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Evolving crack influence on the strength of frozen sand soils

T.A. Gavrilov*, **G.N. Kolesnikov**

Petrozavodsk State University, Petrozavodsk, Russia

* E-mail: gtimmo@mail.ru

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Abstract. The object of the study in this work is the relationship between the elastic modulus, tensile stresses and deformations of frozen sandy soil with evolving cracks using the example of three-point bending. The goal is to develop a methodology for determining the modulus of elasticity and tensile stresses in frozen sandy soil under force indirectly. The choice of the object and the purpose of the study is motivated by the relevance of the soil strength problems during seasonal freezing. To achieve the goal, methods of mathematical modeling of mechanical systems with changing characteristics during the deformation, also the testing methods of samples on the SHIMADZU AGS-X test machine were used. A mathematical model has been developed, the realism of which is ensured by taking into account the evolution of a crack and using the effective geometric characteristics known from fracture mechanics. It has been substantiated that the destruction of the material occurs on the descending branch of the «load - displacement» diagram. The simulation results are consistent with the data known in the scientific literature. The condition for the model application is the existence of an extremum point on the curve «load - displacement». Prospects for the development of the topic are associated with the adaptation of the proposed approach to the analysis of the of frozen soil state, taking into account its rheological properties.

1. Introduction

The object of study in this work is the relationship between the elastic modulus, tensile stresses and deformations of frozen sandy soil with evolving cracks. The choice of the research object is motivated by the relevance of the problem of soil strength during seasonal freezing and the need in continuing the research focused on solving these problems. To develop the topic of the work, we pay attention to the following facts.

Large volumes of sandy soils, which include both sand and gravel mixtures, are used in the construction of roads, buildings and other engineering structures. The strength of the soil under power and temperature influences should be sufficient to ensure the reliability of these objects. Tensile stresses are known to appear in the soil mass during seasonal freezing. These stresses can cause frost cracking in the upper layer of the highway, if the soil strength is insufficient [1, 2]. In this case, tensile stresses are proportional to the elastic modulus; therefore, simple methods for determining its values are necessary. In addition, the modulus of elasticity characterizes the rigidity and strength of frozen soil [3]. Obviously, it is necessary to know the strength and elastic modulus of frozen soil while designing, building, monitoring and forecasting the condition of roads and other construction objects [4–6].

The analysis of the scientific literature showed that intensive studies of the state of frozen soils are currently being conducted taking into account various factors. Studies are usually experimental, and the publication of their contributes to the better understanding of the frozen soils behavior under influences of various kinds. Tests of the frozen soils samples described in scientific works show that the relationship between load and displacement (for example, between force and vertical displacement of the point of its application in three-point bending) is usually non-linear. Nonlinearity is explained by the influence of plastic deformations.

In [7, p. 31], it was shown that plastic deformations and fractures happened when the nucleation of microcracks predominated. It follows that the appearance and evolution of cracks can be considered as the primary cause of the frozen soil destruction. The key issue in this case is the ratio between the damage and the displacement. Then the above nonlinearity of the relationship between load and displacement can be explained by the crack evolution.

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An additional argument in favor of this conclusion can be one of the results of [8], in which a model of the mechanical condition of soil has been proposed on the base of an analysis of tests under uniaxial compression using a logistic equation in this four parameter model, from the results of uniaxial compression tests it becomes possible to determine the elastic modulus of intact material. In addition, the bifurcation phenomenon of cracks in these tests has been explained. In this case, the use of the results of mechanical testing of rocks in the study of the frozen sandy soil strength seem appropriate, since frozen sandy soil can be considered as a kind of composite inhomogeneous material consisting of mineral particles, ice as a binder, unfrozen water inclusions, pores with air and microcracks.

With increasing mechanical stress, the bearing capacity of the soil gradually decreases due to the evolution of microcracks to meso- and macro-damage to the material. Excessive crack evolution significantly reduces the strength and rigidity of frozen ground. However, despite the obvious need, the problem of frozen soils with cracks modeling in order to determine the elastic modulus and evaluate tensile stresses still does not have a sufficiently complete solution [2, 8].

Tensile stresses are known to play the main role in the evolution of cracks. However, uniaxial compression tests dominate in the experimental literature on frozen soils due to the technical difficulties of both specimen preparation and direct measurement of tensile stresses. Therefore, along with direct methods, indirect methods to determining tensile stresses during tests are used, for example, acoustic technologies, three- and four-point bending [7]. In this work three-point bending tests for the indirect determination of the elastic modulus and tensile stresses in a section with an evolving crack were used.

Tests for three-point bending of a frozen soil beam were used by the authors of [7, 9] to determine the stress intensity factor in the neighborhood of the crack tip. According to the Griffiths – Irwin criterion, crack propagation will begin if the stress intensity factor at the crack tip reaches a certain critical value [10]. The influence of evolving cracks on the stress – strain state of sandstone and other brittle rocks during uniaxial compression was studied in [8] using the logistic equation of a three-point bending test. The three-point bending of samples consisting of sand and ice was studied in [11]. The test results showed that such a material is brittle, its strength increases with the decreasing temperature. A model based on a failure arising from the propagation of defects in an ice matrix is presented. Despite the good agreement between the model and experimental results, it was noted that the further work is needed to refine the model.

A modern understanding of the problems of modeling the mechanical state of frozen soils is shown in [1, 2, 12–16, etc.].

The review showed that in literature there is no solution to the problem of determining the elastic modulus of frozen sandy soil and tensile stresses in the cross section of a beam with an evolving crack according to the results of tests for three-point bending.

Objective: to develop a methodology for determining the modulus of elasticity and tensile stresses in frozen sandy soil with an evolving crack under a force.

2. Material and Methods

Object of study: a beam of rectangular cross section with a width of 55 mm, a height of 39 mm; the span of 280 mm (Figure 1, 2).

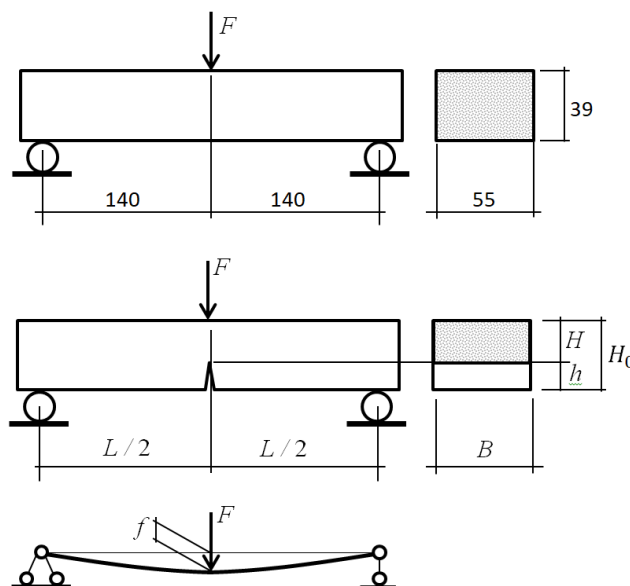


Figure 1. Beam diagrams before and after cracking (dimensions in mm).

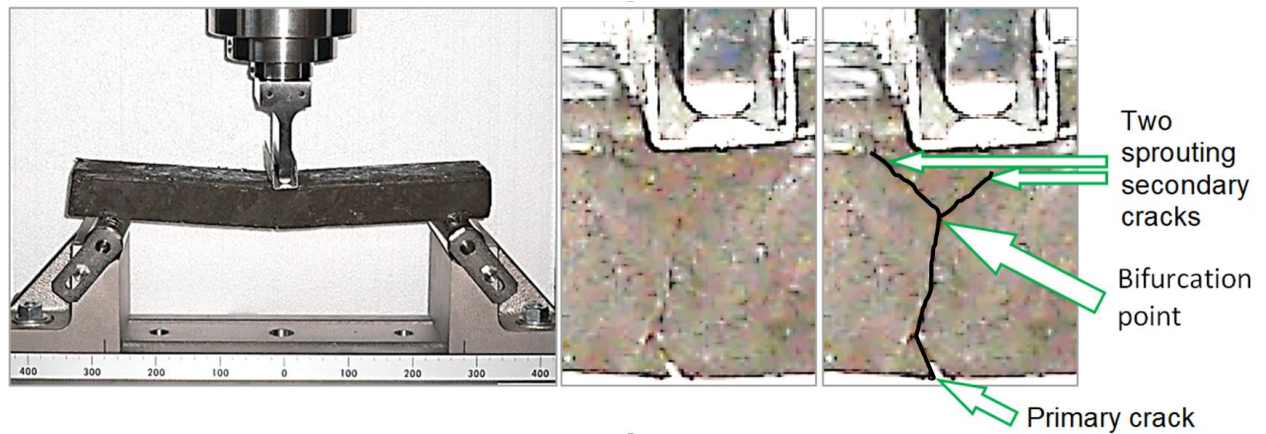


Figure 2. A beam and its central part with an evolving crack.

Samples preparation for testing was carried out by the analogy with [9].

However, there are some particular features. Samples were made from a mixture of sandy soil (with particle sizes not exceeding 2 mm) and gravel (with particle sizes from 2 to 10 mm). This mixture belongs to the first group of fortified sand and gravel mixtures according to Russian State Standard GOST 23735-2014. In the case under discussion, the sand-gravel mixture serves as the material of the upper layer of the dirt road. To predict the bearing capacity of such a road in winter, it is necessary to determine the strength of the above mixture when it freezes.

The particle size distribution of the mix was determined by the sieve method and is characterized by the presence of the soil particles remainder on sieves with openings with a diameter of 10, 7, 5, 3, 2 mm and on the pallet for particles of the size less than 2 mm, respectively, 10.0, 7.5, 5.2, 21.0, 30.1 and 26.2 % (by mass).

The particle size distribution for the sand-gravel mixture is shown in Figure 2.1.

Particle size distribution and the properties of the of soil mixture components affect their strength and stiffness [31]. However, we consider only one soil mixture, but with different water contents. The practical value of the humidity indicator is explained by the fact that freezing water significantly changes the strength of the soil. For example, the Figure (2.2) shows the curves “Displacement f – Force F ” for beams from Figure 2, consisting of the frozen soil mixture mentioned above (Figure 2.1), but different in the water content in the material of the beams.

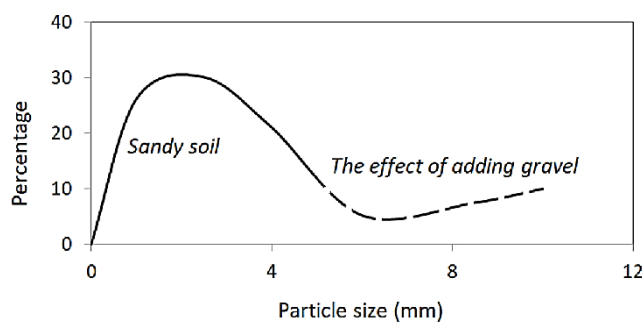


Figure 2.1. The particle size distribution for sand-gravel mixture.

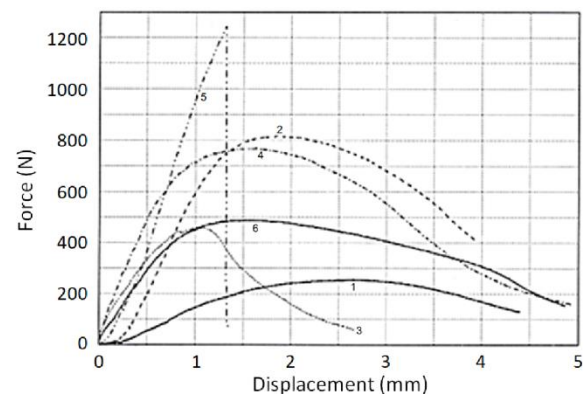


Figure 2.2. Curves «Force F – Displacement f » of the beams according to Figure 2 with different water content in the material of the beams/

Beam 5 (Figure 2.2) contained a large amount of water, which explains the analogy of the dependence “displacement f – force F ” for this beam with the data known on the dependence of the strength on the displacement for the beams consisting of ice [32]. Beam 5 is excluded from the further consideration, since the analysis of the strength of ice beams is beyond the scope of our work.

The water content in the material of each sample was measured using a SHIMADZU MOC-120H moisture analyzer at the temperature of 105 °C in the drying chamber of the analyzer. The decrease in moisture content in the sample during drying was automatically controlled with an accuracy of 0.01 % (by weight) every 30 seconds. The water content (relative humidity) in the material of beams 1, 2, 3, 4 and 6 was equal, respectively, to 7.88, 10.89, 8.75, 12.53 and 14.19 %.

At the moisture value indicated above, the tensile stresses of each of the beams in the section with a crack were calculated (Figures 1 and 2): $\sigma = M / W$, where $M = F_{\max} L / 4$, $W = BH_0^2 / 6$. The calculation results are shown in Figure 2.3. Thus, five series of beams were processed. To determine the number of samples in a series, you can use the method of analogies. Indeed, from a mechanical point of view, ice in frozen ground functions as a cement analogue in concrete [33, 34]. This means that frozen ground can be considered as an analogue of lightweight concrete, for example, cellular concrete. Therefore, to determine a sufficient number of samples for frozen soil testing the recommendations of Russian State Standard GOST 10180-2012 (paragraph 4.1.3 can be used). According to these recommendations the number of samples made from cellular concrete is taken to be 3 in each series. Additional assessments of the of the research results reliability were obtained using the determination coefficient R^2 (Figure 2.3), as well as the elements of the indirect measurements theory (Figures 4, 5).

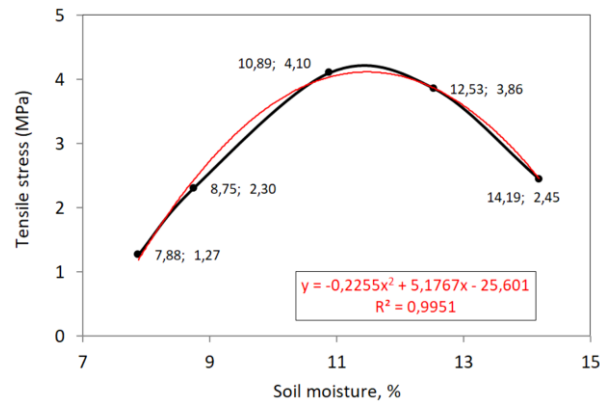


Figure 2.3. Tensile stresses in the beam according to Figure 2 for $F = F_{\max}$.

The data obtained show (Figure 2.3) that the dependence of tensile stresses on humidity in the range from 7.88 to 14.19 % can be described by a polynomial of the second degree with a determination coefficient of almost 1. This means that there is a functional dependence of stresses on the water content in soil. The reliability of the data presented in Figure 2.3 is also confirmed by their consistency to the results known from the scientific literature [33, p. 152–158].

Three-point bending tests were carried out on a SHIMADZU AGS-X testing machine, for which the relative measurement error is not more than 1.0 %. The download speed was chosen equal to 5 mm / min. The temperature of the material on the fracture surface of the samples (minus 4.6 °C) was measured in a non-contact manner using a pyrometer immediately after the test.

Tests have shown that in the process of fracture during three-point bending, a primary crack forms, the length and width of which increase with the load increasing and this is natural. In addition, the signs of bifurcation of the primary fracture and the formation of secondary fractures can be seen (Figure 2).

Note that the bifurcation of a crack during uniaxial compression of sandstone and other brittle materials was predicted in [8, p. 1016] using the logistic equation in a mathematical model of the evolution of the stress-strain state of a material.

In our case (Figures 1 and 2), it is important to pay attention to the fact that with the increasing crack length h , the effective cross-section height H decreases: $0 \leq h \leq H_0$; $H = H_0 - h$.

According to the test results, the diagram “load F – the vertical displacement of the point of force F ” application is obtained. The mathematical processing of the test results is discussed in the following section.

3. Results and Discussion

3.1. Mathematical Processing of the Test Results

Let B and H_0 be, respectively, the width and the height of the cross section of the beam, f is the vertical displacement of the point of force F application, h is the length of the crack (Figure 1). As noted above, with increasing crack length h , the effective cross-sectional height $H = H_0 - h$ decreases. Accordingly, if f changes by some Δf , then the change in effective height is ΔH . With small changes, the dependence of ΔH on Δf can be written as a linear relation with a constant proportionality coefficient K_1 :

$$\Delta H = \frac{\Delta f}{H_0} K_1 H. \quad (1)$$

Let us divide both sides of equality (1) by H_0 and move on to the dimensionless parameters θ and $\Delta\theta$:

$$\theta = \frac{H}{H_0}, \quad \Delta\theta = \frac{\Delta H}{H_0}. \quad (2)$$

The parameter θ can be considered as a geometric characteristic of the effective area. The values of θ vary from 0 to 1, with 0 corresponding to a completely damaged material, and 1 to a material without damage (without cracks).

Equality (1) with $\Delta\theta \rightarrow 0$ transforms to the form:

$$\frac{d\theta}{\theta} = \frac{df}{H_0} K_1. \quad (3)$$

Integrating both sides of the equality (3), we determine the integration constant from the condition: if $f = 0$, then $H = H_0$, i.e. $\theta = 1$. We get:

$$H = H_0 e^{\frac{f}{H_0} K_1}. \quad (4)$$

Using (4), we define I – the effective moment of inertia of the cross section with an evolving crack and W – the effective moment of resistance of the same section:

$$I = \frac{BH^3}{12}, \quad (5)$$

$$W = \frac{BH^2}{6}. \quad (6)$$

For the considered example (Figure 1), we get: $I = 2.719 \cdot 10^{-7} \exp(76.92fK_1)$,
 $W = 0.1394 \cdot 10^{-4} \exp(51.28fK_1)$.

The experiments showed that the ratio “load F – displacement f ” can be represented with sufficient accuracy in the form:

$$F = \frac{48EI f^3}{L^3}. \quad (7)$$

Taking into account equalities (4) and (5), we note that in relation (7) are f , I , F are the variables. In order to obtain explicitly the dependences $F(f)$ and $\sigma(f)$, where σ is the tensile stress in the cross section with a crack, it is necessary to determine the coefficient K_1 (1) indicated above and the elastic modulus E . We will find the values of K_1 and E using the test results. The dependence $F(f)$ obtained in the tests is nonlinear, $F_{extr} = 769$ N and $f_{extr_F} = 1.574$ mm (curve 4 in Figure 2.2). The test results are shown in Figure 3 with markers.

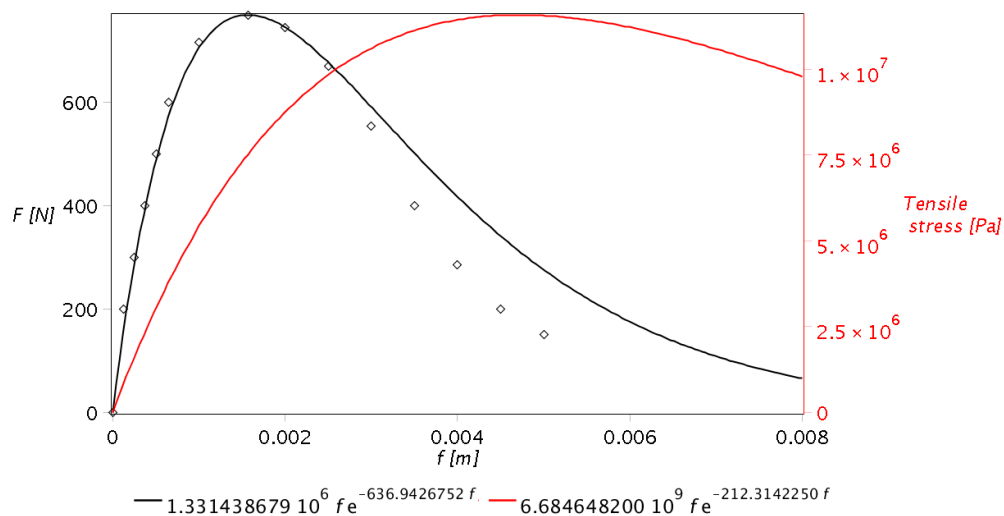


Figure 3. The results of tests (markers) and simulations (red and black lines).

Taking into account the relation (7) and the experimental value f_{extr} , we find coefficient K_1 (1) indicated above from the condition:

$$\frac{dF}{df} = 0. \quad (8)$$

For the example considered (Figure 1) after the transformations we get: $dF / df = E(0.5944 \cdot 10^{-3} + 0.4573 \cdot 10^{-1} f K_1) \exp(76.92 f K_1)$, $K_1 = -8.280$.

Then, again taking into account (7), and using the calculated coefficient K_1 and the experimental values f_{extr} and F_{extr} , we determine the elastic modulus E from the equation $F = F_{extr}$. For the example considered (Figure 1) we find $E = 2240$ MPa after the transformations. Note that this value was obtained at the above temperature (minus 4.6 °C) and is consistent with the data published. For example, in [1] it was shown that the elastic modulus of the upper part of the roadway in winter increases and can reach 10000 MPa.

Using the calculated values of K_1 , E and taking into account relation (7), after the transformations, we find the dependence $F(f)$ explicitly. For the example under consideration, the curve $F(f)$ is presented in Figure 3.

The information obtained is sufficient to determine the maximum tensile stresses σ in the section with a crack. Using the bending moment $M = FL/4$ and the effective moment of resistance (6), we write:

$$\sigma = \frac{FL}{4W}. \quad (9)$$

After the transformations and taking into account (6), (7), (4) and (1), we obtain the dependence $\sigma(f)$ in an explicit form. For the example considered, the curve $\sigma(f)$ is shown in Figure 3. In the example considered, $\sigma_{extr} = 11.58$ MPa, the corresponding displacement is $f_{extr_\sigma} = 4.71$ mm.

Thus, the technique of mathematical processing of the tests results for three-point bending is developed. However, all initial data and calculation results were assumed to be deterministic. In the next section, we will discuss the influence of random deviations of the input data on the simulation results.

3.2. The Random Factors Influence on the Simulation Results

Comparison of the experimental results (markers in Figure 3) and the analytical description of the load – displacement dependence (solid line in the same figure) show that the simulation results do not contradict the experimental data.

Nevertheless, there are many factors (distance between supports, transverse dimensions of the sample, temperature, ice and mineral particles in the soil, particle size distribution, etc.), the quantitative characteristics of which include small random deviations from average values. For engineering practice, it is important to analyze the effect of random deviations on simulation results.

In the case under consideration, the analysis of the influence of random factors is simplified due to the fact that we have obtained analytical expressions of the force F and stress σ in the form of functions f (Figure 3). Assuming that the deflection f is determined with a relative error of not more than 5 %, we use the well-known method for estimating the accuracy of indirect measurements [18], according to which the error in determining the force F is calculated by the formula:

$$\Delta F = \pm \left| \frac{dF}{df} \right| 0.05 f. \quad (10)$$

The derivative dF/df is also used in equation (8).

Taking into account (10), we can calculate the relative error $\varepsilon_F = |\Delta F/f|$.

For illustration, the calculation results of ΔF and ε_F are presented in graphical form in Figure 4, 5, respectively. The markers in Figure 4 show the results of the experiment on Figure 3.

Intensive destruction of the sample (Figure 2) was observed at $f \approx 0.0031 \pm 0.00016$ m = 3.1 ± 0.16 mm. It should be noted that at $f > 0.0031$ m, the difference between the experiment and the simulation increases (Figure 4). This feature indicates the presence of factors that are not taken into account in the simulation. However, it is important for engineering practice to prevent fracture. Hence, in the range $0 \leq f \leq 3.1$ mm, the state of the material is of most interest for practice. It is at this interval that the agreement between the experiment and the simulation is quite high (Figure 5).

So, if $f > 3.1 \pm 0.16$ mm, then the experiment can be terminated. However, we are publishing a full load displacement diagram for possible use in further research.

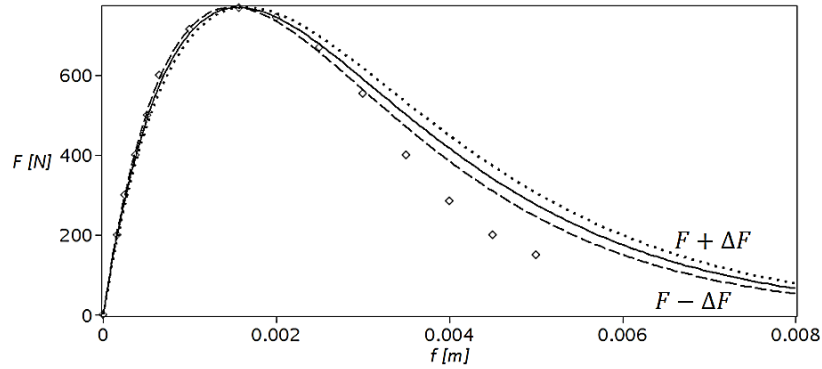


Figure 4. To the analysis of the random factors influence.

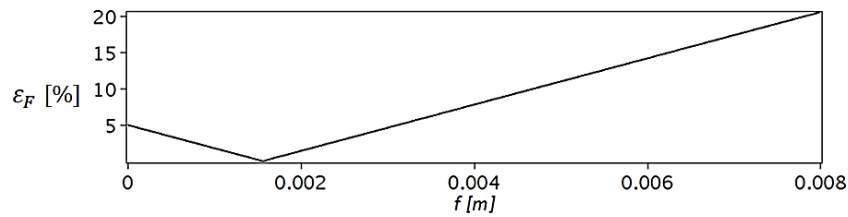


Figure 5. Relative error variation ε_F depending on deflection f .

Similarly, assuming that the deflection f is determined with a relative error of no more than 5 %, we analyze the accuracy of the tensile stress σ calculating. In this case, the error is calculated by the formula

$$\Delta\sigma = \pm \left| \frac{d\sigma}{df} \right| 0.05f. \quad (11)$$

Taking into account (11), we can calculate the relative error $\varepsilon_\sigma = |\Delta\sigma / \sigma|$.

The calculation results of $\Delta\sigma$ and ε_σ are presented in graphical form in Figures 6 and 7, respectively.

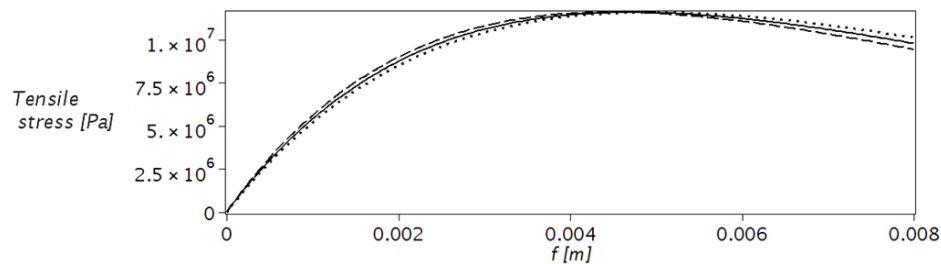


Figure 6. To the analysis of the random factors influence.

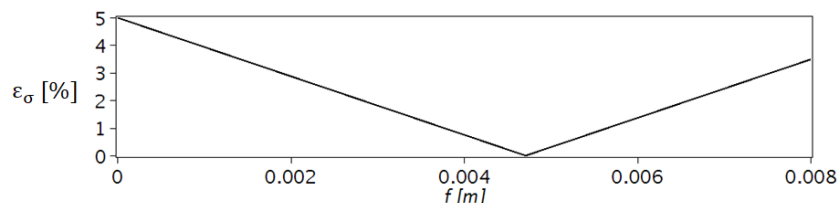


Figure 7. Relative error variation ε_σ depending on deflection f .

We conclude that the influence of random factors on the tensile stress σ in the section with a crack is small (Figure 6) and does not exceed 5 % (Figure 7).

In the example above $F_{extr} = 769$ N, $f_{extr_F} = 1.574$ mm; tensile stress in the section with a crack is 7.52 MPa, if $F = F_{extr}$; $\sigma_{extr} = 11.58$ MPa, $f_{extr_σ} = 4.71$ mm, $F_{extr_σ} = 312$ N.

Using relation (4), we calculate H and the crack length $h = H_0 - H$ (Figure 1) if $f = f_{extr_\sigma}$. We get: $H_{extr_\sigma} \approx 14$ mm, $h_{extr_\sigma} \approx 25$ mm. Then $h_{extr_\sigma}/H_{extr_\sigma} = 25 / 14 \approx 1.8$. Using Figure 2, you can determine the conditional distances from the bifurcation point to the lower edge of the beam, as well as from the bifurcation point to the upper edge of the beam. The ratio of these distances is approximately equal to 2.0. If we take into account that the calculated ratio is approximately equal to the ratio found in the experiment ($1.8 \approx 2$), then we can assume that the bifurcation of the crack is realized if $\sigma \approx \sigma_{extr}$. At the same time, some deviations are explained by the influence of material heterogeneity, small differences in the shape of the real sample from the ideal shape, and other random factors.

In the following section, we will consider the interpretation and comparison of the results of the methodology applied with the results got by other authors.

3.3. Comparison of the Obtained Results with the Results of Other Authors

The model developed is based on the experimental data and relations (1)–(9), describing the full history of the force acting on the beam, and can be assigned to the class of constitutive models that have appeared relatively recently, but are increasingly used in research and prediction of the mechanical condition of frozen soils [14, 17].

The technique developed does not require a large amount of input data and is characterized by the low cost of numerical implementation. From the point of view of practice, it is important to note that for the numerical implementation of the model developed, two test results (F_{extr} and f_{extr_F}) for three-point bending are necessary (Figure 2). In addition, a physically based hypothesis on the dependence of ΔH on Δf is needed. In the case considered, such a hypothesis is presented in the form of equality (1). However, this is not the only option. For example, in [8], a damage variable was used, which varies from 0 to 1, with 0 corresponding to the undamaged (intact) and 1 – completely damaged states of the material under uniaxial compression.

In our case, the geometric characteristics of the effective area (1) were introduced; the form of the equations of the sample state also differs from the relations known from the scientific literature. As a result, a methodology was developed for modeling the state of fairly wide class materials, of state which was confirmed by the practice of applying the technique. For example, using the results of tests for three-point bending of specimen B11 with a notch according to [7, p. 26], as the initial data we will perform their processing according to the methodology discussed above. We will obtain the data presented in Figure 8 by solid lines. Experimental data known from the work [7, p. 26] are shown in Figure 8 by markers.

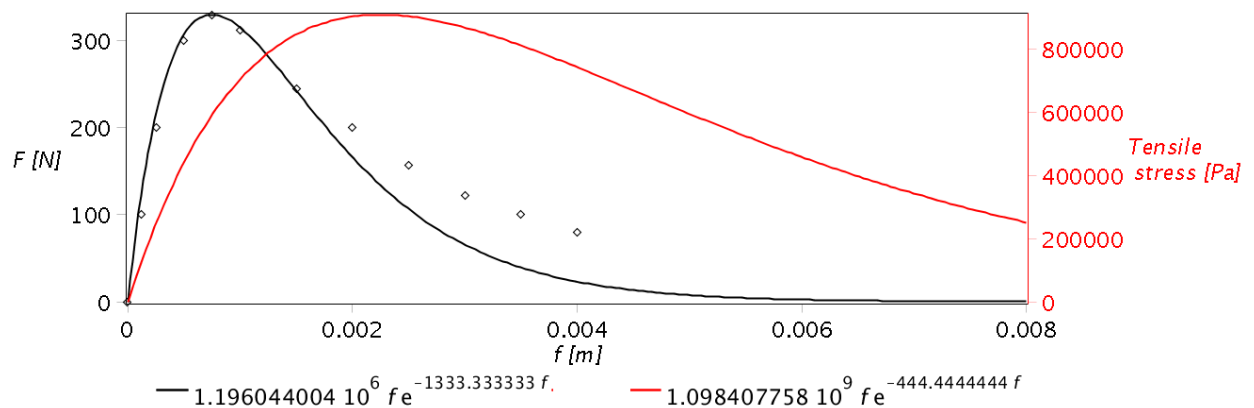


Figure 8. Test results by [7, p. 26] (markers) and our simulations (lines).

For large movements, the predicted values of the load F according to Figure 4 (a beam without a notch) are less than real, and according to Figure 8 (a beam with a notch by [7]) are more. This difference can be explained by the effect of the notch. It is important to note that in the cases considered there are two general patterns, namely: the consistency of the experimental data with the results of mathematical modeling; the increase of random factors influence of the deflection increases after passing the points of extrema (Figures 4 and 8). Thus, the adequacy of the developed model (1-9) is confirmed by the results of the processing the test results, both copyright (Figures 4 and 6), and well-known from the scientific literature (Figure 8).

The influence increase of random factors with an increase in the deflection after passing through the points of extrema (Figures. 4 and 8) can be explained by a decrease in the effective cross-sectional area of the beam due to the cracks evolution. Indeed, crack growth (Figure 2) is accompanied by local fracture and a decrease in the number of frozen soil particles that transmit internal forces from one part of the beam to another.

Formally, according to the statistics, this means a decrease in sample size and, as a consequence, an increase in measurement error. Experimental confirmation can be found in article [19], the authors of which

have tested frozen, thawed and thawing loams using the method of ball stamps of various diameters (0.022; 0.1; 0.3 and 1.0 m). Tests were conducted with controlled precipitation. The rheological properties of the soil have also been taken into account [20].

It was found that the larger the diameter of the stamp is, the higher is the accuracy of the measurements. Obviously, the larger the contact area of the stamp with the soil is, the greater is the number of soil particles in the contact area. Formally, according to statistics, this is equivalent to the sample size increase and the random factors influence on the test results decrease.

In our case, tests at three points bending were carried out with the controlled movement of the loading device (Figure 1). As it was noted above with an increase in the deflection cracks evolve. And it leads to the decrease in the contact amount and, accordingly, and to the increase in the experimental error. This is due to an increase in the random factors influence with an increase in the deflection after passing through the points of extrema (Figures 4 and 8).

The analysis of the results of applying the methodology developed (Figure 3) shows sufficient adequacy of the model in the ascending and in a section of the descending branch of the diagram "load F – displacement f ". With increasing displacement f , the mismatch increases, but is not critical. The search for the causes of this discrepancy may constitute the subject of further experimental and theoretical studies on the topic of the work.

The model developed is nonlinear, and nonlinearity is explained by the evolution of the crack and, accordingly, the change in the effective geometric characteristics (4), (5), (6) of the cross section with the crack. The data obtained are sufficient to conclude that in the full diagram (Figure 3), the deflection f_{extr_F} corresponding to the extremum of the force F_{extr} is less than the deflection $f_{extr_σ}$, which corresponds to the extremum of the tensile stress $σ_{extr}$. From the physical point of view, this means that the destruction of the material occurs on the descending branch of the diagram $F(f)$, i.e. when the force is $F < F_{extr}$, but $σ = σ_{extr}$. Generally speaking, destruction can occur with $σ ≤ σ_{extr}$ and the corresponding displacement $f ≤ f_{extr_σ}$.

As an additional argument confirming the adequacy of the model developed, we refer to the aforementioned work [8], which theoretically substantiates the appearance of a bifurcation of a crack during uniaxial compression of sandstone and other brittle materials.

Limitations on the scope of the model are determined by the properties of soils. From a formal point of view, a necessary condition for applying the model developed is the existence of an extremum on the curve $F(f)$. The class of such materials and the corresponding diagrams obtained in bending tests was studied in [7].

Estimating the results of the work from a practical point of view, it should be noted that using a small amount of initial data, the developed technique allows us to obtain estimates of the elastic modulus of frozen soil and the highest tensile stresses in the section with a crack. The need to obtain such estimates is repeatedly stated in the literature [20, 21]. The analysis showed that the most pressing issues relate to the improving the technology of building roads [1, 3, 4, 23], including logging roads [24], to sufficient (but not excessive) protection of engineering structures from freezing, to designing and forecasting the state of foundations in permafrost regions [25, 26] taking into account temperature deformations [2, 27–29].

Prospects for the development of the present work topic relate to the refinement of the geometric characteristics of the effective area (2) for adapting the approach used in the engineering analysis of soils under the influence of "freeze – thaw" cycles and time factor [1, 30].

4. Conclusions

1. A nonlinear model has been developed for the complete process of deformation of a frozen soil beam with an evolving crack at three-point bending. Nonlinearity is explained by the evolution of the crack and, accordingly, the change in the effective geometric characteristics of a cross section with a crack. For the numerical implementation of the model, it is sufficient to use two parameters: the extremum of the force in the tests for three-point bending before fracture and the value of the vertical displacement of the force application point corresponding to the extremum.

2. The model developed was used for mathematical processing of the of tests results for three-point bending. It was found that the magnitude of the deflection corresponding to the extremum of the force is less than the deflection, which corresponds to the extremum of the tensile stress in the section with a crack. Thus, it is substantiated that the destruction of the material occurs in the descending branch of the «load – displacement» diagram.

3. Using the model proposed a methodology for determining the elastic modulus and the highest tensile stresses has been developed. Analytical dependences of the tensile stress force and extremum in a section with an evolving crack on the deflection value have been obtained. The developed model enables calculating the length of an evolving crack in the entire process of deformation of a beam from frozen soil.

4. The adequacy of the model developed and the reliability of the results of its application are confirmed by consistency with the results of three-point bending on the SHIMADZU AGS-X machine, which provides measurement of force and displacement with a relative error of no more than 1.0 %. The simulation results obtained are also consistent with the experimental and theoretical data known in the literature.

5. Restrictions on the scope of the method are determined by the properties of soils. From a formal point of view, a necessary condition for applying the model is the existence of an extremum on the «load – displacement» curve. Prospects for the study relate to the adaptation of the approach used to the analysis of the mechanical condition of soils under the influence of «freezing – thawing» cycles and time factor.

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Contacts

Timmo Gavrilov, gtimmo@mail.ru

Gennady Kolesnikov, kolesnikovgn@yandex.ru