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Elasto-plastic progressive collapse analysis based on the integration of the equations of motion

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Abstract. This paper considers the progressive collapse analysis of reinforced concrete structures based on the sudden removal of a load-bearing structural element and simulation of the dynamic structural behavior, taking into account the elasto-plastic properties of the material and the degradation of concrete during cracking. A specially developed finite element library is used, which includes triangular and quadrilateral shell finite elements of medium thickness, and a two-node finite element of a spatial frame, which take into account the discrete arrangement of reinforcement and various elasto-plastic material models for concrete and reinforcement. The novelty of the proposed approach lies in the formulation of both: the spatial frame and shell finite elements as a three-dimensional solid body with sequential application of the conventional hypothesis of the for Mindlin-Reissner shells of medium thickness, Timoshenko beams, and the elasto-plastic constitutive models. This makes it possible to achieve sufficiently high reliability of the results for engineering analysis, and on the other hand, a relatively simple implementation, which makes it possible to perform an elasto-plastic dynamic analysis of the entire design model of the structure, and not a separate fragment, in real time from the point of view of practical design. This approach is free from assumptions related to the introduction of a dynamic amplification factor into the quasi-static analysis, which is widely used to solve such problems. The paper provides a numerical example illustrating the effectiveness of using a special structure – an outrigger storey, to prevent progressive collapse, and a comparison of the nonlinear dynamic analysis and the linear one.

1. Introduction

One of the most important problems in the reliability assessment of the entire load-bearing structural system is its resistance evaluation in the case of failure of individual load-bearing structures or in the case of a local defect in the structural system. This problem is sometimes formulated as a structural robustness assessment, which seems to be one of the possible approaches.

Essentially, the problem of resistance evaluation of a load-bearing system in the case of failure of a structural element can be reduced to the analysis of the collapse development in the load-bearing system due to a local cause (failure of an individual structure). This approach corresponds to the modern interpretation of the well-known and commonly used concept of progressive collapse, which is considered as disproportionate collapse due to a failure of the local structure or assembly.

Progressive collapse is a dangerous phenomenon in which the failure of some key load-bearing structural elements leads to the failure of other elements; this in turn leads to a partial or even complete collapse of the structure. This phenomenon attracted much attention in 1968, after the partial collapse of the Ronan Point Building, when an explosion of domestic gas in one of the apartments entailed a chain of collapses throughout the building. This problem, however, became even more acute after the events of 2001, when the twin towers of the World Trade Center (WTC) were destroyed in a terrorist attack, which resulted in a large number of victims and huge economic losses.

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It is quite obvious that the concept of progressive / disproportionate collapse refers to a phenomenon that needs to be prevented, i.e., the collapse development in a load-bearing system due to a failure of an individual structure should not be allowed. In order to ensure the stability of the load-bearing system in the case of failure of a structural element and to provide the resistance of the load-bearing system, building codes now include the requirements for taking into account a possible progressive collapse and preventive measures, which increases the construction costs. One of the components that significantly affect the cost is the factor of the dynamic response of the construction to the failure of its local part. The nonlinear behavior of the system and the dynamic effect can be taken into account in one of three different analytical procedures, i.e., linear static analysis (LS), nonlinear static analysis (NS) or nonlinear dynamic analysis (ND).

The dynamic effect was usually taken into account in quasi-static analyses by introducing a dynamic amplification factor of $k_{dyn} = 2.0$ into the linear static analysis (LS) regardless of the type and mass distribution of the designed structure.

The simplicity of quasi-static analysis and dissatisfaction with such a rough decision prompted the scientists to refine the values of k_{dyn}, and in 2009 the dynamic amplification factor formulas appeared, which allow to replace the nonlinear dynamic analysis with a quasi-static linear or nonlinear one. These factors were empirically derived by A. McKay [1] based on statistical processing of the results of frame structure analyses for column failures and presented in the report written together with K. Marchand and D. Stevens [2]. In the same year, the proposed formulas were included in UFC 4-023-03: Design of Buildings to Resist Progressive Collapse. They were further refined in the works of M. Liu [3, 4], M. Tsai [5], H. Saffari and J. Mashhadi [6] and others, but they still referred only to simple orthogonal frame structures. More complex designs (with additional bracings, outriggers, etc.) were not considered. Moreover, the quasi-static analysis did not provide sufficient accuracy, and sometimes did not even guarantee a conservative solution, which was noted, for example, in [7, 8]. Therefore, nonlinear dynamic analysis remains a very important problem, which is studied by many researchers [9–12].

This paper is devoted to the same problem. The simulation of the progressive collapse process is based on a nonlinear dynamic analysis of the structure as a whole or of its separate part. Nonlinearity is caused by high stress levels in the material leading to partial and/or complete collapse of structural fragments. Not only does this approach have great computational complexity, but it should also be based on a mechanical-mathematical model that adequately describes the processes occurring in structural elements. Partial and complete collapse of structural elements often leads to poor conditioning of the system of governing equations; therefore, it becomes necessary to use specific approaches that ensure the computational stability of the method. The foregoing emphasizes the difficulties of the considered problem and explains the fact that the vast majority of practical calculations today are based on approximate models using certain simplifications. Therefore, creating a method that uses a nonlinear dynamic analysis based on sufficiently advanced mechanical models of physical nonlinearity to solve the progressive collapse problem is still a relevant task. Solutions obtained on the basis of such an approach could possibly improve simpler approaches — static, linear dynamic, or nonlinear dynamic ones based on simplified nonlinearity models.

In the absence of proven calculation methods that take into account the nonlinear behavior of the entire reinforced concrete structure, some generalized partial criteria for individual elements and assemblies are proposed and justified [13, 14]. This approach should be considered as a solution to the problem at a local level, but it cannot be accepted as a generalized methodology for assessing the progressive collapse stability of reinforced concrete load-bearing systems.

Without reducing the generality of the proposed approach, we will consider only reinforced concrete structures with bar or plate-bar load-bearing systems, whose behavior during progressive collapse is much less studied than the behavior of steel structures. There are many different approaches today to simulating the behavior of concrete at high stress levels. We will consider only the most typical ones.

These or other relations of the theory of plasticity are used in [15–23], and in many other works. Degradation of concrete in the tension area during cracking is described by the descending branch of the $\sigma - \varepsilon$ diagram. Usually, in the absence of reinforcement, a finite element solution becomes mesh-dependent: it diverges when the finite element mesh refines [24, 25]. Different approaches were proposed for dealing with this phenomenon, which, as a rule, involved a significant complication of the design model. A nonlocal continuum approach is presented in [18–20], where exact relations for strains at a given point are replaced by weighted average expressions obtained by the integration over a finite neighborhood of this point.

In [26], as well as in a number of other works, derivatives of the higher order stress tensor components are kept in the continuum equilibrium equations, which makes it possible to stabilize the convergence of the numerical solution within the descending branch of the $\sigma - \varepsilon$ diagram. The disadvantage of such approaches is the fact that it is not clear how to choose the values of constants that appear in constitutive relations.

In [27–29] and other works, linear fracture mechanics methods are used to simulate cracking in concrete. A particle method is proposed in [30], which assumes that only particles uniformly distributed over the volume of concrete can have embedded cracks. If the maximum tensile stress exceeds the tensile strength of the material in a unit volume, a discrete crack is initiated in the nearest particle.

The above works, as a rule, considered individual structural elements (a beam, a flat frame, a slab) in a static formulation. There are rare articles, for example, [31], in which the nonlinear dynamic approach is applied to the entire design model. However, these are usually strongly simplified design models, not design models of real structures.

The novelty of this approach is to study the dynamic behavior of a reinforced concrete structure as a whole in an elasto-plastic formulation using governing equations derived from the three-dimensional equations of solid mechanics, taking into account traditional static and kinematic hypotheses of the Mindlin-Reissner theory for shells of medium thickness, Timoshenko beams, and the elasto-plastic constitutive models.

Also, we use an original formulation of the deformation theory of plasticity in the terms of residual strains, allowing us easy to pass from tensile zone to compression one and vice versa. Unlike most approaches published and implemented in modern software, we take into account the stiffness of reinforcement not only on tension-compression but on transverse shear as well, which helps to avoid geometrical instability or poor conditioning of the problem in cases where the finite elements are in the tension area and concrete has significant damages [32, 34, 35].

In order to prevent the divergence of the numerical solution with mesh refinement after passing the yield point of the $\sigma-\varepsilon$ diagram, a simple engineering idea is used: the reinforcement, whose $\sigma-\varepsilon$ diagram does not have a descending branch, should regularize the numerical solution. It is shown in [32, 34, 35] that if the slope of the descending branch of the $\sigma-\varepsilon$ diagram for concrete in reinforced concrete thin-walled structures does not exceed a certain limit value depending on the ratio of the elastic modules of steel and concrete and the reinforcement ratio, then the curve of equilibrium states is monotonically increasing.

On the one hand, the proposed approach demonstrates the reliability of the results acceptable for engineering purposes, which is confirmed by numerous comparisons with the results of well-designed experiments and with reliable numerical solutions [32–35, 37]. On the other hand, the proposed approach is quite simple, which allows performing an elasto-plastic analysis of the entire design model of the structure, and not a separate fragment, on a desktop computer in real time from the point of view of the designer.

2. Methods

2.1. Finite Element Library

The developed finite element library includes triangular and quadrilateral shell finite elements of medium thickness [32–34] (Figure 1), described by the Mindlin-Reissner equations, and a two-node finite element of a spatial frame based on the Timoshenko beam theory [35] (Figure 2).

The stability of the above finite elements against the shear locking is provided. Reinforcement is smeared in the plane of a shell finite element and forms reinforcement layers. The discrete arrangement of rebar along the thickness of the element remains though. Each reinforcement layer is formed by reinforcing bars of the same direction, cross-section and material. The number of reinforcement layers is not limited. The axes of the reinforcement layers s_1-s_4 can be rotated by any angle with respect to the axes of the local coordinate system $OX_1Y_1Z_1$, which allows us to consider structures of complex geometric shape for any configuration of the finite element mesh. Longitudinal reinforcement is taken into account discretely in bar finite elements (Figure 2).

Constitutive relations are based both on the plastic flow theory and on the deformation theory of plasticity. The test problems considered in [32–34] have shown that the application of the deformation theory of plasticity in the case of non-cyclic loading leads to results closer to the experimental ones than the application of the plastic flow theory. We attribute this to the fact that the deformation theory of plasticity takes into account the nonlinear behavior of concrete from the very beginning of loading, while the plastic flow theory assumes the material behavior to be elastic and linear until the image point reaches the yield surface in the space of principal stresses. Another argument for using the deformation theory of plasticity in the progressive collapse analysis is the fact that most design standards, including the Eurocode, are based on this theory, since they govern the form of the $\sigma-\varepsilon$ diagram, but do not provide any information about the shape of the yield surface. Therefore, in this paper we will consider only the constitutive relations derived from the deformation theory of plasticity.

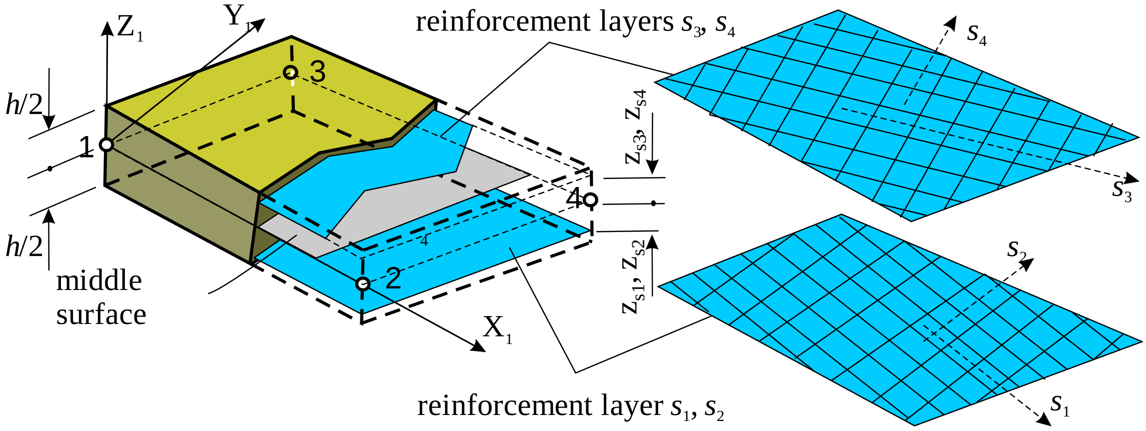


Figure 1. Quadrilateral finite element.

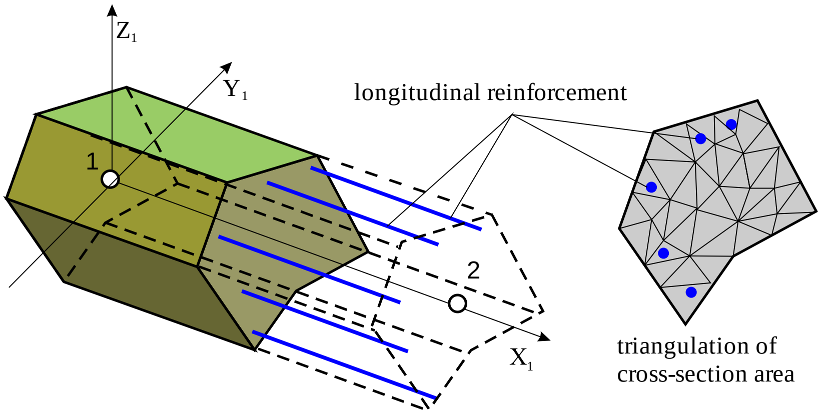


Figure 2. Two-node finite element of a spatial frame.

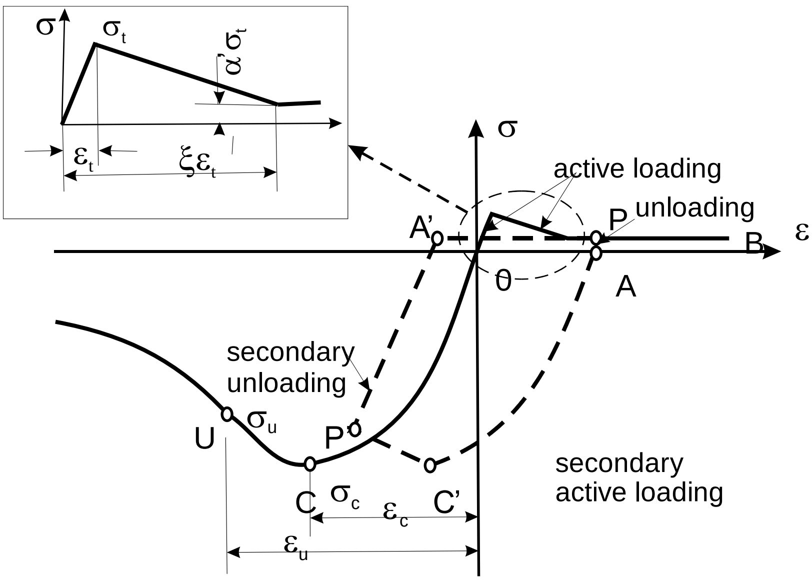


Figure 3. $\sigma - \varepsilon$ diagram for concrete.

We use the $\sigma - \varepsilon$ diagram for concrete proposed in [16] (Figure 3). The stress and strain values in the points C and U (σ_c, ε_c and σ_u, ε_u, respectively), as well as the initial elastic modulus of concrete E define the configuration of the $\sigma - \varepsilon$ curve in the tension area. Here σ_c is the ultimate compressive strength of concrete. A trilinear diagram is used in the tension area, and the descending branch describes the degradation of concrete during cracking. Here σ_t is the ultimate tensile strength of concrete, $\varepsilon_t = \sigma_t/E$, parameter α' defines the residual tensile strength of concrete and is usually equal to zero, and the parameter ξ defines the length of the descending branch and the softening modulus of concrete $E_t = -E/(\xi - 1)$. A distinctive feature of this approach is the fact that the relations of the deformation theory of plasticity are formulated in terms of residual strains [32–34], which allows the transition from the compression area to the tension area and vice versa (Figure 3).

As an example, suppose that from the very beginning there is an active loading in the tension area of the unit volume of concrete – the OP path. Elastic unloading occurs at the point P (the PA path), which then transfers into an active loading in the compression area (the AC´P´ path). The residual strains are calculated at the point A, the origin of the $\sigma - \varepsilon$ diagram is transferred to the point A, and the analysis of the active loading in the compression area begins (the AC´P´ path). The current strain is determined as $\varepsilon - \varepsilon_A$, where ε_A is the residual strain in the point A. It is assumed that concrete with cracks caused by tension can take compressive loads without any damage. Therefore we assume that $\sigma_{C'} = \sigma_C$. Elastic unloading begins at the point P´ (the P´A´ path), the residual strains are determined again at the point A´, the origin of the $\sigma - \varepsilon$ diagram is transferred to this point, and the active loading in the tension area begins. The strains are determined as $\varepsilon - \varepsilon_{A'}$. There are cracks in the tension area of concrete, which formed during the loading stage OP, therefore the stress level is limited by the residual concrete strength $\alpha'\sigma_t$.

The behavior of the reinforcement is also described by the relations of the deformation theory of plasticity formulated in terms of residual strains, and a symmetric bilinear diagram $\sigma - \varepsilon$ with a small hardening is assumed. We denote as E_s the elastic modulus for steel.

2.2. Integration of the Nonlinear Equations of Motion

The problem is solved in three stages. First, a nonlinear static problem is solved for the entire original structure

$$N'\left(\mathbf{u}'_0\right) = \mathbf{f}'_{stat}, \tag{1}$$

where $\mathbf{u}'_0$ is a displacement vector corresponding to the solution of the static problem, $N'\left(\mathbf{u}'_0\right)$ is a nonlinear operator that returns the vector of internal forces of the system, $\mathbf{f}'_{stat}$ is the static load, which includes dead and constant live load for the original system.

Second, one of the columns of the first floor is removed, and its action is replaced by the reaction vector $\mathbf{f}_{dyn}$, which is determined by solving the problem (1):

$$N\left(\mathbf{u}_0\right) = \mathbf{f}_{stat} + \mathbf{f}_{dyn}. \tag{2}$$

All the loads here are applied statically, varying in proportion to the same parameter. Removing the column changes the design model, therefore, at the first stage, the values in the equation (1) are written with a prime. As a result, we obtain the stress-strain state of the considered structure equivalent to that obtained in the first stage before removing the column.

Third, the column is suddenly removed ($\mathbf{f}_{dyn}$ = 0 when $t \geq t^*$, t is a current time and t^* is a moment when given column is removed) and we obtain a Cauchy problem with inhomogeneous initial conditions defined in the second stage:

$$\begin{cases} \mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + N\left(\mathbf{u}\right) = \mathbf{f}_{stat} + \mathbf{f}_{dyn} \\ \mathbf{u}\left(0\right) = \mathbf{u}_0\ ,\ \dot{\mathbf{u}}\left(0\right) = 0 \end{cases}, \tag{3}$$

where $\mathbf{M}$ and $\mathbf{C}$ are mass and dissipation matrices respectively. Proportional damping is used in this paper $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}_t(\mathbf{u})$, where $\mathbf{K}_t(\mathbf{u}) = \partial N(\mathbf{u})/\partial\mathbf{u}$ is the tangent stiffness matrix. The problem (3) is solved by the method presented in [32, 36]. Thus, the problem of progressive collapse is reduced to the integration of nonlinear equations of motion (3) with inhomogeneous initial conditions.

The presented approach is a special case of a more general method [37], which is used for seismic analysis of structures.

3. Numerical Results and Discussion

Two design models of the structure are considered. The first model (model A) corresponds to a traditional design of a multistorey building which is widely used. The second design model (model B) entails the introduction of a special outrigger storey with elements of high stiffness into the load-bearing system of the building (is shown in red in Figure 4) which must stop the propagation of destructions. After the failure of a vertical load-bearing structure (column) in the lower floor of the building, the outrigger storey structures provide suspension of the entire system of vertical load-bearing elements located above the failed column.

The principle of using outrigger structures as elements ensuring the stability of load-bearing systems in the event of failure of structures in the lower floors was proposed at the beginning of the 21st century and is used in the Russian Federation, Ukraine, Belarus and others countries.

However, even with the widespread use of outrigger structures in the design and construction of reinforced concrete buildings, there is currently no detailed reliability substantiation of such structures, taking into account the nonlinear behavior of reinforced concrete under dynamic loading caused by the sudden failure of a local load-bearing structure.

The design model A is obtained from the design model B by removing the reinforcement of the outrigger storey shown in red. The accidental situation is simulated by removing a corner column of the first floor.

The pitch between the columns in the direction OX and OY is 6 m, and the height of the storey is 3 m.

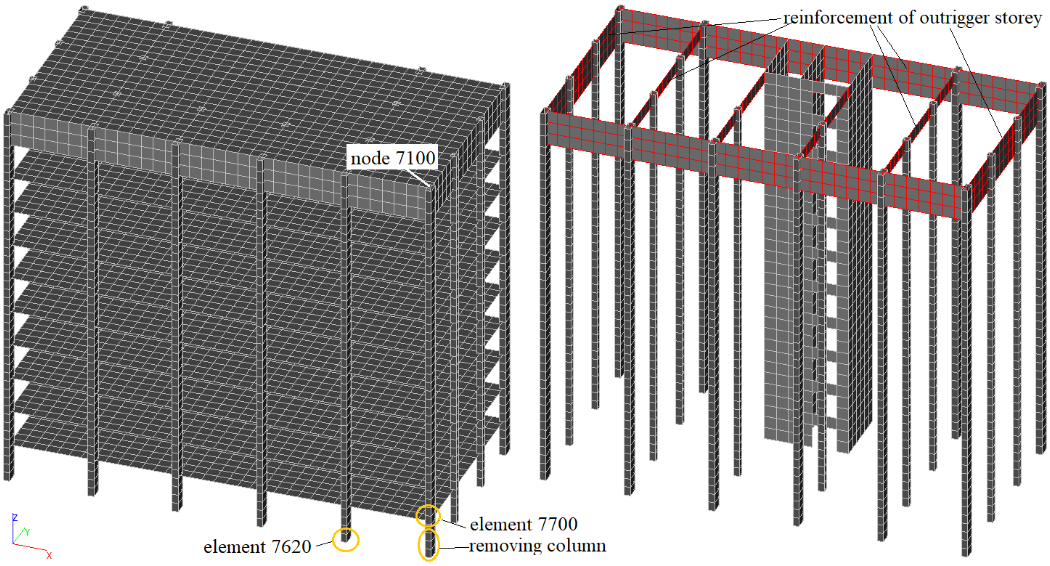


Figure 4. Design model B of a multistorey building with an outrigger storey.

Figure 5 shows the middle column cross-section, and Figure 6 shows the edge column cross-section, the parameter μ denotes the reinforcement ratio. They also show the triangulation mesh necessary for the numerical integration over the volume of the bar finite element to track zones of plasticity, and to calculate the stress and strain tensor components at the centers of gravity of the triangles [32, 35]. 200 mm thick walls are reinforced with Ø22 mm rebar with a spacing of 100 mm, $z_{s1} = z_{s3}$ = 84 mm (vertical reinforcement) and $z_{s2} = z_{s4}$ = 62 mm (horizontal reinforcement) – see Figure 1. The reinforcement ratio both in the vertical and horizontal direction is 2.0%. 200 mm thick floor slabs are reinforced with Ø10 mm rebar with a spacing of 100 mm, $z_{s1} = z_{s3}$ = 90 mm (reinforcement in the direction of the *OX* axis), $z_{s2} = z_{s4}$ = 80 mm (reinforcement in the direction of the *OY* axis). The reinforcement ratio is $\mu_x = \mu_y$ = 0.3%.

The following properties are assumed for concrete: E = 30 018 MPa, ν = 0.2, σ_c = 18.5 MPa, σ_t = 1.55 MPa, ξ = 20 (see Section 2 and Figure 3), and for steel – E_s = 200 000 MPa, ν = 0.3, σ_y = 400 MPa. Here ν is the Poisson's ratio and σ_y is yield stress for steel.

Dissipation parameters α = 0.34 and β = 0.0038 correspond to the modal damping ξ_1 = 0.05 and ξ_2 = 0.1 from the critical value for the frequencies ω_1 = 4 sec^{-1} and ω_2 = 50 sec^{-1}.

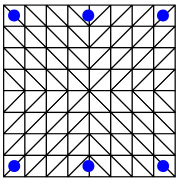


Figure 5. The middle column
50×50 cm, 6Ø32 mm, μ = 1.93 %

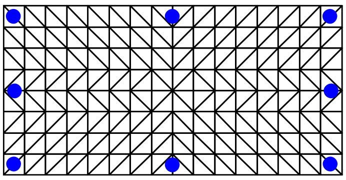
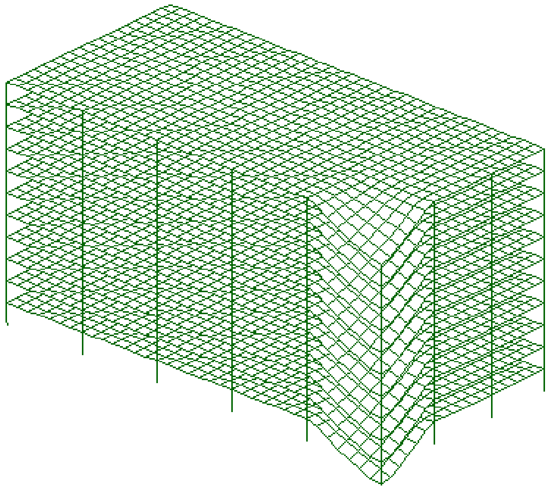


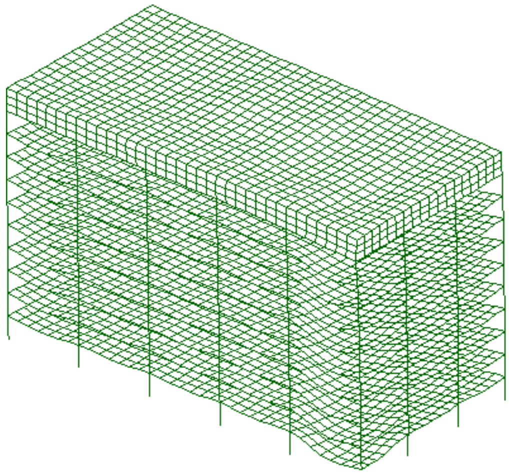
Figure 6. The edge column
100×50 cm, 8Ø40mm, μ = 2.01 %

Figure 7 shows the deformation patterns of models A (left) and B (right), obtained as a result of elasto-plastic (a, b) and elastic (c, d) analysis. Since the displacements of the structural fragment over removed column resulting from the use of nonlinear analysis increase indefinitely in time, and the displacements in linear analysis remain limited in time (Figure 8), displacement scale in Figure 7.c is much more than displacement scale in Figure 7.a. The deformation schemes of both model A and model B obtained using both linear and nonlinear analysis are generally similar. However, the elasto-plastic analysis of model A shows that the displacements of the structural fragment above the remote column increase unlimitedly (Figure 8, b), which indicates collapse. The analysis was interrupted at a time of 2.4 seconds when the vertical displacement of the node above the remote column reached 9 m. Unlike nonlinear analysis, linear analysis cannot naturally establish the fact of destruction – Figure 8, a. When performing linear analysis, hereinafter, the same approach is used as for nonlinear analysis, but a linear diagram $\sigma - \varepsilon$ is set for both concrete and reinforcement. The exception is the classical linear analysis, which uses finite elements that do not take into account the presence of reinforcement, and concrete is considered as a linear elastic material.

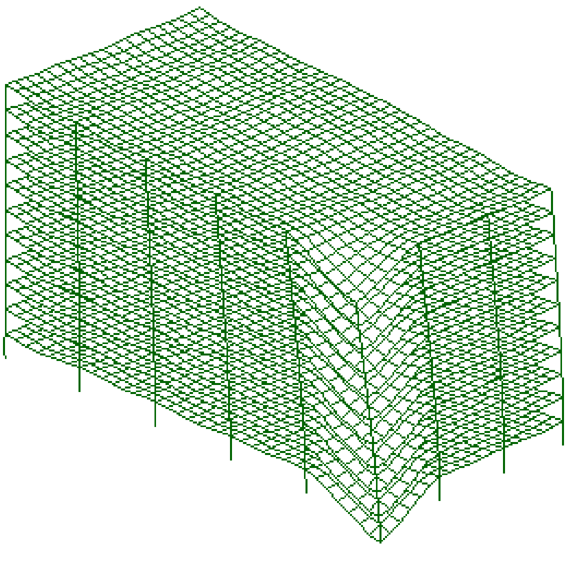
An analysis of the displacements of Model B (Figure 9) confirms the effectiveness of using outrigger structures. The results of the linear analysis do not allow to fully confirm the effectiveness of the reinforcement of the supporting system using the outrigger storey, since the stresses in concrete and reinforcement reach such high values that linear methods of strength analysis become inapplicable. Moreover, the classical linear analysis does not take into account the presence of reinforcement in the finite element, which does not provide even a conservative result of the calculation analysis, which, with the necessary restrictions, could be considered acceptable. Therefore, it is of undoubted interest to compare the results of direct dynamic calculations of model B with the outrigger, performed in a linear and physically nonlinear formulation. The removal of the corner column occurs at time t^* = 0.05 s. If the initial conditions obtained from the solution of the static problem (2) correspond to the given load of the Cauchy problem (3), then the sudden application of all the forces of the problem (2) should not cause oscillations until the column is removed, which is confirmed by the results shown in Figures 9, 10.



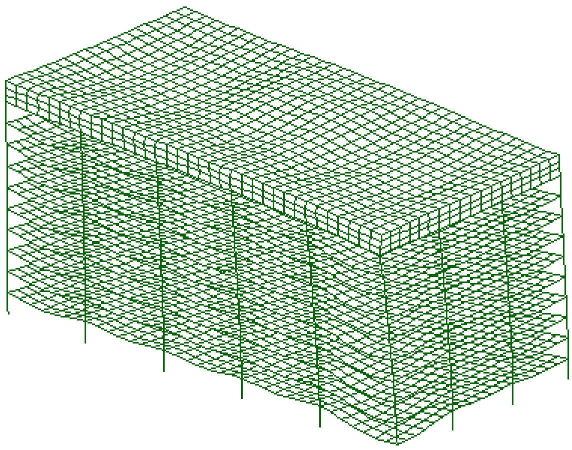
a – nonlinear analysis, model A



b – nonlinear analysis, model B



c – linear analysis, model A



d – linear analysis, model B

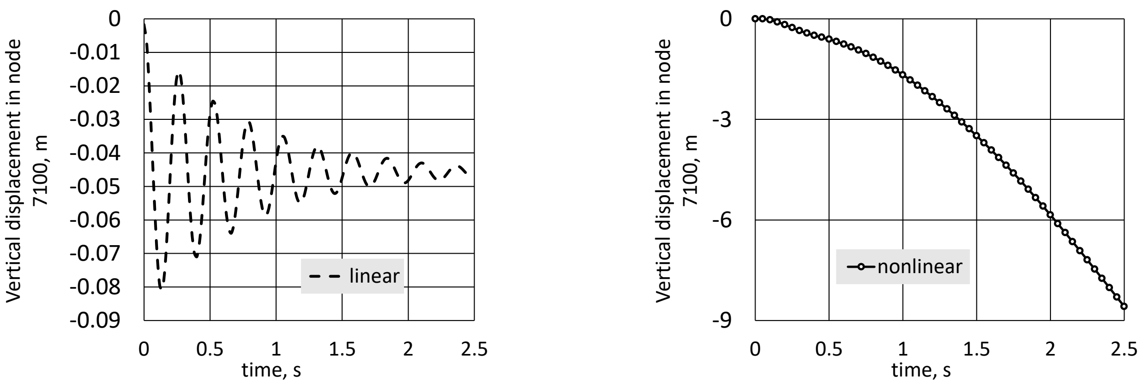
Figure 7. The deformed shape after removing of corner column (a – nonlinear analysis, model A; b – nonlinear analysis, model B; c – linear analysis, model A; d – linear analysis, model B).

A comparison of the linear and nonlinear analysis of model B shows:

- the deformation pattern at the initial stage is almost identical, but the magnitude of the extreme displacements is significantly different (Figure 9) 0.0216 m (elasto-plastic analysis) and 0.0123 m (linear analysis);
- the peak value of the longitudinal force in the nearest surviving column (finite element 7620), obtained as a result of nonlinear dynamic analysis, is 24 % less than the corresponding value obtained as a result of linear analysis (Figure 10).
- bearing system behavior after removal of the corner column is oscillatory (Figure 9, 10);
- the displacements obtained with using elasto-plastic analysis turned out to be much larger than the displacements of the elastic analysis, and the period of the fundamental mode for the nonlinear design model is also significantly longer than for elastic;
- the damping of the elasto-plastic design model due to its dissipative nature with the same viscous friction parameters turned out to be much larger than the damping of the elastic model;
- the w 7100 linear static and N 7620 linear static curves (Figure 9, 10) are given to show how much the displacements and efforts of the linear solution of the static problem with the classical approach that does not take into account the presence of reinforcement in concrete differ from the solution of the elasto-plastic dynamic problem in the proposed formulation since traditional methods based on dynamic amplification factors rely on the solution of the linear static problem;
- the dependence of the longitudinal force on time in the finite element 7700 (Figure 4) over the remote column for linear and nonlinear analysis turned out to be close (Figure 11) – the longitudinal force reaches its maximum for a short time interval (0.01 ÷ 0.015 sec), after which its value stabilizes.

Thus, the presented method of elasto-plastic analysis is an effective numerical approach that allows one to identify the features of stress-strain states of reinforced concrete bearing systems, taking into account dynamic effects.

A detailed design analysis using the direct integration of the equations of motion based on the nonlinear behaviour of reinforced concrete structures allowed us to confirm the effectiveness of outrigger storeys as special structures that prevent the progressive collapse of the load-bearing system. The developed method allows performing the analysis of other types of structures designed to protect the load-bearing system in the event of instantaneous failures of vertical load-bearing elements.



a – linear analysis **b – nonlinear analysis**

Figure 8. Model A. The vertical displacement in node 7100 (a – linear analysis; b – nonlinear analysis)

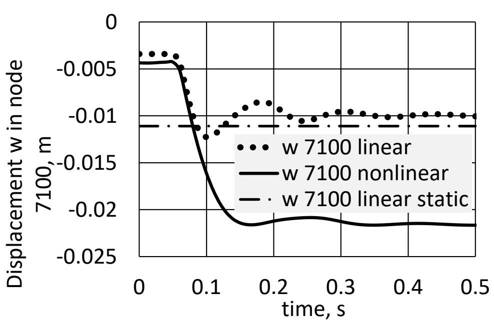


Figure 9. Model B. The vertical displacement in node 7100. Comparison of linear and nonlinear solutions.

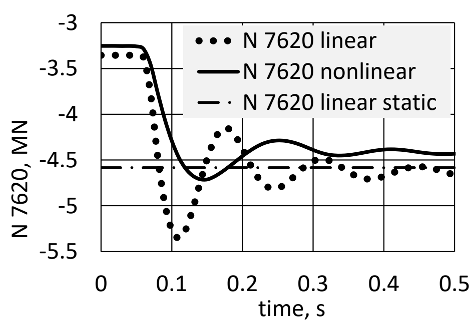


Figure 10. Model B. The longitudinal force in finite element 7620. Comparison of linear and nonlinear solutions.

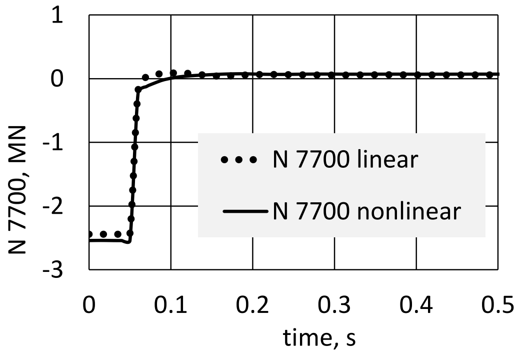


Figure 11. Model B. The longitudinal force in finite element 7700. Comparison of linear and nonlinear solutions.

Discussion. This article presents the results of elasto-plastic dynamic analysis of spatial structure, performed in the accepted formulation of the task of progressive destruction. We have not been able to find any literary source in which, in such a formulation, a large-scale problem for the entire design model would be solved. We know of a few works (for example, [9]), where, in a different formulation (a contact explosion was considered), the dynamic behavior of the entire structure was simulated taking into account the elasto-plastic deformation of its elements. Unfortunately, in the above-mentioned works, data on the impact models or on other parameters of the computational model are not fully presented, which makes it impossible to directly compare our results with the results presented in them. Therefore, the validation of the reliability of the results obtained in this research is based on the solution of individual test problems (benchmarks), producing a comparison of the numerical results obtained by the proposed method with the results of qualitatively performed experiments or high-precision numerical solutions that we trust [32–35, 37].

In a number of works, for individual structural elements, experimental studies on the deformation and destruction of reinforced concrete structures under conditions of support failure have been carried out. However, we are not aware of the publication of the results of experimental studies devoted to the behavior of the supporting system as a whole under conditions of local failure of the supporting structures. Thus, the assessment of the consistency of the results obtained in this work is based on comparison with the results of the general engineering assessment of the consequences of emergency situations associated with structural failures. Analysis of these materials – photographs, descriptions of accidents, etc. - show that the result obtained by us corresponds, in general, to the actual scheme of deformation and damage of reinforced concrete structures under conditions of failure of the supporting elements.

4. Conclusions

Accounting for the dynamic nature of the removal of structural elements allows to perform numerical modeling without using artificially introduced dynamic amplification factors. Taking into account the elasto-plastic properties of the material and the degradation of concrete during cracking leads to a significant increase in displacements and a slight decrease in forces in the structural elements compared to the linear dynamic analysis.

The results of the performed studies demonstrate the effectiveness of the analysis of an entire load-bearing system using the direct integration of the equations of motion taking into account the nonlinear behavior of reinforced concrete. The obtained results also allow to formulate approaches to the justification of deformation criteria for their further use in developing simplified calculation methods for mass application.

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