

DOI: 10.18721/JPM.14208  
UDC 539.125.4

## DIQUARK PARTON DISTRIBUTION FUNCTIONS BASED ON THE LIGHT-FRONT AdS/QCD QUARK-DIQUARK NUCLEON MODEL

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In the paper, we present a phenomenological unpolarized parton distribution functions (PDFs) for diquarks based on the light front soft-wall AdS/QCD quark-diquark nucleon model. From a probed model consistent with the Drell–Yan–West relation and quark counting rule, we have performed a fit of some free parameters using known phenomenological data of quark PDFs. The model considers the entire set of possible valence diquarks within the nucleon. In our investigation, we focused on the spin-0  $((ud)_0)$ , spin-1  $((ud)_1)$  and spin-1  $((uu)_1)$  valence diquarks in the proton. The diquark PDFs obtained can be used in proton-proton collision simulations.

**Keywords:** diquark, parton distribution function, AdS/QCD, holography

**Citation:** Rodriguez-Aguilar B., Berdnikov Ya.A., Diquark parton distribution functions based on the light-front AdS/QCD quark-diquark nucleon model, St. Petersburg Polytechnical State University Journal. Physics and Mathematics. 14 (2) (2021) 90–103. DOI: 10.18721/JPM.14208

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## ПАРТОННЫЕ ФУНКЦИИ РАСПРЕДЕЛЕНИЯ ДИКВАРКОВ, ОСНОВАННЫЕ НА (AdS/QCD)-МОДЕЛИ НУКЛОНА КВАРК-ДИКВАРК

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В работе представлены феноменологические неполяризованные партонные функции распределения для дикварков, основанные на модели с мягкой стеной на световом конусе AdS/QCD кварк-дикваркового нуклона. На основе проверенной модели, согласующейся с соотношением Дрелла – Яна – Веста и правилом счета кварков, в работе определен набор параметров с использованием известных феноменологических данных кварковых функций распределения партонов (ФРП). Модель рассматривает все возможные состояния валентных дикварков. В данном исследовании мы остановились на рассмотрении состояний валентных дикварков  $ud$  со спинами 0  $((ud)_0)$  и 1  $((ud)_1)$ , а также  $uu$  со спином 1  $((uu)_1)$  в протоне. Полученные дикварковые ФРП могут быть использованы при моделировании протон-протонных столкновений.

**Ключевые слова:** дикварк, функция распределения партонов, AdS/QCD, голография

**Ссылка при цитировании:** Родригес-Агилар Б., Бердников Я.А. Партонные функции распределения дикварков, основанные на (AdS/QCD)-модели нуклона кварк-дикварк // Научно-технические ведомости СПбГПУ. Физико-математические науки. 2021. Т. 14. № 2. С. 90–103. DOI: 10.18721/JPM.14208

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## Introduction

Since the second half of the last decade of the 20<sup>th</sup> century, the AdS/CFT correspondence [1] between string theory in anti-de Sitter (AdS) space-time and conformal field theories (CFTs) in physical space-time has been a very active and interesting field of study. Among other things, the wealth of this correspondence stands in the possibility to perform calculations between opposite coupling regimes, strongly coupled theories can be mapped into weakly coupled ones and *vice versa*. CFTs are defined as scale invariant theories, so it is impossible to applicate the AdS/CFT correspondence to the quantum chromodynamics (QCD) itself directly.

It is worth noting that this is because the coupling constants change with the renormalization scale  $\mu$  in QCD that we get the condition under which perturbation theory is valid [2].

Nevertheless, in the strong coupling regime of QCD, the couplings appear to be approximately constant. This is the basis for a light-front holography, an approximation of the AdS/CFT to QCD quantized on the light front (light-front AdS/QCD) [3] that has shown the ability to find analytic solutions in the non-perturbative regime of QCD, like improving predictions of hadron masses and structure properties (see e.g. Ref. [4]).

In this work, we are particularly interested in the fact that light-front AdS/QCD predicts a general form of two particle bound state wave function inside nucleons which cannot be derived simply from valence quarks [4, 5]. This has led to considerable progress in nucleon analytical results considering valence diquarks in their structure, just as light-front wave functions QCD matched with soft-wall AdS/QCD predictions [6 – 8].

Another recent result contemplates the scale evolution of the parton distribution functions (PDFs) for a quark-diquark nucleon model using scale-dependent parameters following the DGLAP (Dokshitzer – Gribov – Lipatov – Altarelli – Parisi) evolution [5], that are consistent with the quark counting rule and Drell – Yan – West relation [9, 10]. Based on these last two results, we have fitted the PDF parameters of the

quark-diquark nucleon model to the available data from NNPDF2.3 QCD + QED NNLO [11] for  $u$  and  $d$  quarks, in order to get the unpolarized PDFs for the spin-0  $(ud)_0$ , spin-1  $(ud)_1$  and spin-1  $(uu)_1$  diquarks. With such parameters available, the diquark PDFs can be used to simulations of proton (and neutron) collisions with participating diquarks.

To consider proton collisions based on a nucleon model with diquark structures inside, it is useful to inspect the properties of the parton model.

## The parton model

The cross section for proton-proton collisions can be expressed by the so called improved parton model formula [12]:

$$\sigma_{(P_1, P_2)} = \sum_{i,j} dx_1 dx_2 f_i^1(x_1, \mu) f_j^2(x_2, \mu) \times \hat{\sigma}_{ij}(x_1 P_1, x_2 P_2, \alpha_s(\mu), \mu), \quad (1)$$

where the scripts 1 and 2 are labels to incoming proton beams carried momentum  $P$ .

In this scenario, the incoming proton beam is equivalent to a beam made of constituent partons. Typically, these partons are taken as the massless-pointlike elementary particles, quarks and gluons [12], with longitudinal momentum distribution characterized by the parton distribution functions  $f_i(x, \mu)$ .

This means, given some proton with momentum  $P$ , the probability to find in such parton  $i$  with momentum between  $xP$  and  $(x + dx)P$  is precisely  $dx f_i(x, \mu)$  being dependent as well of the renormalization scale  $\mu$ .

While represents the parton cross sections, which can be computed with perturbative QCD (pQCD) for sufficiently small running coupling  $\alpha_s(\mu)$  [2].

However, due to the fact that partons cannot be observed as free particles, the PDFs cannot be calculated using pQCD. Nowadays, the simplest way to obtain PDFs is fitting observables to experimental data, among other phenomenological tools (see e.g. Refs. [13, 14]).

Nevertheless, in order to work with a parton model using constituent diquarks, we must ex-

pand this picture beyond quarks and gluons. As we mentioned above, recent results from soft-wall AdS/QCD [4, 7] have shown a phenomenological approach to reproduce unpolarized PDFs of quark-diquark nucleons [5].

In the next section we show how this phenomenological approach has been constructed to finally obtain our parameters that allow us to exhibit our diquark PDFs.

### The soft-wall light front AdS/QCD quark-diquark nucleon model

In this section we intend to outline how to obtain the PDF of a quark-diquark nucleon model using soft-wall light front holographic QCD (for a more detailed analysis see Ref. [5] and its references, from where this section is heavily based).

To construct such a PDF model, it is assumed that a virtual incoming photon interacts with a massless-valence quark. The other two valence quarks are then forming a spectator diquark. In this way, it is ensured that this model is in accordance with the traditional quark-interacting frameworks, from where it is possible to build reliable properties for the nucleon model, so for diquarks. The diquarks can have then either spin-0 (scalar diquark) or spin-1 (vector diquark).

The nucleon state is represented by a spin-flavor SU (4) symmetry. This implies that the possible states are the isoscalar-scalar diquark singlet state, the isoscalar-vector diquark state and the isovector-vector diquark state. Shortly, the diquark can be either scalar or axial-vector.

For the proton state we can write it as

$$|P; \pm\rangle = C_S |uS^0\rangle^\pm + C_V |uA^0\rangle^\pm + C_{VV} |dA^1\rangle^\pm, \quad (2)$$

where, following the original notation in Ref. [5],  $S$  and  $A$  represent the scalar and vector diquark having isospin at their superscript; the subscripts in the coefficients denote the isoscalar-scalar ( $S$ ), the isoscalar-vector state ( $V$ ) and the isovector-vector state ( $VV$ ).

For the neutron, the state is given by the isospin symmetry  $u \leftrightarrow d$ .

Without losing the generality of the model, we

will take the case for the proton, which is what we care about in this work.

Using the light-cone convention  $x^\pm = x^0 \pm x^3$  [15] it is convenient to choose a frame where the proton transverse momentum vanishes, denoted as

$$P \equiv \left( P^+, \frac{M^2}{P^+}, \mathbf{0}_\perp \right),$$

where  $M$  is the proton mass.

So the momentum of the struck quark can be taken as

$$p \equiv \left( xP^+, \frac{p^2 + |\mathbf{p}_\perp|^2}{xP^+}, \mathbf{p}_\perp \right)$$

and the diquark

$$P_X \equiv \left( (1-x)P^+, P_X^-, -\mathbf{p}_\perp \right).$$

We can interpret from this notation that  $x = p^+/P^+$  is the longitudinal momentum fraction carried by the struck quark.

Now, we can express the two particle Fock-state expansion.

For total angular momentum projection  $J^z = \pm 1/2$  with spin-0 diquark is given by

$$|uS\rangle^\pm = \int \frac{dx d^2\mathbf{p}_\perp}{2(2\pi)^3 \sqrt{x(1-x)}} \times \left[ \psi_+^{\pm(u)}(x, \mathbf{p}_\perp) \left| +\frac{1}{2} s; sP^+, \mathbf{p}_\perp \right\rangle + \psi_-^{\pm(u)}(x, \mathbf{p}_\perp) \left| -\frac{1}{2} s; sP^+, \mathbf{p}_\perp \right\rangle \right], \quad (3)$$

where

$$|\lambda_q \lambda_s; sP^+, \mathbf{p}_\perp\rangle$$

is the two-particle state having struck quark of helicity  $\lambda_q$  and a scalar diquark having helicity  $\lambda_s = s$  (spin-0 singlet diquark helicity is denoted by  $s$  to distinguish from triplet diquark).

While, the spin-1 diquark state is given by the following expression [16]:

$$\begin{aligned}
 |vA\rangle^\pm &= \int \frac{dx d^2\mathbf{p}_\perp}{2(2\pi)^3 \sqrt{x(1-x)}} \times \\
 &\times \left[ \Psi_{++}^{\pm(v)}(x, \mathbf{p}_\perp) \left| +\frac{1}{2} + 1; xP^+, \mathbf{p}_\perp \right\rangle + \right. \\
 &+ \Psi_{-+}^{\pm(v)}(x, \mathbf{p}_\perp) \left| -\frac{1}{2} + 1; xP^+, \mathbf{p}_\perp \right\rangle + \\
 &+ \Psi_{+0}^{\pm(v)}(x, \mathbf{p}_\perp) \left| +\frac{1}{2} 0; xP^+, \mathbf{p}_\perp \right\rangle + \\
 &+ \Psi_{-0}^{\pm(v)}(x, \mathbf{p}_\perp) \left| -\frac{1}{2} 0; xP^+, \mathbf{p}_\perp \right\rangle + \\
 &+ \Psi_{+-}^{\pm(v)}(x, \mathbf{p}_\perp) \left| +\frac{1}{2} - 1; xP^+, \mathbf{p}_\perp \right\rangle + \\
 &\left. + \Psi_{--}^{\pm(v)}(x, \mathbf{p}_\perp) \left| -\frac{1}{2} - 1; xP^+, \mathbf{p}_\perp \right\rangle \right].
 \end{aligned} \quad (4)$$

where  $|\lambda_q \lambda_D; sP^+, \mathbf{p}_\perp\rangle$  represents a two-particle state with a quark of helicity  $\lambda_q = \pm \frac{1}{2}$  and a vector diquark of helicity  $\lambda_D = \pm 1, 0$  (triplet). Here  $v = u, d$  is a flavor index.

The light-front (LF) wave functions with spin-0 diquark state,  $\Psi_\pm^{\pm(u)}$  at the initial scale  $\mu_0$  for  $J = \pm \frac{1}{2}$  are given by expressions [8]:

$$J = +\frac{1}{2} : \begin{cases} \Psi_+^{+(u)}(x, \mathbf{p}_\perp) = N_S \phi_1^{(u)}(x, \mathbf{p}_\perp), \\ \Psi_-^{+(u)}(x, \mathbf{p}_\perp) = \\ = N_S \left( -\frac{p^1 + ip^2}{xM} \right) \phi_2^{(u)}(x, \mathbf{p}_\perp); \end{cases} \quad (5)$$

$$J = -\frac{1}{2} : \begin{cases} \Psi_+^{-(u)}(x, \mathbf{p}_\perp) = \\ = N_S \left( \frac{p^1 - ip^2}{xM} \right) \phi_2^{(u)}(x, \mathbf{p}_\perp), \\ \Psi_-^{-(u)}(x, \mathbf{p}_\perp) = N_S \phi_1^{(u)}(x, \mathbf{p}_\perp). \end{cases} \quad (6)$$

In a very similar way, for vector diquarks with  $J = \pm \frac{1}{2}$  the LF wave functions  $\Psi_{\pm\pm}^{\pm(v)}$  at the initial scale  $\mu_0$  can be written as

$$J = +\frac{1}{2} : \begin{cases} \Psi_{++}^{+(v)}(x, \mathbf{p}_\perp) = \\ = N_1^v \sqrt{\frac{2}{3}} \left( \frac{p^1 - ip^2}{xM} \right) \phi_2^{(v)}(x, \mathbf{p}_\perp), \\ \Psi_{-+}^{+(v)}(x, \mathbf{p}_\perp) = N_1^v \sqrt{\frac{2}{3}} \phi_1^{(v)}(x, \mathbf{p}_\perp), \\ \Psi_{+0}^{+(v)}(x, \mathbf{p}_\perp) = N_0^v \sqrt{\frac{1}{3}} \phi_1^{(v)}(x, \mathbf{p}_\perp), \\ \Psi_{-0}^{+(v)}(x, \mathbf{p}_\perp) = \\ = N_0^v \sqrt{\frac{1}{3}} \left( \frac{p^1 + ip^2}{xM} \right) \phi_2^{(v)}(x, \mathbf{p}_\perp), \\ \Psi_{+-}^{+(v)}(x, \mathbf{p}_\perp) = 0, \\ \Psi_{--}^{+(v)}(x, \mathbf{p}_\perp) = 0; \end{cases} \quad (7)$$

$$J = -\frac{1}{2} : \begin{cases} \Psi_{++}^{-(v)}(x, \mathbf{p}_\perp) = 0, \\ \Psi_{-+}^{-(v)}(x, \mathbf{p}_\perp) = 0, \\ \Psi_{+0}^{-(v)}(x, \mathbf{p}_\perp) = \\ = N_0^v \sqrt{\frac{1}{3}} \left( \frac{p^1 - ip^2}{xM} \right) \phi_2^{(v)}(x, \mathbf{p}_\perp), \\ \Psi_{-0}^{-(v)}(x, \mathbf{p}_\perp) = N_0^v \sqrt{\frac{1}{3}} \phi_1^{(v)}(x, \mathbf{p}_\perp), \\ \Psi_{+-}^{-(v)}(x, \mathbf{p}_\perp) = N_1^v \sqrt{\frac{2}{3}} \phi_1^{(v)}(x, \mathbf{p}_\perp), \\ \Psi_{--}^{-(v)}(x, \mathbf{p}_\perp) = \\ = N_1^v \sqrt{\frac{2}{3}} \left( \frac{p^1 + ip^2}{xM} \right) \phi_2^{(v)}(x, \mathbf{p}_\perp). \end{cases} \quad (8)$$

The LF wave functions  $\phi_i^{(v)}$  ( $i = 1, 2$ ) are the twist-3 LF wave functions. These functions can be derived in light-front QCD and in soft-wall AdS/QCD [4, 17–19, 6].

In Ref. [7] a generalized form to  $\phi_i^{(v)}$  was proposed by matching the electromagnetic form factors of the nucleon in soft-wall AdS/QCD and light-front QCD, getting that

Table 1  
The fitted parameters for nucleon valence  $u$  and  $d$  quarks at the initial scale  $\mu_0$  [5]

Parameter	Value	
	$u$	$d$
$a_1^v$	$0.280 \pm 0.001$	$0.5850 \pm 0.0003$
$b_1^v$	$0.1716 \pm 0.0051$	$0.7000 \pm 0.0002$
$a_2^v$	$0.84 \pm 0.02$	$0.9434^{+0.0017}_{-0.0013}$
$b_2^v$	$0.2284 \pm 0.0035$	$0.6400^{+0.0082}_{-0.0022}$
$\delta^v$	1.0	1.0

Notation:  $v = u, d$  – quarks, while  $a_i^v, b_i^v, a_2^v, b_2^v, \delta^v$  – parameters defined in Eq. (9).

$$\begin{aligned} \varphi_i^{(v)}(x, \mathbf{p}_\perp) = & \frac{4\pi}{\kappa} \sqrt{\frac{\log(1/x)}{(1-x)}} x^{a_i^v} (1-x)^{b_i^v} \times \\ & \times \exp\left[-\delta^v \frac{\mathbf{p}_\perp^2 \log(1/x)}{2\kappa^2 (1-x)^2}\right], \end{aligned} \quad (9)$$

where  $\kappa$  is a scale parameter coming from the soft-wall AdS/QCD model.

With this information, it is possible to write the Dirac and Pauli form factors for spin-1/2 composite particle systems [20].

In Ref. [21] it was found, by fitting the proton form factors from the soft-wall AdS/QCD model with experimental data [22 – 26], that the best agreement is given with  $\kappa = 0.4066$  GeV. Furthermore, in Ref. [5] the flavor form factors for  $u$  and  $d$  in this light-front diquark model was fitted with experimental data [27, 28], obtaining the value of the parameters  $a_i^{(v)}$  and  $b_i^{(v)}$  at the initial scale  $\mu_0$  (see Table 1).

In the same way, using the Sachs form factors, the coefficients for the quark-diquark nucleon state (2) were obtained in Ref. [5]:

$$C_S^2 = 1.3872, \quad C_V^2 = 0.6128, \quad C_{VV}^2 = 1.0.$$

Besides, the normalized constants  $N_i$  were found to be

$$\begin{aligned} N_S &= 2.0191, \quad N_0^{(u)} = 3.2050, \quad N_0^{(d)} = 5.9423, \\ N_1^{(u)} &= 0.9895, \quad N_1^{(d)} = 1.1616. \end{aligned}$$

### Quark-diquark unpolarized PDF evolution

The unpolarized parton distribution function is defined as [8, 5]:

$$\begin{aligned} f^{(v)}(x, \mu_0) = & \frac{1}{2} \int \frac{dz^-}{2(2\pi)} \exp\left(\frac{ip^+ z^-}{2}\right) \times \\ & \times \left\langle P; S \left| \bar{\psi}^{(v)}(0) \gamma^+ \psi^{(v)}(z^-) \right| P; S \right\rangle \Big|_{z^+ = z_T = 0}, \end{aligned} \quad (10)$$

which depends only on the light-cone momentum fraction  $x = p^+/P^+$  where the proton state  $|P; S\rangle$  with spin  $S$  is given as in Eq. (2).

Indeed,  $\gamma^+$  is the light-cone representation of the usual  $\gamma^\mu$  matrix, detailed definition is found in Ref. [15].

The leading order QCD evolution of the unpolarized PDF is given as the standard DGLAP expansion [29, 30, 5]:

$$\begin{aligned} \int_0^1 dx x^n f(x, \mu) = & \\ = & \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{\gamma_n^{(0)}}{2\beta_0}} \int_0^1 dx x^n f(x, \mu_0), \end{aligned} \quad (11)$$

where the anomalous dimension is determined by

$$\gamma_n^{(0)} = -2C_F \left( 3 + \frac{2}{(n+1)(n+2)} - 4 \sum_{k=1}^{n+1} \frac{1}{k} \right) \quad (12)$$

and the running coupling constant is given as

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln\left(\frac{\mu^2}{\Lambda_{\text{QCD}}^2}\right)}. \quad (13)$$

In this work we take  $C_F = 4/3$ ,  $\beta_0 = 9$  and  $\Lambda_{\text{QCD}} = 0.226$  GeV.

The initial scale in most of the works on which ours is based is taken to be  $\mu_0 = 0.313$  GeV since it is a value available for pion phenomenology.

Thus, the light-front diquark unpolarized PDFs at scale  $\mu$  are given by [5]:

$$f^{(S)}(x, \mu) = N_S^2(\mu) \times \left[ \frac{1}{\delta^u(\mu)} x^{2a_1^u(\mu)} (1-x)^{2b_1^u(\mu)+1} + x^{2a_2^u(\mu)-2} (1-x)^{2b_2^u(\mu)+3} \times \frac{\kappa^2}{(\delta^u(\mu))^2 M^2 \ln(1/x)} \right]; \quad (14)$$

$$f^{(A)}(x, \mu) = \left( \frac{1}{3} N_0^{(v)2}(\mu) + \frac{2}{3} N_1^{(v)2}(\mu) \right) \times \left[ \frac{1}{\delta^v(\mu)} x^{2a_1^v(\mu)} (1-x)^{2b_1^v(\mu)+1} + x^{2a_2^v(\mu)-2} (1-x)^{2b_2^v(\mu)+3} \times \frac{\kappa^2}{(\delta^v(\mu))^2 M^2 \ln(1/x)} \right]. \quad (15)$$

The parameters  $a_i^v$ ,  $b_i^v$ ,  $\delta^v$  are now dependent on the scale  $\mu$  such that the relation (11) holds, i. e. [5],

$$a_i^v(\mu) = a_i^v(\mu_0) + A_i^v(\mu), \quad (16)$$

$$b_i^v(\mu) = b_i^v(\mu_0) - B_i^v(\mu) \frac{4C_F}{\beta_0} \ln\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right), \quad (17)$$

$$\delta^v(\mu) = \exp\left[\delta_1^v \left(\ln\left(\frac{\mu^2}{\mu_0^2}\right)\right)^{\delta_2^v}\right] \quad (18)$$

where the quantities  $A_i^v(\mu)$  and  $B_i^v(\mu)$  are defined as

$$\Pi_i^v(\mu) = \alpha_{\Pi,i}^v \mu^{2\beta_{\Pi,i}^v} \left[ \ln\left(\frac{\mu^2}{\mu_0^2}\right) \right]^{\gamma_{\Pi,i}^v} \Big|_{i=1,2} \quad (19)$$

for  $\Pi = A, B$ .

The  $a_i^v(\mu)$  and  $b_i^v(\mu)$  are the parameters given in Table 1. It should be noted that the parameter  $\delta^v$  tends to unity while  $\mu \rightarrow \mu_0$ .

In order to find the evolution parameters  $\alpha_{\Pi,i}^v$ ,  $\beta_{\Pi,i}^v$ ,  $\gamma_{\Pi,i}^v$  and  $\delta^v$  it is useful to write the flavor decomposed PDFs  $f^u(x, \mu)$  and  $f^d(x, \mu)$ . It was well discussed in Ref. [8] that for the relation between quark flavors and diquark states should have a linear behavior with free coefficients to be determinate with experimental data. Indeed, in the same way the proton state (2) has to be consistent with the real world under the same coefficients  $C_S$ ,  $C_V$  and  $C_{VV}$ , which was how the flavored form factors were decomposed from the diquarks, and such parameters founded in Ref. [5].

So, the flavor decomposed PDFs are given as

$$f^u(x, \mu) = C_S^2 f^{(S)}(x, \mu) + C_V^2 f^{(V)}(x, \mu), \quad (20)$$

$$f^d(x, \mu) = C_{VV}^2 f^{(VV)}(x, \mu). \quad (21)$$

Then, the flavored PDF  $f^v(x, \mu)$  in the light-front quark-diquark model can be written as

$$f^v(x, \mu) = N^{(v)} \left[ \frac{1}{\delta^v(\mu)} x^{2a_1^v(\mu)} (1-x)^{2b_1^v(\mu)+1} + x^{2a_2^v(\mu)-2} (1-x)^{2b_2^v(\mu)+3} \times \frac{\kappa^2}{(\delta^v(\mu))^2 M^2 \ln(1/x)} \right], \quad (22)$$

where

$$N^{(u)} = \left( C_S^2 N_S^2 + C_V^2 \left( \frac{1}{3} N_0^{(u)2} + \frac{2}{3} N_1^{(u)2} \right) \right) \quad (23)$$

and

$$N^{(d)} = \left( C_{VV}^2 \left( \frac{1}{3} N_0^{(d)2} + \frac{2}{3} N_1^{(d)2} \right) \right), \quad (24)$$

for  $u$  and  $d$  quarks respectively.

In this work, we have followed the fashion of Ref. [5] and we have obtained the values of the evolution parameters by fitting the flavor PDFs (22) with data from NNPDF2.3 QCD + QED NNLO [11].

The fit was performed in gnuplot [31], an open source plotting tool using non-linear least-square theory, taking first a  $f^v$  depending on parameters  $\Pi_i^v(\mu)$  then getting the evolution parameters  $\alpha_{\Pi,i}^v$ ,  $\beta_{\Pi,i}^v$ ,  $\gamma_{\Pi,i}^v$  and  $\delta^v$ .

The unpolarized PDF data was fitted for 100 equal-spaced data points for different  $x \in (0, 1)$  and  $\mu^2 = 2, 4, 8, 16, 32, 64, 128, 256 \text{ GeV}^2$ .

The fitted parameters for  $\alpha_{\Pi,i}^v, \beta_{\Pi,i}^v, \gamma_{\Pi,i}^v$  are shown in Table 2 while the fitted  $\delta^v$  being shown

in Table 3.

In appendix A we show the different fits performed for the scales mentioned above.

With this data applied to the PDFs (14) and (15), we have drawn the functions  $x \cdot f(x)$  of the isoscalar-scalar diquark and isovector-vector diquark for energy scales  $\mu^2 = 10, 10^2, 10^3$  and  $10^4 \text{ GeV}^2$  shown in Fig. 1,  $a, b, c, d$  respectively. The smooth bands show the case of the scalar diquark, while the checkered bands are for the mentioned vector diquark. It is important to note that

$$\frac{1}{3} N_0^{(u)2} + \frac{2}{3} N_1^{(u)2} \approx N_S^2,$$

from values reported in Ref. [5], so the behavior of the  $f^{(S)}$  (isoscalar-scalar) curve and the  $f^{(V)}$  (isoscalar-vector) one is very similar.

PDF evolution parameters with 95% confidence bounds

Table 2

$\Pi_i^v(\mu)$	$\alpha_i^v$	$\beta_i^v$	$\gamma_i^v$	$\chi^2/d.o.f$
$A_1^u$	$-0.196314 \pm 0.002266$	$-0.197209 \pm 0,010210$	$0.927163 \pm 0.036270$	0.09
$B_1^u$	$6.48940 \pm 0.04592$	$0.161127 \pm 0.006494$	$-0.910813 \pm 0.021850$	0.17
$A_2^u$	$-0.441651 \pm 0,002674$	$-0.0389503 \pm 0,0058020$	$0.306214 \pm 0.019020$	0.995
$B_2^u$	$2.58149 \pm 0.26410$	$-0.0548368 \pm 0.0780600$	$-0.807298 \pm 0,277900$	1.54
$A_1^d$	$-0.119059 \pm 0.002517$	$-0.124819 \pm 0.018800$	$0.952914 \pm 0.060100$	0.27
$B_1^d$	$12.84810 \pm 0.09134$	$0.0976609 \pm 0.006134$	$-0.80035 \pm 0.01510$	0.53
$A_2^d$	$-0.514816 \pm 0.000724$	$-0.001555 \pm 0.001244$	$0.171831 \pm 0.003307$	0.41
$B_2^d$	$1.10727 \pm 0.00703$	$0.0844447 \pm 0.005591$	$-0.57190 \pm 0.01486$	0.03

PDF evolution parameters  $\delta_1^v$  and  $\delta_2^v$  for  $v = u, d$

Table 3

$\delta^v(\mu)$	$\delta_1^v$	$\delta_2^v$	$\chi^2/d.o.f$
$\delta^u$	$0.035074 \pm 0.03009$	$0.48314 \pm 0.06732$	10.50
$\delta^d$	$0.406762 \pm 0.007024$	$0.46990 \pm 0.01275$	3.79

Footnote: in Tabs. 2 and 3 the data was used from NNPDF2.3 QCD+QED NNLO [11].

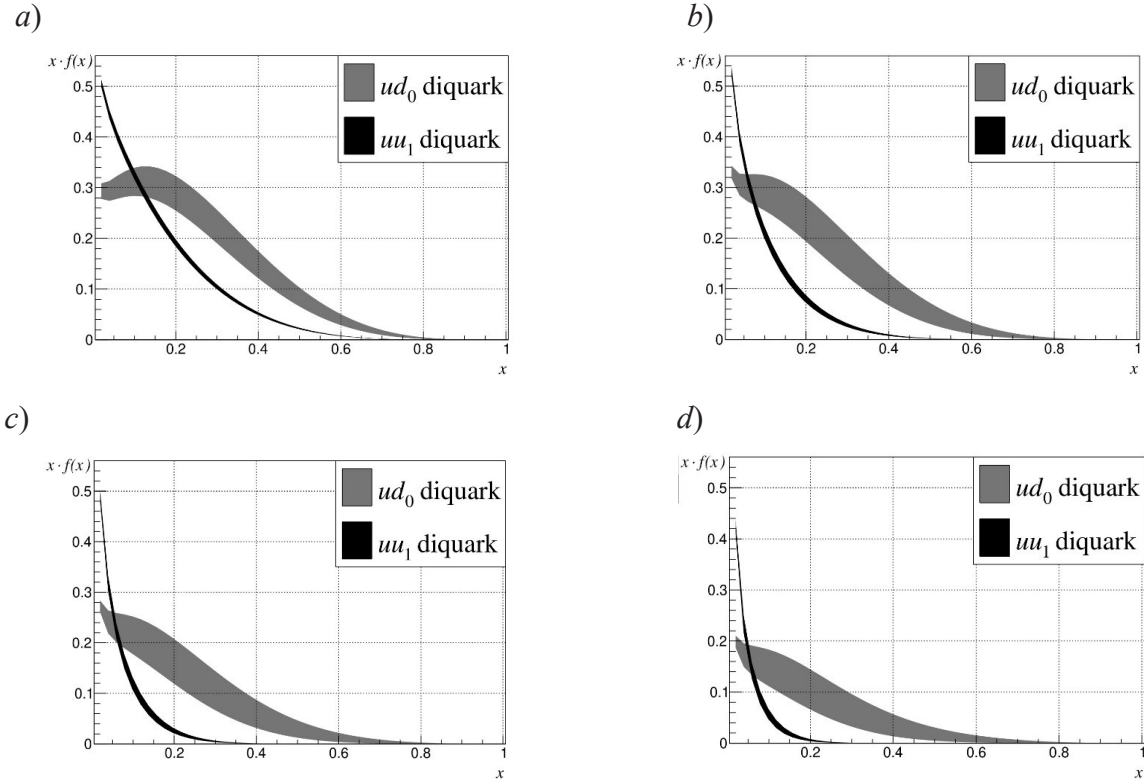


Fig. 1. Graphs of the  $x:f(x)$  functions (diquark PDFs) at different scale energies  $\mu^2$ , (GeV)<sup>2</sup>: 10 (a), 10<sup>2</sup> (b), 10<sup>3</sup> (c), 10<sup>4</sup> (d). The cases of the scalar  $ud_0$  (gray bands) and vector  $uu_1$  (black ones) diquarks are shown

### Conclusions

The soft-wall light front AdS/QCD has allowed us to construct parton distribution functions (PDFs) for diquarks in agreement with the data obtained for quarks phenomenologically. We have particularly taken data from NNPDF2.3 QCD+QED NNLO [11], but the model can be adapted to desired experimental data with  $u$  and  $d$  quark PDF information.

Although the uncertainties for the values in  $\Pi_V^i(\mu)$  reported here should be still improved, an acceptable fit for the functions (14) and (15) is shown in our parameters in Tabs. 2 and 3 looking at  $\chi^2/d.o.f.$

In general terms, the behavior of diquark PDFs observed in Figs. 1 reveals a similarity to the quark PDFs. Such behavior goes in the sense that as the energy scale increases, a shift to  $x = 0$  of the peak of the functions is visible; as well as, while  $x$  approaches 1,  $x f$  tends to vanish exponentially. This fact can be compared with the re-

sults in Ref. [5], where using the same model with NNPDF21(NNLO) [32] data were fitted the  $u$  and  $d$  quark PDFs.

The phenomenological diquark PDFs reported here are intended to be tested within the framework of particle collisions.

Especially for us, it is expected to study the effect in the production of hadrons in collision simulations of the AdS/QCD quark-diquark nucleon model taking into account participant diquarks in hard processes.

### Acknowledgements

B.R. would like to thank to the Secretaría Nacional de Ciencia y Tecnología (Ref. Grant FIN-DECYT/EDUCA CTi 02-2019) of Guatemala for financial support.

Moreover, many thanks to Anatolii Iu. Egorov (HSEP IPNT Peter the Great SPbPU) for many valuable comments and insightful discussions of the conducted studies.



**Appendix**

**Parameter fitting for PDF evolution from NNPDF2.3 QCD+QED NNLO**

The scale evolution of  $A_i^v$  and  $B_i^v$  is parameterized by  $\alpha_i^v$ ,  $\beta_i^v$  and  $\gamma_i^v$ . While  $\delta^v$  is parameterized by  $\delta_1^v$  and  $\delta_2^v$ ,  $f(x, \mu)$  is given by Eq. (22) along with Eqs. (16) – (19). The  $f(x, \mu)$  function depending on the parameters  $A_i^v$ ,  $B_i^v$  and  $\delta^v$  are fitted at 8 different energy scales  $\mu^2$  in Table 4 for  $u$  quark, while the fitted parameters for  $d$  quark are given in Table 5.

Each  $\chi^2/d.o.f$  was evaluated for 100 equally-spaced points for different  $x \in (0, 1)$ . The fitting of the parameters at  $\mu^2 = 2, 4, 8, 16, 32, 64, 128$  and  $256 \text{ GeV}^2$  are shown in Fig. 2 and 3. The data points are extracted from NNPDF2.3 QCD + QED NNLO [11]. It should be noted that the  $\chi^2/d.o.f$  values show that the uncertainty ranges found with the fit are overestimated, this is because for this first instance, uncertainties were not taken from the PDF data of Ref. [11]. An improvement in this fact is expected for future works.

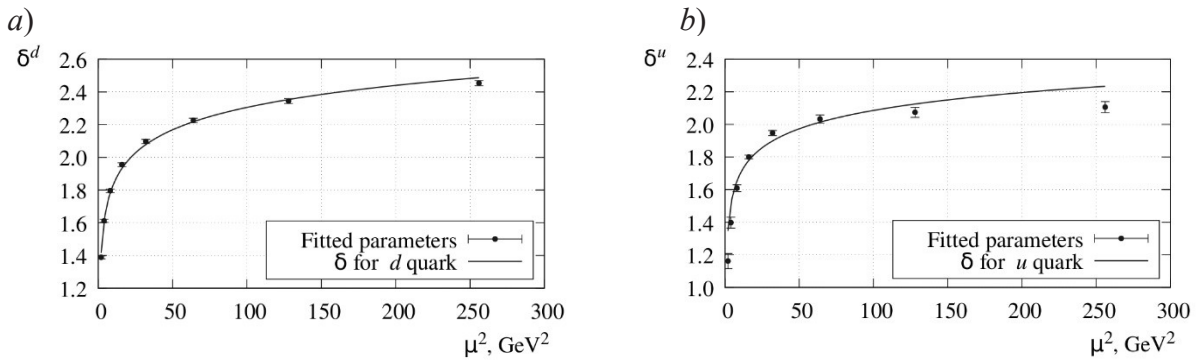


Fig. 2. Plots of  $\delta^u$  (a) and  $\delta^d$  (b) parameters vs. energy scales obtained by fitting the data of Table 3 through varying evolution parameters  $\delta_1^v$  and  $\delta_2^v$  or  $u$  (a) and  $d$  (b) quarks

Table 4

**Fitting of the PDF  $f_1(x)$  at various energy scales for the  $u$  quark**

$\mu^2, \text{GeV}^2$	$A_1^u$	$B_1^u$	$A_2^u$	$B_2^u$	$\delta^u$	$\chi^2/d.o.f$
2.0	$-0.133482 \pm \pm 0.027630$	$9.88657 \pm \pm 0.57650$	$-0.398994 \pm \pm 0.008142$	$2.50897 \pm \pm 0.82540$	$1.16148 \pm \pm 0.04635$	$9.15238e - 06$
4.0	$-0.206116 \pm \pm 0.014570$	$5.97471 \pm \pm 0.18860$	$-0.463197 \pm \pm 0.004770$	$1.64702 \pm \pm 0.29730$	$1.39743 \pm \pm 0.03360$	$2.73019e - 06$
8.0	$-0.257193 \pm \pm 0.006954$	$4.63066 \pm \pm 0.06874$	$-0.508357 \pm \pm 0.002519$	$1.28388 \pm \pm 0.10670$	$1.60899 \pm \pm 0.02080$	$6.44055e - 07$
16.0	$-0.294376 \pm \pm 0.002982$	$3.97296 \pm \pm 0.02359$	$-0.542694 \pm \pm 0.001198$	$1.01137 \pm \pm 0.03058$	$1.80059 \pm \pm 0.01118$	$1.22395e - 07$
32.0	$-0.316551 \pm \pm 0.003503$	$3.63544 \pm \pm 0.02254$	$-0.567223 \pm \pm 0.001527$	$0.774017 \pm \pm 0.020070$	$1.94722 \pm \pm 0.01540$	$1.77367e - 07$
64.0	$-0.325091 \pm \pm 0.005228$	$3.45959 \pm \pm 0.02855$	$0.582866 \pm \pm 0.002362$	$0.620872 \pm \pm 0.018460$	$2.03299 \pm \pm 0.02475$	$4.29130e - 07$
128.0	$-0.325202 \pm \pm 0.006335$	$3.36176 \pm \pm 0.03040$	$-0.592571 \pm \pm 0.002883$	$0.504157 \pm \pm 0.019470$	$2.07403 \pm \pm 0.03057$	$7.16693e - 06$
256.0	$-0.324260 \pm \pm 0.006969$	$3.28934 \pm \pm 0.04935$	$-0.600304 \pm \pm 0.003162$	$0.504157 \pm \pm 0.019470$	$2.10605 \pm \pm 0.03384$	$6.94007e - 06$

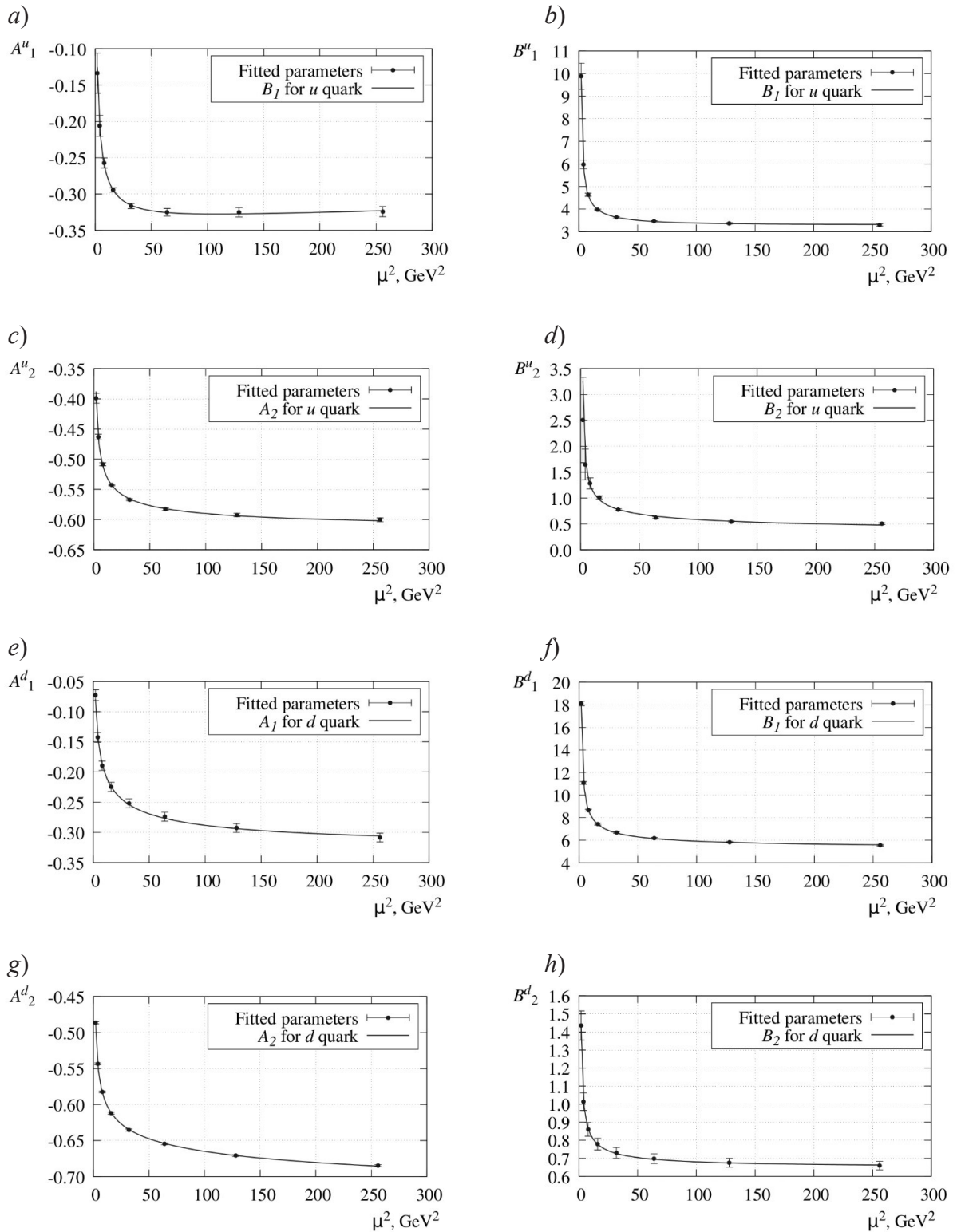


Fig. 3. Plots of  $A_1^u$  (a),  $B_1^u$  (b),  $A_2^u$  (c),  $B_2^u$  (d),  $A_1^d$  (e),  $B_1^d$  (f) and  $A_2^d$  (h) parameters vs. energy scales obtained by fitting the data of Tabs. 4, 5 through using Eq. (19) and varying evolution parameters  $\alpha_{\Pi,i}^v$ ,  $\beta_{\Pi,i}^v$  and  $\gamma_{\Pi,i}^v$  for u (a – d) and d (e – h) quarks

Table 5

Fitting of the PDF  $f_1(x)$  at various scales for the  $d$  quark

$\mu^2, \text{GeV}^2$	$A_1^d$	$B_1^d$	$A_2^d$	$B_2^d$	$\delta^d$	$\chi^2/d.o.f$
2.0	$-0.0726864 \pm 0.0088830$	$18.11530 \pm 0.19740$	$-0.486242 \pm 0.001573$	$1.43562 \pm 0.08108$	$1.388050 \pm 0.008694$	$1.54472e-06$
4.0	$-0.142581 \pm 0.008094$	$11.07900 \pm 0.11820$	$-0.543380 \pm 0.001465$	$1.01348 \pm 0.04888$	$1.612030 \pm 0.009794$	$1.42185e-06$
8.0	$-0.189572 \pm 0.007766$	$8.65931 \pm 0.09223$	$-0.582432 \pm 0.001427$	$0.860135 \pm 0.037990$	$1.79567 \pm 0.01090$	$1.35531e-06$
16.0	$-0.224539 \pm 0.007578$	$7.42345 \pm 0.07912$	$-0.611889 \pm 0.001410$	$0.778533 \pm 0.032410$	$1.95505 \pm 0.01194$	$1.29922e-06$
32.0	$-0.251850 \pm 0.007459$	$6.67682 \pm 0.07126$	$-0.635289 \pm 0.001402$	$0.729967 \pm 0.028960$	$2.09703 \pm 0.01291$	$1.24493e-06$
64.0	$-0.274124 \pm 0.007376$	$6.17642 \pm 0.06598$	$-0.654558 \pm 0.001398$	$0.697843 \pm 0.026600$	$2.22594 \pm 0.01381$	$1.19305e-06$
128.0	$-0.292732 \pm 0.007316$	$5.81723 \pm 0.06219$	$-0.654558 \pm 0.001398$	$0.675414 \pm 0.024860$	$2.34434 \pm 0.01466$	$1.14381e-06$
256.0	$-0.308610 \pm 0.007272$	$5.54758 \pm 0.05934$	$-0.684823 \pm 0.001397$	$0.659191 \pm 0.023530$	$2.45415 \pm 0.01547$	$1.09728e-06$

Secrearía Nacional de Ciencia y Tecnología of Guatemala FINDECYT/EDUCA CTi 02-2019.

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*Received 15.05.2021, accepted 27.05.2021.*

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*Статья поступила в редакцию 15.05.2021, принята к публикации 27.05.2021.*

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