and realized with the help of "disconnection" and the deformation effect, which also takes into account the effect of disturbing the continuity of concrete.

In this case, the distributed reinforcement is replaced by two (for the plane model) and four (for the spatial model) by rod finite elements in each mutually perpendicular direction, respectively. The displacement of nodes is determined from the calculation of a two-element design model with the loads specified in the nodes (nodal finite element forces). In this case, the anchoring of two nodes in a flat model and four nodes in a spatial model (alternating with pivotally fixed and pivotally movable supports), for the purpose of averaging, it is necessary to set left, right, front, and bottom-top. It is also important that along with the node loads in the two-element model, deformation effects associated with the width of the crack opening are also specified, which in turn is related to the discontinuity effect [23, 24].



a) b)

c) d)

e)

Figure.3. The construction of a two-console flat and spatial model:
a – flat, without "disconnection"; b – the same, after " disconnection";
c –three-dimensional, before "disconnection"; d – the same as (c), after " disconnection";
e – deformation effects; 1 – 255 FE before the "disconnection"; 2 – 201 FE;
3 – 255 FE after the "disconnection"; 4 – 233 FE

The deformation action is set in each node (except the reference ones) in three increments in accordance with Figure 3, e, where l, m and n are the direction cosines of the principal crack opening vector at one or another point to the x, y and z axes, respectively.

Then, having the applied efforts and movements in the nodes of the consoles, the values of the work in two states are compared: "before the disconnection" and "after the disconnection" of the two-element model. From the condition of equality of these works, the thickness of the finite elements in the state "before the disconnection" decreases. This procedure is performed for all pairs of FE adjacent to the fracture (along horizontal, vertical or their lateral surfaces) from different sides. As a result, along the imaginary crack, the thickness of the finite elements decreases, which provokes the formation and development of cracks according by the criterion of regular disperse cracks, without resorting to FE.

The average forces in the nodes in different directions for the two-element console model are determined from the physically non-linear calculation of the entire structure. For this purpose, nodal forces are used in the corresponding FE of concrete and reinforcement.

In the places where the horizontal sections of simulated cracks move vertically and laterally, the robots of the angular finite elements are determined by averaging them.

As a result, a new thickness of the finite elements adjacent to the crack is found by the formula:

$$b = \frac{W_1}{W_2} \cdot b_1, \tag{2}$$

where W_1 and W_2 the work of the two-element model "before disconnection" and "after disconnection", respectively.

The proposed algorithm provides for an iterative process controlled by the achieved accuracy of the thickness of the marked finite elements that are adjacent to imaginary cracks and the dynamic characteristics of the reinforced concrete construction (building or structure).

It is appropriate to note here that the rigidity of the core reinforced concrete structures in areas with inclined cracks, including those that intersect (characteristic for seismic actions for the landing sites and interfaces) is replaced by equivalent rigidity:

$$B(\lambda) = \frac{M^2 \cdot \Delta x}{2 \cdot W_3}, \tag{3}$$

where W_3 – work of forces of selected area.

Here, the iterative process ends after reaching a predetermined error in the determination $B_1(\lambda)$.

In areas with normal cracks, the rigidity of the core reinforced concrete structures is determined using the values of the bending moment M and the radius of curvature ρ and according to the normative technique [22] for the corresponding stretched zone (the section with normal cracks is recommended to be divided into 4 to 6 zones):

$$B_i(\lambda) = M_i \cdot \rho_i \tag{4}$$

5. Projections of different cracks on the horizontal (vertical) are sought on the basis of the block model with the estimated cross sections passing through the beginning and end of the crack (specified in the iteration process, one of these sections, as a rule, is attached to the strongest force – the supporting reaction R_{sup} or goes to one of the faces of the construction) with the use of analytic dependencies, which are based on the extremum of a function of many variables using the Lagrange $F_{1,2} = f\left(q_{sw}, x_B, \sigma_s, x, \sigma_b, \sigma_{s,I}, \sigma_{b,1}, C_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7\right)$ multipliers and $F_3 = f\left(q_{sw}, x_{B,2}, \sigma_{s,3}, c_2, \ , \lambda_1, \lambda_2, \lambda_3\right)$, respectively, and the resulting condition for the equality of the zero partial derivatives [16, 24]:

$$\left.\begin{aligned} &\frac{\partial f}{\partial x_1}+\lambda_1\frac{\partial \varphi_1}{\partial x_1}+\lambda_2\frac{\partial \varphi_2}{\partial x_1}+...+\lambda_m\frac{\partial \varphi_m}{\partial x_1}=0\\ &\frac{\partial f}{\partial x_2}+\lambda_1\frac{\partial \varphi_1}{\partial x_2}+\lambda_2\frac{\partial \varphi_2}{\partial x_2}+...+\lambda_m\frac{\partial \varphi_m}{\partial x_2}=0\\ &...\\ &\frac{\partial f}{\partial x_n}+\lambda_1\frac{\partial \varphi_1}{\partial x_n}+\lambda_2\frac{\partial \varphi_2}{\partial x_n}+...+\lambda_m\frac{\partial \varphi_m}{\partial x_n}=0 \end{aligned}\right\} \quad (5)$$

The result is the formula:

$$(k_1'k_2'k_{21}'+k_1'k_{21}'+k_1'k_{23}')C_2^2+C_2+k_1'k_{22}'-k_1'k_2'k_{21}'=0. \quad (6)$$

Parameters k_1', k_2', $k_{21}'-k_{23}'$ depend on the geometric characteristics of the reinforced concrete composite structures, the geometric and mechanical characteristics of concrete and reinforcement, the adhesion parameters, the parameters of the stress-strain state of the calculated sections I-I and II-II that pass through the beginning and end of the inclined (spatial) crack, respectively – S, B', $B_{a,1}'$, $B_{a,2}'$, A_{sw}, E_{sw}, $q_{sw}, q_{sw,hor}, Q_{s,3}'$, h_0, τ_b, x $x_{B,2}$, $\sigma_b, \sigma_S, \sigma_{S,1}$, $\tau_{xy,2}$, a, b, $R_{\sup}$, $A_{S,i}$, α, ψ_S, ν_b.

6. In the flat-stressed and spatial reinforced concrete (including composite) constructions, a multi-level process of development of various spatial cracks takes place [16, 17]. It is determined by the calculated models of their level appearance in the form of representative volumes of concrete with a reinforcing bar long Δx, commensurable with the distance between the clamps (Figures 2 a, b – for plane-stressed and Figure 4 – for spatial reinforced concrete constructions), which allow us to find the distances between the cracks and the width of their opening, taking into account the discontinuity effect.

Then

$$\begin{aligned} \varepsilon_{bt}(y) &= \sigma_{sw}A_{sw}\cdot\frac{1}{D_{13}'}-\sigma_{sw}(y)A_{sw}\cdot\frac{1}{D_{13}'}+\frac{D_{14}'}{D_{13}'}\cdot y+\frac{D_{15}'}{D_{13}'}= \\ &= \varepsilon_{sw}\cdot E_{sw}\cdot A_{sw}\cdot\frac{1}{D_{13}'}-\varepsilon_{sw}(y)\cdot E_{sw}\cdot A_{sw}\cdot\frac{1}{D_{13}'}+\frac{D_{14}'}{D_{13}'}\cdot y+\frac{D_{15}'}{D_{13}'}, \end{aligned} \quad (7)$$

where the parameters $D_1'...D_{12}'$ are expressed as functions of the forces in the sections that cut out a representative volume (Figure 4) with the parameters of concrete, reinforcement and clutch.

The level model is used to determine the deformations of stretched concrete $\varepsilon_{bt}(y)$ along the axis of the transverse reinforcement of the i-th level of formation of different spatial cracks, the distance between them and the width of their opening in a reinforced concrete composite structure.

The nature of the diagram $\varepsilon_{sw}(y)$, performed by experiments of other authors [4, 6, 15, 16], shows that for a certain strain load, the deformations on the areas adjacent to the cracks begin to decrease and even change sign, while deformations in the middle of the section between the cracks continue to increase until at this point new crack. Analysis of the nature of the diagram $\varepsilon_{bt}(x)$ shows the necessity (Figure 5) of taking into account the deformation effect in crack [23].

7. After finding different cracks and determining the deformations in the concrete along the axis of the transverse and longitudinal reinforcement, it is possible, according to the accepted criterion for the formation of cracks, to proceed to the calculation of the distance between the spatial cracks.

In this case, knowing the deformations in the transverse reinforcement or the deformation in the longitudinal reinforcement in the section with the y coordinate and the deformation in the concrete (determined using the calculated model of the *i*-th level (Figure 4) by the formula (8), it is possible to find the relative mutual displacements of the reinforcement and concrete:

$$\varepsilon_g(y)=\varepsilon_{sw}(y)-\varepsilon_{bt}(y), \quad (8)$$

where on the basis of [23]:

$$\varepsilon_{sw}(y) = \varepsilon_{sw} + \frac{\Delta T}{E_{sw} A_{sw}} - \frac{S}{A_{sw} E_{sw}} \int_0^y \tau(y) dy. \quad (9)$$

Here S_s is the perimeter of the cross section of the reinforcement; ε_s – deformation of the reinforcement in crack; ΔT – resultant conditional tangential stresses in the local zone adjacent to the crack [23, 24]; $\tau(x)$ – conditional tangential stresses.

After substituting (8) into (9) and taking into account the accepted criterion for the formation of cracks, after differentiating and solving the inhomogeneous first-order differential equation, we may have:

$$\varepsilon_g(y) = C \cdot e^{-By} + \frac{D_{14}}{D_{13} \cdot B}. \quad (10)$$

where B – is the parameter of bond reinforcement with concrete, determined in accordance with the formula $B = \dfrac{S_S \cdot G}{K \cdot A_S \cdot E_S}$. The integration constant *C* is found from the boundary condition, according to which, for $y = 0$, $\varepsilon_g(y) = \varepsilon_{sw} + \dfrac{\Delta T}{A_{sw} E_{sw}} - \dfrac{\sigma_{bt,c}}{\nu_b E_b}$.

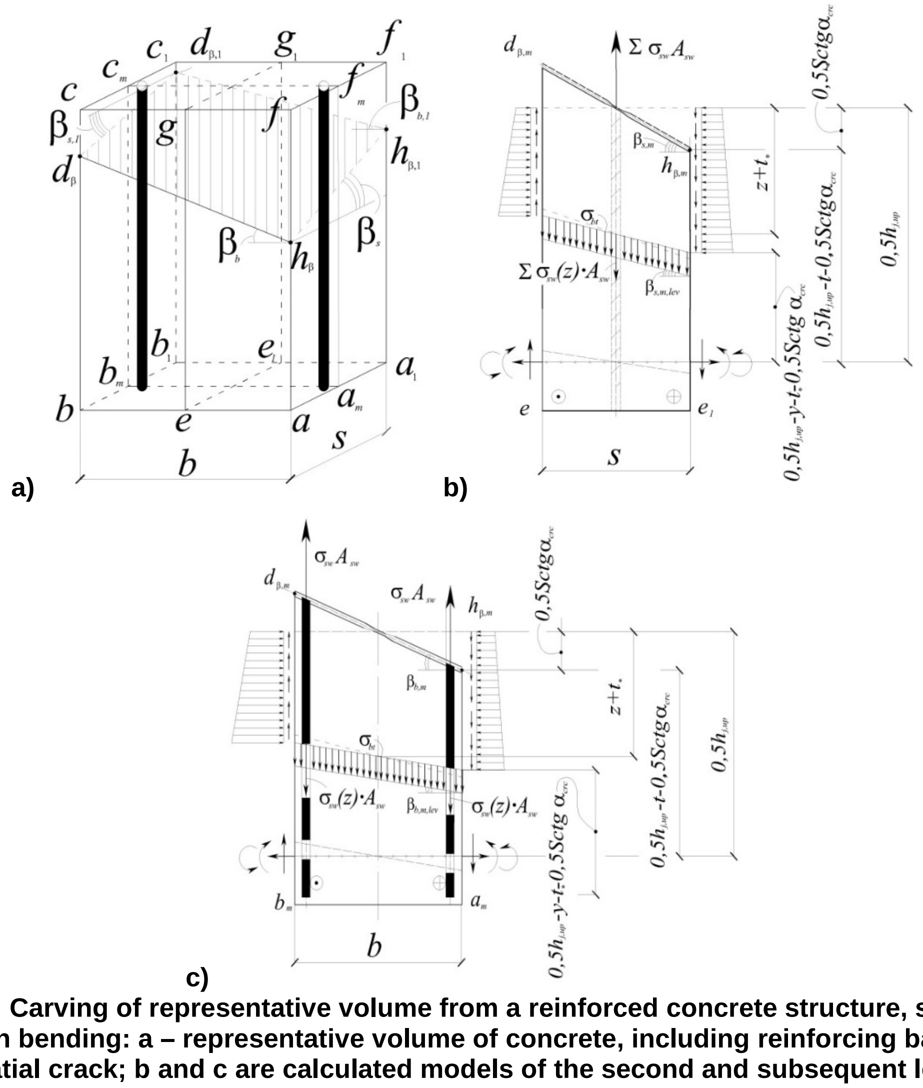


Figure 4. Carving of representative volume from a reinforced concrete structure, subject to torsion with bending: a – representative volume of concrete, including reinforcing bars and part of a spatial crack; b and c are calculated models of the second and subsequent levels for determining deformations of stretched concrete $\varepsilon_{bt}(y)$ between spatial cracks with their longitudinal and transverse sections, respectively

Then

$$C = \varepsilon_{sw} + \frac{\Delta T}{A_{sw} E_{sw}} - \frac{\sigma_{bt,c}}{\nu_b E_b} - \frac{D_{14}}{D_{13} \cdot B}. \tag{11}$$

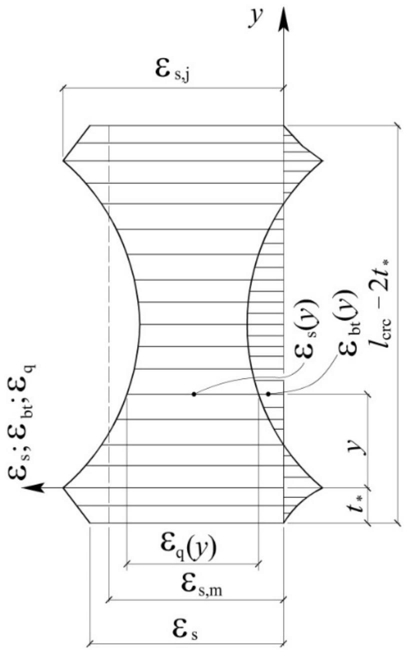


Figure 5. The diagrams of concrete deformation $\varepsilon_{bt}(y)$, reinforcement $\varepsilon_{sw}(y)$ and their relative mutual displacements $\varepsilon_g(y)$ in the area between inclined (for a flat model) or spatial (for a spatial model) cracks in RCC

Here $\frac{\sigma_{bt,c}}{\nu_b E_b}$ corresponds to concrete deformations $\varepsilon_{bt}(y)$ in a section located at a distance t_* from the section with a crack (Figure 5). In this case, the values $\sigma_{bt,c}$ are taken with a “minus” sign here and in all the below formulas.

Then,

$$\varepsilon_g(y) = \left(\varepsilon_{sw} + \frac{\Delta T}{E_{sw} A_{sw}} - \frac{\sigma_{bt,c}}{\nu_b E_b} - \frac{D_{14}}{D_{13} \cdot B} \right) e^{-By} - \frac{D_{14}}{D_{13} \cdot B}. \tag{12}$$

From the solution of the differential equation (12) and taking into account the criterion for the formation of cracks, distances between cracks of the following level along the axes of the transverse (longitudinal) reinforcement are sought:

$$l_{crc} = \frac{2\left(\ln B_4' - B' t_*\right)}{-B'}. \tag{13}$$

where the parameters B', B_2', B_3', B_4' are functions of the boundary deformations of the elongation of concrete, the parameters taking into account the effect of discontinuity (through $\sigma_{bt,c}$ and ΔT – the compressive stress and the resultant conditional tangential stress in the local zone adjacent to the crack (they are determined in accordance with the proposals of V.M. Bondarenko and Vl.I. Kolchunov [23]), geometric parameters, the parameters of bond reinforcement with concrete.

Thus, the cracking continues until the moment of destruction. In this case, not one is allocated (as is customary in a number of known techniques), but several levels of crack formation [25]:

$$\left.\begin{array}{c} l_{crc} > l_{crc,1} - no \;\; cracks \\ l_{crc,1} \geq > l_{crc} > l_{crc,2} \;\; - first \;\; level \\ l_{crc,2} \geq l_{crc} > l_{crc,3} - second \;\; level \\ \dots \;\; \dots \;\; \dots \\ l_{crc} \geq 6 \cdot t_{*} - last \;\;\; level \end{array}\right\} . \quad (14)$$

Comparing the functional and level values of l_{crc}, an analysis is performed of the possible realization of the appearance of subsequent fracture levels.

Having the levels of crack formation along the longitudinal and transverse reinforcement of the reinforced concrete structure, a complete picture of the various cracks adjacent to the concentrated force and to the support is constructed.

The refined and most complete degree of realization of the cracks (whether these cracks intersect the lateral reinforcement or it will be crossed only by a dangerous spatial crack) are determined from the analysis of the stress-strain state along the clamps on the basis of the design scheme, the subsequent level shown in Figure 4, which "closes" to the multi-iterative process provided by the PC.

Crack opening is considered as the accumulation of relative mutual displacements of reinforcement and concrete in areas located on both sides of the crack; this takes into account the effect associated with the violation of the continuity of concrete (the modernized Thomas' hypothesis) [23, 24].

In accordance with this hypothesis, the problem by definition reduces to finding the relative mutual displacements $\varepsilon_g(y)$ of reinforcement and concrete in different sections between the cracks:

$$a_{crc} = 2\int_{0}^{t_*} \varepsilon_g(y_1)dy_1 + \int_{0}^{\eta \cdot l_{crc}} \varepsilon_g(y)dy + \int_{\eta \cdot l_{crc}}^{l_{crc}} \varepsilon_g(y)dy . \quad (15)$$

After integration and some simplifications, we get:

$$a_{crc} = -\frac{2\Delta T}{G} - \frac{2B_{a,2}}{B} - \frac{2B_2}{B}\ln\left(1 + \frac{B_{a,2} \cdot A_{sw}E_{sw}}{q_{sw}S + B_{a,1}A_{sw}E_{sw}}\right), \quad (16)$$

where G – conditional modulus of deformation of bond reinforcement with concrete; S – perimeter of the cross section of the reinforcement; ε_s – deformation of the armature in the crack; A_{sw} – the cross-sectional area of the clamps; other parameters have already been expanded above.

3. Results and Discussion

The algorithm in accordance with the proposed methodology for calculating the rigidity of flat-stressed and spatial reinforced concrete composite structures with the use of the software complex of the PC "LIRA-CAD" was considered in two variants [14, 16, 17, 21, 22].

In the first variant, the rigidity is determined with using the special technique of simulating explicit cracks-gaps when they are opened and closed (Figure 2), taking into account the effect of discontinuity and inconsistency of deformations of concrete and reinforcement. Reinforcing bars are modeled with an additional 201 FE, and a possible closure of the crack is a concrete 255 FE.

The second option involves performing the calculation without changing the order and numbers of the FE. In the FEs adjacent to implicit cracks, their thickness decreases. The work of each pair of FEs is calculated twice using a two-element console model (Figure 3): before "disconnection" FE (W_1) and after "disconnection" FE with applied nodal forces and deformation effects from crack opening, taking into account the discontinuity effect (with distributed the reinforcement is replaced by two (for a planar model) and four (for a spatial model) by rod fibrous FEs.

Efforts at the nodes of the two-element console model are determined from the nonlinear calculation of the entire reinforced concrete construction for the given force and deformation effects. The movements of the nodes are determined from the calculation of the two-element console model (TeCM) with the forces applied and stresses in the nodes (Figure 3). When solving the inverse problem associated with the need to determine the width of the discrete crack opening, deformation effects are not specified, but a gap with its minimum possible width is modeled. In this case, the displacement of its banks along three mutually perpendicular directions as a result of the calculation of the TeCM determines the corresponding components of the width of the discrete crack opening between this FE pair.

As a result, the new thickness of the FEs adjacent to the crack is determined by formula (2). The calculation algorithm assumes the presence of an iterative process, which is controlled by the achieved thickness accuracy of the indicated FEs. In this case, it is also possible to analyze the width of the crack opening, obtained in existing software systems in the finite elements with reduced thickness, which adjoin the imaginary discrete crack [14, 21, 22]. Such a comprehensive comparative analysis of the widths of the disclosure will undoubtedly contribute to an in-depth study of this quantity, which in the theory of reinforced concrete is "obstinate," which is not amenable to description in the form of an acceptable theoretical formula, has been for many decades.

Thus, a method is proposed for calculating the resistance of reinforced concrete structures with the combined effect of transverse force, bending and twisting moments and longitudinal force for the second stage of the stress-strain state, which allows us to reveal the actual stress-strain state in the presence of spatial cracks, with the distance between the cracks and the width of their disclosure.

4. Conclusions

1. It is substantiated the urgency and necessity of technique development for modeling spatial discrete cracks in the case of complex resistance of reinforced concrete structures of buildings and structures, under the action torsion with bending, which have a significant effect on their stress-strain state, and, first of all, on the change in rigidity.

2. The working calculation prerequisites have been developed on the basis of analysis and generalization of experimental and theoretical studies. They allow one to simulate discrete cracks and evaluate the rigidity of reinforced concrete structures, with complex resistance – torsion with bending, which includes:

- modeling of different spatial cracks by bilinear surfaces;
- the proposed classification of basic spatial cracks;
- block calculation model with working sections, to determine the stress-strain state of which the capabilities of existing software complexes are attracted, taking into account the inter-medial disturbance in the seam between the layers of concrete and the incompatibility of deformations of concrete and reinforcement;
- modeling cracks by "shaving" the final elements with the help of cracked cracks and setting the deformation effect, taking into account the effect of discontinuity;
- involvement of a special two-element cantilever model for analyzing the resistance along the fracture path and along the seam between the concrete layers;
- change in the thickness of the final elements adjacent to the crack and the introduction of equivalent rigidities;
- finding the projections of adjacent spatial cracks on the horizontal (vertical) with the use of analytic dependencies, which are based on the extremum of functions of many variables using Lagrange multipliers;
- a special design model for multilevel crack formation, which allows us to search for the distances between the cracks and the width of their openings, taking into account the effect of discontinuity, and to display a multilevel crack development process involving inequalities.

3. The algorithm is developed for calculating the stiffness of plane-stressed and spatial reinforced concrete structures with complex resistance – torsion with bending and the presence of discrete cracks involving existing computer systems, including:

- two options for special modeling of opening and closing of cracks, taking into account the effect of discontinuity and inconsistency of deformations of concrete and reinforcement;
- the iterative process of applying forces in the nodes of the two-element console model, determined from the nonlinear calculation of the entire reinforced concrete structure (or its block design model with working sections-spatial and normal, passing through the end of the spiral-

shaped crack) to given force and deformation effects (in solving the inverse problem , associated with the need to determine the width of the opening of a discrete spiral-like crack, deformation effects are not specified, but a spatial gap is modeled minimally possible opening width, thus moving it coasts along three mutually perpendicularly directions define components width of the cracks between the respective pair of finite elements), which allows you to significantly refine their stress-strain state in the case of force, deformation (including seismic) impacts and, as a consequence, significantly improve the efficiency of reinforced concrete design.

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