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Analytical calculation of deflection of a planar truss with a triple lattice

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Abstract. We propose a scheme of a statically determinate truss with straight chord s and four supports, one of which is a fixed hinge, the other movable hinges. The task is to obtain the dependence of the deflection of the middle span of the structure on the number of panels. The problem is solved by induction using operators of the Maple computer mathematics system. The deflection is determined by the Maxwell – Mohr's formula, the forces in the rods are found from the solution of the joint system of the equation of equilibrium of nodes, the unknowns of which include the reaction of the supports. of nodes, whose unknowns include the reaction of the supports. The inclusion of support reactions in the system of equilibrium equations allows us to reveal the external static indeterminability of the structure. Generalizing a number of solutions for trusses with a consistently increasing number of panels, the desired dependence is obtained. To do this, we create recurrent equations that satisfy the terms of the sequences of coefficients in the deflection formula. The resulting homogeneous linear recurrent equations have a degree no higher than the eighth in the case of a problem with a load distributed over the upper chord and the sixth for a concentrated load in the middle of the span. Solving these equations in the Maple environment using the rsolve operator gives expressions of the dependence of the coefficients of the desired formula on the number of panels. The asymptotics of solutions are found. The dependence of the horizontal shift of the mobile support on the action of distributed and concentrated load is also obtained. Formulas are derived for the dependence on the number of panels of forces in some elements in the middle of the span. The obtained solutions can be used for preliminary evaluation of the designed structure and for evaluating the accuracy of numerical solutions.

1. Introduction

Rod structures that carry loads or perform fencing or decorative functions are widely used in construction [1–6]. The most common planar schemes of such structures, consisting of pivotally connected rods that experience only compression or tension (trusses). On the basis of such models, more complex schemes with arbitrary (rigid or elastic) nodal connections are also built. Among all truss schemes, statically determinate ones are singled out separately, the forces in which are found from the static equations. Trusses differ in the shape of chords and the grid pattern. Despite a fairly large variety of schemes for statically determinate planar trusses, the search for new designs continues. This search is not an end in itself. Different working conditions of structures suggest different schemes. In some cases, a well-chosen scheme gives an advantage in terms of structural rigidity, where this is especially important, for example, in railway bridges or industrial buildings with suspended crane equipment. In other cases, the truss is used as a decorative element and must have a non-standard architectural expression. In [7], spatial periodic hinge – rod structures are developed for modeling materials with a cellular structure. Very often, the structure requires more strength than rigidity. In all these variants, you can not do with one simple scheme, for example, a truss with parallel chords and a triangular lattice. A variety of schemes is required. In 2005, Hutchinson and Fleck even called the search for new statically determinate schemes "hunting" [8, 9]. In the future, the direction of studying the features of calculating regular plane and spatial structures is continued [10, 11]. Statically determinate regular schemes of arch-type trusses were proposed and analytically calculated in [12–18]. More than 70 schemes of regular trusses with formulas for the deflection dependence on the number of panels are contained in the reference [19]. The relevance of this research is determined by the need to have a reliable alternative method

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of obtaining a solution for numerical calculations of building structures. This method is the analytical method used in this paper. In addition, analytical solutions can be used for truss optimization problems [20–24].

The first paper aim is the task of developing a new architectural and expressive scheme of a statically determinate truss in order to expand the class of statically determinate schemes of regular type trusses, and second – the analysis of this scheme and obtaining an analytical solution for the deflection of the structure, whose parameters include the number of panels, which significantly expands the applicability of the desired formula.

The paper proposes a scheme of an externally statically indeterminate truss with straight belts and four supports, one of which is a fixed hinge, the other three are movable hinges. Triple lattice of parallel struts and resemble in structure members of a cable-stayed bridge Harp design or a Bolman truss with an additional lower chord. The task is to get a formula for calculating the dependence of the deflection of the middle span of the truss on the number of panels, the load and the size of the structure. The problem is solved by induction using operators of the Maple computer mathematics system.

The main novelty of the work consists in the proposed scheme of a statically determinate regular truss scheme, for which a simple analytical solution obtained by the induction method is admissible.

2. Methods

2.1. The calculation of the forces in the members

A distinctive feature of the structure under consideration is its external static indeterminate, defined by four supports, one of which is pivotally fixed, the other ones pivotally movable. However, this scheme is not only statically determinate, but also allows an analytical solution of the deflection problem for an arbitrary number of panels. The total length of all truss rods with n panels is $2(4n-1)a+3hn+12cn$, where $c=\sqrt{a^2+h^2}$. Each panel consists of six braces, three struts h high each, and six upper chord rods a long. Adjacent panels intersect with each other along two common members of the upper chord and are connected along the lower chord by a rod of length $4a$. Thus, shorter rods are located in the upper chord that is subject to compression, and the lower stretched chord contains long members. The truss consists of $m=14n+6$ rods, including five rods that model supports.

The truss is externally statically indeterminate, which means that the standard calculation scheme with independent determination of support reactions does not pass here. The reactions of the supports can only be determined from solving the joint system of equilibrium equations of all nodes together with the forces in the rods.

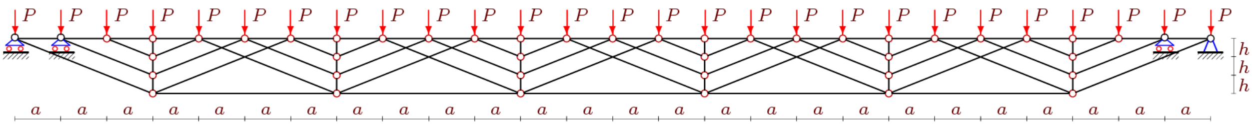

Figure 1. Load distributed over the upper chord, *n* = 6.

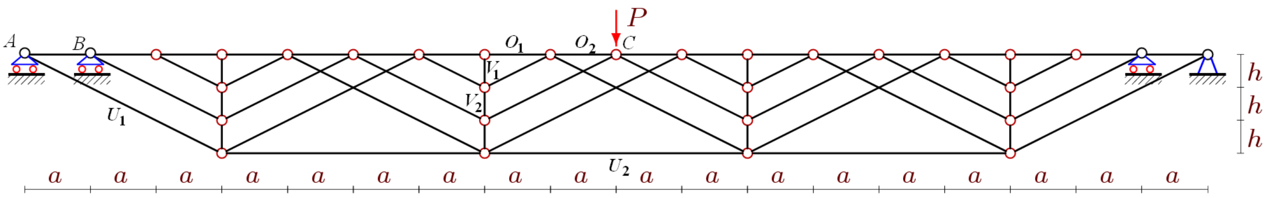

Figure 2. Concentrated force, *n* = 4.

Since the goal is to derive a formula for deflection, the forces in the rods must also be found in symbolic form. In [12, 14], an algorithm for calculating forces in planar truss rods in the Maple computer mathematics system is proposed. Nodes and rods of the truss are numbered (Fig. 3). The program code for the values of the coordinates of the nodes of the structure:

$$x_i=a(i-1),\ y_i=0,\ i=1,...,4n+3,$$
$$x_{i+4n+3j}=x_{4j},\ \ y_{i+4n+3j}=-hi,\ i=1,2,3,\ \ j=1,...,n. \quad (1)$$

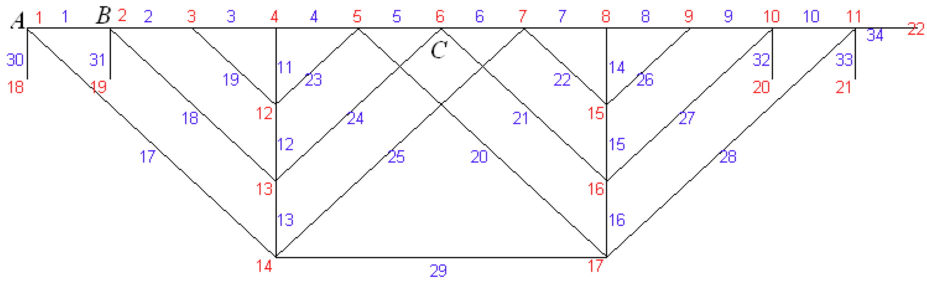

Figure 3. Numbering of nodes and rods, *n* = 2.

Each element is assigned an ordered list of end numbers. For upper chord, for example, these lists have the form: $N_i = [i, i+1], \; i = 1,...,4n+2.$ For the lower chord:

$$N_{13n+2+i} = [3i+3+4n, 3i+6+4n], \; i = 1,...,n-1. \quad (2)$$

The matrix of the system G of node equilibrium equations consists of guide cosines of forces, which are calculated through projections of members $l_{1,i} = x_{N_{2,i}} - x_{N_{1,i}}, \; l_{2,i} = y_{N_{2,i}} - y_{N_{1,i}}, \; i = 1,...,m,$ and their lengths $l_i = \sqrt{l_{1,i}^{\;2} + l_{2,i}^{\;2}}$. Even (j = 2) rows of the matrix contain guide cosines with a horizontal x axis, and odd (j = 1) rows contain guide cosines with a vertical y axis:

$$G_{k,i} = -l_{j,i} / l_i, \; k = 2N_{i,2} - 2 + j, \;\; k \le m, \; j = 1,2, \; i = 1,...,m,$$
$$G_{k,i} = l_{j,i} / l_i, \;\; k = 2N_{i,1} - 2 + j, \;\; k \le m, \;\; j = 1,2, \; i = 1,...,m. \quad (3)$$

Solving a system of equilibrium equations using standard operators of the Maple system gives forces in the rods. The scheme of distribution of forces in the rods in the case of a distributed load (Fig. 1) is shown in Fig. 4

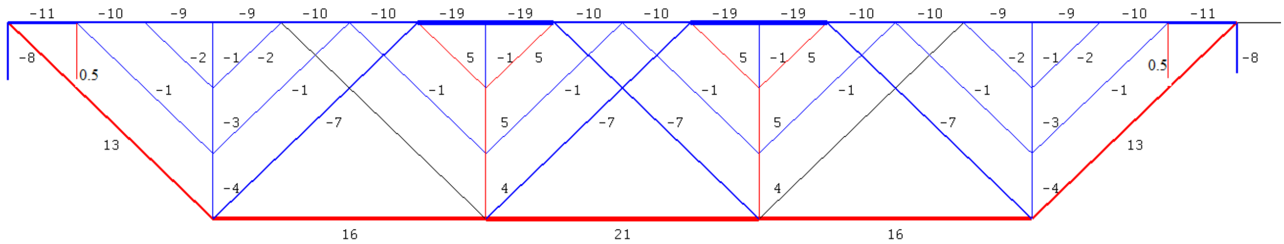

Figure 4. Forces in the rods in the case of distributed load action, *n* = 4, *a* = 4m, *h* = 3m.

Compressed members are marked in blue, and tension members are marked in red. The upper chord is compressed, the lower chord is stretched. It is significant that the extreme racks of the truss are compressed, the lower middle ones are stretched. The thickness of the line is proportional to the modules of forces related to the load P. In the case of concentrated force, not all members of the upper chord are compressed (Fig. 5), and many members, including side extensions, are not loaded.

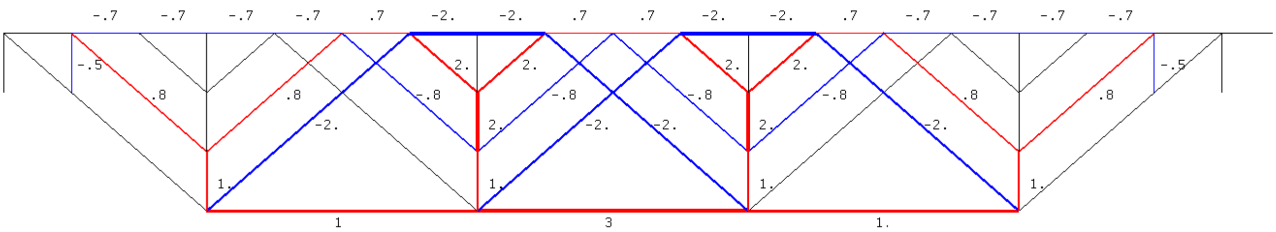

Figure 5. Forces in the rods under the action of a concentrated load, *n* = 4, *a* = 4m, *h* = 3m.

It is noted that for an odd number of panels, the determinant of the system of equilibrium equations turns to zero. This indicates the kinematic variability of the system. This is confirmed by the diagram of possible node velocity. For n = 1 it is given in Fig. 6:

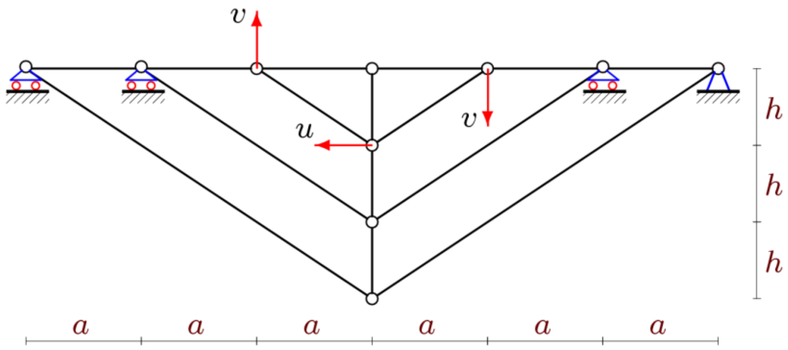

Figure 6. Diagram of possible velocity of nodes for instantly changeable truss, *n* = 1, *v/a* = *u/h*.

If the calculations are made not in analytical form, but numerically, then due to rounding errors, the effect of turning the determinant of the system to zero may not be noticed. Consider trusses with an even number of panels $n = 2k$.

2.2. Calculation of deflection

The deflection of the middle of the truss is calculated using the Maxwell – Mohr's formula

$$\varDelta = \sum_{j=1}^{m-5} \frac{S_j \tilde{s}_j l_j}{EF}, \tag{4}$$

where E and F are the elastic modulus of the members and their cross-sectional area, l_j and S_j are the length and force in the j-th element from the action of a given load, $\tilde{s}_j$ is the force from the unit load applied to the upper chord nodes in the middle of the span.

Calculations show that the deflection of the truss for any number of panels has the form

$$\varDelta = P(C_{1,k}a^3 + C_{2,k}c^3 + C_{3,k}h^3)/(h^2 EF), \tag{5}$$

where the coefficients $C_{j,k}$, $j = 1,2,3$ depend only on the number k. The main task at this stage is to find these dependencies. Sequential calculation of trusses with increasing number k = 1,2,... gives the sequence of coefficients. Common terms of these sequences can be found by composing recurrent equations for sequence members using the rgf_findrecur operator. To determine the coefficient $C_{1,k}$, it was necessary to consistently find the deflection of 16 trusses at k = 1,2,..., 16 and obtain a linear homogeneous equation of the eighth order:

$$C_{1,k} = 2C_{1,k-1} + 2C_{1,k-2} - 6C_{1,k-3} + 6C_{1,k-5} - 2C_{1,k-6} - 2C_{1,k-7} + C_{1,k-8}. \tag{6}$$

The solution of the equation is given by the rsolve operator:

$$C_{1,k} = (40k^4 + 2(9 - 8(-1)^k)k^2 + ((-1)^k - 1)(2k - 1))/2. \tag{7}$$

The other coefficients are determined in the same way:

$$\begin{aligned} C_{2,k} &= (40k^4 + 16(2(-1)^k - 5)k^3 + 2(37 - 24(-1)^k)k^2 + (1 - (-1)^k)(38k + 15))/6, \\ C_{3,k} &= (40k^4 + 16(2(-1)^k - 5)k^3 + 2(7 - 24(-1)^k)k^2 + (1 - (-1)^k)(26k + 3))/6. \end{aligned} \tag{8}$$

Coefficients $C_{2,k}$ and $C_{3,k}$ differ only in a few terms. Formula (5) with these coefficients is the main solution of the problem. It can be supplemented by calculating the truss for other loads and by output formulas for the forces in some of the most critical rods in the relation to the stability and strength of the rods. So, for the case of the action of a concentrated force (Fig. 2), we have the following coefficients, obtained also by induction

$$C_{1,k}=4k^3+2(2-(-1)^k)k+(-1)^k-1,$$
$$C_{2,k}=(4k^3+6((-1)^k-1)k^2+(20-6(-1)^k)k-3(-1)^k+3)/3, \quad (9)$$
$$C_{3,k}=2k(2k^2+3((-1)^k-1)k-3(-1)^k+4)/3.$$

In the case of a load distributed over the nodes of the lower chord (Fig. 7), the coefficients of the solution (3) have the form

$$C_{1,k}=5k^4+(3-2(-1)^k)k^2,$$
$$C_{2,k}=(10k^4+4(2(-1)^k-5)k^3+4(5-3(-1)^k)k^2+8(1-(-1)^k)k-3(-1)^k+3)/6, \quad (10)$$
$$C_{3,k}=k(5k^3+2(2(-1)^k-5)k^2+(7-6(-1)^k)k+2((-1)^k-1))/3.$$

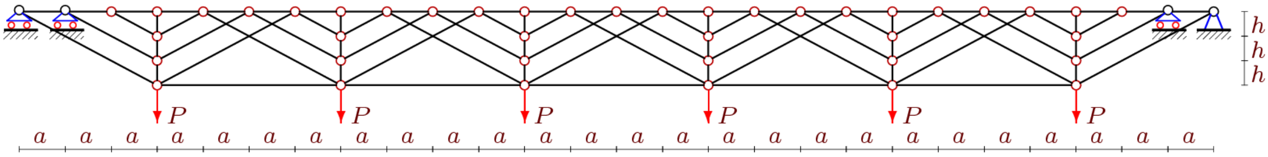

Figure 7. Load distributed over the lower chord, *n* = 6.

2.3. The forces in the members

In the process of calculating the deflection in analytical form, formulas for the forces in all rods and supports were obtained. We will write out only the most interesting results related to the most compressed and stretched rods (Fig. 2). For a distributed load, we have the following expressions:

$$O_1=-Pa(8k^2-3)/(2h),\ O_2=-Pa(8k-1)/(2h),$$
$$U_1=4Pck/h,\ \ U_2=4Pak^2/h,\ V_1=-P,\ V_2=P(2k+1)(2k-3). \quad (11)$$

At $k>1$, the most compressed rod has a force of O_1. To approximate the stability of the structure, we can use Euler's formula, taking the coefficient of reduced length equal to 1:

$$Pa(8k^2-3)/(2h)=\pi^2EJ/a^2, \quad (12)$$

where J is the moment of inertia of the cross section.

Reactions of supports of an externally statically indeterminate structure have the form:

$$Y_A=4Pk,\ Y_B=-P/2. \quad (13)$$

It is interesting to note that the reaction of support B does not depend on the number of panels or the size of the structure. Judging by the sign, the reaction of this support is directed downward, the support does not support the structure from below, but holds it. In the case of a concentrated load for the same rods and support reactions the formulas are obtained:

$$O_1=-Pa(2k-1)/(2h),\ O_2=Pa(-1)^k/(2h),$$
$$U_1=-Pc((-1)^k-1)/(2h),\ \ U_2=Pak/h,\ V_1=0,\ V_2=P(2k+(-1)^k-1)/2, \quad (14)$$
$$Y_A=P(1-(-1)^k)/2,\ Y_B=P(-1)^k/2.$$

Here the signs of reactions and some forces depend on the parity of the number of panels in the half span.

2.4. Shifting of the movable support

In addition to the deflection of the middle of the span, an equally important operational characteristic of the truss is the shift of the movable supports under the influence of an external vertical load. Of course, in girder trusses, this value is not as large as in arched or frame trusses, but to calculate the support structures, the engineer must have an estimate of the amount of support shift under load. When calculating the horizontal displacement in the Maxwell – Mohr's formula (4), $\tilde{s}_j$ it is the forces from the action of the horizontal unit force on the movable support *A*. Omitting the intermediate calculations, we present the results of an inductive generalization of solutions. In the case of distributed load action the offset of the support *A* has the following formula depending on the number of panels:

$$\delta = 16Pa^2k(k+1)(2k+1)/(3EFh). \tag{15}$$

For the case of a concentrated load, this dependence has the form:

$$\delta = 2Pa^2k(1+k-(-1)^k)/(EFh). \tag{16}$$

For the case of a load distributed over the lower belt, the dependence of the shift of the movable support on the number of panels has the form:

$$\delta = 2Pa^2k(4k^2+6k+5)/(3EFh). \tag{17}$$

3. Results and Discussion

The features of the obtained solution in the case of distributed load are studied on the graphs of the dependence of the dimensionless deflection $\Delta' = \Delta EF/(P_{sum}L)$, where $P_{sum} = (4n-1)P$ is the total load on the structure. Let us assume that the span length is fixed $L = 2(2n+1)a$. As the number of panels increases, the length of each panel decreases, and the relative deflection increases (Fig. 8). Note that this growth is uneven and strongly depends on the height h. The intersection of curves shows that for different heights, but the same number of panels in this setting, the relative deflection can be the same. The deflection – height ratio also depends on the number of panels. At the beginning of the graph, for small k, the order of the curves is natural. The higher the height of the truss, the smaller the deflection. Starting from a certain value of k, the curves are reversed. Note that the effect of self-intersection of the curves on the graph and the alternation of their order is associated with the nonlinearity of the problem and the accepted assumption about the constancy of the load and the span length. If the deflection is attributed not to the total load and the panel size rather than the span is fixed, then the self-intersection effect disappears.

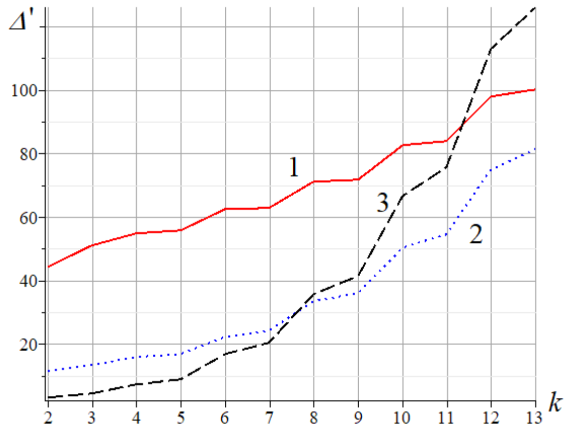

Figure 8. Dependence of dimensionless deflection on the number of panels,

1 – h = 1 m; 2 – h = 2 m; 3 – h = 4 m; L = 100 m.

Curves of deflection dependence on the number of panels are non-monotonic. The presence of jumps in such curves is typical for trusses with a complex lattice [14]. Trusses with a lattice consisting of triangles, these curves are smooth and have no jumps [15, 16].

In the case of a load applied to the lower belt, the curves are almost the same with the same effects of intersecting the curves and changing their order.

The asymptotics of the solution turns out to be nonlinear. By means of Maple system, we get that the deflection dependence on the number of panels with increasing k tends to be cubic:

$$\lim_{k\to\infty} \Delta'/k^3 = 5h/(3L). \tag{18}$$

The same order is obtained for a concentrated load: $\lim_{k\to\infty} \Delta'/k^3 = 8h/(3L)$. The cubic character of the deflection growth with an increase in the number of panels is also found in the arch-type truss [14]. However, the deflection growth in this problem is much slower. The growth factor is $5h/(384L)$.

The dependence of the deflection on the height reveals a minimum (Fig. 9). As the number of panels increases, the extreme point on the h axis shifts to lower values. The presence of a minimum deflection with a change in height is very typical for such problems [14–16] and is associated with the form of the solution. The quantity h in (5) has the second degree in the denominator and cubic in the numerator. Functions of this kind usually have a minimum. A less obvious minimum is found in a similar statement of the problem for the arch [14]. In [8–11] such curves were not constructed.

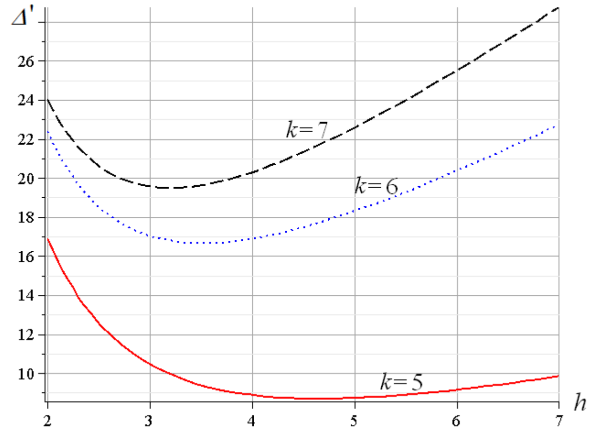

Figure 9. Dependence of the deflection on the height of the truss, L = 100 m.

In addition to the usual calculations of forces and deflections in truss problems, this paper reveals a somewhat unexpected feature of the proposed scheme – kinematic degeneration of the structure. This is shown in the degeneration of the matrix of the system of equilibrium equations for certain values of the panel numbers. This effect was not found in [1–3, 8–11], where regular trusses were also studied. The effect of kinematic degeneration of many regular truss schemes with a complex lattice at a certain number of panels was previously noted in the reference [19].

4. Conclusions

- A new scheme of a statically determinate truss is proposed and its mathematical model is constructed, which allows calculating the forces in the members and obtaining the deflection dependence on the number of panels in an analytical form.
- Kinematic degeneration of the structure was found for an odd number of panels. The result is confirmed by a picture of the distribution of virtual node velocities.
- By generalizing a series of solutions for trusses with a successively increasing number of panels, a formula is obtained for the dependence of the deflection and forces in some rods on the truss size, load and number of panels. Formulas have the form of polynomials in the number of panels of degree no higher than the fourth.
- The dependence of the horizontal displacement of the mobile support under the action of three types of loads is found. An asymptotic analysis of the solution of the deflection problem was performed, which revealed the cubic character of the deflection growth with an increase in the number of panels.
- The solution graphs constructed for specific dimensions of the structure showed the presence of a minimum deflection depending on the height of the truss and a jumpy, non-monotonous increase in the deflection with an increase in the number of panels.

5. Acknowledgements

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