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Analytical dependence of the deflection of the spatial truss on the number of panels

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Abstract. A statically definable girder truss consists of two flat side trusses with parallel belts and descending braces, connected along the upper and lower belt by horizontal ties. The truss at the corners has four supports, the vertical reaction of one of which is selected from the equilibrium condition of the structure as a whole. Several types of external load are considered: the load in the middle of the span, vertical, evenly distributed over the nodes of the upper or lower belt, and uniform horizontal (wind), applied to the nodes of the upper belt. The forces in the rods and supports are determined by the method of cutting out the nodes. The deflection is found by the Maxwell-Mohr formula under the assumption of linear elasticity of the rods. The dependence of the deflection on the number of panels is obtained from solving problems for trusses with a consistently increasing number of panels. Generalization of these solutions to an arbitrary number of panels is found using special operators of the Maple computer mathematics system. Using the Maple operators, we derive and solve homogeneous linear recurrent equations that satisfy the coefficients of the desired formula. The found dependence has the form of a polynomial in the number of panels. Curves of the deflection dependence on the number of panels, the load and the size of the structure are constructed. Some asymptotic properties of the obtained solution are found. Formulas for the dependence of forces in the most stretched and compressed rods on the size of the structure, the load, and the number of panels that can be used to analyze the strength and stability of the structure are derived.

1. Introduction

In most analytical and numerical calculations of spatial trusses, the structure is replaced by separate flat trusses that carry the main load. The work of links is not taken into account. Modern computer mathematics systems allow for analytical calculations of spatial structures taking into account the forces in all rods.

The task is to determine the analytical dependence of the deflection of the spatial truss (Fig. 1) on the number of panels under different loading of the truss. The construction of a mathematical model of the structure, the calculation of forces in the rods, and the inductive derivation of the desired formulas are performed in the system of analytical transformations Maple [1]. Previously, a similar problem for flat trusses in analytical form was solved by the induction method in works [2–9], spatial — in works [10, 11]. The original algorithm for analytical calculation of lattice systems (planar, spatial, statically definable and statically indeterminate) was developed in [12]. The only drawback of the method is that it is not possible to obtain compact calculation formulas for the dependence, for example, of the deflection on the number of panels or any periodicity elements. In [13], non-linearity is taken into account in the calculation of frames.

The used calculation algorithm can be transferred almost without changes to other computer mathematics systems, which have a block for determining the General terms of sequences and solving a system of linear algebraic equations in symbolic form. In addition to Maple, the Mathematica system meets these requirements most of all, and to a lesser extent, the Reduce, Derivative, and Maxima systems.

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2. Methods

2.1. Truss scheme. The calculation of the forces in the bars

The construction under consideration consists of two inclined flat trusses with parallel belts and descending struts of length $c=\sqrt{a^2+d^2+h^2}$ (Fig. 1, 2), connected on the lower and upper panel by tie rods on the long sides.

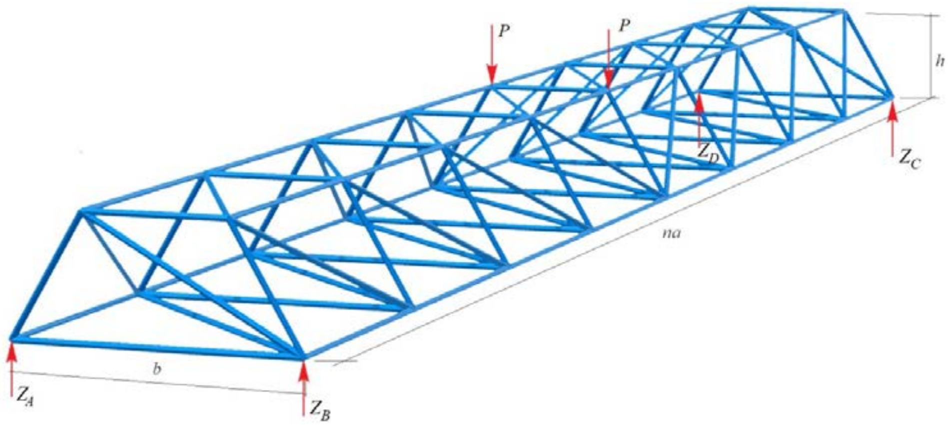

Figure 1. Vertical load in the middle of the span, *n*=2*k*=8.

The design is statically definable with n panels and contains $n_s=12(n+1)$ rods. Supports located at three corners of the truss are modeled with six rods. Three rods simulate a spherical support A, two rods-a cylindrical D, one vertical — a movable in the plane hinge B. All connections of the rods in the design are hinged. The upper face of the spatial truss is a rectangular truss with length na and width $b-2d$ with struts of length $d=\sqrt{a^2+(b-2d)^2}$ (Fig. 2). Instead of the fourth support, a vertical force $Z_C=P_{sum}/4$ is applied to node C in the case of a symmetrical load P_{sum} (centered in the middle or evenly distributed across all nodes).

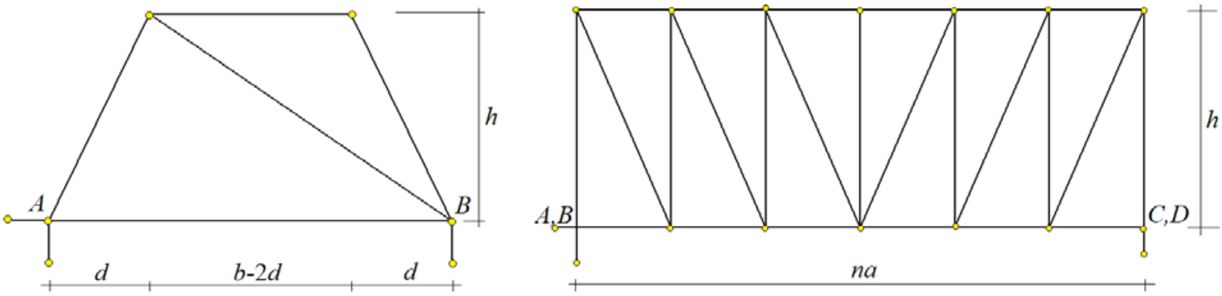

Figure 2. The size of the truss, *n*=6.

If the corner C were fixed to a real hinge support, the structure would become statically indeterminate once, but the result of solving the static indeterminability problem would give the same result, provided that all vertical supports have the same stiffness and length. The number of panels we choose is even $n=2k$, so that the deflection of the truss can be judged by the vertical displacement of the nodes of its middle section.

To determine the forces in the rods, we use the program [1], which is based on drawing up and solving a system of equilibrium equations for nodes in symbolic form. The geometry of the truss is based on the coordinates of nodes at specified lengths of bars and the number of panels. The origin of coordinates is placed in the corner A, the axis x directed from A to B, y — longitudinal, z — vertical (Fig. 3):

$$x_i=b,\ x_{i+n+1}=0,x_{i+2n+2}=b-d,x_{i+3n+3}=d,$$
$$y_i=y_{i+n+1}=y_{i+2n+2}=y_{i+3n+3}=a(i-1), \qquad (1)$$
$$z_i=z_{i+n+1}=0,\ z_{i+2n+2}=z_{i+3n+3}=h,\ \ i=1,..,n+1.$$

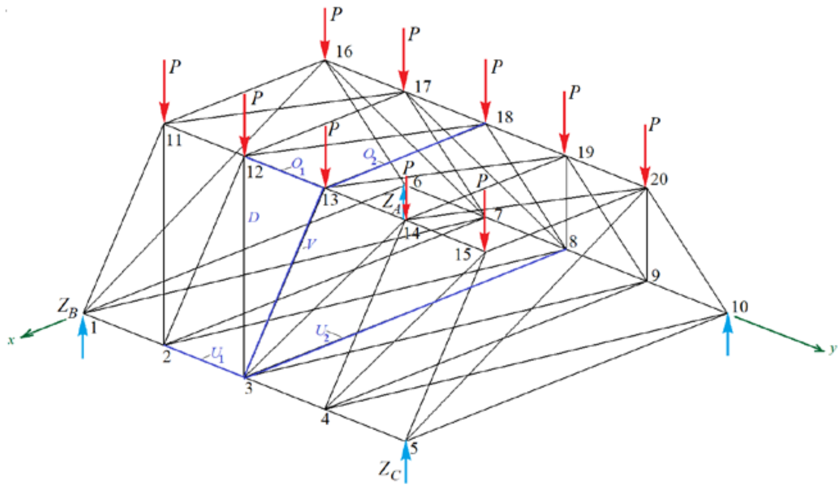

Figure 3. Numbering of nodes and rods. Uniform vertical load on the upper belt, *n*=2*k*=4.

The spatial structure of the truss that determines the order of connecting rods is defined by conditional vectors that have rod numbers directed along the rods and contain the numbers of the ends of the rods (the numbers of the corresponding nodes). The longitudinal bars of the lower truss belt, for example, are defined by the following vectors

$$N_i = [i, i+1],\ N_{i+n} = [i+n+1, i+n+2],\ i = 1,...,n.$$

Other bars of the structure are set in the same way.

The square matrix of the equilibrium equations of nodes size $n_s \times n_s$ is formed from the guiding cosines of forces, which are calculated from the coordinates of their ends. For a statically defined truss $n_s = 3m$, where $m = 4(n+1)$ is the number of nodes. The matrix G is divided into three lines for each node. These lines record the guide cosines of the rods attached to the node. Information about the rods in the node is contained in the vectors N_i, $i = 1,...,n_s$. Rows of the matrix with numbers $3i-2$, $i = 1,...,n_s$ correspond to the projection of forces on the x axis, rows $3i-1$ — projections on the y axis, rows of the type $3i$ — projections on the vertical z axis:

$$G_{3N_{i,1}-3+j,i} = l_{j,i} / L_i,\ G_{3N_{i,2}-3+j,i} = -l_{j,i} / L_i,\ j = 1,2,3,\ i = 1,..,n_s,$$

where $l_{1,i} = x_{N_{i,2}} - x_{N_{i,1}}$, $l_{2,i} = y_{N_{i,2}} - y_{N_{i,1}}$, $l_{3,i} = z_{N_{i,2}} - z_{N_{i,1}}$ are conditional projections of rods on the coordinate axis — L_i, $i = 1,...,n_s$ the length of the rods. Projections of the external load are recorded in the right part of the system of equilibrium equations. Solving a system of equations gives forces in all rods (including support rods).

To analyze the stability of compressed truss rods, you can use the calculation data to get the dependencies of the forces in them on the number of panels. Consider the six rods in the middle of the truss span that are potentially the most dangerous in terms of loss of stability or tensile strength (Fig. 3). Inductive analysis provides the following solutions for uniform load across the upper belt:

$$O_1 = Pak^2 / (2h),\ O_2 = -Pd / h,\ U_1 = Pa(k^2 - 1) / (2h),\ \ U_2 = 0,\ V = -Pg / h,\ D = Pc / (2h).$$

In the case of forces in the middle of the span (Fig. 1):

$$O_1 = -Pak / (2h),\ O_2 = 0,\ U_1 = Pa(k-1) / (2h),\ \ U_2 = -Pd / h,\ V = 0,\ \ D = Pc / (2h).$$

To determine the force dependence on the number of panels, first calculate a series of trusses with a consistently increasing number of panels. The **rgf_findrecur** operator of the Maple system finds recurrent equations for the sequence of obtained analytical expressions of effort, and the **rsolve** operator, solving these equations, gives the already sought dependencies. The most complex equation was found when determining the force in the longitudinal rod of the lower belt U_1 the action of a uniform load:

$$U_{1,k} = 3U_{1,k-1} - 3U_{1,k-2} + U_{1,k-3}.$$

In all other cases, the forces are constant or the dependencies of the forces on k are fairly obvious, and you do not need to use a computer math system to derive the formulas. Numerical analysis of the obtained solution shows that under the condition of constant span and total load $P_{sum}=2P(n+1)$ for a changing number of panels n=2k, all relative forces $U_1'=U_1/P_{sum}$, $O_1'=O_1/P_{sum}$, $O_2'=O_2/P_{sum}$, $D'=D/P_{sum}$, $V'=V/P_{sum}$ in the rods asymptotically tend to their limit: $\lim_{k\to\infty} U_1'=L/(16h)=\tilde{S}$, $\lim_{k\to\infty} O_1'=-\tilde{S}$. The curves in Figure 4 are constructed at L = 100 m, h = 5 m, and d = 1 m. Note that the forces in this section of the truss do not depend on the width b of the truss along the lower belt.

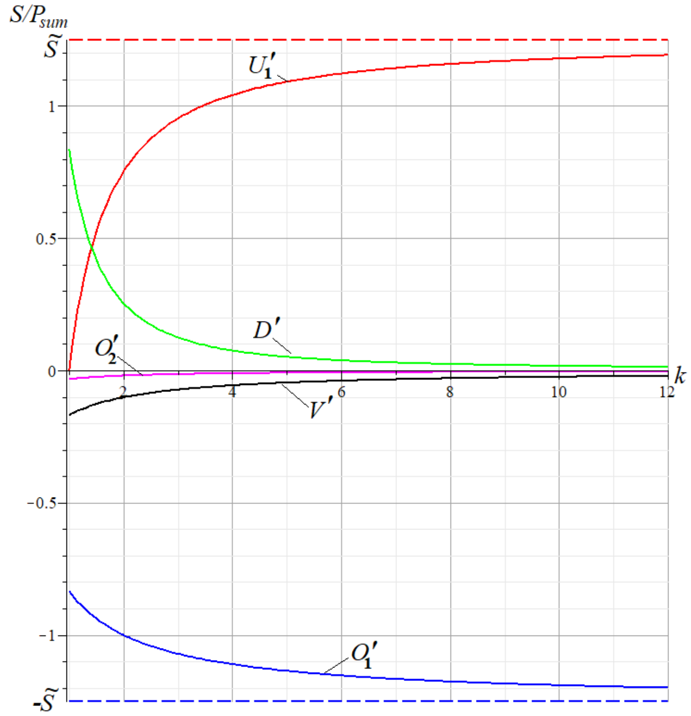

Figure 4. Dimensionless critical stress in the rods, depending on the number of panels.

In addition, the Maple graphical tools allow you to get a picture of the distribution of forces on the rods for a fixed number of panels. In figure 5, the calculation results for k = 4, a=b=4m, h=2m, d=1m show compressed and unloaded rods in blue, and stretched rods in red. The thickness of the lines is proportional to the force modulus. The number indicates the value of the force in the rod related to the load P. The calculation shows that, as expected, the forces in the rods of the belts increase towards the middle of the span. The longitudinal bars of the upper belt are compressed, the lower — stretched. In the side panels (inclined), the reverse pattern is found somewhat unexpectedly. All rods in these panels are most loaded at the ends of the structure near the supports. The connections of the upper belt have very small compressive forces, while in the lower belt all the rods (except the extreme ones) are not strained at all.

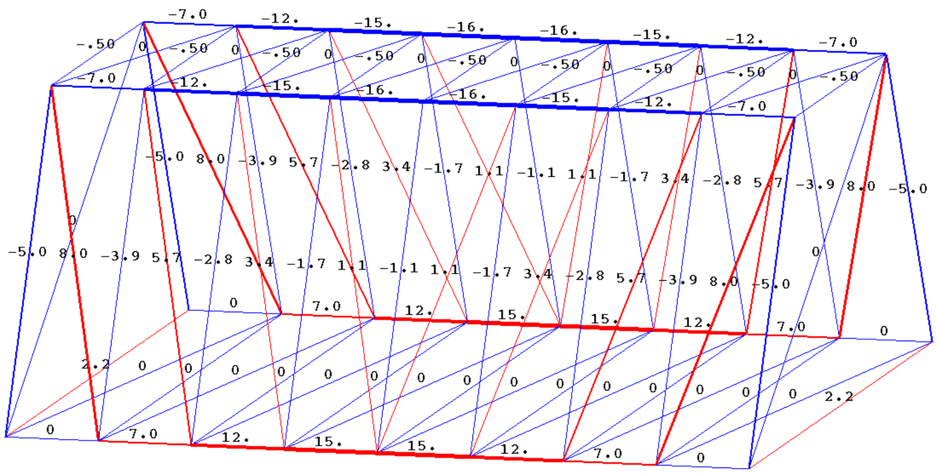

Figure 5. The forces in the rods, *k*=4.

2.2. Deflection from the action of a vertical load

A more difficult problem arises in the derivation of the formulas for deflection. Consider three types of vertical load on the truss. To calculate the deflection, use the Maxwell-Mohr formula

$$\Delta = \sum_{j=1}^{n_s-6} \frac{S_j s_j l_j}{EF}, \quad (2)$$

where E and F is the elastic modulus of the rods and their cross-sectional area, l_j and S_j is the length and force in the *j*-th rod from the action of a given load, s_j is the force from the unit load applied to the upper belt nodes in the middle of the span. Calculations of deflection of trusses with a consistently increasing number of panels using the formula (2) show that the type of expression for any values of k does not change. This is typical for regular systems [14] and is manifested not only in static problems [2–10], but also in dynamics [15]. For the case of a load applied to the upper belt nodes (Fig. 3), we have:

$$\Delta_I = P(C_{1,k}a^3 + C_{2,k}c^3 + C_{3,k}g^3 + C_{4,k}bd^2)/(2h^2EF). \quad (3)$$

The coefficients in this relationship form sequences whose common members can be determined using the already mentioned **rgf_findrecur** and **rsolve** operators of the Maple system. For the coefficient at a^3, the **rgf_findrecur** operator returns a fifth-order equation

$$C_{1,k} = 5C_{1,k-1} - 10C_{1,k-2} + 10C_{1,k-3} - 5C_{1,k-4} + C_{1,k-5}.$$

The **rsolve** operator gives a solution to this equation

$$C_{1,k} = k^2(5k^2+1)/6.$$

Other coefficients have a simple form and are obtained in the same way:

$$C_{2,k} = k^2,\ C_{3,k} = k(k+2),\ C_{4,k} = k + 1/2:$$

The calculation shows that in the case of a load applied uniformly to the nodes of the lower belt, the type of expression (2) for the deflection does not change much:

$$\Delta_{II} = P(C_{1,k}a^3 + C_{2,k}(c^3+g^3) + C_{3,k}bd^2)/(2h^2EF). \quad (4)$$

The coefficients here have the form $C_{1,k} = k^2(5k^2+1)/6,\ C_{2,k} = k^2,\ C_{3,k} = k+1/2.$

Similarly, for a concentrated load in the middle of the span, we have

$$\Delta_{III} = P(C_{1,k}a^3 + C_{2,k}(c^3+g^3) + C_{3,k}bd^2)/(2h^2EF), \quad (5)$$

where $C_{1,k} = k(2k^2+1)/3,\ C_{2,k} = k,\ C_{3,k} = 3/2.$

A linear combination of three solutions, Δ_I, Δ_{II} and Δ_{III} gives a formula for calculating a truss with a fairly wide set of loads, including the weight of the structure itself.

2.3. Deformations from the action of a lateral load

One of the advantages of the applied algorithm for solving the problem of truss deformation, in addition to its analytical form, is the ease with which it can be reconfigured to another load. The main costs for generating formulas are spent on modeling the truss grid and debugging the solution. Replacing the load is reduced only to replacing the vector of the right part of the system of equilibrium equations. Consider the wind effect on structures [16]. It is obvious that due to the low windage of the structure, this load has a very small effect on the deformation of the truss [17]. Let us model this effect with a lateral horizontal load evenly distributed over the nodes of the upper belt (Fig. 8). As in the previous cases of loading, we will apply a load modeling the support to the free corner C of the lower belt truss. We find the load value from the equation of moments relative to the y axis: $Z_C = -P(n+1)h/(2b)$. With this action, we actually replaced the procedure for revealing static uncertainty if there was a vertical support rod in this corner. As a result of induction we have the following simple solution for the vertical displacement of the point M:

$$\Delta_{IV} = P(2k+1)(g^3+bd^2)/(2hbEF). \qquad (6)$$

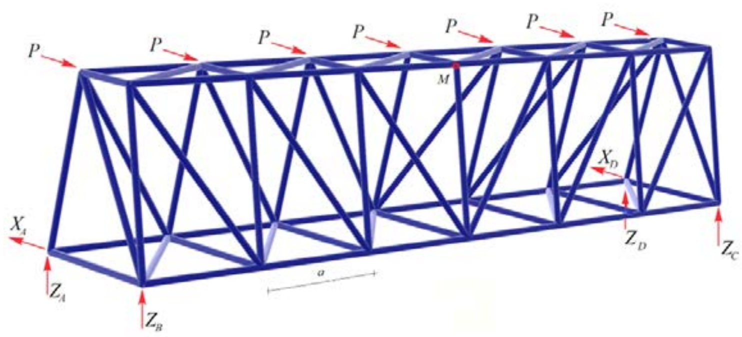

Figure 6. Lateral (wind) load, *k*=3.

3. Results and Discussion

It is best to evaluate the obtained solutions based on numerical calculations performed using the found formulas for specific tasks.

The curves of the dependence of the dimensionless deflection $\Delta' = \Delta_I EF/(P_{sum}L)$ on the number of panels constructed using the formula (3) for the case of a uniform vertical load along the upper belt, provided that the span of the structure $L = na = 100\,m$ is constant, and the total load $P_{sum} = 2(n+1)P$ on the truss show the presence of a pronounced extreme (Fig. 7). Calculations were made for h=3 m, b= 4m. the oblique asymptote is also Observed. Using the Maple methods, you can calculate the slope of the asymptote. We get: $\lim_{k\to\infty} \Delta'/k = f^3/(4h^2L)$, where $f = \sqrt{h^2+d^2}$. The expression for the extreme point itself cannot be obtained in analytical form, but the figure shows that the position of this point on the axis k is almost independent of the size d.

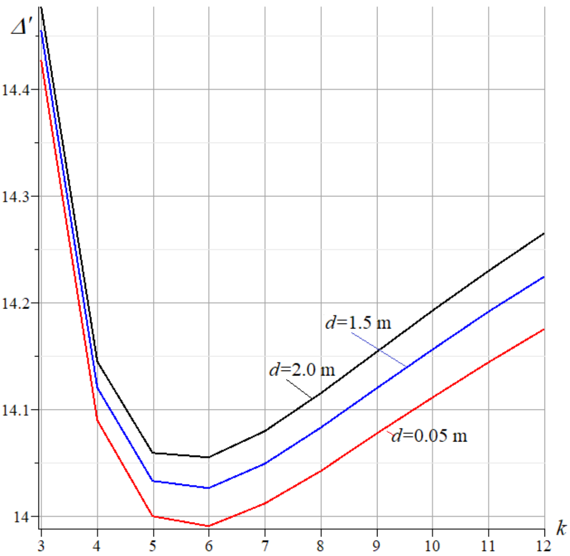

Figure 7. Dependence of dimensionless deflection on the number of panels.

If we trace the dependence of the deflection of the truss under the action of a uniform load on the height of the truss h (Fig. 8), then you can also notice the extremum (h = 8 m), although very weakly expressed. These curves are constructed at $L = na = 60\,m$, b =8 m, k = 40. If the cross-section of the truss is rectangular (the width at the top is equal to the width at the bottom, or d = 0), the deflection is expected to be smaller. The other extreme is the degenerate case b = 2d, or the triangular section. The deflection here is greater, but from the point of view of saving material, this case is also interesting for the designer.

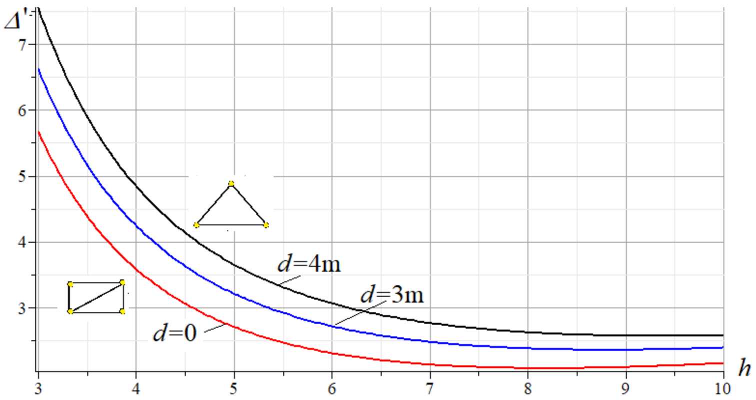

Figure 8. Dependence of the deflection on the height of the truss.

Formulas (4) and (5) for other vertical loads give similar curves, differing only in scale. For the horizontal load, as expected, the deflection is several orders of magnitude less than that of the vertical one. The minimum deflection is also detected, however, at unrealistically low truss heights (9). With a span length of L = 100 m, d = 1 m, this minimum corresponds to 1.5 m. In this setting, the width of the truss b plays a significant role. As the width increases, the truss becomes more rigid.

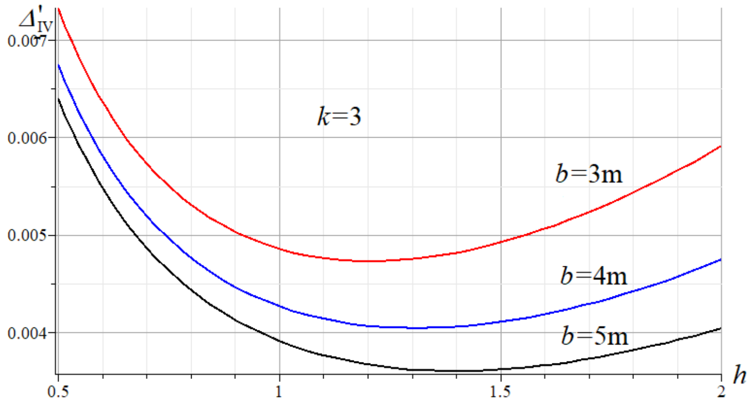

Figure 9. Dependence of the deflection of the truss on the action of the wind load on the height.

The main advantage of analytical solutions that include not only the size of the structure and load, but also the number of panels, is the ability to apply calculation formulas to structures with a large or very large number of rods. If numerical methods inevitably begin to accumulate counting errors when the number of rods in an object increases [18–26], then the accuracy of the calculation formula obtained by induction does not change and is determined only by the adequacy of the design model used. Another advantage of the obtained solution is the ability to perform optimization [27–35] of the truss by mathematically analyzing the formula, quickly compare options for building parameters and find optimal combinations of sizes and number of panels. Extreme points are found on the graphs based on the found formulas, which suggest the appropriate optimal design parameters to the designer. Analysis of forces in the truss also revealed an unexpected and unusual distribution of forces in the bars of the side panels of the truss.

4. Conclusions

A mathematical model of a spatial beam truss is constructed. An algorithm for displaying accurate analytical solutions for the forces in the rods and deflections of the structure under the action of various types of loads has been developed. The obtained solutions proved to be quite compact, suitable both for evaluating the performance of real structures of this type, and for testing numerical solutions performed in standard packages based on the finite element method. The applied modeling and analysis algorithm can be applied to other regular systems, both planar and spatial.

References

1. Kirsanov, M.N. Analysis of the buckling of spatial truss with cross lattice. Magazine of Civil Engineering. 2016. 64. Pp. 52–58. DOI: 10.5862/MCE.64.5
2. Voropay, R.A., Domanov, E.V. The dependence of the deflection of a planar beam truss with a complex lattice on the number of panels in the system Maple. Postulat. 2019. No. 1. P. 12
3. Arutyunyan, V.B. Analytical calculation of the deflection street bracket for advertising. Postulat. 2019. No. 1 http://e-postulat.ru/index.php/Postulat/article/download/2300/2340
4. Voropay, R.A., Domanov, E.V. Analytical solution of the problem of shifting a movable support of a truss of arch type in the Maple system. Postulat. 2019. No. 1. http://e-postulat.ru/index.php/Postulat/article/download/2345/2386

5. Ilyushin, A.S. The formula for calculating the deflection of a compound externally statically indeterminate frame. Structural mechanics and structures. 2019. Vol. 3. No. 22. Pp. 29–38.
6. Egorov, S.S. The inductive method of solving the problem of deflection of the symmetric core structures of complex shape in the system Maple for arbitrary number of panels. Science Almanac. 2017. 3-3 (29). Pp. 254–257. DOI: 10.17117/na.2017.03.03.254 http://ucom.ru/doc/na.2017.03.03.254.pdf
7. Tinkov, D.V. Comparative analysis of analytical solutions to the problem of truss structure deflection. Magazine of Civil Engineering. 2015. No. 5. Pp. 66–73. (rus) DOI: 10.5862/MCE.57.6
8. Smirnova, A.A., Rakhmatulina, A.R Analytical calculation of the displacement of the truss support. Science Almanac. 2017. No. 2-3(28). Pp. 275–278. https:// elibrary.ru/download/elibrary_28913800_87609841.pdf
9. Rakhmatulina, A.R., Smirnova, A.A. The dependence of the deflection of the arched truss loaded on the upper belt, on the number of panels. Science Almanac. 2017. No. 2-3(28). Pp. 268–271. https://elibrary.ru/download/elibrary_28913798_61471993.pdf
10. Kirsanov, M.N. Analytical calculation of deflection of rectangular spatial roof structure. Proceedings of the Moscow State University of Civil Engineering. 2018. Vol. 13. No. 5(116). Pp. 579–586. DOI: www.dx.doi.org/10.22227/1997–0935.2018.5.579-586]
11. Kirsanov, M.N. The deflection of spatial coatings with periodic structure. Magazine of Civil Engineering. 2017. 76. Pp. 58–66. DOI: 10.18720/MCE.76.6
12. Rybakov, L.S. Linear theory of a flat orthogonal grid. Mekhanika tverdogo tela. 2001. No. 4. Pp. 106–118. (rus)
13. Galishnikova, V.V., Dunaiski, P., Pahl, P.J. Geometrically Nonlinear Analysis of Plane Trusses and Frames SUN MeDIA, Stellenbosch (Republic of South Africa), 2009. Pp. 382. doi:10.18820/9781920109998.
14. Hutchinson, R.G., Fleck, N.A. Microarchitectured cellular solids – the hunt for statically determinate periodic trusses. ZAMM Z. Angew. Math. Mech. 2005. Vol. 85. No. 9. Pp. 607–617 DOI: 10.1002/zamm.200410208
15. Kirsanov, M.N. The property of embedding of natural frequencies spectra of regular mechanical systems. Structural mechanics and structures. 2019. No. 2(21). Pp. 40–49.
16. Petersen, O.W., Oiseth, O., Lourens, E. Investigation of dynamic wind loads on a long-span suspension bridge identified from measured acceleration data. Journal of Wind Engineering and Industrial Aerodynamics. 2020. Vol. 196. Pp. 104045. https://doi.org/10.1016/j.jweia.2019.104045
17. Zhou, Q., Ma, B., Zhu, Q., Zhang, H. Investigation on wind loads on angle-steel cross-arms of lattice transmission towers via direct force measurement. Journal of Wind Engineering and Industrial Aerodynamics. 2019. Vol. 191. Pp. 117–126. doi: 10.18820/9781920109998.
18. Rybakov, V.A., Gamayunova, O.S. Stress-state elements frame structures from thin-walled rods. Construction of Unique Buildings and Structures. 2013. No. 7(12). Pp. 79–123. (rus)
19. Rybakov, V.A., Al Ali, M., Panteleev, A.P., Fedotova, K.A., Smirnov, A.V. Bearing capacity of rafter systems made of steel thin-walled structures in attic roofs. Magazine of Civil Engineering. 2017. No. 8. Pp. 28–39. doi: 10.18720/MCE.76.3.
20. Vatin, N.I., Havula, J., Martikainen, L., Sinelnikov, A.S., Orlova, A.V., Salamakhin, S.V. Thin-walled cross-sections and their joints: tests and fem-modelling. Advanced Materials Research. 2014. No. 945–949. Pp. 1211–1215. DOI: 10.4028/www.scientific.net/AMR.945-949.1211
21. Tusnina, V.M. Semi-rigid steel beam-to-column connections. Magazine of Civil Engineering. 2017. 73. Pp. 25–39. doi: 10.18720/MCE.73.3.
22. Tusnina, O.A., Danilov, A.I. The stiffness of rigid joints of beam with hollow section column. Magazine of Civil Engineering. 2016. 64. Pp. 40–51. doi: 10.5862/MCE.64.4
23. Gusakova, N.V., Filyushina, K.E., Gusakov, A.M., Minaev, N.N. Selection criteria of space planning and structural solutions of low-rise buildings. Magazine of Civil Engineering. 2017. 75. Pp. 84–93. doi: 10.18720/MCE.75.8.
24. Travush, V.I., Fedorova, N.V. Survivability of structural systems of buildings with special effects. Magazine of Civil Engineering. 2018. 81. Pp. 73–80. doi: 10.18720/MCE.81.8
25. Siriguleng, B., Zhang, W., Liu, T., Liu Y. Z. Vibration modal experiments and modal interactions of a large space deployable antenna with carbon fiber material and ring-truss structure. Engineering Structures. 2019. Pp. 109932. https://doi.org/10.1016/j.engstruct.2019.109932
26. Cheng, J., Xu, M., Xu, H. Mechanical performance analysis and parametric study of double-deck plate-truss composite steel girders of a three-tower four-span suspension bridge. Engineering Structures. 2019. Vol. 199. Pp. 109648. https://doi.org/10.1016/j.engstruct.2019.109648
27. Serpik, I.N., Alekseytsev, A.V. Optimization of flat steel frame and foundation posts system. Magazine of Civil Engineering. 2016. 61. Pp. 14–24. doi: 10.5862/MCE.61.2
28. Topping, B.H.V. Shape optimization of skeletal structures: a review. Journal of Structural Engineering. 1983. Vol. 109. No. 8. Pp. 1933–1951 https://doi.org/10.1061/(ASCE)0733-9445(1983)109:8(1933)
29. Kaveh, A., Laknejadi, K. A hybrid evolutionary graph-based multi-objective algorithm for layout optimization of truss structures. Acta Mechanica. 2013. Vol. 224. No. 2. Pp. 343–364. DOI: 10.1007/s00707-012-0754-5
30. Tinkov, D.V., Safonov, A.A. Design Optimization of Truss Bridge Structures of Composite Materials. Journal of Machinery Manufacture and Reliability, 2017. Vol. 46. No. 1. Pp. 46–52. DOI: 10.3103/S1052618817010149
31. Pholdee, N., Bureerat, S. Comparative performance of meta-heuristic algorithms for mass minimisation of trusses with dynamic constraints. Advances in Engineering Software. 2014. Vol. 75. Pp. 1–13.
32. Lieu, Q.X., Do, D.T.T., Lee, J. An adaptive hybrid evolutionary firefly algorithm for shape and size optimization of truss structures with frequency constraints. Computers & Structures. 2018. Vol. 195. Pp. 99–112. https://doi.org/10.1016/j.compstruc.2017.06.016
33. Alekseytsev, A.V. Evolutionary optimization of steel trusses with the nodal joints of rods. Magazine of Civil Engineering. 2013. 40. Pp. 28–37. DOI: 10.5862/MCE.40.3
34. Alekseytsev, A.V., Akhremenko, S.A. Evolutionary optimization of prestressed steel frames, Magazine of Civil Engineering. 2018. 73. Pp. 32–42. DOI: 10.18720/MCE.81.4.
35. Serpik, I.N., Alekseytsev, A.V., Balabin, P.Y. Mixed Approaches to Handle Limitations and Execute Mutation in the Genetic Algorithm for Truss Size, Shape and Topology Optimization. Periodica Poly-technica Civil Engineering. No. 61(3). 2017. Pp. 471–482.

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