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## Stress level in beams with sinusoidal perforation

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**Abstract.** The problem of determining of stress state in beams with sinusoidal perforation under concentrated force was considered. An empirical expression has been obtained for the stress distribution in dependence on parameters of perforation and force factors – bending moment and shear force. Von Mises equivalent stresses near the edge of openings is presented by the sum of two items caused by two types of deformations – shear and bending. The numerical coefficients of this dependence were determined with help of the finite element method calculations. The obtained empirical relation was verified by FEM calculations using the ANSYS software. It was studied effect of the fillet radius in corners of openings on stress state of beams with sinusoidal perforation in wide range of relative height of openings. Obtained results allow to conclude that calculation of equivalent stresses in beams with sinusoidal perforation bring to divergence not exceeding 5%.

### 1. Introduction

In perforated beams there are the most common circular and hexagonal openings. Such beams differ from each other in level of stress concentration and manufacturing cost. The stress concentration in beams with hexagonal openings is much higher than in beams with circular openings. But they are cheaper due to significant reduction in volume of cutting. The desire of designers to reduce the stress level in beams with hexagonal openings led to production of beams with sinusoidal perforation. In fact, these are the same beams with hexagonal openings obtained by non-waste technology, but with fillet corner radius extended. The beams with sinusoidal openings are using in frameworks of different constructions (Figure 1). The range of design conceptions of such beams is wide.



Figure 1. Framework with sinusoidal perforated beams.

The strength and optimum design of perforated beams were studied in many works of foreign [1–21] and Russian [22–28] authors. In most of them, the stress distribution is considered near of hexagonal openings of regular shape using finite element method [1, 3, 8, 9, 11, 12, 14–24]. In researches [4–7, 10, 13, 26, 27] it was studied beams with sinusoidal perforation and in researches [2,25,28] beams with circular openings are investigated.



Budi et al. [1] performed the optimization analysis using FEM and laboratory tests on 225 mm height castellated steel beams with 3 m length for verification. All tested models have 150 mm height holes with various opening angles in range  $45^\circ$ – $70^\circ$  and different distances between holes. A summary analysis of castellated beams showed that maximum stress concentration in models with  $45^\circ$  opening angle is located in the weld joint area of web-post, but in all other models maximum stresses take place at the corner area of hexagonal holes. Strength analysis in [1] allow to conclude that optimum angle size is  $60^\circ$  with distance between holes (0.186 – 0.286) mm. Durif et al. [4-7] research load capacity of cellular beams with sinusoidal perforation. The study is focused on the experimental and numerical analyses of isolated web-posts specimens with four opening quarters taken from whole cellular beams. The stress measurements near contours of openings were also made using resistance wire strain gauges. Chhapkhane at al. [15] deals with finding the elastic stress distribution in castellated beams by FEM and experimentally on steel models. Strain gauge location was determined using the initial finite element models. The tests were carried out on simply supported short castellated beams containing only 4 openings with central concentrated loading. The stress level obtained with the any strain gauges located around the interior cells was the same. No stress concentration was fixed. Wakchaure at al. [18] observed from the finite element analysis that as the depth of hexagonal opening increases, stress concentrations increases at the hole corners. So by taking corrective measures, i.e. by rounding hole corners, the strength of castellated beams can be improved in practice. Wang et al. [21] investigated by FEM the failure of castellated beams with fillet corner web openings. Compared to beams with traditional hexagonal openings the proposed beams with fillet corner enjoy a higher load bearing capacity. Pritykin et al. [23, 25] conducted series of studies of the stress distribution in beams near hexagonal openings of regular shape, stress concentration in cellular beams with circular opening and obtained some empirical relations for estimating of the stress values. Later Pritykin et al. [26, 27] studied and obtained relations for deformations and stress concentration in castellated beams with sinusoidal perforation but only for one size of fillet radius. Dobrachev et al. [24] try to describe stress distribution in the web-post of castellated beam analytically by approximate function.

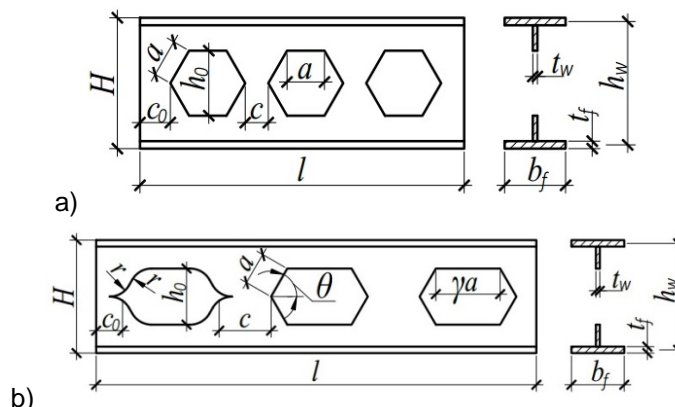
The purpose of the study is to obtain an empirical relation for evaluating of von Mises equivalent stress in beams with sinusoidal web perforation. A simply supported beams loaded with concentrated force in the middle of span were considered.

## 2. Methods

### 2.1. Dependence for Equivalent Stresses in the Zone of T-shaped Flanges

According to the geometry of perforations the highest stress level can be either in area of reentering angle of openings on neutral axis, where maximum shearing stresses act, or near an opening corners, where stress level is determined by action of bending moment and shear force. We derive an empirical dependence for the definition of the stresses in the region of openings in the zone of T-section.

When deriving dependences for stresses, we consider dimensions of hexagonal openings with a fillet radius of the corner  $r = 0$  as basic values (Figure 2a). In general, web perforation of a beam with sinusoidal openings is determined by three linear parameters (Figure 2b): openings height  $h_0 = 2a \sin \theta$ , web-post width  $c = \gamma a$ , representing minimum distance between edges of two neighbor openings and fillet radius of the corner  $r$ . For beam with sinusoidal perforation the width of web-post is always equal to the horizontal side of opening. One more parameter is angle  $\theta$  of inclination of hexagonal sides to neutral axis of beam. It can vary within  $45^\circ \leq \theta \leq 70^\circ$ , but in the research it will be equal  $\theta = 60^\circ$ .



**Figure 2. Perforated beam geometry:**  
**a) basic perforation; b) perforation with sinusoidal openings.**

As well known, in calculations for different beams comparing, relative parameters are convenient to use. Therefore, a relative height of openings is denoted as  $\beta = h_0 / h_w$  ( $h_w$  is height of the beam web,  $h_w = H - t_f$ ). To indicate the dimensions of beam the following abbreviated form of notation  $l - h_w - t_w - b_f - t_f - \beta - \gamma - \rho$  was used in research. It completely determines the beam geometry. The interpretations are:  $l$  – length of beam,  $h_w$  and  $t_w$  – height and thickness of web,  $b_f$  and  $t_f$  are width and thickness of flanges;  $\rho = r / a$  is relative fillet radius ( $a$  is inclined side of opening). The dimensions of the beam are indicated in centimeters.

In lateral bending, both bending moment  $M$  and shear force  $V$  play an important role in magnitude of  $\sigma_{\max}^{eqv}$ . Taking into account the above notation, we can approximately represent stresses  $\sigma_{\max}^{eqv}$  near of edge  $n^{\text{th}}$  opening of beam with two components: from action of shear force  $V$  and bending moment  $M_n$

$$\sigma_{\max}^{eqw} = \alpha_V \frac{V}{h_w t_w} + \alpha_M \frac{M_n}{W}, \quad (1)$$

where  $\alpha_V$  and  $\alpha_M$  are force and moment coefficients unknown here yet and determining by using the FEM calculations; modulus of section  $W$  is calculated approximately as

$$W = b_f t_f h_w + h_w^2 t_w / 6. \quad (2)$$

Assume that contour of the first opening is located at a distance  $c$  from the support. Then bending moment  $M_n$  in region of  $n^{\text{th}}$  opening, where von Mises equivalent stress  $\sigma_{\max}^{eqv}$  is determined, can be represented in the form

$$M_n = V x_n = V(2n - 1)(\cos \theta + \gamma)a. \quad (3)$$

Substitution of (2) and (3) into (1) taking into account accepted notation  $a = h_0 / 2 \sin \theta$  and  $\beta = h_0 / h_w$  leads to the expression

$$\sigma_{\max}^{eqv} \approx \left( \alpha_V + \alpha_M^* \frac{(n - 0.5)(\cos \theta + \gamma)\beta}{\sin \theta (b_f t_f / h_w t_w + 0.167)} \right) \frac{V}{h_w t_w} \quad (4)$$

Relation (4) in general form is valid for any angle of inclination angle but in particular case, when  $\theta = 60^\circ$  it is simplified to view

$$\sigma_{\max}^{eqv} \approx \left( \alpha_V + \alpha_M \frac{(n - 0.5)(0.5 + \gamma)\beta}{6 b_f t_f / h_w t_w + 1} \right) \frac{V}{h_w t_w}. \quad (5)$$

The influence of fillet radius is taken into account using the coefficients  $\alpha_V$  and  $\alpha_M$ . The value  $b_f t_f / h_w t_w$  represents ratio of areas of flanges and web. For practical application of relation (5), it remains to determine coefficients of force  $\alpha_V$  and moment  $\alpha_M$ .

## 2.2. Coefficient of Stress Concentration

To compare efficiency of different designs, it is convenient to use stress concentration factor  $\alpha_\sigma$ , which in the study was defined as ratio of maximum equivalent stresses in the opening zone  $\sigma_{\max}^{eqv}$  to the level of maximum stresses in beam with a solid web  $\sigma_{\max}^{TT}$  under action of external load. The value  $\alpha_\sigma$  was estimated by formula

$$\alpha_\sigma = \sigma_{\max}^{eqw} / \sigma_{\max}^{TT}, \quad (6)$$

where  $\sigma_{\max}^{TT}$  is stresses in flange, determined according to the technical theory of beam as

$$\sigma_{\max}^{TT} = M_{\max} / W. \quad (7)$$

Here  $M_{\max}$  is the maximum bending moment,  $W$  is the section modulus of beam without openings in web, determined by (2).

For possibility of practical application of relations (5) and (6) first of all is necessary to determine coefficients of force  $\alpha_V$  and moment  $\alpha_M$  with help of FEM calculations.

As known the accuracy of FEM calculations is largely determined by size of FE: smaller finite elements lead to more accurate calculation as a rule. However, it is impractical to use a small size FE mesh for entire structure because of calculation time. For example, on a computer with 4 GB of RAM, calculating of a system of equations with 400000 unknowns takes about a minute. To reduce the size of equations system and, relatively, calculation time, different approaches can be used: the method of superelements; accounting for symmetry of structure, allowing only half of beam to be considered; the use of irregular mesh of FEs and others. The last two approaches turn out to be the simplest and quite effective. However, the question arises what the size of finite elements would be sufficient to obtain the required calculation accuracy. It is quite difficult to theoretically justify the optimal sizes of FEs, so in the most cases this is done on the basis of successive analysis with a decrease in size of elements until difference in results becomes negligible (for example, 0.1% – 0.2%).

In study a fine mesh was used at the edge of openings through one and only on that part where the level of maximum stresses was estimated (Figure 3). It was considered one part of beam to the section where concentrated force is applied. The resulting system of equations turned out to be optimal in terms of calculation time. It is clear that the size of the FE should be linked to fillet radius of corners of hexagonal opening. After all, the smaller radius, the smaller finite elements should be, otherwise calculation of average stresses within FE reduce accuracy. In the calculations below, the radius  $r$  was taken equal to  $r = 0.25a$ ,  $r = 0.5a$  and  $r = 0.75a$ , where  $a$  – inclined side of opening. The analysis showed that satisfactory accuracy is achieved with FE sizes of  $(0.02 - 0.03)r$ , therefore, in the calculations, FE sizes near edge of opening were equal to  $\Delta_{FE} = 2$  mm, and in the rest of beam –  $\Delta_{FE} = 20$  mm, with overall opening height  $h = 500 - 600$  mm.

The simply supported beams loaded with concentrated force applied at an arbitrary point of span were initially considered.

### 3. Results and Discussion

#### 3.1. Dependence for Equivalent Stresses in the Zone of T-section

The series of calculations of perforated beams made from №60 rolled section with different opening parameters were carried out. The values of relative fillet radius of openings  $\rho$  in range  $0.25 \leq \rho \leq 0.75$ , the relative height of openings in the range  $0.667 \leq \beta \leq 0.73$  with a fixed relative width of web-post  $\gamma = 1.5$  and the angle of inclination of sides  $\theta = 60^\circ$ , were varied.

Under manufacturing a perforated beam using non-waste technology there is a relationship between the initial height of beam with a solid web  $H_0$  and height of the perforated beam  $H$ , depending on the relative height of openings  $\beta$

$$H = H_0 / (1 - 0.5\beta). \quad (8)$$

Below it is considering beams manufacturing of pattern beam with height  $H_0 = 600$  mm as the initial one, then for values of  $\beta = 0.667$ ;  $\beta = 0.7$  and  $\beta = 0.73$  in accordance with (8) the height of perforated beams will be  $H = 900$  mm;  $H = 923$  mm and  $H = 945$  mm respectively.

The FEM calculations of the simply supported beam loaded with force  $F = 10$  kN in the middle of span, according to the program developed by the authors using the ANSYS software solution, lead to the values shown in Figure 3.

The analysis of results revealed that for beams with the above parameters, acceptable values of coefficients  $\alpha_V$  and  $\alpha_M$  correspond to the expressions:

$$\alpha_V = 60(2.17\beta - 1)(1.25 - \rho); \tag{9}$$

$$\alpha_M = 3.8(5.15\beta - 1). \tag{10}$$

As can be seen from (9) and (10), the coefficient  $\alpha_V$  can be approximately considered linearly dependent on  $\rho$  and  $\beta$ , and the coefficient  $\alpha_M$  – linearly dependent only on  $\beta$ . The stress distributions in the beams with parameters  $l-h_w-1.2-19-1.78\text{cm}-\beta-1.5-0.75$  shown in Figure 3 are linear, i.e., the values of von Mises equivalent stress  $\sigma_{\max}^{eqv}$  near the edge of openings are proportional to magnitude of the bending moment. The given empirical dependence (5), taking into account the relations (9) and (10), allows us to compare the maximum equivalent stresses in the region of sinusoidal openings with an arbitrary number  $n$ , obtained by the FEM and analytically (Table 1).



**Figure 3. Stress state of simply supported beams  $l-h_w-1.2-19-1.78\text{cm}-\beta-1.5-0.75$  with different opening height:**

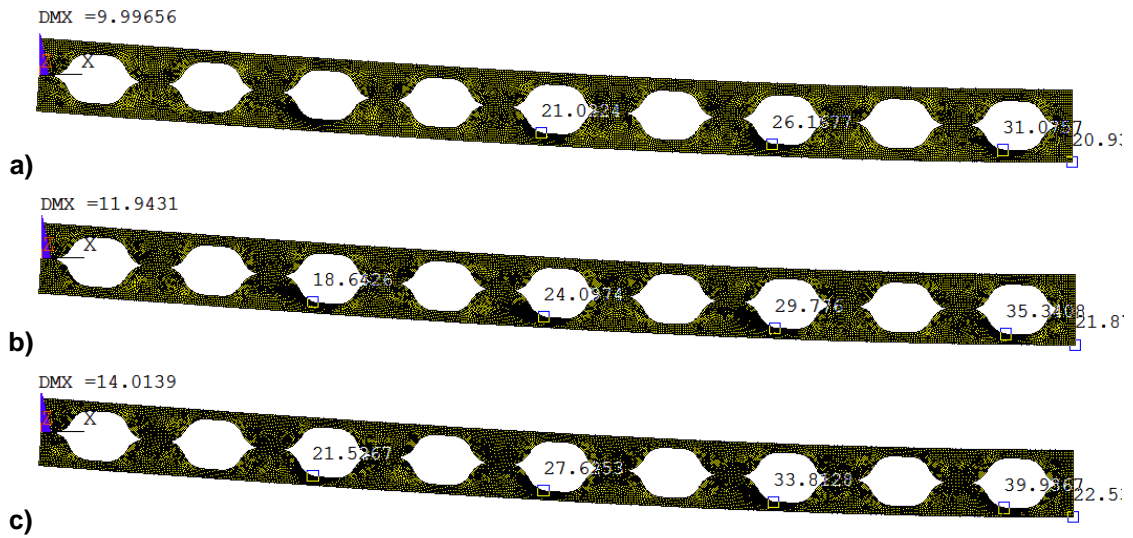
**a)**  $h_w = 88.22 \text{ cm}; \beta = 0.667$ ; **b)**  $h_w = 90.52 \text{ cm}; \beta = 0.7$ ; **c)**  $h_w = 92.72 \text{ cm}; \beta = 0.73$

**Table 1. Comparative analysis value of stresses  $\sigma_{\max}^{eqv}$  in beams  $l-h_w-1.2-19-1.78\text{cm}-\beta-1.5-0.75$  at different relative height of openings.**

Opening number, $n$		5	7	9	
$\beta$	0.667	FEM	15.1	19.0	22.7
		by (5)	15.3	19.3	23.3
Divergence $\delta$ , %		1.3	1.6	2.6	
Beam		$l-88.22-1.2-19-1.78 \text{ cm}-0.667-1.5-0.75$			
$\beta$	0.7	FEM	17.4	21.7	25.9
		by (5)	17.2	21.6	26.1
Divergence $\delta$ , %		1.1	0.5	0.8	
Beam		$l-90.52-1.2-19-1.78 \text{ cm}-0.7-1.5-0.75$			
$\beta$	0.73	FEM	19.9	24.6	29.2
		by (5)	18.9	23.7	28.6
Divergence $\delta$ , %		5.0	3.7	2.1	
Beam		$l-92.72-1.2-19-1.78 \text{ cm}-0.73-1.5-0.75$			

As can be seen from the results presented in Table 1, the divergence of stresses calculated on (5) and by FEM does not exceed 5%.

The similar calculations performed by the FEM for perforated beams made from the initial profile №50 (Russian State Standard GOST 8239-89) are presented in Figure 4. The divergence according to (5) in this case is also less than 5% (Table 2).



**Figure 4. Stress state of simply supported beams  $l-h_w-1.0-17-1.52\text{cm}-\beta-1.5-0.75$  with different opening height:**

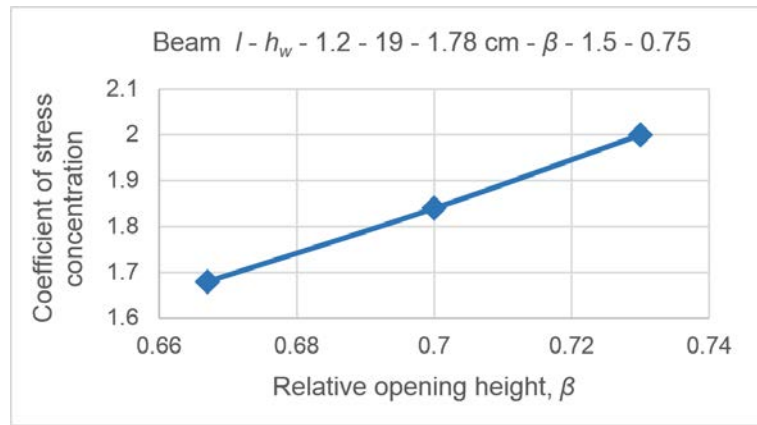
**a)  $h_w = 73.48 \text{ cm}$ ;  $\beta = 0.667$ ; b)  $h_w = 75.38 \text{ cm}$ ;  $\beta = 0.7$ ; c)  $h_w = 77.22 \text{ cm}$ ;  $\beta = 0.73$**

**Table 2. Comparative analysis value of stresses  $\sigma_{\max}^{eqv}$  in beams  $l-h_w-1.0-17-1.52\text{cm}-\beta-1.5-0.75$  at different relative height of openings.**

Opening number, n		5	7	9	
$\beta$	0.667	FEM	21.0	26.1	31.1
		by (5)	21.3	26.7	32.1
Divergence $\delta$ , %		1.4	2.3	3.2	
Beam $l-73.48-1.0-17-1.52 \text{ cm}-0.667-1.5-0.75$					
$\beta$	0.7	FEM	24.0	29.8	35.3
		by (5)	23.9	29.9	35.9
Divergence $\delta$ , %		0.4	0.3	1.7	
Beam $l-75.38-1.0-17-1.52 \text{ cm}-0.7-1.5-0.75$					
$\beta$	0.73	FEM	27.6	33.8	39.9
		by (5)	26.2	32.8	39.4
Divergence $\delta$ , %		5	3.0	1.3	
Beam $l-77.22-1.0-17-1.52 \text{ cm}-0.73-1.5-0.75$					

### 3.2. Stress concentration factor

The strength calculation of any structures, including beams, is estimated by the level of maximum stresses arising in it. Considering the data presented in Figure 3 with a constant fillet radius of corner, the value  $\alpha_\sigma$  calculated on (6) varies linearly depending on the relative height of openings  $\beta$  (Figure 5).



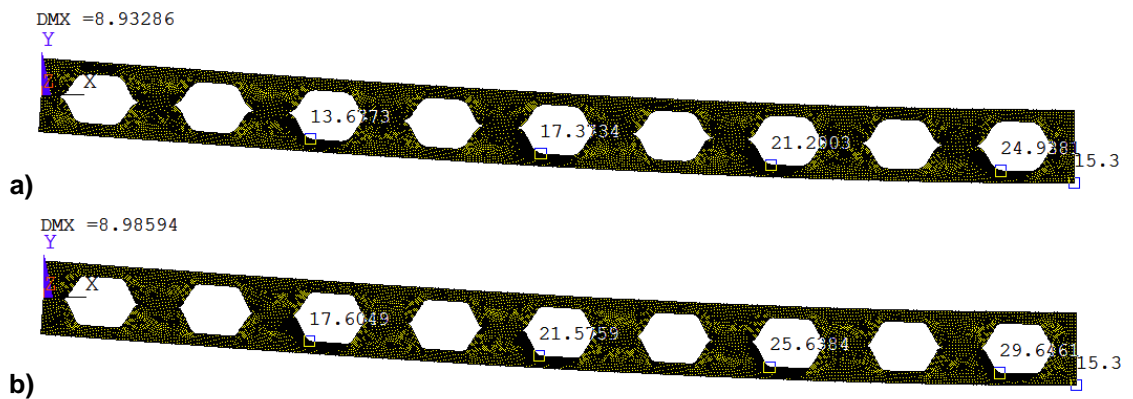
**Figure 5. Dependence value  $\alpha_\sigma$  on relative height of openings  $\beta$ .**

The stress concentration level is significantly affected not only by the relative height of the openings  $\beta$ , but also by the fillet radius of their corners. In practice, openings with relative fillet radius of corners from  $\rho = 0.25$  (Figure 6a) to  $\rho = 0.75$  (Figure 6b) are used. Therefore, beams with different fillet radius were investigated.



**Figure 6. Perforated beams with different fillet radius of corner: a)  $r = 0.25a$ ; b)  $r = 0.75a$ .**

At relative fillet radius of corner  $\rho = 0.25$  and  $\rho = 0.5$  for beams made from  $H_0 = 600$  mm profile, at  $\beta = 0.667$ , the stress distribution pattern shown in Figure 7 indicates that a decrease of  $\rho$  lead to an increase of stress concentration. So, when the radius decreases from  $r = 0.75a$  (Figure 3a) to  $r = 0.5a$  (Figure 7a), the stress level increases by about 10%, and additional reducing of radius from  $r = 0.5a$  (Figure 7a) to  $r = 0.25a$  (Figure 7b), lead to augmentation of stresses by another 20%. For radius  $r = 0.25a$  and  $\beta = 0.667$ , the value of  $\alpha_\sigma = 2.15$ , while for  $r = 0.75a$  the coefficient  $\alpha_\sigma = 1.65$ .

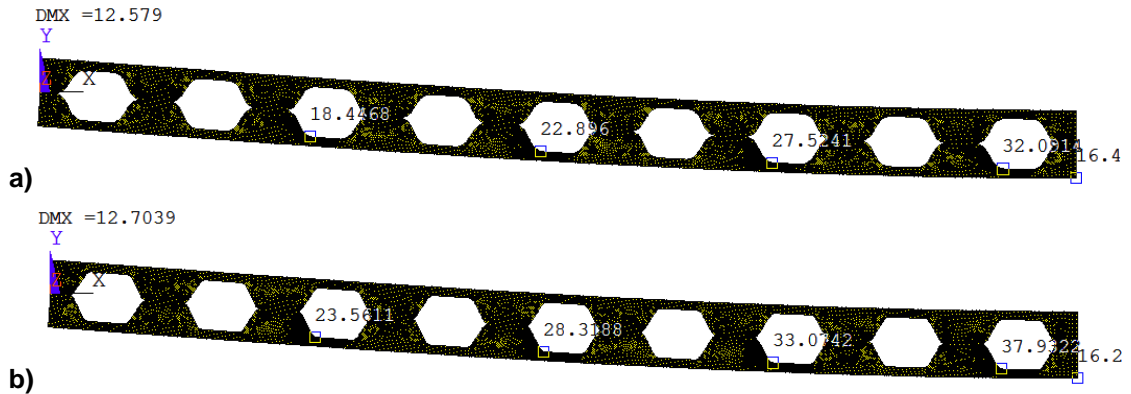


**Figure 7. Stress level in beams  $l - 88.22 - 1.2 - 19 - 1.78\text{cm} - 0.667 - 1.5 - \rho$  with:**

**a)  $\rho = 0.5$ ; b)  $\rho = 0.25$ .**

Similar calculations by FEM of beam  $l-92.72-1.2-19-1.78\text{cm}-0.73-1.5-\rho$  with the fillet radius  $r = 0.5a$  and  $r = 0.25a$  lead to a picture of the stress distribution shown in Figure 8. The value of  $\alpha_\sigma$  for beam with  $\rho = 0.5$  (Figure 8a) does not exceed value  $\alpha_\sigma = 2.17$ , and with a further decrease of radius to  $\rho = 0.25$  (Figure 8b), the stress concentration level increases to  $\alpha_\sigma = 2.56$ .

For beam shown in Figure 8b, the stress level calculated according to (5) gives a value of  $\sigma_{\max}^{eqv} = 36.5$  MPa in the 9<sup>th</sup> opening, which indicates divergence with calculation by FEM of 3.7%.



**Figure 8. Stress level in beams  $l-92.72-1.2-19-1.78\text{cm}-0.73-1.5-\rho$  with:**  
**a)  $\rho = 0.5$ ; b)  $\rho = 0.25$ .**

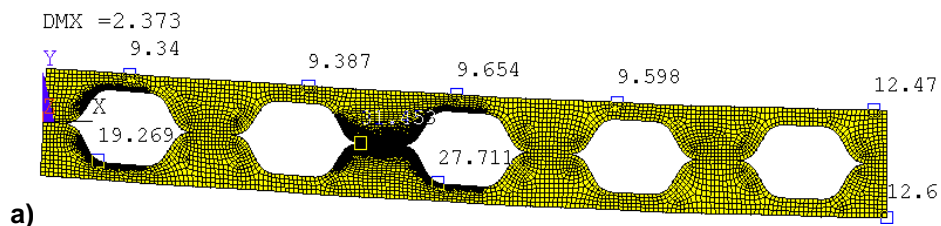
The obtained results are applicable for case of loading a simply supported beam with a force  $F$  in an arbitrary section of span, taking as the shear force  $V$  the reaction of support in corresponding section of beam. Applying the superposition principle, it is possible to determine the stress concentration under action of a few concentrated forces.

Since in beams with sinusoidal openings, potentially dangerous zones can also be the welded zones of the perforated teeth at the level of neutral axis, where the reentering angles are located, these zones were also studied.

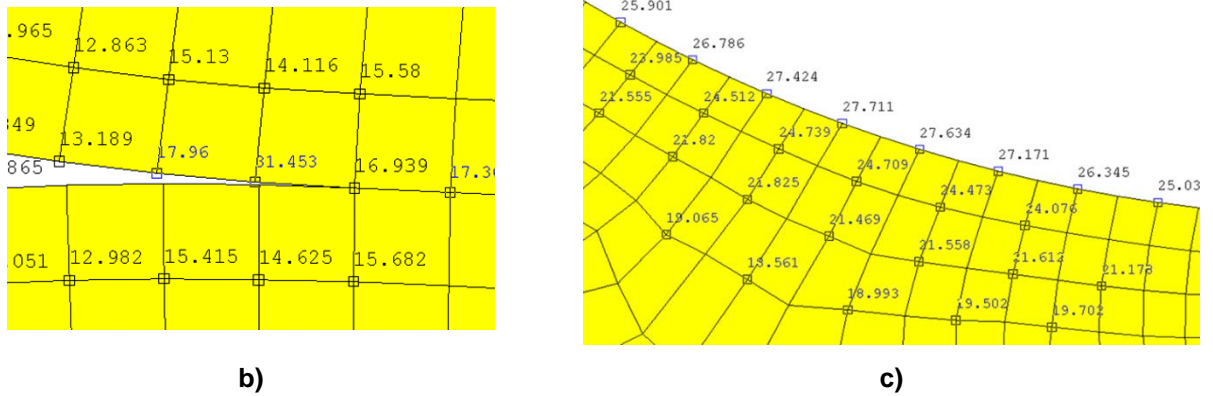
### 3.3. Equivalent stress in web-post zone

Consider a simply supported beam  $15.4H-73.48-0.6-17-1.52\text{cm}-0.667-1.5-0.5$  loaded with concentrated force in the middle of span. Calculation of stress level by FEM using ANSYS software (Figure 9a) shows stress state of the web-post zones at the level of neutral axis.

As can be seen from Figure 9b, the danger zone of stress state is the entering acute angle in web-post: maximum stresses in this zone near axis of 2<sup>nd</sup> hole reach value  $\sigma_{\max}^{eqv} = 31.453$  MPa, while the maximum stress in area of rounded zone of 3<sup>rd</sup> hole is only  $\sigma_{\max}^{eqv} = 27.711$  MPa (Figure 9c), i.e. near neutral axis a stress level is higher. In flanges at a stated load the maximum stresses are  $\sigma_{flange} = 12.6$  MPa (Figure 9a). Thus, the stresses near neutral axis are more than 2.5 times higher than stresses in flanges. Based on this it can be concluded that decreasing the stress concentration in the region of opening corners lead to augmentation of stresses near the neutral axis (Figure 9b).

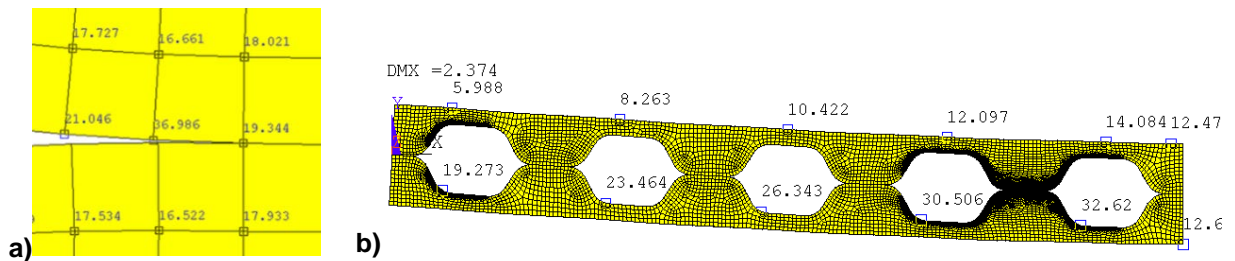






**Figure 9. Stress state of beam 15.4H – 73.48 – 0.6 – 17 – 1.52cm – 0.667 – 1.5 – 0.5 under force  $F = 10$  kN applied in mid-span: a) general picture of stress distribution; b) near axis of 2<sup>nd</sup> hole; c) in rounded zone of 3<sup>rd</sup> hole.**

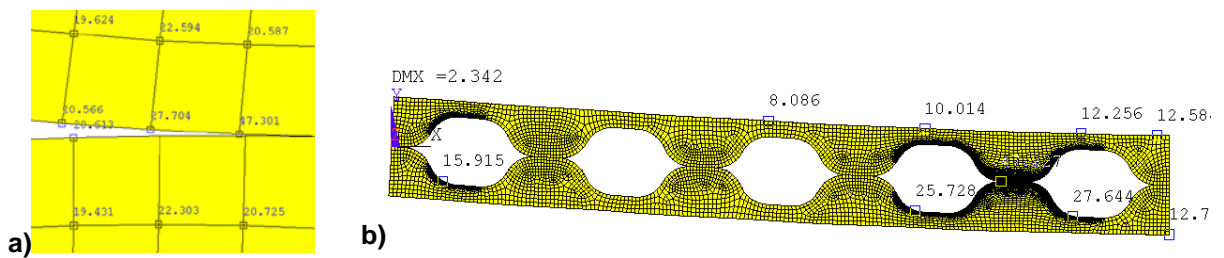
Stress state near axis of 4<sup>th</sup> hole (Figure 10a) show level  $\sigma_{max}^{eqv} = 36.916$  MPa, while maximum stresses at contour of 5<sup>th</sup> hole (Figure 10b) is only  $\sigma_{max}^{eqv} = 32.62$  MPa.



**Figure 10. Stress state of beam 15.4H – 73.48 – 0.6 – 17 – 1.52cm – 0.667 – 1.5 – 0.5 under force  $F = 10$  kN applied in mid-span: a) near axis of 4<sup>th</sup> hole; b) general picture of stress distribution.**

As can be seen from Figure 10, at a constant shear force, the stresses at the reentering edge of opening increase with expansion of bending moment from 31.5 MPa at the second web-post to 36.916 MPa at the fourth web-post (Figure 10a). The stress concentration factor at edge of opening in the region of the 4<sup>th</sup> web-post take a value  $\alpha_{\sigma} = 36.916 / 11.9 \approx 3.11$ .

Let us now consider a beam with an increased fillet radius up to  $r = 0.75a$ . From Figure 11 it follows that growth of the fillet radius of corner practically does not affect the stress level in the beam flange near of application of concentrated force. But the level of maximum equivalent stresses  $\sigma_{max}^{eqv}$  decreases from 32.62 MPa (Figure 10b) to 27.64 MPa in the region of the 5<sup>th</sup> openings (Figure 11b). At the same time, the level  $\sigma_{max}^{eqv}$  in the zone of reentering angles increases almost 1.3 times from 37 MPa to 47.3 MPa (Figure 11a). Thus, the level of stress concentration increases to  $\alpha_{\sigma} = 47.3 / 11.9 \approx 3.97$ .



**Figure 11. Stress state of beam 15.4H – 73.48 – 0.6 – 17 – 1.52cm – 0.667 – 1.5 – 0.75 under force  $F = 10$  kN applied in mid-span: a) near axis of 4<sup>th</sup> hole; b) general picture of stress distribution.**

In fact, the level of stresses in the region of the reentering angles at the level of the neutral axis is much lower due to the presence of welds, which somewhat smooth the sharp reentering angles (Figure 6b).

Note that in case of sinusoidal perforation the zone of increased stresses includes only one node in the zone of the reentering angle at the level of neutral axis (see, for example, Figure 10a or Figure 11a). At the same time, in the region of neighboring nodes, the stress level decreases by 1.7–2 times, but in the zone of the T-shaped of flanges, the region of increased stresses is quite extensive, covering several nodes (Figure 9c). Hence, it can be concluded that using of beams with sinusoidal perforation allows to reduce the stress level approximately 1.7–2.3 times compare with regular hexagon perforation. As for the reentering nodes, as can be seen from Figure 6b, the presence of welds decrease sharpness of corners and thereby reduce the actual level of maximum stresses to an acceptable value. In study of Durif et al. [4] it is noted the formation of four plastic hinges takes place at the opening corners. Our research confirms that namely at corners level of stresses is highest. In addition to results of work [26] this study allows estimate level of stresses depending on radius of fillet. In work of Wakchaure [17] from the finite element analysis results, it is also concluded that the castellated steel beam corners of openings should be rounded to reduce the stress level.

## 4. Conclusions

1. An empirical dependence is obtained for stresses  $\sigma_{\max}^{eqv}$  near the edge of openings in the form of the sum of two items which makes it possible to differentiate the role of shear force and bending moment.
2. With a constant shear force, the maximum von Mises stresses  $\sigma_{\max}^{eqv}$  near the edge of the hexagonal openings are linearly distributed in proportion to the magnitude of the bending moment.
3. For perforated beams, the value of  $\alpha_{\sigma}$  should be calculated as the ratio of the maximum equivalent stresses at the edge of openings to the maximum stresses in the flange, determined by the technical theory of beam bending.
4. With a fixed fillet radius of the corners, the stress concentration at the openings increases in proportion to their relative height. The value  $\alpha_{\sigma}$  with a relative fillet radius  $\rho = 0.75$  does not exceed 2, decreasing with diminishing height of openings.
5. Although the level of stress concentration in the regions of the reentering angles in the web-posts exceeds the level  $\alpha_{\sigma}$  in regions of the rounding of openings, the presence of welds reduces this concentration.

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