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A probabilistic approach to estimation of the ultimate load of end-bearing piles on settlement criterion

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Abstract. Pile foundation is one of the most common types of foundations in the presence of soft soils. The safety of the entire structure depends on the pile foundation safety. Stochastic (probabilistic) modeling of load-bearing elements is a modern trend in the quantitative structural safety assessment. The article describes the probabilistic approach to estimate the bearing capacity (ultimate load) of end-bearing piles on settlement criterion. Design load and elastic modulus of a pile material are the main stochastic (random) parameters that determine the probability of end-bearing pile failure. The graphical model of failure and safety areas is proposed, on the basis of which conclusions can be drawn about the allowable load during the design and inspection of pile foundations. The numerical example shows that the method of pile reliability analysis based on FOSM (First Order Second Moment) gives similar results as the method on the basis on FORM (First Order Reliability Method). Therefore, for an accurate evaluation of the reliability index (according to the considered mathematical model), the first iteration of the FORM method is sufficient. Experimental modeling of pile behavior showed good convergence of the theoretical result with the experimental result based on the Monte Carlo method. For a comprehensive reliability analysis of end-bearing piles, it is necessary to consider a pile as a system (in terms of reliability theory).

1. Introduction

Pile foundations are one of the most common types of foundations in the presence of weak soils. There are two types of piles: friction piles and end-bearing piles. End-bearing piles transfer the load to the soil only by the pile's toe (lateral friction is ignored). The safety of whole structure depends on end-bearing piles safety. There is an important task to evaluate the maximum allowable load on the pile. In addition, the value of the allowable load should provide the pile reliability.

By Eurocode 0 “Basis of structural design”, the reliability is the ability of a structure or a structural member to fulfill the specified requirements, including the design working life, for which it has been designed. Reliability covers safety, serviceability and durability of a structure and is usually expressed in probabilistic terms. The measure of reliability is the failure probability or safety probability. Structural reliability is a fundamental part of buildings and structures; reliability combines design problems, work planning, production and operation of buildings and structures [1]. As noted in [2], “reliability analysis has been a hot research topic in recent years, as the influences of uncertainty arising on loads, material properties, dimensions, and geometries become more and more profound”. The research in [3] notes that reliability theory demonstrates the rapid growth and recognized importance for structural safety issues over the past decades. Reliability analysis allows to quantify the safety level of structure or structural element.

Reliability analysis has found applications in pile foundation engineering. The paper [4] proposes a two-dimensional axisymmetric numerical probabilistic modeling of an earth platform over clayey sand improved by stiff vertical piles using a finite-difference continuum approach; only the soil parameters are considered as random variables. The article [5] reviews the problems of piles probabilistic design in the North Sea. In [6] it is noted, that “reliability-based design (RBD), a hot issue in pile foundation engineering, has attracted more and more attention from engineers and researchers”. The paper [6] also proposed a methodology to calculate the optimal ultimate base and shaft resistance factors for reliability-based design of driven piles by considering

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the setup effects that can significantly enhance the ultimate shaft resistance after initial installation. T.V. Ivanova, I.U. Albert, B.G. Kaufman et al. [7] presented the probabilistic analysis of the friction pile behavior on the pile material and soil strength criterion.

There are several limit state criteria for axial loaded end-bearing pile: pile material strength, pile stability, strength of soil under a pile's toe and pile settlement. The calculation of the end-bearing pile settlement was often allowed not carried out because of its small value: only the pile material deformation was considered. For this reason, there is a lack of information about the reliability analysis of end-bearing piles by the deformation criterion. With the development of high-rise construction, the load on the piles has increased significantly, and end-bearing piles settlement can affect the stress-strain state or functioning of the structure.

Let us consider approaches to the estimation of end-bearing pile settlement. Generally, the settlement of end-bearing pile can be represented as the sum of two displacements:

$$s = s_s + s_p, \tag{1}$$

where s_s is pile settlement from soil deformation under the pile toe; s_p is pile settlement from pile material deformation (shortening).

Such an approach is implemented in mathematical models in various researches. A similar equation was proposed by X. Chen in [8]:

$$s = s_s + s_p = \frac{N}{AC_0} + \frac{Nl}{EA}, \tag{2}$$

where C_0 is the vertical reaction coefficient of the pile toe when the single axial limit compression strength of the rock sample is 1000 kPa, C_0=300000 kPa/m; s is pile tip settlement; N is an axial load; l is a pile length; E is an elastic module of pile body; A is the pile cross-section area;

In [9], the empirical equation was proposed for determining pile settlement:

$$s = \frac{d}{100} + \frac{Nl}{EA}, \tag{3}$$

where d is pile diameter.

In [10], the following approach was proposed to assess the settlement of a large diameter long pile:

$$s = \frac{2Nl}{5EA}. \tag{4}$$

Taking into account the notes in [11], the settlement of end-bearing pile also can be calculated by the equation:

$$s = s_s + s_p = \frac{1-v}{Gd} + \frac{Nl}{EA}, \tag{5}$$

where v is soil Poisson's ratio under the pile toe and G is soil shear modulus under the pile toe.

In [12], G. Mylonakis proposed the extended dependence for determining the settlement of end-bearing pile:

$$s(z) = \frac{2N}{EAl} \sum_{m=0}^{\infty} \frac{K_0(\eta \alpha_m d/2) \cos \alpha_m z}{\alpha_m^2 \left[K_0(\eta \alpha_m d/2) + \frac{\pi d \eta G}{EA\alpha_m} K_1(\eta \alpha_m d/2) \right]}, \tag{6}$$

where K_0, K_1 are modified Bessel functions of zero and first order; $\eta^2 = \frac{2}{1-v}$ is the later displacement factor; $\alpha_m = \frac{\pi}{2l}(2m+1)$, $m = 0, 1, ..., n$;

From an analysis of the above equations (2) - (6), it can be concluded about the generality of the proposed approaches.

Equation (6) has a similar dependence of variables as equation (5), if we transform model (5) to the form $N \le \left(s_u - \frac{1-\nu}{G \cdot d} \right) \frac{EA}{l}$. However, using complex mathematical dependencies in the analysis of structural reliability can lead to a complex nonlinear problem that complicates its use in the engineering practice. Also, the complex nonlinear relationship between random (stochastic) parameters in (6) can lead to an underestimation of the actual reliability level due to big uncertainty (compared to semi-probabilistic approaches). In this regard, the equation (5) will be used in the further analysis. The algorithm considered below can be easily projected, for example, onto model (2) or similar mathematical models of limit state.

An important problem in end-bearing piles design is the presence of cracks in the rock base. The paper [13] analyzed a compilation of previously published and new data on the uniaxial compressive strength (UCS) of various rock types using the pore-emanated crack and the wing crack models. The presence of cracks in the rock base affects the pile deformability under axial load. The paper [14] described and analyzed an international experience in piles designing in a rock base with cracks.

New numerical methods for piles behavior modeling are also noteworthy. The research [15] proposed to determine pile settlement based on various three-dimensional finite element models. The paper [16] presents modeling of pile settlement using neural networks. For some geotechnical sites, it needs to consider the stability of pile as Timoshenko beam [17, 18]. In this paper, we consider only one criterion of the limit state – the settlement. Fundamental experimental studies of pile settlements are also considered in the A.A. Bartholomew research [19].

In this paper, the expansion and development of the provisions in [7] is proposed to consider a probabilistic approach to estimate end-bearing piles ultimate load on settlement criterion with design reliability level. The aim of the paper is a probabilistic analysis of ultimate load on axial loaded end-bearing pile on settlement criterion. It is required to solve such problems as: to determine which parameters are random (stochastic); to choose the method of reliability analysis; to analyze the influence of the variability of individual parameters on the overall pile reliability level and to compare the analytical solution with experimental data by Monte-Carlo simulation.

2. Methods

The values of stochastic (random) parameters in (5) are determined by tests and represent some statistical subset of the data. Stochastic parameters are indicated by a wavy line above the symbol. The mathematical model of the limit state (5) can be written as:

$$\tilde{N} \le \left(s_u - \frac{1-\nu}{\tilde{G} \cdot d} \right) \frac{\tilde{E} \cdot A}{l}, \tag{7}$$

where s_u is ultimate value of pile settlement.

The elasticity modulus of the pile material and the shear modulus of soil under the pile toe are determined by the repeated tests, which allows to describe them by the normal distribution. The axial load on the pile is described by a combination of different probability distributions, which can also be generically reduced to normal. There are different approaches to assign the maximum allowable pile settlement [20–22].

Denote $\tilde{N} = X$ and $\left(s_u - \frac{1-\nu}{\tilde{G} \cdot d} \right) \frac{\tilde{E}A}{l} = Y$. The parameters of the normal distribution function for random variable X are taken as a mathematical expectation $E[X]$ and standard deviation σ_X of axial load $\tilde{N} = X$.

The expected value $E[Y]$ and standard deviation σ_y for Y on the first stage can be found by decomposing the function into a Taylor series:

$$E[Y] = \left(s_u - \frac{1-\nu}{E[G] \cdot d} \right) \frac{E[E] \cdot A}{l}; \; \sigma_y = \sqrt{\left(\frac{\partial Y}{\partial G} \right)^2 \sigma_G^2 + \left(\frac{\partial Y}{\partial E} \right)^2 \sigma_E^2}.$$

The probability of pile non-failure by model (7) can be calculated as:

$$P = \Pr(X \le Y) = \Phi\left(\frac{E[Y]-E[X]}{\sqrt{\sigma_y^2+\sigma_x^2}}\right) = \Phi(\beta), \quad (8)$$

where $\Phi(\beta)$ is determined by the table values of the integral Laplace function (*z*-table); β is the reliability index.

Such an approach is called FOSM (First Order Second Moment). As $\sigma_x = E[X]\cdot CV_x$, where CV_x is axial load coefficient of variation, then safety condition by reliability index β of the end-bearing pile can be written as:

$$\beta \ge \frac{E[Y]-E[X]}{\sqrt{\sigma_y^2+\left(E[X]\cdot CV_x\right)^2}}. \quad (9)$$

Reliability index β can be specified in a design assignment.

If condition (9) is not met, the design parameters are adjusted by reducing the allowable axial load or its variability; or by increasing the class of pile material, by changing pile geometric parameters or by conduction more stringent quality control of the piles material with rejection of piles which variation coefficient of elastic modulus is higher than required values.

After assigning the final design parameters of the pile and fulfilling the condition (9), it is necessary to clarify the value of the reliability index β. Since the limit state model is nonlinear, it is necessary to use FORM (First Order Reliability Method) algorithm to more accurately estimate the reliability index β of the pile. The FOSM method also fails to be invariant with different mathematically equivalent formulations of the same problem. As noted in [23], the first-order reliability method (FORM) is considered to be one of the most reliable computational methods and has become a basic method for structural reliability analysis.

The mathematical model of the limit state (7) can be written as:

$$g = \left(s_u - \frac{1-\nu}{\tilde{G}\cdot d}\right)\frac{\tilde{E}\cdot A}{l} - \tilde{N} \le 0. \quad (10)$$

Then reliability index is:

$$\beta = \frac{E[g]}{\sigma_g}, \quad (11)$$

where the mathematical expectation of limit state function g is: $E[g] = g\left(E[N],\ E[G],\ E[E]\right) = \left(s_u - \dfrac{1-\nu}{E[G]\cdot d}\right)\dfrac{E[E]\cdot A}{l} - E[N]$; and standard deviation is: $\sigma_g = \sqrt{\left(\dfrac{\partial g}{\partial G}\right)^2\sigma_G^2 + \left(\dfrac{\partial g}{\partial E}\right)^2\sigma_E^2 + \left(\dfrac{\partial g}{\partial N}\right)^2\sigma_N^2}$.

For generality, denote: $\tilde{G}_2 = x_1$, $\tilde{E} = x_2$, $\tilde{N} = x_3$.

"Sensitivity factors" are calculated by equation:

$$\alpha_i = -\frac{\dfrac{\partial g}{\partial x_i}\sigma_{x_i}}{\left[\sum_{i=1}^{n}\left(\dfrac{\partial g}{\partial x_i}\sigma_{x_i}\right)^2\right]^{1/2}}. \quad (12)$$

New x - and u - coordinates are calculated for the limit state function (10) as:

$$x_i^* = E[x_i] + \beta \cdot \sigma_{x_i} \cdot \alpha_i \,, \tag{13}$$

$$u_i^* = \frac{x_i^* - E[x_i]}{\sigma_{x_i}} . \tag{14}$$

New function values are calculated in the new coordinates: $g\left(x_i^*\right)$ and $\dfrac{\partial g\left(x_i^*\right)}{\partial x_i^*}$.

After that, a new reliability index $\beta*$ is determined by the equation:

$$\beta* = \frac{g(x*) - \sum \dfrac{\partial g(x*)}{\partial x_i} \sigma_{x_i} u_i}{\left[\sum_{i=1}^{n} \left(\dfrac{\partial g(x*)}{\partial x_i} \sigma_{x_i} \right)^2 \right]^{1/2}} . \tag{15}$$

Then iterations are repeated until the reliability index β converges.

3. Results and Discussion

Let us consider the proposed approach in example. Design parameters (and its statistical parameters) are given in Table 1.

Table 1. Parameters for pile stochastic analysis.

Parameter	Expected value	Standard deviation
Soil Poisson's ratio	0.35	-
Soil shear modulus, MPa	50	1.5
Pile diameter, m	0.35	-
Pile length, m	6	-
Elastic modulus, MPa	30000	800
Pile cross-section area, m^2	0.096	-
Allowable settlement, m	0.010	-

The Y expected value and standard deviation calculated by above equations as:

$$E[Y] = \left(s_u - \frac{1-v}{E[G] \cdot d} \right) \frac{E[E] \cdot A}{l} = \left(0.01 - \frac{1-0.35}{50 \cdot 0.35} \right) \frac{30 \cdot 10^3 \cdot 0.096}{6} = 4.811 \text{ MN}.$$

$$\sigma_y = \sqrt{\left(\frac{\partial Y}{\partial G} \right)^2 \sigma_G^2 + \left(\frac{\partial Y}{\partial E} \right)^2 \sigma_E^2} = \sqrt{0.287 + 16.16 \cdot 10^9} = 0.128 \text{ MN}.$$

As can be seen from the calculations, the variability of the soil shear modulus under the pile toe does not practically affect the overall variability of the pile bearing capacity according to the settlement criterion.

Let us take design reliability index β=3. There are different approaches to assign a design reliability index [2, 24, 25, etc.]. The Table 2 shows the values of allowable mathematical expectations of load $E[X]$ depending on the load coefficient of variation CV_x at a given reliability index β.

Table 2. Expected values of ultimate loads $E[X]$ at the different levels of load variation coefficients CV_x.

CV_x	0.01	0.03	0.05	0.10	0.15	0.20	0.25	0.30	0.35
$E[X]$, MN	4.404	4.267	4.087	3.650	3.284	2.981	2.728	2.515	2.332

Fig. 1 presents a diagram which reflects safety and failure areas in the appointment of the load statistical parameters: $E[X]$ and CV_x. Such diagrams can be used to estimate the allowable value of the load in the inspection or design of pile foundations.

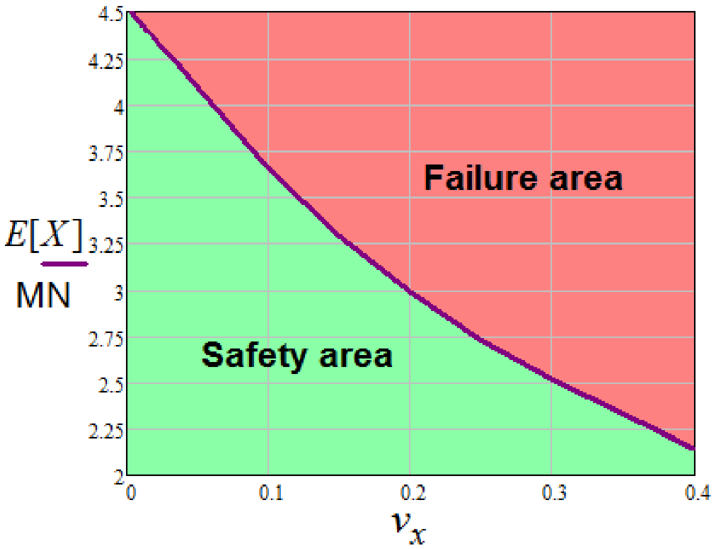

Figure 1. Diagram for ultimate load estimation.

Assume that the load is given by the following statistical parameters: $E[X]$ = 3.900 MN; CV_x = 0.07; σ_x = 0.273 MN. As seen in Fig. 1, the pile is in the safety area by the settlement criterion. The reliability index in this case is:

$$\beta = \frac{E[Y] - E[X]}{\sqrt{\sigma_y^2 + (E[X] \cdot CV_x)^2}} = \frac{4.811 - 3.900}{\sqrt{0.128^2 + (3.900 \cdot 0.07)^2}} = 3.019 .$$

The variability of the elastic modulus of the pile material contributes greatly to the solution uncertainty. The effect of the modulus change is analyzed in the table 3 at different coefficients of variation.

Table 3. Reliability index β at the different levels of pile elastic modulus expected value $E[E]$ and variation coefficient CV_E.

Coefficient of variation CV_E	Expected vale of elastic modulus $E[E]$, MPa·10³								
	25	26	27	28	29	30	31	32	33
0.03	0.365	0.896	1.421	1.938	2.447	2.949	3.443	3.929	4.407
0.05	0.321	0.784	1.233	1.669	2.092	2.503	2.901	3.287	3.660
0.07	0.278	0.673	1.053	1.417	1.766	2,100	2.421	2.729	3.024
0.10	0.224	0.540	0.839	1.123	1.391	1.646	1.888	2.118	2.337
0.15	0.165	0.394	0.610	0.812	1.002	1.180	1.349	1.508	1.658
0.20	0.128	0.307	0.473	0.628	0.774	0.910	1.039	1.159	1.273

The Table 3 reflects the important role of quality control of physical and mechanical properties of pile materials (for example, concrete). Graphically, the table 3 data are shown in Fig. 2.

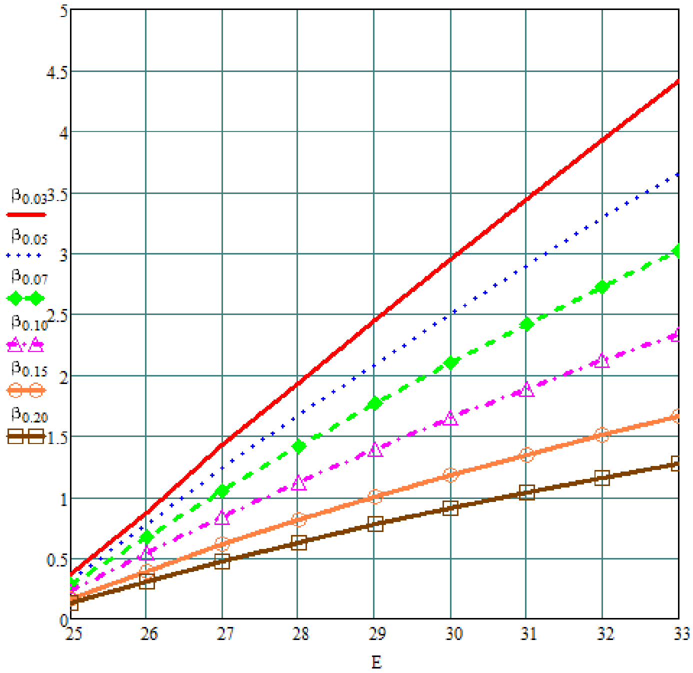

Figure 2. β - $E[E]$ diagrams at the different levels of variation coefficients for pile material elastic modulus E.

Fig. 2 shows that in order to provide a higher level of reliability, it is necessary to assign a greater expectation of the elasticity with an increase in its coefficient of variation. Diagram of the Fig. 2 type allows us to set the next parameters: 1) reliability index β (based on the results of nondestructive testing of existing piles); 2) the required minimum parameters of the designed piles at a given reliability index β.

Let us specify the value of β by the FORM approach.

Expected value and standard deviation of limit state function are:

$$E[g]=\left(s_u-\frac{1-v}{E[G]d}\right)\frac{E[E]A}{l}-E[N]=\left(0{,}01-\frac{1-0.35}{50\cdot 10^6\cdot 0.35}\right)\frac{30\cdot 10^9\cdot 0.096}{6}-3.9\cdot 10^6=9.105\cdot 10^5\text{ N.}$$

$$\sigma_g=\sqrt{\left(\frac{\partial g}{\partial G}\right)^2\sigma_G^2+\left(\frac{\partial g}{\partial E}\right)^2\sigma_E^2+\left(\frac{\partial g}{\partial N}\right)^2\sigma_N^2}=$$

$$=\sqrt{\left(\frac{AE[E](v-1)}{d\cdot l\cdot E[G]^2}\right)^2\sigma_G^2+\left(\frac{A\left(s_{ult}-[v-1]/[d\cdot E[G]]\right)}{l}\right)^2\sigma_E^2+\sigma_N^2}.$$

$$\sigma_g=3.016\cdot 10^5\text{ N.}$$

The reliability index (or Cornell index) is:

$$\beta=\frac{E[g]}{\sigma_g}=\frac{9.105\cdot 10^5}{3.016\cdot 10^5}=3.019.\text{ Dimensionless}$$

Specify the parameters values for the next iteration by (12), (13) and (14) in table 4.

Table 4. Parameters for reliability index $\beta*$ evaluation.

	G	E	N
α	$1.777\cdot 10^{-6}$	0.425	–0.905
$x*$	$5.000\cdot 10^{7}$ Pa	$3.142\cdot 10^{10}$ Pa	$2.687\cdot 10^{6}$ N
$u*$	$7.432\cdot 10^{-6}$	1.779	–3.785

New reliability index $\beta*$ by (15) is:

$$\beta* = \frac{g(x*) - \sum \frac{\partial g(x*)}{\partial x_i} \sigma_{x_i} u_i}{\left[\sum_{i=1}^{n} \left(\frac{\partial g(x*)}{\partial x_i} \sigma_{x_i} \right)^2 \right]^{1/2}} \rightarrow \beta* = 3.019 .$$

Based on the results of the FORM approach, we obtain the same reliability index as in the FOSM approach: $\beta = \beta* = 3.019$. Consequently, the accuracy of the FOSM approach is enough to estimate the reliability index β even under different mathematical models of limit states - (7) or (10).

Statistical tests were carried out in MathCAD by the Monte Carlo method. Random values were given through the function $rnorm(1000,\ E[X],\ \sigma_x)$. According to the results of 1000 statistical tests, exceeding the limit state was recorded in 2 cases. The theoretical probability of pile non-failure in frequency is (1000-2)/1000=0.9980. This probability value is extremely close to the theoretical probability value of the reliability index $P = \Phi(\beta = \beta* = 3.019) \approx 0.9987$.

Table 5. Comparison of reliability levels by FOSM, FORM and Monte-Carlo methods.

Method	FOSM	FORM	Monte-Carlo
Reliability level	0.9987	0.9987	0.9980

For a comprehensive assessment of reliability, it is necessary to identify the pile reliability according to other criteria of limit states (pile material, soil base strength, pile stability, etc.) and consider the pile foundation as a structural system [26, 27, etc.].

Target values for the reliability index β for various design situations, and for reference periods of 1 year and 50 years, are indicated in Appendix C, Eurocode 0 “Basis of structural design”. For example, reliability index for serviceability limit state (for reference periods of 1 year) is $\beta = 2.9$. Joint Committee on Structural Safety (JCSS) Probabilistic Model Code sets target values for the reliability index β in dependence with a comparative cost of safety measures and failure consequences. However, the reliability index should be calculated individually for each structure (or structural element) based on the value of the acceptable risk [28].

4. Conclusions

1. The article describes the methods for end-bearing pile bearing capacity estimation and reliability analysis of settlement criterion;

2. The diagram of safety and failure areas is proposed for assigning and checking design of axial load statistic parameters on end-bearing piles;

3. It is shown by numerical example, that design load and elastic modulus of the pile materials are the main stochastic (random) parameters that determine the probability of end-bearing pile failure

4. Different mathematical models of limit states were analyzed by FOSM and FORM approaches. Results show that there is no “invariant” problem for FORM approach in that case;

5. Experimental modeling of random (stochastic) parameters of the end-bearing pile was carried out to verify the proposed approaches. A good convergence of theoretical results with experimental simulation data is established.

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