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Improved gradient projection method for parametric optimisation of bar structures

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Abstract. Numerical optimization and the finite element method have been developed together to make possible the emergence of structural optimization as a potential design tool. The main research goal of this paper is the development of mathematical support and a numerical algorithm to solve parametric optimization problems of structures with orientation on software implementation in a computer-aided design system. The paper considers parametric optimization problems for bar structures formulated as nonlinear programming ones. The method of the objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations has been used to solve the parametric optimization problem. Equivalent Householder transformations of the resolving equations of the method have been proposed. They increase numerical efficiency of the algorithm developed based on the considered method. Additionally, proposed improvement for the gradient projection method also consists of equivalent Givens transformations of the resolving equations. They ensure acceleration of the iterative searching process in the specified cases described by the paper due to decreasing the amount of calculations. The comparison of the optimization results of truss structures presented by the paper confirms the validity of the optimum solutions obtained using proposed improvement of the gradient projection method. The efficiency of the proposed improvement of the gradient projection method has been also confirmed taking into account the number of iterations and absolute value of the maximum violation in the constraints.

1. Introduction

Over the past 50 years, numerical optimization and the finite element method have individually made significant advances and have together been developed to make possible the emergence of structural optimization as a potential design tool [1]. In recent years, great efforts have been also devoted to integrate optimization procedures into the CAD facilities. With these new developments, lots of computer packages are now able to solve relatively complicated industrial design problems using different structural optimization techniques [2].

Applied optimum design problems for bar structures in some cases are formulated as parametric optimization problems, namely as searching problems for unknown structural parameters, which provide an extreme value of the specified objective function in the feasible region defined by the specified constraints. In this case, structural optimization is performed by variation of the structural parameters when the structural topology, cross-section types and node type connections of the bars, the support conditions of the bar system, as well as loading patterns and load design values are prescribed and constants.

Kibkalo et al. in the paper [3] formulated a parametric optimization problem for thin-walled bar structures and considered methods to solve them. The searching for the optimum solution has been performed by varying the structural parameters providing the required load-carrying capacity of structural members and the minimum value of manufacturing costs.

Alekseytsev has described the process of developing a parametrical-optimization algorithm for steel trusses in the paper [4]. Parametric optimization has been performed taking into account strength, stability and stiffness constraints formulated for all truss members. The objective function has been formulated depending on the specific manufacturing of the truss panel joints in term of the manufacturing cost calculated based on the labor costs and materials used.

Serpik et al. in the paper [5] developed an algorithm for parametric optimization of steel flat rod systems. The optimization problem has been formulated as a structural weight minimization problem taking into account strength and displacement constraints, as well as overall stability constraints. The cross-sectional dimensions of the truss members and the coordinates of the truss panel joints have been considered as design variables. The structural analysis of internal forces and displacements for considered structures has been performed using the finite element method. An iterative procedure for searching for optimum solution has been proposed in [6].

Sergeyev et al. in the paper [7] formulated a parametric optimization problem with constraints on faultless operation probability of bar structures with random defects. The weight of the bar structures has been considered as the objective function. Initial global imperfections have been considered as small independent random variables distributed according to normal distribution law, as well as buckling load value has been also considered as a random variable.

The mathematical model of the parametric optimization problem of structures includes a set of design variables, an objective function, as well as constraints, which reflect generally non-linear dependences between them [8]. If the objective function and constraints of the mathematical model are continuously differentiable functions, as well as the search space is smooth, then the parametric optimization problems are successfully solved using gradient projection non-linear methods [9]. The gradient projection methods operate with the first derivatives or gradients only both of the objective function and constraints. The methods are based on the iterative construction of such a sequence of the approximations of design variables that provides convergence to the optimum solution (optimum values of the structural parameters) [10].

Additionally, a sensitivity analysis is a useful optional feature that could be used in scope of the numerical algorithms developed based on the gradients methods [11]. Thus, in the paper [12] Sergeyev et al. formulated a parametric optimization problem of linearly elastic space frame structures taking into account the stress and multiple natural frequency constraints. The cross-sectional parameters of structural members as well as node positions of the bar structures has been considered as design variables. The sensitivity analysis of multiple frequencies has been performed using analytic differentiation with respect to the design variables. The optimal design of the structure has been obtained by solving a sequence of quadratic programming problems.

Although many papers are published on the parametric optimization of structures, the development of a general computer program for the design and optimization of structures according to specified design codes remains an actual task. Therefore, the main *research goal* of this paper is the development of mathematical support and a numerical algorithm to solve parametric optimization problems of structures with orientation on software implementation in a computer-aided design system.

One of the effective methods to solve parametric optimization problems for structures is gradient projection methods, as shown by the review of scientific researches presented above. That is why, in this paper, a gradient projection method is considered as *investigated object*. The following research tasks are formulated: to propose an improvement of the gradient projection method that ensures the increase of the numerical efficiency of the algorithm developed based on the considered method, as well as the acceleration of the iterative searching process due to decreasing the amount of calculations.

2. Methods

2.1. Parametric optimization problem formulation

Let us consider a parametric optimization problem of a structure consisting of bar members. It can be formulated as presented below: to find optimum values for geometrical parameters of the structure, bar's cross-section dimensions and initial pre-stressing forces introduced into the redundant members of the bar system, which provide the extreme value of the determined optimality criterion and satisfy all load-bearing capacities and stiffness requirements. We assume, that the structural topology, cross-section types and node type connections of the bars, the support conditions of the bar system, as well as loading patterns and load design values are prescribed and constants.

The formulated parametric optimization problem can be stated as a non-linear programming task in the following mathematical terms: to find unknown structural parameters $\vec{X} = \{X_\iota\}^T$, $\iota = \overline{1, N_X}$, providing the least value of the determined objective function:

$$f^* = f\left(\vec{X}^*\right) = \min_{\vec{X} \in \Im_{\mathbb{N}}} f\left(\vec{X}\right), \tag{1.1}$$

in a feasible region (search space) $\Im$ defined by the following system of constraints:

$$\boldsymbol{\psi}\left(\vec{X}\right)=\left\{\psi_{\kappa}\left(\vec{X}\right)=0 \mid \kappa=\overline{1,N_{EC}}\right\}; \quad (1.2)$$

$$\boldsymbol{\varphi}\left(\vec{X}\right)=\left\{\phi_{\eta}\left(\vec{X}\right)\leq 0 \mid \eta=\overline{N_{EC}+1,N_{IC}}\right\}; \quad (1.3)$$

where $\vec{X}$ is the vector of the design variables (unknown structural parameters); f, ψ_{κ}, φ_{η} are the continuous functions of the vector argument; $\vec{X}^{*}$ is the optimum solution or optimum point (the vector of optimum values of the structural parameters); f^{*} is the optimum value of the optimum criterion (objective function); N_{EC} is the number of constraints-equalities $\psi_{\kappa}\left(\vec{X}\right)$, whose define hyperplanes of the feasible solutions; N_{IC} is the number of constraints-inequalities $\varphi_{\eta}\left(\vec{X}\right)$, whose define a feasible region in the design space $\Im$.

The vector of the design variables can consist of a set of unknown geometrical parameters of the structure, a set of unknown cross-sectional dimensions of the structural members, as well as a set of unknown initial pre-stressing forces introduced into the specified redundant members of the structure.

The specific technical-and-economic index (material weight, material cost, construction cost etc.) or another determined indicator can be considered as the objective function Eq. (1.1) taking into account the ability to formulate its analytical expression as a function of design variables $\vec{X}$.

Load-bearing capacities constraints (strength and stability inequalities) for all design sections of the structural members subjected to all design load combinations at the ultimate limit state as well as displacements constraints (stiffness inequalities) for the specified nodes of the bar system subjected to all design load combinations at the serviceability limit state should be included into the system of constraints Eqs. (1.2) – (1.3). The design internal forces in the bar structural members used in the strength and stability inequalities of the system Eqs. (1.2) – (1.3) are considered as state variables depending on design variables $\vec{X}$ and can be calculated from the linear equations system of the finite element method [13]. The node displacement of the bar system used in stiffness inequalities of the system Eqs. (1.2) – (1.3) are also considered as state variables depending on design variables $\vec{X}$ and can be also calculated from the linear equations system of the finite element method [13]. Additional requirements, whose describe structural, technological and serviceability particularities of the considered structure, as well as constraints on the building functional volume can be also included into the system Eqs. (1.2) – (1.3).

2.2. An improved gradient projection method for solving the parametric optimization problem

The parametric optimization problem stated as non-linear programming task by Eqs. (1.1) – (1.3) can be solved using a gradient projection method. The method of *objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations* ensures effective searching for solution of the non-linear programming tasks occurred when optimum designing of the structures [14].

The gradient projection method operates with the first derivatives or gradients only of both the objective function Eq. (1.1) and constraints Eqs. (1.2) – (1.3). The method is based on the iterative construction of such sequence Eq. (2.1) of the approximations of design variables $\vec{X}=\left\{X_{\iota}\right\}^{T}$, $\iota=\overline{1,N_{X}}$, that provides the convergence to the optimum solution (optimum values of the structural parameters):

$$\vec{X}_{k+1}=\vec{X}_{k}+\Delta\vec{X}_{k}, \quad (2.1)$$

where $\vec{X}_{k}=\left\{X_{\iota}\right\}^{T}$, $\iota=\overline{1,N_{X}}$ is the current approximation to the optimum solution $\vec{X}^{*}$ that satisfies both constraints-equalities Eq. (1.2) and constraints-inequalities Eq. (1.3) with the extreme value of the objective function Eq. (1.1); $\Delta\vec{X}_{k}=\left\{\Delta X_{\iota}\right\}^{T}$, $\iota=\overline{1,N_{X}}$, is the increment vector for the current values of the design variables $\vec{X}_{k}$; k is the iteration's index. The start point of the iterative searching process $\vec{X}_{k=0}$ can be assigned as engineering estimation of the admissible design of the structure.

The active constraints only of constraints system Eqs. (1.2) – (1.3) should be considered at each iteration.

A set of active constraints numbers $\mathbf{A}$ calculated for the current approximation $\vec{X}_k$ to the optimum solution (current design of the structure) is determined as:

$$\mathbf{A} = \boldsymbol{\kappa} \cup \boldsymbol{\eta},\ \boldsymbol{\kappa} = \left\{ \kappa \middle| \left| \psi_\kappa \left(\vec{X}_t \right) \right| \geq -\varepsilon \right\},\ \boldsymbol{\eta} = \left\{ N_{EC} + \eta \ \middle|\ \phi_\eta \left(\vec{X}_t \right) \geq -\varepsilon \right\}. \quad (2.2)$$

where ε is a small positive number introduced here in order to diminish the oscillations on movement alongside of the active constraints surface.

The increment vector $\Delta\vec{X}_k$ for the current values of the design variables $\vec{X}_k$ can be determined by the following equation:

$$\Delta\vec{X}_k = \Delta\vec{X}_\perp^k + \Delta\vec{X}_\parallel^k, \quad (2.3)$$

where $\Delta\vec{X}_\perp^k$ is the vector calculated subject to the condition of elimination the constraint's violations; $\Delta\vec{X}_\parallel^k$ is the vector determined taking into consideration the improvement of the objective function value. Vectors $\Delta\vec{X}_\parallel^k$ and $\Delta\vec{X}_\perp^k$ are directed parallel and perpendicularly accordingly to the subspace with the vectors basis of the linear-independent constraint's gradients, such that:

$$\left(\Delta\vec{X}_\perp^k \right)^T \Delta\vec{X}_\parallel^k = 0. \quad (2.4)$$

The values of the constraint's violations for the current approximation $\vec{X}_k$ of the design variables are accumulated into the following vector:

$$\mathbf{V} = \left(\psi_\kappa \left(\vec{X} \right) \forall \kappa \in \boldsymbol{\kappa};\ \phi_\eta \left(\vec{X} \right) \forall \eta \in \boldsymbol{\eta} \right).$$

Let us introduce a set $\mathbf{L}$, $\mathbf{L} \subseteq \mathbf{A}$, of the constraint's numbers, such that the gradients of the constraints at the current approximation $\vec{X}_k$ to the optimum solution are linear-independent.

Component $\Delta\vec{X}_\perp^k$ is calculated from the equation presented below:

$$\Delta\vec{X}_\perp^k = \left[\nabla\varphi \right] \vec{\mu}_\perp. \quad (2.5)$$

where $\left[\nabla\varphi \right]$ is the matrix that consists of components $\dfrac{\partial\psi_\kappa}{\partial X_\iota}$ and $\dfrac{\partial\phi_\eta}{\partial X_\iota}$, here $\iota = \overline{1, N_X}$, $\kappa \in \mathbf{L}$, $\eta \in \mathbf{L}$; $\vec{\mu}_\perp$ is the column-vector that defines the design variables increment subject to the condition of elimination the constraint's violations. Vector $\vec{\mu}_\perp$ can be calculated as presented below.

In order to correct constraint's violations $\mathbf{V}$, vector $\Delta\vec{X}_\perp^k$ to a first approximation should also satisfy Taylor's theorem for the continuously differentiable multivariable function in the vicinity of point $\vec{X}_k$ for each constraint from set $\mathbf{L}$, namely:

$$-\mathbf{V} = \left[\nabla\varphi \right]^T \Delta\vec{X}_\perp^k. \quad (2.6)$$

With substitution of Eq. (2.5) into Eq. (2.6) we obtain the system of equations to determine column-vector $\vec{\mu}_\perp$:

$$\left[\nabla\varphi \right]^T \left[\nabla\varphi \right] \vec{\mu}_\perp = -\mathbf{V}. \quad (2.7)$$

Component $\Delta\vec{X}_\parallel^k$ is determined using the following equation:

$$\Delta\vec{X}_\parallel^k = \xi \times \vec{p} = \xi \left(\nabla\vec{f} - \left[\nabla\varphi \right] \vec{\mu}_\parallel \right), \quad (2.8)$$

where $\nabla\vec{f}$ is the vector of the objective function gradient in the current point (current approximation of the design variables) $\vec{X}_k$; $\vec{p}$ is the projection of the objective function gradient vector onto the active constraints surface in the current point $\vec{X}_k$; $\vec{\mu}_\parallel$ is the column-vector that defines the design variable's increment subject to the improvement of the objective function value. Column-vector $\vec{\mu}_{\|}$ can be calculated approximately using the least-square method by the following equation:

$$[\nabla\varphi]\vec{\mu}_{\|} \approx \nabla\vec{f}\,, \tag{2.9}$$

or from the equation presented below:

$$[\nabla\varphi]^T[\nabla\varphi]\vec{\mu}_{\|} = [\nabla\varphi]^T\nabla\vec{f}\,; \tag{2.10}$$

where ξ is the step parameter, which can be calculated subject to the desired increment Δf of the objective function on movement along the direction of the objective function anti-gradient. The increment Δf can be assign as 5...25% from the current value of the objective function $f\left(\vec{X}_t\right)$:

$$\Delta f = \xi\left(\nabla\vec{f}\right)^T\nabla\vec{f}\,,\ \xi = \frac{\Delta f}{\left(\nabla\vec{f}\right)^T\nabla\vec{f}}\,, \tag{2.11}$$

where in case of minimization Eq. (1.1) Δf and ξ accordingly have negative values. The parameter ξ can be also calculated using the dependency presented below:

$$\xi = \frac{\Delta f}{(\vec{p})^T\nabla\vec{f}}\,, \tag{2.12}$$

that follows from the condition of attainment the desired increment of the objective function Δf on the movement along the direction of the objective function anti-gradient projection onto the active constraints surface. Step parameter ξ can be also selected as a result of numerical experiments performed for each type of the structure individually [15, 16].

Fig. 2.1 presents a graphical illustration for step to the point $\vec{X}_{k+1}$ depending on location of the current approximation $\vec{X}_k$ in the two-dimension search space.

Using Eq. (2.5) and Eq. (2.8), Eq. (2.3) can be rewritten as presented below:

$$\Delta\vec{X}_k = [\nabla\varphi]\,\vec{\mu}_{\perp} + \xi\left(\nabla\vec{f} - [\nabla\varphi]\,\vec{\mu}_{\|}\right), \tag{2.13}$$

or

$$\Delta\vec{X}_k = \xi\,\nabla\vec{f} + [\nabla\varphi]\left(\vec{\mu}_{\perp} - \xi\,\vec{\mu}_{\|}\right), \tag{2.14}$$

where column-vectors $\vec{\mu}_{\perp}$ and $\vec{\mu}_{\|}$ are calculated using Eq. (2.7) and Eq. (2.9) or Eq. (2.10), respectively.

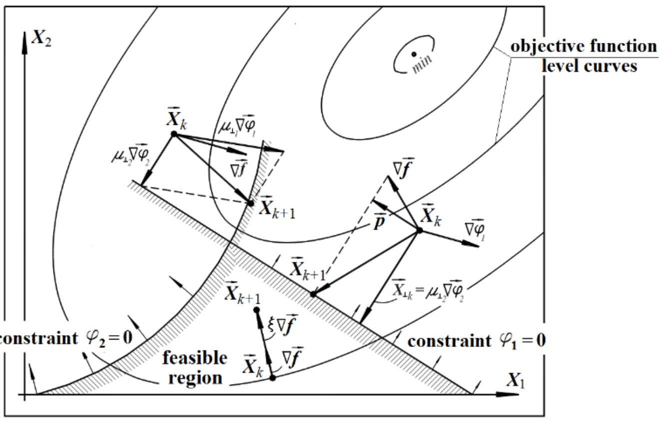


Figure 2.1. Step to the next point $\vec{X}_{k+1}$ depending on location of the current approximation $\vec{X}_k$ for two design variables X_1 and X_2.

The linear-independent constraints of the system Eqs. (1.2) – (1.3) should be detected when constructing the matrix of the active constraints gradients $[\nabla\varphi]$ used by Eq. (2.7) and Eq. (2.9) or Eq. (2.10). Selection of the linear-independent constraints can be performed based on the equivalent transformations of the resolving equations of the gradient projection method using the non-degenerate transformation matrix $\mathbf{H}$, such that the sub-diagonal elements of the matrix $\mathbf{H}[\nabla\varphi]$ equal to zero. Besides,

$$\mathbf{H}^T\mathbf{H} = \mathbf{I};\tag{2.15}$$

$$\mathbf{H} = \mathbf{H}_t \times \ldots \times \mathbf{H}_i \times \ldots \times \mathbf{H}_2 \times \mathbf{H}_1;\tag{2.16}$$

where $\mathbf{I}$ is the unit matrix; t is the total number of the linear-independent gradients of the active constraints, $\mathbf{H}_i$ is the transformation matrix, such that $\mathbf{H}_i^T\mathbf{H}_i = \mathbf{I}$, at the same time the sub-diagonal element are equal to zero in matrix $\mathbf{H}_i \times \mathbf{H}_{i-1} \times \ldots \times \mathbf{H}_2 \times \mathbf{H}_1 \times [\nabla\varphi]$ for column's numbers $\overline{1,\ i}$. Described conditions are satisfied by the orthogonal matrix of the elementary mapping (Householder's transformation) [17, 18].

Let us present here the following algorithm to form set $\mathbf{L}$ and to construct matrix $\mathbf{H}[\nabla\varphi]$.

1. $i = 0$, $\mathbf{L} = \varnothing$ and $[\nabla\mathbf{\Phi}]_0 = [\nabla\varphi]$ should be assumed, where $[\nabla\varphi]$ is the matrix that comprises from the column-gradients of all active constraints. All columns of matrix $[\nabla\mathbf{\Phi}]_0$ should be marked as 'not used' (or linear-independent).

2. $i = i + 1$.

3. Among all 'not used' columns of matrix $[\nabla\mathbf{\Phi}]_{i-1}$, whose correspond to the constraints-equalities Eq. (1.2), one j^{th} column with extreme value of the specified criterion should be selected (e.g., the following criterion $\ell_j^2 = \sum_{k=i}^{N_X} g_{kj}^2$ can be considered as such criterion, where g_{kj} are the j^{th} column's components of matrix $[\nabla\mathbf{\Phi}]_{i-1}$). At the same time, all k^{th} columns of matrix $[\nabla\mathbf{\Phi}]_{i-1}$, for whose the following inequality $\ell_k^2 \le \varepsilon_1$ met, should be marked as 'used', here ε_1 is a small positive number. In case when no constraints-equalities exist or all constraints-equalities Eq. (1.2) are marked as 'used', the selection of j^{th} column should be performed among all 'not used' columns of matrix $[\nabla\mathbf{\Phi}]_{i-1}$, whose correspond to the constraints-inequalities Eq. (1.3). If $\ell_j^2 \le \varepsilon_1$, then generation of set $\mathbf{L}$ and matrix $\mathbf{H}[\nabla\varphi]$ is finished. $\mathbf{H}[\nabla\varphi] = [\nabla\varphi]_{i-1}$. In case of $\ell_j^2 \le \varepsilon_1$ and $i = 1$ (i. e. $\mathbf{L} = \varnothing$), there is a contradiction in the system of constraints Eqs. (1.2) – (1.3). In other case, moving to the next step performs.

4. k^{th} number of the constraint, that corresponds to the j^{th} column number, should be included into set $\mathbf{L}$, $\mathbf{L} \leftarrow \mathbf{L} + \{k\}$.

5. Calculate $[\nabla\mathbf{\Phi}]_i = \mathbf{H}_i[\nabla\mathbf{\Phi}]_{i-1}$. It is reasonable to execute the multiplication only for 'not used' columns. It should be noted, when using Householder's transformation matrix $\mathbf{H}_i$ is not constructed evidently [18]. At the same time, matrix $[\nabla\mathbf{\Phi}]_i$ can be constructed within the ranges of matrix $[\nabla\mathbf{\Phi}]_{i-1}$ when no additional memory is needed.

6. If $i = 1$, then $[\nabla\varphi]_i = \vec{q}_j$, where $\vec{q}_j$ is j^{th} column-vector of matrix $[\nabla\mathbf{\Phi}]_i$. When $i > 1$ $[\nabla\varphi]_i$ is constructed using extension of the matrix $[\nabla\varphi]_{i-1}$ by the column-vector $\vec{q}_j$. j^{th} column of matrix $[\nabla\mathbf{\Phi}]_i$ is selected as 'used', then moving to the step 2 performs.

Using Householder's transformations described above triangular structure of the nonzero elements of matrix $\mathbf{H}[\nabla\varphi]$ is formed step-by-step. Besides, Eq. (2.7) and Eq. (2.9) can be rewritten as follow:

$$\left([\nabla\varphi]^T\mathbf{H}^T\right)\left(\mathbf{H}[\nabla\varphi]\right)\vec{\mu}_{\perp} = -\mathbf{V};\tag{2.17}$$

$$\mathbf{H}[\nabla\varphi]\vec{\mu}_{||} \approx \mathbf{H}\nabla\vec{f}.\tag{2.18}$$

In order to calculate column-vectors $\vec{\mu}_{\perp}$ and $\vec{\mu}_{||}$, it is required only to perform forward and backward substitutions in Eq. (2.17) and Eq. (2.18).

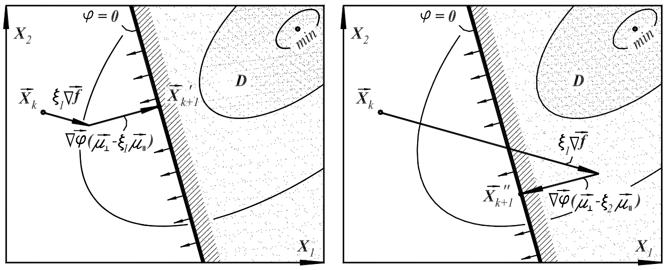
To accelerate the convergence of the minimization algorithm presented above, h th columns should be excluded from matrix $\mathbf{H}[\nabla\varphi]$. These columns correspond to those constraints from Eq. (1.3), for which the following inequality satisfies:

$$\mu_{\perp h} - \xi \times \mu_{||h} > 0 . \qquad (2.19)$$

Actually, if $\mu_{\perp h} - \xi_2 \mu_{||h} > 0$, then the return onto the active constraints surface from the feasible region $\Im$ is performed with simultaneous degradation of the objective function value (see Fig. 2.2, *b*). At the same time, in case of:

$$\mu_{\perp h} - \xi_1 \mu_{||h} < 0 , \qquad (2.20)$$

both the improvement of the objective function value and the return from the inadmissible region onto the active constraints surface are performed (see Fig. 2.2, *a*).



a ***b***

Figure 2.2. The selection of the constraints-inequalities:
***a* –** $\mu_{\perp h} - \xi_1 \mu_{||h} < 0$**;** ***b* –** $\mu_{\perp h} - \xi_2 \mu_{||h} > 0$**.**

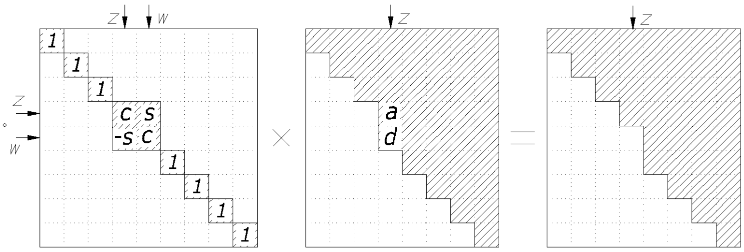
When excluding h th columns from matrix $\mathbf{H}[\nabla\varphi]$ corresponded to those constraints for whose Eq. (2.13) is satisfied, the matrix $(\mathbf{H}[\nabla\varphi])_{red}$ with a broken (non-triangular) structure of the non-zero elements is obtained. The set $\mathbf{L}$ of the linear-independent active constraints numbers transforms into the set $\mathbf{L}_{red}$ respectively. At the same time, the vector of the constraint's violations $\mathbf{V}$ reduced into the vector $\mathbf{V}_{red}$ accordingly.

In order to restore triangular structure of the matrix $(\mathbf{H}[\nabla\varphi])_{red}$ with zero sub-diagonal elements, Givens transformations (Givens rotations) [19, 18] can be used. Givens transformations for the matrix $(\mathbf{H}[\nabla\varphi])_{red}$ consist of construction such square matrix $\mathbf{G}_{wz}$, for which corresponded wz th element of matrix $\mathbf{G}_{wz}(\mathbf{H}[\nabla\varphi])_{red}$ returns zero (see Fig. 2.3) [20]. Since $c^2 + s^2 = 1$ by definition, it follows:

$$(\mathbf{G}_{wz})^T \mathbf{G}_{wz} = \mathbf{I} . \qquad (2.21)$$

An obvious method to calculate c and s for d th non-zero sub-diagonal element and for a th diagonal element is presented below:

$$c = \frac{a}{\sqrt{a^2 + d^2}}, \quad s = \frac{d}{\sqrt{a^2 + d^2}}; \qquad (2.22)$$



$\mathbf{G}_{wz}$ $(\mathbf{H}[\nabla\varphi])_{red}$ $\mathbf{G}_{wz}(\mathbf{H}[\nabla\varphi])_{red}$

Figure 2.3. Scheme for Givens rotations (non-zero elements of the matrixes are hatched).

The Givens matrix $\mathbf{G}$ can be calculated similarly to the matrix $\mathbf{H}$ using the following equation:

$$\mathbf{G} = \mathbf{G}_{\gamma} \times \ldots \times \mathbf{G}_{i} \times \ldots \times \mathbf{G}_{2} \times \mathbf{G}_{1}. \tag{2.23}$$

where γ is the number of the Givens transformations. So, Givens transformations should be executed several times (with different values z and w), while the matrix $\mathbf{G}_{wz}\left(\mathbf{H}\left[\nabla\varphi\right]\right)_{red}$ has no all zero sub-diagonal elements (for example presented by Fig. 2.3, $\gamma = 5$).

Taking into account Givens transformations, Eq. (2.17) and Eq. (2.18) for column-vectors $\left(\vec{\mu}_{\perp}\right)_{red}$ and $\left(\vec{\mu}_{\parallel}\right)_{red}$ can be rewritten as:

$$\left(\left[\nabla\varphi\right]^{T}\mathbf{H}^{T}\right)_{red}\mathbf{G}^{T}\mathbf{G}\left(\mathbf{H}\left[\nabla\varphi\right]\right)_{red}\left(\vec{\mu}_{\perp}\right)_{red} = -\mathbf{V}_{red}; \tag{2.24}$$

$$\mathbf{G}\left(\mathbf{H}\left[\nabla\varphi\right]\right)_{red}\left(\vec{\mu}_{\parallel}\right)_{red} \approx \mathbf{GH}\nabla\vec{f}; \tag{2.25}$$

and the main resolving equation of the gradient projection method Eq. (2.13) and Eq. (2.14) can be rewritten as presented below:

$$\Delta\vec{X}_{k} = \left(\mathbf{H}\left[\nabla\varphi\right]\right)_{red}\left(\vec{\mu}_{\perp}\right)_{red} + \xi\left(\nabla\vec{f} - \left(\mathbf{H}\left[\nabla\varphi\right]\right)_{red}\left(\vec{\mu}_{\parallel}\right)_{red}\right), \tag{2.26}$$

or

$$\Delta\vec{X}_{k} = \xi\ \nabla\vec{f} + \left(\mathbf{H}\left[\nabla\varphi\right]\right)_{red}\left(\left(\vec{\mu}_{\perp}\right)_{red} - \xi\left(\vec{\mu}_{\parallel}\right)_{red}\right). \tag{2.27}$$

The proposed improvement for the method of the objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations consists of equivalent transformations of the resolving equations using Householder transformations. The transformations with matrix $\mathbf{H}$ presented by Eq. (2.24) and Eq. (2.25) of the resolving equations of the gradient projection method Eq. (2.7) and Eq. (2.9) increase the numerical efficiency of the algorithm developed based on the gradient projection method described above.

Additionally, the proposed improvement for the gradient projection method includes equivalent transformations of the resolving equations using Givens rotations. The transformations with matrix $\mathbf{G}$ presented by Eq. (2.24) and Eq. (2.25) ensure acceleration of the iterative searching process Eq. (2.1) in case when Eq. (2.19) takes into account due to decreasing the amount of calculations.

It should be noted that the lengths of the gradient vectors for the objective function Eq. (1.1), as well as for constraints Eqs. (1.2) – (1.3), remain as they were in scope of the proposed equivalent transformations ensuring the dependability of the optimization algorithm.

The determination the convergence criterion is the final question when using the iterative searching for the optimum point Eq. (2.1) described above. Considering the geometrical content of the gradient steepest descent method, we can assume that at the permissible point $\vec{X}_{k}$ the component of the increment vector $\Delta\vec{X}_{\parallel}^{k}$ for the design variables should be vanish, $\Delta\vec{X}_{\parallel}^{k} \to 0$, in case of approximation to the optimum solution of the non-linear programming task presented by Eqs. (1.1) – (1.3). So, the following convergence criterion of the iterative procedure Eq. (2.1) can be assigned:

$$\left\|\Delta\vec{X}_{\parallel}^{k}\right\| = \sqrt{\sum_{\iota=1}^{N_X}\left(\Delta X_{\parallel,\iota}^{k}\right)^{2}} < \varepsilon_{1}, \tag{2.28}$$

where ε_{1} is a small positive number.

Taking into consideration Eq. (2.28), let us formulate the following stop criteria in the iterative searching procedure of Eq. (2.1).

Stop criterion 1: when the objective function gradient in the current approximation $\vec{X}_{k}$ is close to zero indicating on extreme character of the current approximation, as well as there are no violated constraints:

$$\begin{cases} \mathbf{\Sigma} = \varnothing, \\ -\varepsilon \geq \nabla\vec{f} \geq +\varepsilon; \end{cases} \tag{2.29}$$

where $\mathbf{\Sigma}$ is the set of the violated constraints numbers, $\mathbf{\Sigma} = \left\{ s \middle| \left| \psi_s \left(\vec{X}_k \right) \right| > \varepsilon; \ \phi_s \left(\vec{X}_k \right) > \varepsilon \right\}$;

Stop criterion 2: when the projection of the objective function gradient in the current approximation $\vec{X}_k$ onto the active constraints surface is close to zero (objective function gradient is perpendicular to the active constraints surface) indicating impossible further improvement of the objective function value, as well as there are no violated constraints:

$$\begin{bmatrix} \mathbf{\Sigma} = \varnothing; \\ -\varepsilon \geq \vec{p} \geq +\varepsilon; \end{bmatrix} \quad (2.30)$$

Stop criterion 3: when in the current approximation $\vec{X}_k$ of the iterative searching procedure Eq. (2.3) the total number of the active constraints t equals to the number of design variables N_X, as well as all active constraints are ε-active (both not violated constraints and those ones for whose inequality Eq. (2.13) met):

$$\begin{bmatrix} \mathbf{\Sigma} = \varnothing; \\ t = N_X; \\ \mu_{\perp f} - \xi \times \mu_{\| f} < 0, \forall f \in \mathbf{L}. \end{bmatrix} \quad (2.31)$$

This stop criterion for the iteration process Eq. (2.1) corresponds to the case when the current approximation $\vec{X}_k = \left(X_\iota^k \right)^T, \iota = \overline{1, N_X}$, to the optimum solution is located at the intersection of the constraints (i.e., vertex). In this case, no correction of the constraints violations is needed and further improvement of the objective function value is not possible.

Stop criterion 4: when the objective function values within two consecutive iterations are the same with acceptable accuracy subject to the absence of the violated constraints:

$$\begin{bmatrix} \mathbf{\Sigma} = \varnothing; \\ f\left(\vec{X}_{k-1} \right) \approx f\left(\vec{X}_k \right). \end{bmatrix} \quad (2.32)$$

3. Results and Discussion

In order to estimate an efficiency of the new methods or algorithms, a comparison with alternative methods or algorithms presented by other authors using different optimization techniques should be performed. Criteria to implement such comparison are described, e.g. by Haug & Arora [15] and Crowder et al. [21]. Many of these criteria, such as robustness, amount of functions calculations, requirements to the computer memory, numbers of iterations etc. cannot be used due to lack of corresponded information in the technical literature. Therefore, an efficiency estimation of the method of objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations presented above will be based on the comparison of the optimization results obtained using proposed improvement of the gradient projection method, as well as of the results presented by the literature and widely used for testing. The initial data and mathematical models of the parametric optimization problems considered below were assumed as the same as described in the literature.

3.1. Parametric optimization of a three-bar truss

Optimization of a three-bar truss (see Fig. 3.1) has been firstly solved by Schmit [22] using a non-linear programming method. Besides, the task has been also considered by Haug and Arora [15]. The parametric optimization problem is formulated as searching for optimum cross-sectional areas b_1, b_2 and b_3 of the truss bars providing the least value of the truss weight subject to normal stresses and flexural stability constraints, as well as displacements and eigenvalue constraints. The load cases for the considered truss are presented in Table 3.1. Initial data for optimization of the truss are shown in Table 3.2.

Table 3.1. Load cases for considered truss.

Load case j	1	2	3
θ_j, °	45	90	135
$P_j \times 10^3$ [pound-force]	40	30	20
P_j [kN]	177.9897	133.4922	88.9948

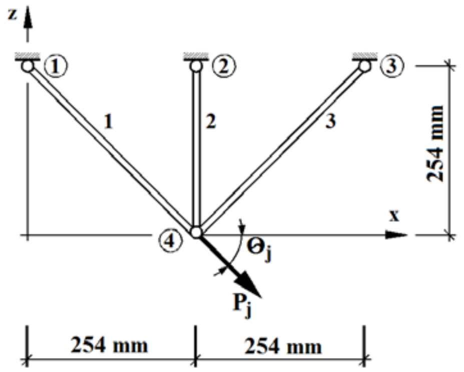


Figure 3.1. Three-bar truss.

Table 3.2. Initial data for optimization of the truss.

Unit weight of the truss material ρg	0.1 pound/inch^3 = 0.027154 N/cm^3
Modulus of elasticity E	10^7 pound/inch^2=6.8971×10^6 N/cm^2
Allowable stresses value σ_1^a, σ_3^a for the 1st and 3rd truss members	5000 pound-force/inch^2=3.4486 kN/cm^2
Allowable stresses value σ_2^a for the 2nd truss member	2000 pound-force/inch^2=1.3794 kN/cm^2
Non-dimensional factor β used to calculate second moment area of inertia for each truss member, $I_i = \beta b_i$	1.0
Ultimate vertical z^a and horizontal x^a displacements of the truss nodes	0.005 inch=0.0127 cm
Lower limit value for eigenvalue	$\zeta_0 = 1.872 \cdot 10^8$

The objective function can be written as presented below:

$$\psi_0 = \rho g l\left(b_1\sqrt{2} + b_2 + b_3\sqrt{2}\right) \to \min; \tag{3.1}$$

where b_1, b_2 and b_3 are cross-sectional areas of the truss bars; l is the truss height, $l = 25.4$ cm (see Fig. 3.1). Let us formulate strength constraints for each truss members for all load cases as follows:

$$\psi_{3(i-1)+j} = \frac{\left|N_i^j\right|}{b_i \sigma_i^a} - 1 \le 0; \tag{3.2}$$

where N_i^j is the axial force for i th truss member subjected to j th load case, $i = \overline{1,3}$, $j = \overline{1,3}$. Besides, let include to the system of constraints the inequalities for the positive values of the design variables:

$$\psi_{9+i} = -b_i \le 0; \tag{3.3}$$

Flexural buckling constraints for all truss members can be written using Hooke's law as presented below:

$$\psi_{12+3(i-1)+j} = -\frac{\left(x_4^j + z_4^j\right) l}{\pi^2 \beta b_i} - 1 \le 0; \tag{3.4}$$

where x_4^j, z_4^j are linear displacements for node 4 of the truss subjected to j th load case along the directions of $0x$ and $0z$ axes respectively. The constraints on the minimum values of the eigenvalues can be written analytically using calculation results of the eigenvalues stability problem for the considered truss:

$$\psi_{22} = \frac{2\sqrt{2}\rho\, l^2 \zeta_0 \left(\frac{b_1 + b_3}{b_2}\sqrt{2} + 1\right)}{3E\left(\frac{b_1 + b_3}{b_2} + \sqrt{2} - \sqrt{\left(\frac{b_1 - b_3}{b_2}\right)^2 + 2}\right)} - 1 \le 0. \tag{3.5}$$

Let also formulate displacements constraints for 4th truss node in the plane $x0z$:

$$\psi_{22+j} = -1 - \frac{x_4^j}{x^a} \le 0\,; \tag{3.6}$$

$$\psi_{25+j} = \frac{x_4^j}{x^a} - 1 \le 0\,; \tag{3.7}$$

$$\psi_{28+j} = -1 - \frac{z_4^j}{z^a} \le 0\,; \tag{3.8}$$

$$\psi_{31+j} = \frac{z_4^j}{z^a} - 1 \le 0\,. \tag{3.9}$$

Starting from start values of the design variables $\vec{b}^0 = \left(64.516,\ 32.258,\ 32.258\right)^T$ cm² with truss weight $G^0 = 116.602$ N, an optimum solution $\vec{b}^* = \left(57.4878,\ 12.4482,\ 27.4299\right)^T$ cm² with the optimum weight $G^* = 91.383$ N has been obtained. The comparison of the optimization results for the considered three-bar truss obtained by Haug and Arora [15] and in this article is presented in Table 3.3. Good correlation of obtained optimization results with the results of the other authors confirms the validity of the optimum solutions calculated using the proposed improvement of the gradient projection method.

The step-by-step characteristics of the iterative searching for optimum design of the three-bar truss are presented in Table 3.4. Eleven iterations have been performed. The iterative searching process for the optimum point was stopped due to the following stop criterion: increment of the design variables within two consecutive iterations was less than 0.0001, whereas there were no violated constraints.

Table 3.3. Comparison of the optimization results for the three-bar truss.

Source	Start and optimum cross-section areas [cm²] for truss member			**Truss weight [N]**
	1	2	3	
Start values by Haug and Arora [15]	64.516	32.258	32.258	**116.602**
Optimum values by Haug and Arora [15]	59.2257	13.9355	24.8387	**91.588**
Optimum values by this paper	57.4878	12.4482	27.4299	**91.383**

[*]truss member's numbers are indicated in Fig. 3.1

Table 3.4. Step-by-step characteristics of the iterative searching for optimum design of the three-bar truss.

Iteration number	Current values of the design variables [cm²]			Objective function value [kN]	Numbers of the active constraints	Maximum violation of the constraints
	b_1	b_2	b_3			
0	64.5160	32.2580	32.2580	0.1166	–	–
1	44.5160	22.2580	22.2580	0.0805	15	0.35835
2	60.7847	12.2580	12.2580	0.0797	10, 15, 18, 22, 24, 33, 34	0.46276
3	40.7847	14.8577	22.2580	0.0717	15, 18, 22, 26	0.42237
4	55.5763	15.8613	21.5250	0.0861	15, 22	0.10076
5	57.3934	13.0115	26.3367	0.0906	15, 22	0.01189
6	57.5871	12.5536	27.2368	0.0914	15, 22	0.00027
7	57.4967	12.4569	27.4140	0.0914	15, 22	$9.67\cdot10^{-6}$
8	57.4885	12.4489	27.4288	0.0914	15, 22	$7.55\cdot10^{-8}$
9	57.4878	12.4483	27.4299	0.0914	15, 22	$1.43\cdot10^{-9}$
10	57.4878	12.4483	27.4299	0.0914	15, 22	$8.74\cdot10^{-11}$
11	57.4878	12.4483	27.4299	0.0914	15, 22	$6.55\cdot10^{-12}$

3.2. Optimization of a ten-bar cantilever truss

A parametric optimization problem of a ten-bar cantilever truss (see Fig. 3.2) is widely used in the literature [15, 23, 24, 25] in order to compare different methods for solving optimization problems. The parametric optimization problem is formulated as follows: to find unknown cross-sectional areas for each truss member $\vec{b} = (b_i)^T$, $i = \overline{1,10}$, with weight minimization of the truss subjected to stresses constraints in all truss bars, node displacements constraints, as well as constraints on the minimal cross-section areas. A start value $b_0 = 1.0$ inch²=6.4516 cm² was used as a start approximation for variable cross-sections areas for all bars of the considered truss.

The considered truss undergoes two load cases, as shown in Fig. 3.2 and presented in Table 3.5. Initial data for optimization of the truss are presented in Table 3.6.

Table 3.5. Load cases for ten-bar cantilever truss.

Load case number	Node number (Fig. 3.2)	Point load along axis $0z$, $\times 10^3$ [pound-force]	Point load along axis $0z$ [kN]
1	2	-100.0	-444.82
	4	-100.0	-444.82
2	1	50.0	222.41
	2	-150.0	-667.23
	3	50.0	222.41
	4	-150.0	-667.23

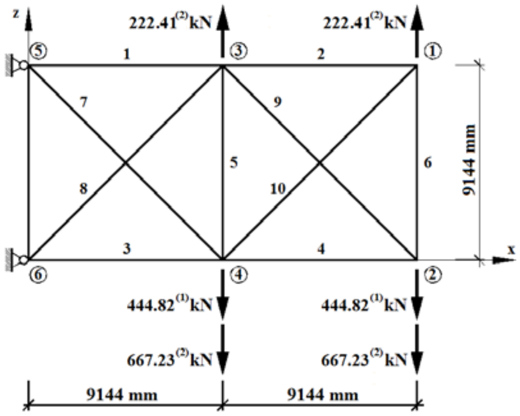


Figure 3.2. Ten-bar cantilever truss.

Table 3.6. Initial data for optimization of the ten-bar truss.

Unit weight of the truss material ρg	0.1 pound/inch³=0.027143 N/cm³
Modulus of elasticity E	10⁷ pound/inch²=6.8948×10⁶ N/cm²
Non-dimensional factor β used to calculate second moment area of inertia for each truss member, $I_i = \beta b_i^2$	1.0
Lower limit value b^L for cross-sectional areas for all truss bars	0.10 inch²=0.64516 cm²
Allowable stresses value for the all truss member σ^a	25000 pound-force/inch²=17.236 kN/cm²
Ultimate vertical z^a displacements of the truss nodes	±2 inch=$\pm$ 5.08cm

Variable cross-section areas for each truss member $\vec{b} = (b_i)^T$, $i = \overline{1,\ 10}$, were considered as design variables. The objective function can be written as presented below:

$$\psi_0 = \rho g l\left(b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + \sqrt{2}\left(b_7 + b_8 + b_9 + b_{10}\right)\right) \rightarrow \min\,; \tag{3.10}$$

where l is the truss height, $l = 914.4$ cm (see Fig. 3.2). Constraints on lower limit value for variable cross-sectional areas for all truss bars are written as follows:

$$\psi_i = 1 - \frac{b_i}{b_i^L} \le 0\,. \tag{3.11}$$

Stresses constraints can be formulated as presented below:

$$\psi_{10+i} = \frac{|N_i|}{b_i \sigma_i^a} - 1 \le 0\,. \tag{3.12}$$

where N_i is the axial force in the i th truss member. Displacement constraints for the truss nodes are written as follows:

$$\psi_{20+j} = -1 - \frac{z_j}{z^a} \le 0\,; \tag{3.13}$$

$$\psi_{24+j} = \frac{z_j}{z^a} - 1 \le 0\,; \tag{3.14}$$

where x_j, z_j are linear displacements of j th truss node, $j = \overline{1,4}$.

Starting from the initial truss design with start weight $G^0 = 1.867$ kN, an optimal solution with the optimum weight of $G^* = 22.514$ kN has been obtained for the truss subjected to the first load case. Additionally, starting from the initial truss design with the start weight of $G^0 = 1.867$ kN, an optimal solution with the optimum weight of $G^* = 20.806$ kN has been obtained for the truss subjected to the second load case. For both loaded cases, the iterative searching process for the optimum point was stopped due to the following stop criterion: the increment of the design variables within two consecutive iterations was less than 0.0001, as well as there were no violated constraints.

Table 3.7. Comparison of the optimization results for the ten-bar cantilever truss.

Bar number, i	Design variable	Start values for design variables [cm^2]	Optimal cross-section area for i th truss member [cm^2]			
			for the first load case		for the second load case	
			Haug&Arora [15]	This paper	Haug&Arora [15]	This paper
1	b_1	6.4516	193.7480	196.7185	152.0255	151.8842
2	b_2	6.4516	0.6452	0.6452	0.6452	0.6452
3	b_3	6.4516	150.1545	149.6227	163.0771	163.1232
4	b_4	6.4516	98.6192	98.1744	92.5353	92.7578
5	b_5	6.4516	0.6452	0.6452	0.6452	0.6452
6	b_6	6.4516	3.5903	3.5536	12.7084	12.7079
7	b_7	6.4516	48.1825	48.0919	80.0063	79.8721
8	b_8	6.4516	136.7610	135.7063	82.9031	82.7963
9	b_9	6.4516	139.4707	138.8988	130.8707	131.1797
10	b_{10}	6.4516	0.6452	0.6452	0.6452	0.6452
Truss weight [kN]		**1.867**	**22.523**	**22.511**	**20.808**	**20.807**
Number of active constraints			4	5	4	6
Numbers of active constraints			2, 5, 10, 22	2, 5, 10, 15, 21	2, 10, 15, 22	2, 5, 10, 15, 16, 22
Modulus of the maximum violation in the constraints			$0.27 \cdot 10^{-4}$	$6.35 \cdot 10^{-7}$	$0.17 \cdot 10^{-3}$	$1.19 \cdot 10^{-7}$

Table 3.8. Comparison of the optimization results for the ten-bar cantilever truss.

Source of information	Optimum weight [kN]			
	Load case 1		Load case 2	
	Stresses constraints only	All constraints	Stresses constraints only	All constraints
This paper	7.087	22.511	7.404	20.807
Haug & Arora [15]	7.089	22.523	7.411	20.808
Schmit & Miura [25]	7.089	22.591	7.407	20.811
Rizzi [24]	7.089	22.590	7.407	20.811
Dobbs & Nelson [23]	7.217	22.605	–	22.514

Table 3.7 presents comparison of the optimization results for considered ten-bar truss obtained by Haug and Arora [15] and in this article. Table 3.8 shows a comparison of the optimization results for the ten-bar cantilever truss obtained using the proposed improved method of objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations with optimization results presented by the literature [15, 23, 24, 25].

Good correlation of obtained optimization results with the results of the other authors confirms the validity of the optimum solutions calculated using proposed improvement of the gradient projection method. The efficiency of the proposed improvement of the gradient projection method has been also confirmed taking into account the number of iterations and absolute value of the maximum violation in the constraints. The deviations available in some presented results can be explained by using a numerical approach to the iterative searching with specified accuracy.

3.3. Optimization of a 24-bar transmission tower

A parametric optimization problem for a transmission tower (see Fig. 3.3) has been considered by Haug and Arora [15]. The transmission tower is subjected to 2 load cases, as shown in Table 3.9. The initial data for optimization of the tower are presented in Table 3.10. Taking into account the symmetry of the structural form, the vector of the design variables has been reduced to 7 variable cross-section areas for 25 structural members of the considered tower (see Table 3.11). The parametric optimization problem is formulated as searching for optimum cross-sectional areas $\vec{X} = \left(X_i\right)^T$, $i = \overline{1,7}$, of the tower structural members, whose provide the least weight of the tower subjected to stresses constraints, node displacements constraints, as well as constraints on the minimal cross-section areas.

A start value $A_0 = 1.0$ inch^2=6.4516 cm^2 was used as a start approximation for the variable cross-sections areas for all members of the considered tower. The dimensions of the optimization problem comprised 7 design variables and 129 constraints.

Table 3.9. Load cases for the transmission tower.

Load case number	Node number (Fig. 3.3)	Point load along axis [kN]		
		$0x$	$0y$	$0z$
1	1	2.2241	–	–
	2	2.2241	–	–
	3	4.4482	–22.241	44.4822
	4	–	–22.241	44.4822
2	3	–	–22.241	88.9644
	4	–	–22.241	–88.9644

The positive direction of the point loads coincides with the positive direction of the corresponded axes

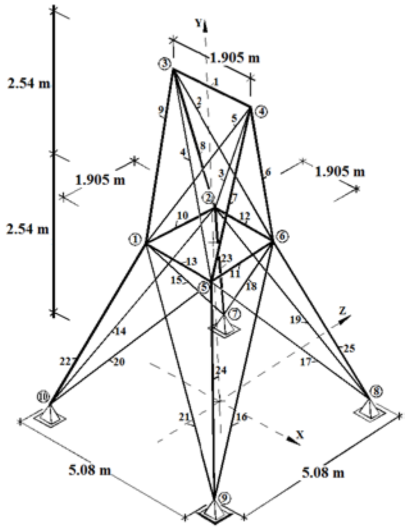


Figure 3.3. Design model of the transmission tower.

Table 3.10. Initial data for optimization of the tower.

Unit weight of the tower material ρg	0.1 pound/inch^3=0.027154 N/cm^3
Modulus of elasticity E	10^7 pound/inch^2=6.8971×10^6 N/cm^2
Non-dimensional factor β used to calculate second moment area of inertia for each tower structural member $I_i = \beta b_i^2$	1.0
Lower limit value A^L for cross-sectional areas for all tower members	0.01 inch^2=0.0645 cm^2
Ultimate node displacements of the tower x^a, y^a, z^a	0.35 inch=0.889 cm
Allowable stresses value for the all tower member σ^a	± 40000 pound/inch^2= ± 27.5885 kN/cm^2

Table 3.11. Comparison of the optimization results for the transmission tower.

Design variable	Tower structural members*	Optimal cross-section areas for tower members [cm2]	
		Haug and Arora [15]	This paper
A_1	1	0.0645	0.0655
A_2	2, 3, 4, 5	13.2103	13.1754
A_3	6, 7, 8, 9	19.3322	19.3654
A_4	10, 11, 12, 13	0.0645	0.0645
A_5	14, 15, 16, 17	4.4213	4.4056
A_6	18, 19, 20, 21	10.4626	10.4669
A_7	22, 23, 24, 25	17.2335	17.2343
Tower weight [kN]		**2.4250**	**2.4245**

**bar's numbers are indicated in Fig. 3.3*

At the continuum optimum point there were 5 active constraints: constraints on lower limit value for variable cross-sectional area for 10th, 11th, 12th and 13th tower members; 3rd and 4th node displacement constraint of the tower along axis $0z$ for both load cases. The iterative searching process for the optimum point was stopped due to the following stop criterion: increment of the design variables within two consecutive iterations was less than 1×10^{-6}, as well as there were no violated constraints.

The comparison of the optimization results for the transmission tower is presented by Table 3.11. Good correlation of obtained results with the results of the other authors confirms the validity of the optimum solutions calculated using the proposed improvement of the gradient projection method. Start values of the design variables have no influence on the optimum solutions for the considered non-linear problems, validating thus the accuracy of the obtained optimum solutions. The efficiency of the proposed improvement of the gradient projection method has been also confirmed taking into account the number of iterations and absolute value of the maximum violation in the constraints.

4. Conclusion

The paper considers parametric optimization problems for the bar structures formulated as nonlinear programming task. The method of the objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations has been used to solve the parametric optimization problem.

Equivalent Householder transformations of the resolving equations of the method have been proposed. They increase numerical efficiency of the algorithm developed based on the considered method.

Equivalent transformations (Givens rotations) of the resolving equations of the method have been also proposed. They ensure acceleration of the iterative searching process in the specified cases described by the paper due to decreasing the amount of calculations.

Lengths of the gradient vectors for objective function, as well as for constraints remain as they were in scope of the proposed equivalent transformations ensuring the reliability of the optimization algorithm.

In order to estimate an efficiency of the proposed improvement of the gradient projection method, a comparison of the obtained optimization results with the results presented by the literature and widely used for testing has been performed. Good correlation of obtained results with the results of the other authors confirms the validity of the optimum solutions calculated using the proposed improvement of the gradient projection method. The efficiency of the proposed improvement of the gradient projection method has been also confirmed taking into account the number of iterations and absolute value of the maximum violation in the constraints.

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