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Cold-formed steel joints with partial warping restraint

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Abstract. The article investigates the influence of joints on the warping torsion of cold-formed steel bars. The modern warping torsion theory suggests cold-formed steel bars to be simply supported or fixed at the ends. Simple support provides zero warping restraint. Fixed support provides fool warping restraint at the joint of the bar. In real constructions cold-formed steel joints are partial warping restrained. Not considering the partial restraint of deplanations by real joints leads to an incorrect assessment of the twist angles and the stress state of thin-walled steel bars in warping torsion. This article deals with an experimental and analytical investigation of warping torsion of cold-formed steel bars with bolted joints. Considered 142C16, 142C20, 262C23 and 262C29 sections. Five types of joints considered: a wall and both flanges of the bar end sections are fixed; the upper and lower flanges are fixed; the wall is fixed; the wall and the lower flange are fixed; the lower flange is fixed. First, analytical expressions for twist angles and bimoments for warping torsion for bars with partial warping restraints obtained. Analytical results are compared with the results of the warping torsion experiment conducted at Moscow State University of Civil Engineering. The cold-formed steel specification is shown to be a poor predictor for the twist angle and bimoment value of twisting members. The warping factor coefficient is recommended for the estimation of the degree of the joint warping constraint. Experimental values of warping factors for different joint types are obtained. The influence of partial warping restraints and cross-section deformation on the work of the tested cold-formed steel bars are evaluated.

1. Introduction

The technical theory of torsion of thin-walled rods of an open profile was created in the 30s of the 20th century in [1]. One of the main assumptions used in its development is the hypothesis of a rigid contour. It is assumed that the cross-section contour maintains its shape when the bar is twisted. Torsion leads not only to cross-section rotation about the center of twist but at the same time the points of the section undergo different displacements along the longitudinal axis. These displacements, called deplanations, lead to warping of cross-sections of a thin-walled bar in the torsion. Because of warping restraint, additional sectorial normal stresses arise in the bar. The basic theory of thin-walled members with open cross-sections was developed by Vlasov [1]. To determine the twist angles and bimoments in the rod, Vlasov proposed the differential equation:

$$\theta^{IV} - k^2\theta'' = \frac{m(z) - b'(z)}{E J_\omega}, \tag{1}$$

where θ is the angle of twist of the rod; $m(z)$ is the intensity of external distributed twisting moments; $b'(z)$ is the derivative with respect to z of the intensity of external distributed bimoments; J_ω is the warping constant of the cross-section; J_d is the torsion constant of the cross-section; E is the modulus of elasticity; G is the shear modulus; $k = \sqrt{\frac{G J_d}{E J_\omega}}$ is the elastic flexural-torsional characteristic of the thin-walled rod.

Equations of the elastic line of twist angles and bimoments for different types of load arrangements are presented in tabular form by Bychkov [2]. Solutions presented in [1] and [2] are valid for rods subjected to zero or completely warping restraints in the absence of a rod twisting in the supported sections.

A number of research studies have been carried out over the past decades to determine the influence of restraints on the work of cold-formed steel bars. A significant part of the studies is devoted to the study of the carrying capacity of cold-formed thin-walled steel bars, working in bending, compression, and torsion. In the studies reviewed issues of strength, overall and local stability. In [3–8], the results of theoretical and numerical studies of the strength and stability of cold-formed bars are presented. Solved problems of determining forces and displacements using rod and shell finite elements, propose theoretical solutions for determining bimoments taking into account bending moments, consider the problem of distortional buckling. The stress-strain state of cold-formed steel bars under torsion was studied experimentally [9] and theoretically [10]. In [9], considering the experimental data obtained, an expression for the bearing capacity of an eccentrically loaded C-profile bar was obtained. In [10], three methods are compared, the theory of calculating thin-walled constructions [1], the method of representing constrained torsion by bending with torsion, and the method of representing torsion by bending the shelves of a thin-walled rod by a pair of forces. The performed work confirmed the effectiveness of the theoretical and numerical methods used in the calculation of thin-walled structures experiencing bending, compression, and torsion.

Extensive experience has been gained in experimental studies of thin-walled systems [11–25]. The articles show the results of tests for bending and compression of C and Z shaped profiles, solid, perforated, with simple and complex edge stiffeners. The obtained data on strength and stability were compared with theoretical and numerical solutions. Based on the tests carried out in [12], an expression was given to evaluate the carrying capacity of thin-walled Z-profiles in biaxial bending. In [15] the method of selecting the effective width of the section elements for calculating the effective section properties was justified. In [16] the method for determining the effective width of the elements sections and method of determining the bearing capacity of the rod. The experience of experimental studies can be successfully adapted to solve the problems of the operation of cold-formed steel bars with various joints.

The Consideration of Saint-Venant's principle of using the Vlasov theory [1] is the work of [26]. It shows the influence of boundary conditions on the operation of thin-walled rods. In [27], the effect of conjunction flexibility on the critical load of cold-formed C-shaped profiles was studied. In addition to analytical data, tests of the considered structures are given.

Experimental and theoretical studies of the behavior of trusses made of thin-walled members were carried out in [28]. Taking into account the research carried out, a technique for strengthening the upper chord of the truss has been proposed, and an eaves joint has been developed.

The results of numerical and experimental studies of frame structures made of cold-formed steel members are presented in [29–31]. In [32], the results of experimental studies of bolted joints of cold-formed steel trusses are given. Bolted joints are considered semi-rigid to bend. The authors proposed a technique for numerical modeling of bolted joints, which is based on previous studies [33] and [34]. In [35, 36], experimental and numerical studies of structures made of thin-walled rods were carried out, and the rotation stiffness of semi-rigid nodal joints was determined. In [36], recommendations were given on numerical modeling, taking into account the initial imperfections and features of bolted joints.

The results of cyclic tests of the cold-formed frame joints and the method for determining the bearing capacity of bolted joints are presented in [37]. The refined method for calculating the bolted connections of thin-walled elements [38] is based on the results of experimental studies. Authors give recommendations on the determining the carrying capacity of bolted joints. Experimental studies of bolted connections for cold-formed frames were carried out in [39].

The common feature for these studies is that the joints supporting the rods are “idealized”, considered to be zero or completely warping restraint. The degree of restriction of deplanations with real joints is not considered. The effect of the degree of deplanation restriction is not taken into account. Also, in addition to [1] and [2], little attention is paid to the separately constrained torsion of the rod. Thus, despite extensive studies of the operation of thin-walled structures, the issues of accurately determining the forces and deformations in such structures, taking into account their actual joints, are of definite practical and scientific interest.

This article discusses the warping torsion of cold-formed steel bars with different bolted joint types. The Autors carried out a series of warping torsion tests of cold-formed C-shaped profiles of different lengths and sizes under different joint conditions. The Experimental results were compared with analytical solutions. The main goal of the study is to provide a method for bolted joints warping stiffness calculation. This method will be used to specify the influence of partial warping restraints on the twist angles and bimoments value of cold-formed steel bars. According to the warping torsion experimental results, a part of the torsion angles of the cold-formed bar, obtained by deforming the contour of the cross-section, will be estimated. The range of the flexural-torsional characteristic kl will be determined beyond which the cross-section contour deformation can be neglected.

2. Methods

Practically used joints of cold-formed steel bars impose partial warping restraint in accordance with complete twist restraint. They do not provide zero or complete deplanations restriction at supported sections. Considering the joints to be zero or completely warping restraint leads to significant errors in determining the twist angles and the bimoments. Experimental and theoretical studies have been carried out at Moscow State University of Civil Engineering aimed at identifying the features of the behavior of cold-formed steel bars under various boundary conditions.

A series of warping torsion tests were conducted. A total of 40 cold-formed C-shaped bars of four sections 142C16, 142C20, 262C23, and 262C20 under different joint conditions were tested. Steel grade of cold-formed steel sections is S450GD EN 1036:2015. The elastic modulus is E = 210000 N/mm^2, the shear modulus is $G \approx$ 81000 N/mm^2, the yield stress is fy = 450 N/mm^2. The lengths, dimensions of the cross-sections and the values of the flexural-torsional characteristics *kl* of the tested bars are given in Table 1. Warping torsion test arrangements are given in Fig. 1 and Fig. 2. the Bars were fixed at the ends. Torque was applied to the middle of the span of the bar. At the distance of 260 mm from the central section of the bar four LVDTs were located (see Fig. 1). Two on the upper flange and two on the lower flange. Torque moment values are listed in Table 2. The bars ends were bolted to a rigid fixed support structure. Five types of joints were considered

1. fixed wall and both flanges;
2. fixed upper and lower flange;
3. fixed wall;
4. fixed wall and a bottom flange;
5. fixed bottom flange.

Table 1. Section geometrical characteristics.

Profile	h (mm)	b (mm)	c (mm)	t (mm)
142C16	142	60	13	1.6
142C20	142	60	13	2.0
262C23	262	65	13	2.3
262C29	262	65	13	2.9

where: h is the wall height. b is the flange width. c is the flange stiffeners height. t is the wall thickness.

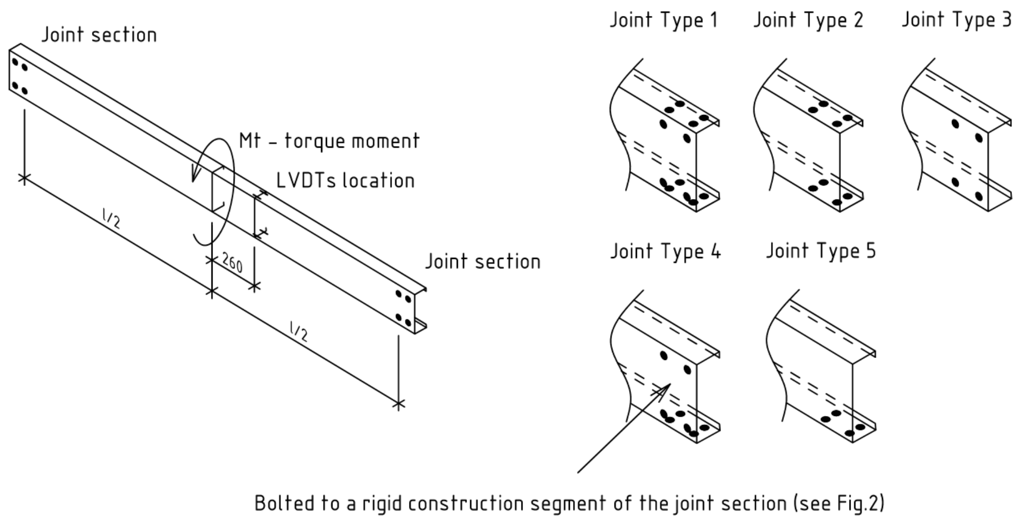


Figure 1. Warping torsion test arrangement. types of joints.

Table 2. Torque moment values.

Profile	l (mm)	kl	M_t (kN mm)				
			Type1	Type2	Type3	Type4	Type5
142C16	1955	0.864	146.6	146.8	859.6	703.0	20.0
142C20	1955	1.094	156.7	146.7	933.0	104.2	31.1
262C23	1955	0.698	229.1	229.1	205.9	229.2	49.2
262C29	1955	0.895	377.1	377.1	357.3	377.2	54.1
142C16	3940	1.706	380.9	337.9	298.7	360.0	17.8
142C20	3940	2.160	419.4	419.5	250.9	419.4	21.7
262C23	3940	1.379	133.8	133.8	113.8	133.9	54.5
262C29	3940	1.767	145.8	145.8	146.1	145.8	62.2

where: l is the length of the bar, kl is the dimensionless flexural-torsional characteristic of the bar, $kl = l\sqrt{\frac{GJ_t}{EJ_\omega}}$. J_t is the St. Venan torsion constant of the bar cross section. J_ω is the warping constant of the bar cross section. Torque moment M_t value is calculated as product of numbers of a load applied at the end of a loading device console to a distance from load application point to an axis of rotation of the loading device (see Fig. 2). Type 1, 2, 3, 4, 5 indicates joint types (see Fig. 1).

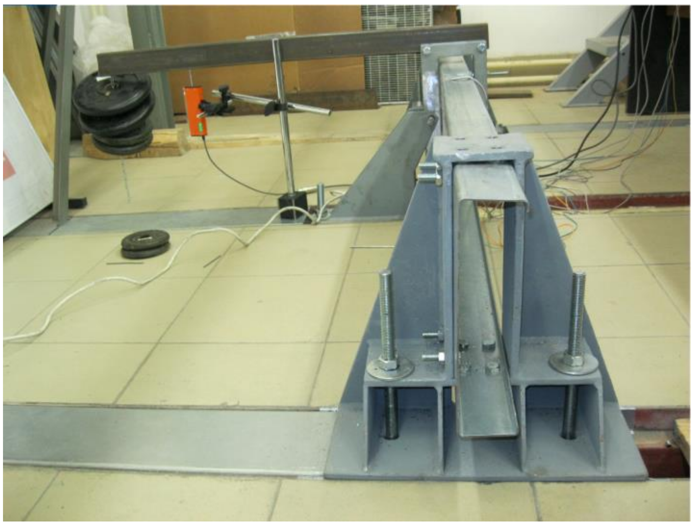


Figure 2. Warping torsion of cold-formed C-shaped steel bar. Joint type 3.

Table 3. Experimental angles of twist θ_{exp}. The ratios of experimental and theoretical values of angles of twist.

Profile	l (mm)	kl	θ_{exp} (grad)					θ_{exp} / $\theta_{complete}$				
			Type 1	Type 2	Type 3	Typ e 4	Type 5	Type 1	Type 2	Type 3	Type 4	Type 5
142C16	1955	0.864	3.44	3.17	2.69	2.02	1.15	1.72	1.58	2.29	2.11	4.22
142C20	1955	1.094	2.74	2.95	2.05	2.11	1.27	1.45	1.66	1.81	1.68	3.38
262C23	1955	0.698	1.06	1.07	1.85	1.26	2.22	2.06	2.08	4.01	2.45	20.9
262C29	1955	0.895	1.43	1.45	1.87	1.57	1.47	2.07	2.10	2.86	2.27	14.9
142C16	3940	1.706	4.39	4.48	4.50	4.37	4.49	1.09	1.26	1.43	1.15	2.38
142C20	3940	2.160	3.55	3.55	3.05	3.55	4.27	0.98	0.98	1.41	0.98	2.29
262C23	3940	1.379	2.77	2.80	3.65	3.11	4.03	1.24	1.26	1.93	1.39	4.44
262C29	3940	1.767	2.38	2.43	3.33	2.56	3.25	1.22	1.25	1.71	1.32	3.92

Table 3 employs the following notations: θ_{exp} is experimental angles of twist at the central section of the bar. $\theta_{complete}$ is theoretical angles of twist calculated for bars at the central section of the bar with joints providing complete warping restriction (23).

Table 4. Experimental bimoment values $B_{\omega,exp}$. The ratios of experimental and theoretical values of bimoments.

Profile	l (mm)	kl	$B_{\omega,exp}$ / 10^4 (kN mm^2)					$B_{\omega,exp}$ / $B_{\omega,complete}$				
			Type 1	Type 2	Type 3	Type 4	Type 5	Type 1	Type 2	Type 3	Type 4	Type 5
142C16	1955	0.864	1.39	1.53	1.34	0.86	0.19	0.85	0.93	1.40	1.10	0.84
142C20	1955	1.094	1.79	1.85	1.71	1.55	0.46	0.99	1.09	1.58	1.28	1.28
262C23	1955	0.698	2.93	3.02	4.36	3.64	1.19	1.09	1.12	1.80	1.35	2.05
262C29	1955	0.895	4.88	4.98	7.02	5.52	1.22	1.11	1.13	1.68	1.25	1.93
142C16	3940	1.706	1.17	1.03	1.07	1.16	0.78	0.92	0.91	1.07	0.96	1.31
142C20	3940	2.160	1.37	1.39	1.32	1.47	0.97	1.01	1.02	1.62	1.08	1.38
262C23	3940	1.379	4.99	5.01	5.42	5.31	3.11	1.08	1.08	1.38	1.15	1.65
262C29	3940	1.767	5.13	5.22	6.18	5.34	3.43	1.05	1.07	1.26	1.09	1.64

where: $B_{\omega,exp}$ is experimental bimoment value. $B_{\omega,complete}$ is theoretical bimoment value calculated for bars with joints providing complete warping restriction (24).

As a result, the angles of twist θ_{exp} in the central section of the bar, Table 3, and bimoments $B_{\omega,exp}$ in the LVDTs location section, Table 4, were determined. A significant difference between the experimental and theoretical results calculated for joints with zero and complete warping restriction was found, Table 3, Table 4. To identify the causes of this mismatch, the effects of contour and form of cross-section deformation on the supports and in the place where the load was applied were studied. As a result, theoretical expressions were obtained for twisting angles and bimoments for bars with partial torsional and warping restraints. Theoretical expressions for the twisting angles and the bimoments described below. The level of joint warping restraint was determined on the reference bimoment perceived by the joint.

The theoretical solution to the effect of the influence of partial torsional and warping restraints on the behavior of cold-formed steel bars is based on Vlasov theory [1]. Consider the case when, on the supports (due to the flexibility of the joint), the twist angle and the deplanation are not equal to zero. In this case, on the supports, the twist angle θ is proportional to the level of torque moment on the joint section M_θ, and the degree of deplanation δ is proportional to the level of bimoment on the joint section $B_{\omega 0}$. In this case, the relationship between internal forces and deformations for twist angles and deplanations on the supports can be written in the form:

$$\begin{cases} \theta_{01} = k_{\theta 1} M_{\theta 1}; \\ \theta_{02} = k_{\theta 2} M_{\theta 2}; \\ \delta_{01} = k_{\delta 1} B_{\omega 01}; \\ \delta_{02} = k_{\delta 2} B_{\omega 02}, \end{cases} \tag{2}$$

where k_θ and k_δ are the twist and deplanation flexibility of the joint, respectively. Index 0 represents the cross-section of the bar, and indices 1 and 2 represent the beginning and the end of the bar.

The solution of Eq. (1) can be written as:

$$\theta = A \cdot sh\ kz + B \cdot ch\ kz + C \cdot z + D + f(z) \tag{3}$$

where $f(z)$ is the particular solution of Eq (3). Instead of $ch\ kz$, $sh\ kz$, z and 1 in Eq. (3), we introduce the partial integrals $\psi_1(z)$, $\psi_2(z)$, $\psi_3(z)$, $\psi_4(z)$ which are linear combinations of the first:

$$\begin{cases} \psi_1 = a_1 sh\ kz + a_2 ch\ kz + a_3 z + a_4; \\ \psi_2 = b_1 sh\ kz + b_2 ch\ kz + b_3 z + b_4; \\ \psi_3 = c_1 sh\ kz + c_2 ch\ kz + c_3 z + c_4; \\ \psi_4 = d_1 sh\ kz + d_2 ch\ kz + d_3 z + d_4, \end{cases} \tag{4}$$

Then the solution of Eq. (1) can be written as:

$$\theta = A \cdot \psi_1 + B \cdot \psi_2 + C \cdot \psi_3 + D \cdot \psi_4 + f(z), \tag{5}$$

The constants A, B, C, and D are evaluated from the boundary conditions for the bar with partial torsional and warping restraints:

at z = 0:

$$\theta = k_{\theta1} M_{01} \text{ and } \theta'' = -\frac{B_{\omega01}}{EJ_{\omega}},$$

at $z = l$:

$$\theta = k_{\theta2} M_{02} \text{ and } \theta'' = -\frac{B_{\omega02}}{EJ_{\omega}} \tag{6}$$

To simplify further calculations, we take the partial integrals of the Eq. (5) so that they satisfy the following conditions:

$$\begin{cases} \psi_1(0)=1;\ \psi_1''(0)=0;\ \psi_1(l)=0;\ \psi_1''(l)=0; \\ \psi_2(0)=0;\ \psi_2''(0)=1;\ \psi_2(l)=0;\ \psi_2''(l)=0; \\ \psi_3(0)=0;\ \psi_3''(0)=0;\ \psi_3(l)=1;\ \psi_3''(l)=0; \\ \psi_4(0)=0;\ \psi_4''(0)=0;\ \psi_4(l)=0;\ \psi_4''(l)=1 \end{cases} \tag{7}$$

From these conditions we find:

$$\begin{cases} \psi_1 = 1-\frac{z}{l}; \\ \psi_2 = \frac{1}{k^2}\left(\frac{z}{l}-1+\frac{sh\ kl\cdot ch\ kz - ch\ kl\cdot sh\ kz}{sh\ kl}\right); \\ \psi_3 = \frac{z}{l}; \\ \psi_4 = \frac{1}{k^2}\frac{sh\ kz}{sh\ kl}-\frac{z}{k^2\ l}, \end{cases} \tag{8}$$

Consequently,

$$\theta = A\left(1-\frac{z}{l}\right)+\frac{B}{k^2}\left(\frac{z}{l}-1+\frac{sh\ kl\cdot ch\ kz - ch\ kl\cdot sh\ kz}{sh\ kl}\right)+C\frac{z}{l}+\frac{D}{k^2}\left(\frac{sh\ kz}{sh\ kl}-\frac{z}{l}\right)+f(z). \tag{9}$$

Choose the particular solution of Eq. (3) f(z) such that:

$$f(0)=0;\ f'(0)=0;\ f''(0)=0;\ f'''(0)=0. \tag{10}$$

Using the boundary conditions Eq. (6) for Eq. (9), using Eq. (10), we find four non-zero constants A, B, C and D:

$$A = k_{\theta1}M_{01};\ B = -\frac{B_{\omega01}}{EJ_{\omega}};\ C = k_{\theta2}M_{02} - f(l);\ D = -\frac{B_{\omega02}}{EJ_{\omega}} - f''(l). \tag{11}$$

Substituting Eq. (11) into Eq. (9), we obtain the equation of the elastic line of the twist angles for a bar with partial torsional and warping restraints:

$$\begin{aligned} \theta(z) = &-f(l)\frac{z}{l}+\frac{f''(l)}{k^2}\left(\frac{z}{l}-\frac{sh\ kz}{sh\ kl}\right)+f(z)+k_{\theta1}M_{01}\left(1-\frac{z}{l}\right) \\ &+k_{\theta2}M_{02}\frac{z}{l}-\frac{B_{\omega01}}{EJ_{\omega}k^2}\left(\frac{z}{l}-1+ch\ kz-\frac{ch\ kl}{sh\ kl}sh\ kz\right)-\frac{B_{\omega02}}{EJ_{\omega}k^2}\left(\frac{sh\ kz}{sh\ kl}-\frac{z}{l}\right). \end{aligned} \tag{12}$$

Upon double differentiating the Eq. (12) with the respect to the z and multiplying it by -EJ_{ω}, we obtain the equation of bimoments for a bar with partial torsional and warping restraints:

$$B_{\omega} = EJ_{\omega}\frac{f''(l)}{sh\ kl}sh\ kz - EJ_{\omega}f''(z)+B_{\omega01}\left(ch\ kz-\frac{ch\ kl}{sh\ kl}sh\ kz\right)+B_{\omega02}\frac{sh\ kz}{sh\ kl}. \tag{13}$$

It can be seen that when $k_{\theta 1} = k_{\theta 2} = 0$ and $B_{\omega 01} = B_{\omega 02} = 0$, Eq. (12) will be transferred to the equation of the elastic line of the twist angles for the bar, with a complete twisting and zero warping restraints.

$$\theta(z) = -f(l)\frac{z}{l} + \frac{f''(l)}{k^2}\left(\frac{z}{l} - \frac{sh\,kz}{sh\,kl}\right) + f(z). \tag{14}$$

From the same boundary conditions, Eq. (13) will be transferred to the equation of the bimoments for the bar, with a complete twisting and zero warping restraints.

$$B_\omega(z) = EJ_\omega \frac{f''(l)}{sh\,kl} sh\,kz - EJ_\omega f''(z). \tag{15}$$

Similarly, taking the twist angles on the supports equal to zero $k_{\theta 1} = k_{\theta 2} = 0$ and assuming that the warping deformations are completely restrained, that is, $B_{\omega 01}$ and $B_{\omega 02}$ are equal to the support bimoments for a bar with a complete warping restraints, the Eq. (12) will be transformed into an equation of the elastic line of the twist angles for a bar with a complete warping and torsion restraints.

$$\begin{aligned}\theta(z) = & -f(l)\frac{ch\,k(l-z) - ch\,kz + kz \cdot sh\,kl - ch\,kl + 1}{kl \cdot sh\,kl - 2ch\,kl + 2} - \\ & -f'(l)\frac{kl \cdot ch\,kz - kz \cdot ch\,kl + sh\,kl - sh\,kz - sh\,k(l-z) - k(l-z)}{k(kl \cdot sh\,kl - 2ch\,kl + 2)} + f(z).\end{aligned} \tag{16}$$

Eq. (13), becomes the equation of bimoments for a bar with similar boundary conditions.

$$\begin{aligned}B_\omega(z) = & EJ_\omega k^2 f(l)\frac{ch\,k(l-z) - ch\,kz}{kl \cdot sh\,kl - 2ch\,kl + 2} + \\ & + EJ_\omega k f'(l)\frac{kl \cdot ch\,kz - sh\,kz - sh\,k(l-z)}{kl \cdot sh\,kl - 2ch\,kl + 2} - EJ_\omega f''(z).\end{aligned} \tag{17}$$

Substituting into Eq. (12) and Eq. (13) the expressions for the particular solution $f(z)$, one can obtain the values of the twist angles and the bimoments for different load arrangement.

3. Results and Discussion

Using the proposed theoretical solutions, the influence of distortion on the behavior of the tested cold-formed steel bars was evaluated. Particular solution $f(z)$ tested bars with torque applied to the center section of the bar can be written as:

$$f(z) = \frac{M}{k^3 EJ_\omega}\left[sh\,k\left(z - \frac{l}{2}\right) - k\left(z - \frac{l}{2}\right)\right], \tag{18}$$

where M is the applied torque moment. The values of particular solutions $f(z)$ for different load arrangement are listed in [2]. Substituting Eq. (18) into Eq. (12) and Eq. (13), taking into account that the right and left ends of the bar are fixed equally, $k_{\theta 1} = k_{\theta 2}$ and $k_{\delta 1} = k_{\delta 2}$, we obtain the expressions for the twist angles and the bending-twisting bimoment for tested partially torsion and warping restrained bars with a torque applied in the central section:

$$\theta(z) = \frac{M}{2k^3 EJ_\omega}\left(kz - \frac{sh\,kz}{ch\frac{kl}{2}}\right) - \frac{B_{\omega 0}}{k^2 EJ_\omega}\left[\frac{sh\,kz}{sh\,kl}(1 - ch\,kl) - 1 + ch\,kz\right] + k_\theta M_0, \tag{19}$$

$$B_\omega(z) = \frac{M}{2k}\frac{sh\,kz}{ch\frac{kl}{2}} + B_{\omega 0}\left[chkz + \frac{sh\,kz}{sh\,kl}(1 - ch\,kl)\right], \tag{20}$$

where $B_{\omega 0}$ is the bimoment on the joint section; M_0 is the torque moment on the joint section.

When $B_{\omega 0} = 0$ and, $k_\theta = 0$ Eq. (19) and Eq. (20) will be transferred to the equations for the bar, with a complete twisting and zero warping restraints.

$$\theta(z)=\frac{M}{2k^3EJ_\omega}\left(kz-\frac{sh\,kz}{ch\frac{kl}{2}}\right), \tag{21}$$

$$B_\omega(z)=\frac{M}{2k}\frac{sh\,kz}{ch\frac{kl}{2}}. \tag{22}$$

If $B_{\omega 0}=\frac{M}{2k}\frac{1-ch\frac{kl}{2}}{sh\frac{kl}{2}}$ and k_θ=0, Eq. (19) and Eq. (20) will be transferred to the equations for the bar, with a complete twisting and complete warping restraints.

$$\theta(z)=\frac{M}{k^3EJ_\omega}\frac{\frac{kz}{2}sh\frac{kl}{2}-sh^2\frac{kz}{2}-sh\frac{kz}{2}sh\frac{k(l-z)}{2}}{sh\frac{kl}{2}}, \tag{23}$$

$$B_\omega(z)=\frac{M}{2k}\frac{ch\,kz-ch\,k\left(\frac{l}{2}-z\right)}{sh\frac{kl}{2}}. \tag{24}$$

Let us compare the experimental values of the twist angles and bimoments with the theoretical values obtained for the bar with complete twisting and warping restraints. Calculations of the theoretical values of the twist angles and bimoments for bars with complete twisting and warping restraints are carried out using Eq (23) and Eq. (24). To obtain the values of the twist angles θ and the bimoments B_ω for bars with partial restraints, using Eq. (19) and Eq. (20), it is required to know the value of the bimonent on the joint section $B_{\omega 0}$ corresponding to the type joint.

Let us agree that the cross-section of the bar during the torque can be deformed in two ways – the cross-section contour can be deformed out of a plane, which leads to warping displacements. Cross-section in this case does not remain planar after deformation. And the cross-section form can be deformed in-plane without violating the flat section hypothesis. Warping deformations are considered when calculating the twist angles and bimoments according to the Eq. (19) and Eq. (20). In-plane cross-section deformations can be estimated from the results of the experiment.

During the experiment, stresses were measured at four points of the cross-section, located at 260 mm from the center of the bar. Placing the strain gauges at some distance from the point of application of the load minimizes the effect of cross-section in-plane deformation on the value of the measured bimoment. The value of the bimoment $B_{\omega 0}$ on the joint section is obtained from the processing of experimental data. $B_{\omega 0}$ was calculated using the experimental value of the bimoment B_ω on the strain gauges section, using Eq. (20).

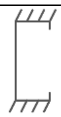
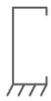
The degree of constraint by the joint of the warping deformations, so-called deplanations, can be described by the warping factor coefficient:

$$K_\delta=\frac{B_{\omega 0,partial}}{B_{\omega 0,complete}}, \tag{25}$$

where K_δ is the warping factor; $B_{\omega 0,partial}$ is the bimoment on the joint section for the bar with partial warping restraint; $B_{\omega 0,complete}$ is the bimoment on the joint section for the bar with complete warping restraint.

A decrease in the joint bimoment is proportional to the warping factor coefficient K_δ, which leads to an increase in the value of the bimoment in the span of the bar. The values of the warping factor K_δ, for examined joint types, are given in Table 5.

Table 5. Mean values of the warping factors.

Warping factor	Joint type				
	 type 1	 type 2	 type 3	 type 4	 type 5
K_δ	0.98	0.96	0.7	0.83	0.55
$B_{\omega 0,partial} / B_{\omega 0,complete}$	1.03	1.04	1.24	1.11	1.35

From Table 5, only two of the examined joint types, type 1 and 2, can be considered as complete warping restraint. And none of the examined joints can be considered as zero warping restraint. So, when only one flange is fixed, the bimoment on the joint is about half of the bimoment for the joint with complete warping restraint.

Using Eq. (25) for the warping factor K_δ, Eq. (19) and Eq. (20) for bimoment and twist angles can be written as:

$$\theta(z) = \frac{M}{2k^3 EJ_\omega}\left(kz - \frac{sh\,kz}{ch\frac{kl}{2}}\right) - \frac{K_\delta B_{\omega 0,complete}}{k^2 EJ_\omega}\left[\frac{sh\,kz}{sh\,kl}(1-chkl)-1+ch\,kz\right]+k_\theta M_0, \quad (26)$$

$$B_\omega(z) = \frac{M}{2k}\frac{sh\,kz}{ch\frac{kl}{2}} + K_\delta B_{\omega 0,complete}\left[ch\,kz + \frac{sh\,kz}{sh\,kl}(1-ch\,kl)\right]. \quad (27)$$

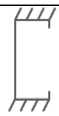
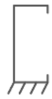
Considering the real joint conditions with the help of the proposed method leads to a more accurate, in comparison with the traditional method [1, 2], the assessment of the stress-strain state with warping torsion of cold-formed steel bars.

It should be noted that the question of determining the warping factor K_δ requires additional studies. When processing the experiment, the values of K_δ determined from the experimental results were used, which leads to the complete coincidence of the experimental values of the bimoment B_ω with the values obtained from Eq. (27). In the absence of a sufficient amount of experimental data, the K_δ values for different joint types can be preliminarily taken from Table 5.

Considering the experimental and theoretical studies carried out, the influence of the cross-section in-plane deformation on the behavior of the tested cold-formed bars was evaluated. The experimental angle of twist was estimated by the rotation of the loading device Fig. 2. The total twist angle is found as the sum of three values, the theoretical value taking into account the additional warping deformation of the bar on the joints Eq. (26), the additional angle of rotation of the loading device due to the in-plane deformation of the bar cross-section at the point of application of the load and the additional angle of twist due to in-plane bar cross-section deformation on the joints.

Comparing the value of the obtained angle of twist of the bar according to Eq. (26) with the experimental value θ_{exp}, it is possible to calculate the part of the angle of twist θ_χ obtained by in-plane cross-section deformation of the cold-formed bar at the joints and in the place of load application. The obtained values of the θ_χ/θ_{exp} relations for the tested bars with different types of joints are listed in Table 6.

Table 6. θ_χ/θ_{exp}.

Profile	l, mm	kl	θ_χ/θ_{exp}				
			 type 1	 type 2	 type 3	 type 4	 type 5
262C23	1955	0.684	0.43	0.42	0.41	0.37	0.88
142C16	1955	0.876	0.43	0.35	0.27	0.37	0.73
262C29	1955	0.895	0.43	0.41	0.32	0.39	0.85
142C20	1955	1.072	0.26	0.27	0.13	0.14	0.54
262C23	3940	1.379	0.10	0.10	0.06	0.09	0.46
142C16	3940	1.766	0.17	0.19	0.10	0.18	0.26
262C29	3940	1.933	0.16	0.15	0.13	0.13	0.40
142C20	3940	2.161	0.00	0.03	0.11	0.05	0.20

Where θ_{exp} is the experimental value of the bar twist angle; $\theta_\chi = \theta_{exp} - \theta$ is the twist angle due to in-plane cross-section deformation; θ is the twist angle of the bar with partial warping restraints.

From Table 6 it can be seen that for joint types 1, 2, 3, and 4, depending on the flexural-torsional characteristic kl, up to 43 % of the angle of twist of a cold-formed bar, occurs due to in-plane cross-section deformation at the load application point and on the joint sections. For joint type 5, the contribution of the in-plane cross-section deformation to the twist angle of the cold-formed bar can be up to 88 %.

For joint types 1, 2, 3, and 4, with kl > 2, the magnitude of the twist angles of the cold-formed bar can be determined by the Eq. (26) with sufficient accuracy for practical use. The joint number 5 requires additional research, but it can be said that taking into account the partial warping restraint of the joint allows using Eq. (26) with a value of kl > 3. When joint types are 1, 2, 3, 4 and kl < 2, as well as when joint type 5 and kl < 3, the cross-section in-plane deformation cannot be neglected.

4. Conclusions

Based on the research conducted, it was established:

1. The degree of warping restriction with real joint types can be described by the value of the bimoment on the joint section $B_{\omega 0}$. When $B_{\omega 0}$=0, the joint is zero warping restrained. When $B_{\omega 0}=M/2k(1\text{-}ch\ kl/2)sh\ kl/2$ the joint is complete warping restrained.

2. Not considering the partial warping restriction by the joint leads to a significant error in the determination of the twist angles of the bar and bimoments. generally, the magnitudes of the twist angles θ and the bimoments B_ω with warping torsion of a thin-walled cold-formed steel bar can be determined by Eq. (12) and Eq. (13). In the case of a load in the form of a torque applied to the central section of the bar according to Eq. (26) and Eq. (27). To consider the type of joint, it is possible to use experimentally obtained values of the warping factor K_δ according to Table 5.

3. According to Eq. (13), additional twisting angles on the bar joints due to deformation of the bar ends or insufficient rigidity of the joint assembly design do not affect the values of bimoments along the bar length.

4. An indicator of the deformability of the cross-sectional contour is the value of the flexural-torsional characteristic of the bar, kl. The smaller it is, the greater the influence of the contour deformations on the operation of the bar during torsion.

5. For cases of fastening a cold-formed bar along two flanges and a wall, along a flange and a wall, along two flanges, as well as for fastening along a wall, the in-plane cross-section deformation can be neglected for kl > 2. The case of fastening a cold-formed bar along one flange requires additional study. Previously, we can say that in this case, the in-plane cross-section deformations can be neglected when kl > 3.

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