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Static accounting of highest modes in problems of structural dynamics

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Abstract. The calculation of building structures for dynamic effects is usually performed according to the method of decomposition by its own forms of vibrations. However, the problem is that such a method gives an exact solution of the dynamic problem with full consideration of the entire spectrum of modes. Moreover, when solving practical problems with the use of software systems, dynamic calculations are performed approximately taking into account a limited number of the first natural modes of oscillation. The contribution to the dynamic response of the structure of unaccounted higher forms of oscillations, as a rule, is not evaluated at all. The results show that the error of such a solution to a dynamic problem can be significant. Consequently, this paper is devoted to the method of static registration of higher forms of oscillations in problems of the dynamics of building structures. The description of the main provisions of the method is given, examples of its implementation in the calculation of spatial structures under the action of an external harmonic load are given. With the help of a computational program complex, the displacements of nodes and internal forces in the elements of the structures under consideration are determined. Various parameters of the dynamic effect and the number of vibration modes taken into account were set. The adopted method of static accounting for higher forms of oscillations requires solving one dynamic problem and two auxiliary static problems. An important circumstance of the approach is that one of the static problems should be solved by the method of decomposition in its own forms of vibrations. The approach proposed in the article allows to significantly reduce the computational costs of dynamic calculation in comparison with the classical approach. This result can be of great importance when solving problems for complex dynamic effects and for structures that are not uniform in hardness.

1. Introduction

Buildings and structures in the process of their construction and operation are affected by various dynamic loading, such as wind load, earthquake, operation of process equipment, impact loads, emergency destruction of a structural element, and so on. Many studies are devoted to the development of constructive solutions for buildings and their elements, methods for calculating under the dynamic loads [1–15]. In depends on the reasons, that led to the oscillations of the structure, its work can be viewed as a superposition of oscillations according to its modes [16–24]. Traditionally, in the dynamic calculation of building structures, only the first few modes of natural oscillations are taken into account, since the contribution of precisely these terms to displacements and internal efforts is the main one. It is rather difficult to take into account the other modes, since the resources of computing devices are limited, and there is no need to include in the calculation higher modes with low energy value. This fact is confirmed by large in number (numerous) studies, for example, in the works [25–28].

However, in some cases, the calculation of buildings and structures on the dynamic load may require consideration of the contribution of higher modes of oscillations. The need for this may arise, for example, when calculating a structure for seismic load [29–34]. In these books are considered the applied and prospective approaches to the calculation of dynamic interaction, the basic principles of seismic design and key aspects of calculations. In the article [35], a study of the vibrations of a building structure was carried out

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and it was concluded that, as the frequency increases, the more rigid and (or) less significant parts of the system will be included in the oscillatory process corresponding to highest modes, while some parts of the structure can be excluded from movement. All these facts lead to the destruction of the elements and, as a result, of the whole system.

In the conditions of a real design process, when continuous changes are made to the project, editing the design scheme and recalculating it can take a significant amount of time and resources. Therefore, there are important and topical questions to reduce the number of modes taken into account in the calculation without losing accuracy of the result, as well as the contribution of the discarded modes to the local high-frequency oscillations of individual elements. Thus, the work [36] is devoted to the analysis of the accuracy of determining the response of a structure to a dynamic effect when taking into account a limited number of modes.

In the article [37], a concept was entered, according to which it is proposed to add an estimate of the total response for all high-frequency modes, determined from a static load, to the standard dynamic calculation. It is noted that with an increase in the number of modes taken into account, the convergence of the result in efforts is lower than in displacements, therefore the inclusion of highest modes can significantly affect local efforts.

The question of estimating the contribution of the rejected tones of vibrations for mechanical systems was also considered in [38], where the authors propose to add to the calculation the correction obtained when solving the auxiliary task of statics. This has greatly reduced the number of modes taken into account to achieve the required accuracy.

In addition, the issue of introducing a quasistatic correction to the dynamic calculation to improve the convergence of the results was considered in [39, 40]. The authors proposed to introduce as an addition the quasistatic component of unrecorded modes of oscillations, which is defined as the difference between the solution of the quasistatic problem and the quasistatic component of the modes of oscillations taken into account in the calculation.

The method of static accounting for the highest vibration modes was proposed in [41–43] for determining the natural vibration frequencies. In our work, it is extended to problems of forced oscillations and involves solving a dynamic problem by the method of spectral expansion in a series of modes with a small number of modes taken into account, as well as solving auxiliary static problems in an exact and approximate formulation.

The purpose of this work is to solve the problem of the dynamics of forced oscillations, taking into account the contribution of the highest modes. The following tasks are solved:

1. The theoretical description of the method under consideration and the formulation of auxiliary problems.

2. Construction of design schemes of spatial structures and determination of their stress-strain state from the action of dynamic and static loads.

3. Comparison of the results obtained in solving the problem of dynamics by the classical method and the method of static accounting for the contribution of the highest modes of oscillations.

2. Methods

Let us consider the standard way to solve a dynamic task. Let the system undergo forced oscillations, its equation of motion is:

$$\rho\ddot{u} = L(u) + q, \tag{1}$$

where $u(x, t)$ is the desired displacement;

ρ is the density of system elements;

q is external dynamic load;

$L(u)$ is an operator of a static task, depending on the nature of the work of the structure;

It can be demonstrated the following examples of the mode of the operator of the static task $L(u)$:

a) $L(u) = EAu''$ is for tasks in tension and compression;

b) $L(u) = -EIu^{IV}$ is for the task on bending rods;

c) $L(u) = -D(\partial^4 u / \partial x^4 + 2\partial^4 u / \partial x^2 \partial y^2 + \partial^4 u / \partial y^4)$ is for tasks of plate bending;

Let the external harmonic load $p(t) = P_0 \sin(\theta t)$ acts on the system, then the equation of motion will look like (2):

$$L(u)+\rho\theta^2 u+P_0=0. \quad (2)$$

The desired displacement is determined using the method of spectral decomposition in a row according to the modes of natural oscillations by the formula (3):

$$u_n(x)=\sum_{k=1}^{n} a_k U_k(x), \quad (3)$$

where n is the number of considered modes;

a_k is the amplitude value of the k-mode $U_k(x)$;

Thus, the solution of a dynamic problem by the formula (3) implies the choice of such a number n of the system's natural oscillations taken into account, which will be enough to find the desired displacement with the necessary accuracy. This number can be quite large, which will lead to a significant investment of time and resources.

Therefore, it is possible to approach the solution of this problem in another way: take in the formula (3) a small number of the terms $N(N << n)$, and the rest (higher modes of oscillations) to take into account in the calculation statically.

To do this, the first step is to consider the solution of the auxiliary static task from the action of the static force P_0. The exact static displacement is determined by solving the differential equilibrium equation (4):

$$L(u)+P_0=0. \quad (4)$$

The next step is to solve another auxiliary static problem by using the method of spectral decomposition in a row according to the modes of natural oscillations. Similar to solving a dynamic problem, you can write an expression for this static task in the next mode (5):

$$u_{n,st}(x)=\sum_{k=1}^{n} b_k U_k(x). \quad (5)$$

Having solved auxiliary static problems using formulas (4) and (5), one can find the desired displacement of a dynamic problem when taking into account a small number N modes of natural oscillations in the following formula (6):

$$u(x)=u_N(x)+\left[u_{ex,st}(x)-u_{N,st}(x)\right], \quad (6)$$

where $u_N(x)$ is the solution of a dynamic task by the formula (3);

$u_{ex,st}(x)$ is the exact solution of the static task by the formula (4);

$u_{N,st}(x)$ is an approximate solution of the static problem by the formula (5).

Thus, the difference $[u_{ex,st}(x)-u_{N,st}(x)]$, obtained in formula (6), is the static contribution of the rejected natural oscillations in formula (3). Figure 1 shows a graphical illustration of the use of this approach to solving problems.

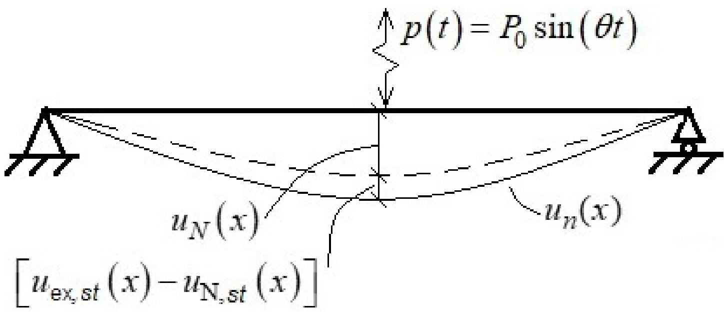

Figure 1. Beam deflection when solving a dynamic problem with a static account of the contribution of highest modes.

As is well known, the movement of a point mass with steady-state oscillations has the formula (7):

$$u(t) = u_{st}\beta(t), \tag{7}$$

here u_{st} is the static deflection of the beam at the location of the point mass from the statically applied force P_0; $\beta(t)$ is the dynamic coefficient equal to the ratio of the dynamic displacement $u(t)$ to the static displacement u_{st} at a given time.

If an external harmonic load acts on the system, then the maximum value of the coefficient β in case of forced damped oscillations can be found using formula (8):

$$\beta = \frac{1}{\sqrt{\left(1-\frac{\theta^2}{\omega^2}\right)^2 + \gamma_{in}^2\frac{\theta^2}{\omega^2}}}. \tag{8}$$

As noted above, the auxiliary static task should be solved by the method of expansion in a row in the modes of natural oscillations. For this, a static calculation is carried out as a solution to a dynamic problem with an external load in the mode $p(t) = P_0 \sin(\theta t)$, where a very small value of the angular frequency is specified. From formula (8), $\beta \to 1$ as $\theta \to 0$ and according to (1) the solution of the dynamic problem will tend to solve statically.

3. Results and Discussion

In order to verify the method of static accounting of the highest modes of oscillations, two calculation schemes were built in the SCAD Office 21.1 software package.

2.1. The space frame

The system consists of rod finite elements (Figure 2). The overall dimensions of the design scheme are 12×10×14 m. The lower nodes of the system are rigidly clamped. The material is B25 concrete, the dimensions of the cross sections of the elements are 20×20, 35×40, 50×50 cm × cm (Figure 3). The harmonic dynamic load $p(t) = P_0 \sin(\theta t)$ is applied at node 15 in the X direction. The amplitude of the force P_0 is assumed to be 100 T, the angular frequency was set in two variants: θ_1 = 8 rad/s and θ_2 = 13 rad/s. According to the results of the modal analysis, the first, second and third natural frequencies (eigenfrequencies) of the system are respectively 10.7, 11.4 and 15.5 rad/s. For reinforced concrete structures adopted the coefficient of inelastic resistance of the material γ_{in} = 0.09.

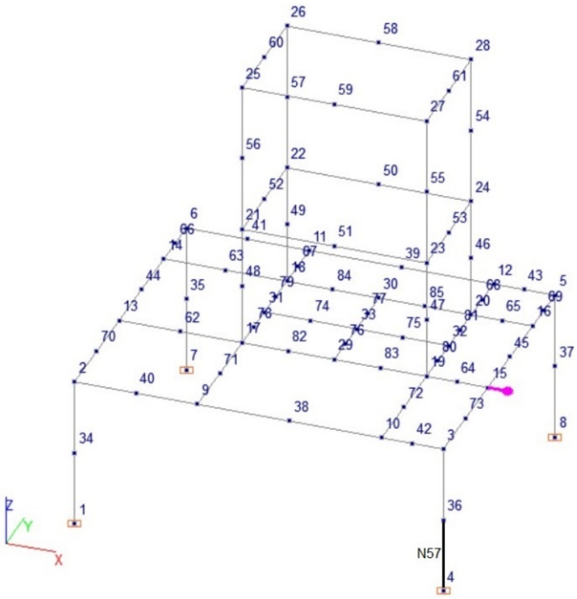

Figure 2. Spatial frame scheme.

Figure 3. Stiffness of scheme elements.

The movement of nodes 2 and 25 in the X direction is considered, as well as the bending moment relative to the Y axis in the N57 element (in Figure 2, it is highlighted with a thick line). The result of the calculation of the frame on the dynamic load is shown in Table 1.

Table 1. Dynamic frame calculation.

Number of the mode	θ = 8 rad/s			θ = 13 rad/s		
	u_2, mm	u_{25}, mm	M_{57}, T·m	u_2, mm	u_{25}, mm	M_{57}, T·m
1	–2.535	–148.860	–5.672	–2.368	139.011	–5.296
2	–2.563	–149.577	–5.841	–2.396	139.730	–5.529
3	–2.744	–158.363	–6.234	–1.942	117.410	–4.544
5	–41.725	–118.532	–99.719	–47.466	164.646	–113.912
7	–48.053	–116.483	–113.526	–54.400	166.908	–128.890
10	–48.909	–125.380	–115.641	–55.327	157.519	–131.180
15	–49.291	–125.775	–116.287	–55.732	157.101	–131.877
30	–45.554	–126.341	–108.006	–51.945	156.542	–123.485
60	–45.372	–126.000	–106.461	–51.763	156.879	–121.933
100	–45.332	–126.004	–106.378	–51.723	156.874	–121.849
140	–45.373	–126.005	–106.620	–51.763	156.874	–122.091
180	–45.373	–126.005	–106.620	–51.763	156.874	–122.091
220	–45.373	–126.005	–106.619	–51.763	156.874	–122.090
250	–45.373	–126.005	–106.619	–51.763	156.874	–122.090

As can be seen, the desired values cease to change significantly already taking into account the 60 modes.

Further, the auxiliary static problem was exactly solved by formula (4) from a force equal to the amplitude of the force P_0. Exact static displacements of nodes 2 and 25 are –39.203 and –44.940 mm respectively, and the bending moment in the element N57 is 92.139 T·m.

The next step is also the static problem was solved by the dynamic method at the frequency of the forcing force θ = 0.0001. As shown in the previous paragraph, at such a frequency, the solution of the dynamic problem should practically coincide with the solution of the static problem. The results of the calculation with a different number of considered modes are given in Table 2.

From Table 2 it can be seen that the method of solving a static problem proposed in this paper by solving a dynamic problem with a very low frequency of the forcing force really converges to an exact solution to the static problem with an increase in the number of vibrational modes taken into account.

Table 2. Static calculation of the frame through modes of oscillations.

Number of modes	u_2, mm	u_{25}, mm	M_{57}, T·m
1	–1.138	–66.839	–2.547
2	–1.151	–67.166	–2.630
3	–1.278	–73.316	–2.904
4	–35.924	–37.643	–85.993
5	–35.895	–37.660	–85.924
6	–36.095	–36.962	–85.183
8	–42.687	–43.642	–101.025
10	–42.721	–44.329	–101.112
12	–43.129	–45.372	–102.213
14	–41.300	–44.713	–97.782
16	–41.893	–44.609	–99.060
200	–39.203	–44.940	–92.139
250	–39.203	–44.940	–92.139

As follows from Table 1, a practically exact solution to the problem of dynamics was obtained when taking into account 140 modes of natural oscillations. In Table 3, this exact solution is compared with the solution proposed by the proposed method, taking into account 5 modes.

Table 3. Frame calculation results.

No.	Solution	θ = 8 rad/s			θ = 13 rad/s		
		u_2, mm	u_{25}, mm	M_{57}, T·m	u_2, mm	u_{25}, mm	M_{57}, T·m
(1)	Dynamic Solution with N = 5	–41.725	–118.532	–99.719	–47.466	164.646	–113.912
(2)	Dynamic Solution with N = 10	–48.909	–125.380	–115.641	–55.327	157.519	–131.180
(3)	Dynamic Solution with N = 15	–49.291	–125.775	–116.287	–55.732	157.101	–131.877
(4)	Dynamic Solution with N = 30	–45.554	–126.341	–108.006	–51.945	156.542	–123.485
(5)	Static exact solution	–39.203	–44.94	–92.140	–39.203	–44.940	–92.140
(6)	Static solution with N = 5	–35.895	–37.660	–85.924	–35.895	–37.660	–85.924
(7)	Static contribution of higher modes	–3.308	–7.280	–6.216	–3.308	–7.280	–6.216
(8)	Dynamic solution with static accounting mode with N = 5	–45.033	–125.812	–105.935	–50.774	157.366	–120.128
(9)	Dynamic exact solution	–45.373	–126.005	–106.619	–51.763	156.874	–122.090

Analysis of the results shown in Table 3 allows us to find the errors of various methods. These errors are given in Table 4, where the following is indicated: Δ is absolute error; ε is relative error.

Table 4. Comparison of results between different methods.

Difference between	Type	θ = 8 rad/s			θ = 13 rad/s		
		u_2, mm	u_{25}, mm	M_{57}, T·m	u_2, mm	u_{25}, mm	M_{57}, T·m
(8) and (9)	Δ	–0.340	–0.193	–0.684	–0.989	–0.492	–1.962
	ε, %	0.75	0.15	0.64	1.91	0.31	1.61
(1) and (9)	Δ	3.648	7.473	6.900	4.297	7.772	8.178
	ε, %	8.04	5.93	6.47	8.30	4.95	6.70
(2) and (9)	Δ	–3.536	0.625	–9.022	–3.564	0.645	–9.090
	ε, %	7.79	0.50	8.46	6.89	0.41	7.45
(3) and (9)	Δ	–3.918	0.230	–9.668	–3.969	0.227	–9.787
	ε, %	8.64	0.18	9.07	7.67	0.14	8.02
(4) and (9)	Δ	–0.181	–0.336	–1.387	–0.182	–0.332	–1.395
	ε, %	0.40	0.27	1.30	0.35	0.21	1.14

Analysis of the results in Table 4, allows to formulate the following conclusions:

1. The use of the method of static accounting of the highest forms of oscillations can significantly improve the accuracy of the results both in terms of displacements and in terms of internal efforts, compared to the standard method with comparable computational costs. For example, the standard method when accounting for 10 modes gives an error of 7–8 %; the proposed method when accounting for 5 modes gives an error of 1–2 %; at the same time, in the proposed method it is necessary to solve the dynamic problem twice, taking into account 5 modes, which is less in computational costs than one solution with 10 modes.

2. The standard method only when taking into account 30 modes gives errors comparable to the errors of the proposed method when taking into account 5 modes. Thus, the proposed method can significantly reduce computational costs to achieve the same accuracy of the solution.

4. The bendable plate

The plate consists of plates (Figure 4). The overall dimensions of the design scheme are 4×5 m. The right and left nodes of the system are rigidly clamped. The material is concrete B25, slab thickness is 15 cm. Forcing dynamic load $p(t) = P_0 \sin(\theta t)$ is applied at node N68 in the Z direction. The amplitude of the forcing force P_0 is assumed to be 100T, the angular frequencies were also set in two variants: θ_1 = 110 rad/s and θ_2 = 150 rad/s. According to the results of the modal analysis, the first, second and third natural frequencies of the system are 129.9, 170.2 and 323.3 rad/s. For reinforced concrete structures adopted the coefficient of inelastic resistance of the material γ_{in} = 0.09.

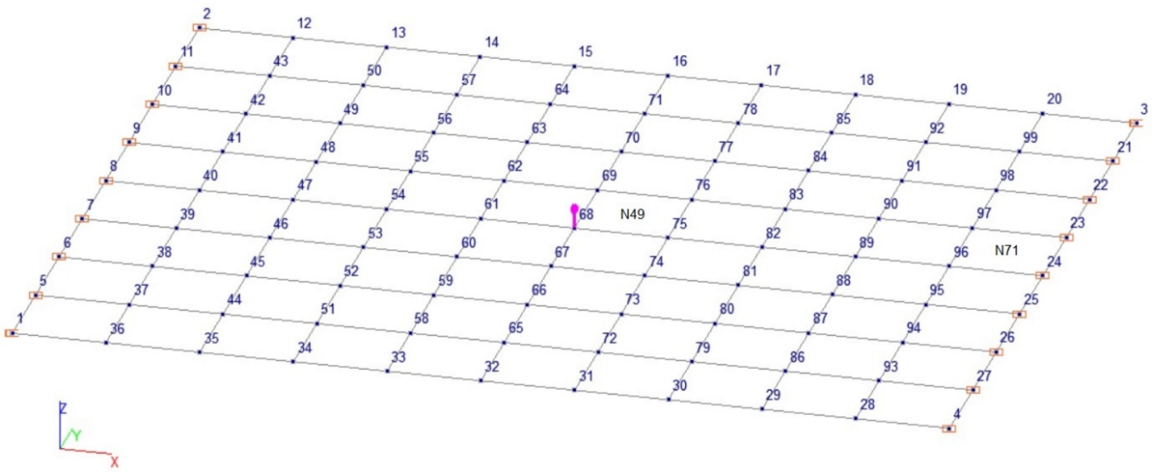

Figure 4. Plate design scheme.

In this case, the vertical movement of the node 68, the bending moment in the N49 element and the shear force in the N71 element (Figure 4) were considered. The result of the calculation of the plate on the dynamic load is given in Table 5.

Table 5. Dynamic plate calculation.

Number of modes	θ = 110 rad/s			θ = 150 rad/s		
	u_{68}, mm	M_{49}, T·m/m	Q_{71}, T/m	u_{68}, mm	M_{49}, T·m/m	Q_{71}, T/m
1	–56.892	32.970	–52.300	48.060	–27.854	44.118
2	–56.892	32.973	–52.226	48.060	–27.854	44.118
3	–61.325	31.318	–61.721	43.304	–24.278	34.091
5	–61.325	36.318	–61.721	43.304	–24.278	34.091
7	–61.325	36.318	–61.721	43.304	–24.278	34.090
10	–61.742	37.799	–56.096	42.896	–22.843	39.515
15	–62.584	39.725	–50.689	42.079	–21.020	44.834
30	–62.883	40.976	–56.467	41.791	–19.859	39.182
60	–63.228	41.663	–54.094	41.459	–19.229	41.485
100	–63.352	41.695	–54.401	41.340	–19.200	41.185
140	–63.352	41.695	–54.403	41.340	–19.200	41.184
180	–63.352	41.695	–54.402	41.340	–19.200	41.184
220	–63.352	41.695	–54.402	41.340	–19.200	41.184
240	–63.352	41.695	–54.402	41.340	–19.200	41.184

As can be seen, the desired values cease to change significantly already taking into account 100 modes.

The exact static displacement of the node 68 from the amplitude forcing force of 100 T is 22.793 mm, the bending moment in the element N49 is 18.155 T·m/m, the lateral force at the node N71 is 16.624 T·m/m.

The next step is also the static problem was solved by the dynamic method at the frequency of the forcing force θ = 0.0001 rad/s. As was proved in the previous paragraph, at such a frequency, the solution of the dynamic problem practically coincides with the solution of the static task.

As follows from Table 5, a practically exact solution of the dynamics problem was obtained with account of 100 modes of natural oscillations. In Table 6, this exact solution is compared with the solution proposed by the proposed method, taking into account 5 modes.

Table 6. The results of the calculation of the plate.

No.	Solution	θ = 110 rad/s			θ = 150 rad/s		
		u_{68}, mm	M_{49}, T·m/m	Q_{71}, T/m	u_{68}, mm	M_{49}, T·m/m	Q_{71}, T/m
(1)	Dynamic Solution with N = 10	–61.742	37.799	–56.096	42.896	–22.843	39.515
(2)	Dynamic Solution with N = 15	–62.584	39.725	–50.689	42.079	–21.020	44.834
(3)	Dynamic Solution with N = 30	–62.883	40.976	–56.467	41.791	–19.859	39.182
(4)	Dynamic Solution with N = 60	–63.228	41.663	54.094	41.459	–19.229	41.485
(5)	Static exact solution	–22.793	18.155	–16.624	–22.793	18.155	–16.624
(6)	Static solution with N = 5	–20.732	12.717	–23.931	–20.732	12.717	–23.931
(7)	Static contribution of higher modes	–2.061	5.438	7.307	–2.061	5.438	7.307
(8)	Dynamic solution with static accounting mode with N = 5	–63.386	41.756	–54.414	41.243	–18.840	41.398
(9)	Dynamic exact solution	–63.352	41.695	–54.402	41.340	–19.200	41.184

Analysis of the results shown in Table 6 allows us to find the errors of various methods. These errors are listed in Table 7, where: Δ is the absolute error; ε is relative error.

Analysis of the results shown in Table 7, allows to formulate the following conclusions:

1. The use of the method of static accounting of the highest modes of oscillations can significantly improve the accuracy of the results both in terms of displacements and in terms of internal efforts, compared to the standard method with comparable computational costs. For example, the standard method when accounting for 15 modes gives errors of 2–10 %; the proposed method when accounting for 5 modes gives an error of 0.02–2 %; at the same time, in the proposed method it is necessary to solve the dynamic problem twice, taking into account 5 modes, which is less in computational costs than one solution with 15 modes.

2. The standard method only when accounting for 60 modes gives an error comparable to the error of the proposed method when accounting for 5 modes. Thus, the proposed method can significantly reduce computational costs to achieve the same accuracy of the solution.

Table 7. Comparison of results between different methods.

Difference between	Type	θ = 110 rad/s			θ = 150 rad/s		
		u_{68}, mm	M_{49}, T·m/m	Q_{71}, T/m	u_{68}, mm	M_{49}, T·m/m	Q_{71}, T/m
(8) and (9)	Δ	–0.034	–0.061	–0.012	0.097	0.360	0.214
	ε, %	0.05	0.15 %	0.02	0.23	1.88	0.52
(1) and (9)	Δ	1.610	–3.896	–1.694	1.556	–3.643	–1.669
	ε, %	2.54	9.34	3.11	3.76	18.97	4.05
(2) and (9)	Δ	0.768	–1.970	3.713	0.739	–1.820	3.650
	ε, %	1.21 %	4.72	6.83	1.79	9.48	8.86
(3) and (9)	Δ	0.469	–0.719	–2.065	0.451	–0.659	–2.002
	ε, %	0.74	1.72	3.80	1.09	3.43	4.86
(4) and (9)	Δ	0.124	0.032	0.308	0.119	0.029	0.301
	ε, %	0.20	0.08	0.57	0.29	0.15	0.73

5. Conclusions

1. In this paper proposed a method for solving a dynamic problem from the action of a harmonic load on structures, based on a static account of higher modes of oscillation. The method requires solving two dynamic problems and one static task.

2. As shown in the examples, the proposed method gives a high accuracy of the solution at lower computational costs compared with the standard method of decomposition in its own modes of oscillation. High accuracy of the solution is obtained not only by movements, but also by efforts.

3. Solving a dynamic problem with a static account of the highest modes of oscillations can significantly save computation time and computer resources, since it is necessary to solve one dynamic problem with a small number of modes and two static tasks, instead of solving one dynamic problem with a large number of modes taken into account in the calculation. As is known, the relationship between the number of forms taken into account and computational costs is non-linear. With a decrease in the dimension of the problem, its solution is an order of magnitude easier and faster.

4. In the future, it is planned to extend the method of static accounting of the highest modes of oscillations to tasks with an arbitrary time-dependent load, as well as to problems of calculating structures from seismic effects.

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Статический учет высших мод колебаний в задачах динамики конструкций

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Ключевые слова: динамика сооружений, спектральный метод, моды колебаний, высшие формы, статический учет, пространственные конструкции

Аннотация. Расчёт строительных конструкций на динамические воздействия обычно выполняется по методу разложения по собственным формам колебаний. Однако, проблема состоит в том, что такой метод дает точное решение динамической задачи при полном учёте всего спектра мод. Более того, при решении практических задач с использованием программных комплексов динамические расчеты выполняются приближенно с учетом ограниченного количества первых собственных форм колебаний. Вклад в динамическую реакцию сооружения неучтенных высших форм колебаний, как правило, никак не оценивается. Результаты показывают, что, погрешность такого решения динамической задачи может оказаться значительной. Следовательно, настоящая работа посвящена способу статического учета высших форм колебаний в задачах динамики строительных конструкций. Приведено описание основных положений метода, даны примеры его реализации при расчете пространственных конструкций под действием внешней гармонической нагрузки. С помощью расчетного программного комплекса определены перемещения узлов и внутренние усилия в элементах рассматриваемых конструкций. Задавались различные параметры динамического воздействия и число учитываемых мод колебаний. Принятый метод статического учета высших форм колебаний требует решения одной динамической задачи и двух вспомогательных статических задач. Важным обстоятельством подхода является то, что одна из статических задач должна быть решена методом разложения по собственным формам колебаний. Предлагаемый в статье подход позволяет значительно снизить вычислительные затраты на динамический расчёт в сравнении с классическим подходом. Этот результат может иметь большое значение при решении задач на сложные динамические воздействия и для неоднородных по жесткости конструкций.

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