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SYMMETRY TRANSFORMATION PARAMETERS AS CLASSICAL DYNAMIC VARIABLES IN THE GAUGE THEORY

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A modification of the gauge theory is proposed, in which the set of generalized coordinates is supplemented with symmetry transformation parameters, and the condition is additionally imposed on the latter, which ensures the classical character of their dynamics in the quantum theory. As a result, additional dynamic variables and transverse physical degrees of freedom in the Hamiltonian become separated. The classical theory of the Yang–Mills field is considered.

Keywords: gauge theory, quantization, symmetry transformation parameters, Hamiltonian

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ПАРАМЕТРЫ ПРЕОБРАЗОВАНИЯ СИММЕТРИИ КАК КЛАССИЧЕСКИЕ ДИНАМИЧЕСКИЕ ПЕРЕМЕННЫЕ В КАЛИБРОВОЧНОЙ ТЕОРИИ

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Предложена модификация калибровочной теории, в которой множество обобщенных координат дополняется параметрами конечных преобразований симметрии. На последние наложено дополнительно условие, обеспечивающее классический характер их динамики после квантования. Как результат, дополнительные динамические переменные и физические поперечные степени свободы в гамильтониане разделяются. Рассмотрена классическая теория поля Янга – Миллса.

Ключевые слова: калибровочная теория, квантование, параметр симметрии, гамильтониан

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Introduction

A simple and natural understanding of the dynamic structure of the gauge theory is that the initial set of dynamic variables in its mathematical apparatus is redundant and some of them should be eliminated by solving the equations of constraints and additional gauge conditions [1]. However, in the subsequent development, this reduction procedure has been replaced by the expansion of the phase space of the theory with the addition of Lagrange multipliers with the corresponding canonical momenta and ghosts, as well as the expanded BRST-symmetry [2 – 6]. Another option for expanding the phase space of the gauge theory was proposed in Ref. [7], where the parameters of finite symmetry transformations were added to the original dynamic variables. These finite shifts in the group space are constructed in the form of integrals of infinitesimal shifts generated by constraints. However, such an extension alone does not solve the problem of separating physical degrees of freedom and pure calibrations and a dynamic interpretation of the theory. It should be supplemented with a structure that allows connecting finite shifts in the group space with observations. In the case of the dynamics of relativistic particles with reparametrization invariance of their world lines, the intrinsic parameter of the symmetry group plays the intrinsic time of each particle. This invariant parametrization also arises naturally in the BRST-invariant representation of the propagator of covariant quantum theory for a relativistic particle [8] and reproduces the Fock [9] and Schwinger [10] formalism based on the introduction of proper time. In Ref. [11], the introduction of this parameter was proposed to be supplemented with the condition of its classical dynamics with the corresponding modification of the initial action. This addition allows us to connect the proper time with observations and get a dynamic interpretation of covariant quantum theory. It can be assumed that such a two-stage modification of the singular theory (adding finite symmetry transformations to dynamic variables and an additional condition for their classical dynamics) will be an effective way to separate physical degrees of freedom and pure gauges in

the general case.

In this paper, this modification has been shown as an example of a free Yang – Mills field. The result should be a separation of the dynamics of the physical transverse components of the Yang – Mills field and the “motion” of the longitudinal components (pure calibrations) in the group space.

Modification of the Yang – Mills’ action

The proposed modification of the gauge theory action is divided into two stages. We proceed from the canonical form of action, namely,

$$I = \int dt [p_i \dot{q}_i - \lambda_a \varphi(p, q) - h(p, q)] \quad (1)$$

where the constraints obey the commutation relations:

$$\begin{aligned} \{\varphi_a, \varphi_b\} &= C_{abd} \varphi_d, \\ \{\varphi_a, h\} &= h_a \varphi_{ad} \end{aligned} \quad (2)$$

(we consider the case $h_a = 0$), and variations of the Lagrangian multipliers ensuring the invariance of the action (1) with respect to the infinitesimal symmetry transformations

$$\begin{aligned} \delta q_i &= \varepsilon_a \{q_i, \varphi_a\}, \\ \delta p_i &= \varepsilon_a \{p_i, \varphi_a\} \end{aligned} \quad (3)$$

have the form

$$\delta \lambda_a = \dot{\varepsilon}_a - C_{abd} \lambda_b \varepsilon_d. \quad (4)$$

At the first stage, according to Ref. [7], we replace the Lagrangian multipliers with explicit functions of the parameters defining the finite symmetry transformation ($\delta s_a = \varepsilon_a$):

$$\lambda_a = \dot{s}_b \Lambda_{ab}(s) \quad (5)$$

which are integrals of functional-differential Eqs. (4). At the second stage, according to Ref. [11], we add a variation generated by the infinitesimal shift of new dynamic variables to the action. We call this step a condition of classical dynamics, since it allows one to remove integration over new dynamic variables in the functional-integral rep-

resentation of the propagator of covariant quantum theory. We carry out these constructions as an example of a free Yang – Mills field $A_{\mu a}$ where μ is a space-time index ($\mu = 0, 1, 2, 3$), and a is an internal index of the gauge theory. Here, A_{0a} are the Lagrange multipliers, so, at the first stage, the original Yang – Mills Lagrangian function takes the form

$$L = \frac{1}{2} \left[\left(\dot{A}_{ia} - \nabla_i (\dot{s}_b \Lambda_{ba}) \right)^2 - B_{ia}^2 \right], \quad (6)$$

where B_{ia} is the Yang – Mills “magnetic field” tension, and the covariant derivative is determined by the following relation [1]:

$$\nabla_i F_a = \partial_i F_a - ig T_{bad} A_{ib} F_d. \quad (7)$$

We will not have need of the explicit form of functions Λ_{ab} here. Now, following Ref. [8], we will still expand the set of new variables by adding infinitesimal shifts ε_a to them, as well to Lagrangian function (6) we add its variation generated by these infinitesimal shifts:

$$\begin{aligned} \tilde{L} = & \frac{1}{2} \left[\left(\dot{A}_{ia} - \nabla_i (\dot{s}_b \Lambda_{ba}) \right)^2 - B_{ia}^2 \right] + \\ & + \left(\dot{A}_{ia} - \nabla_i (\dot{s}_b \Lambda_{ba}) \right) \left(\nabla_i \left(\dot{\varepsilon}_c \Lambda_{ca} + \dot{s}_c \frac{\partial \Lambda_{ca}}{\partial s_d} \varepsilon_d \right) \right). \end{aligned} \quad (8)$$

The canonical form of the modified Yang – Mills’ action

Now we turn to the canonical form of the modified action (8). Let us find the canonical momenta:

$$\begin{aligned} \pi_{ia} = & \dot{A}_{ia} - \nabla_i (\dot{s}_b \Lambda_{ba}) + \\ & + \nabla_i \left(\dot{\varepsilon}_c \Lambda_{ca} + \dot{s}_c \frac{\partial \Lambda_{ca}}{\partial s_d} \varepsilon_d \right), \end{aligned} \quad (9)$$

conjugated to A_{ia} , and

$$\begin{aligned} p_{s_b} = & -\Lambda_{ab} \nabla_i \left(\dot{A}_{ia} - \nabla_i (\dot{s}_c \Lambda_{ca}) \right) - \\ & - \nabla_i \left(\dot{A}_{ic} - \nabla_i (\dot{s}_q \Lambda_{qc}) \right) \frac{\partial \Lambda_{bc}}{\partial s_d} \varepsilon_d - \\ & - \Lambda_{ab} \Delta \left(\dot{\varepsilon}_c \Lambda_{ca} + \dot{s}_c \frac{\partial \Lambda_{ca}}{\partial s_d} \varepsilon_d \right), \end{aligned} \quad (10)$$

conjugated to s_a ($\Delta = \nabla_i \nabla_i$), and

$$P_{\varepsilon_b} = -\Lambda_{ab} \nabla_i \left(\dot{A}_{ia} - \nabla_i (\dot{s}_c \Lambda_{ca}) \right), \quad (11)$$

conjugated to ε_b .

From here we immediately obtain the constraint equations,

$$p_{s_b} = -\Lambda_{ab} \nabla_i \pi_{ia} + P_{\varepsilon_a} \Lambda_{ac}^{-1} \frac{\partial \Lambda_{cb}}{\partial s_d} \varepsilon_d, \quad (12)$$

and generalized velocities in the following combination:

$$\begin{aligned} \dot{\varepsilon}_c \Lambda_{ac} + \dot{s}_c \frac{\partial \Lambda_{ca}}{\partial s_d} \varepsilon_d = \\ = \Delta^{-1} \left(\nabla_i \pi_{ia} + \Lambda_{ab}^{-1} p_{\varepsilon_b} \right). \end{aligned} \quad (13)$$

Now we find the Hamilton function of the modified theory:

$$\begin{aligned} \tilde{h} = & \frac{1}{2} \left[\pi_{ia}^2 + B_{ia}^2 \right] - \\ & - \frac{1}{2} \left[\nabla_i \Delta^{-1} \left(\nabla_k \pi_{ka} + \Lambda_{ab}^{-1} p_{\varepsilon_b} \right) \right]^2, \end{aligned} \quad (14)$$

where we used (13).

Let us see what we have got as a result. Obviously, the constraints (12) commute with the Hamiltonian (14). The Hamiltonian does not contain ε_a which means that canonical momenta P_{ε_a} (color density of a charge) are integrals of motion. We perform the orthogonal longitudinal-transverse splitting of the canonical momenta:

$$\pi_{ia} = \nabla_i \left(\chi_{0a}^L + \chi_a^L \right) + \pi_{ia}^T, \quad (15)$$

with

$$\Lambda_{ab} \Delta \chi_{0b}^L = -P_{\varepsilon_a}. \quad (16)$$

As a result, the quadratic form of the momenta in the Hamiltonian contains only the transverse components:

$$\tilde{h} = \frac{1}{2} \left[\pi_{ia}^{T^2} + B_{ia}^2 \right]. \quad (17)$$



Thus, the longitudinal components of the Yang – Mills are completely excluded from the dynamics in time. For them, only the “dynamics” in the group space described by the constraints (12) remains. Here, the evolution parameters s_a are supplemented by dynamic variables P_{ε_a} which can be eliminated by choosing the origin of the longitudinal component of the momentum according to Eq. (16). In the gauge theory with the constraints linear in canonical momenta, these quantities do not have a dynamic meaning.

Summary

Thus, in the Yang – Mills theory, and generally in the theory with the constraints linear in canonical momenta, the introduction of the classical parameters of symmetry transformations as additional dynamic variables allows us to separate the physical transverse and gauge

longitudinal degrees of freedom. At the same time, classical external sources which are generators of classical symmetry transformations are also added as dynamic variables. These sources themselves can be set equal to zero, as long as the separation of the physical degrees of freedom is done. In theories with quadratic on the canonical momenta constraints, such as the theory of gravity, in which there is a time problem, the modification proposed here introduces the concept of proper time, which also has its own classical source – energy. In contrast to the case considered here, this energy can have a dynamic meaning. This issue will be considered separately.

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