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Modelling of contact interaction of structures with the base under dynamic loading

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Abstract. Constructively nonlinear problems with unilateral constraints are frequent in the calculation of various structures. At the same time, certain difficulties cause problems with the contact friction, as well as with the dynamic action of the load. In such cases, the contact problem becomes more complicated in terms of mathematics and its numerical solution becomes more complicated as well. This article is devoted to the construction of calculation models and methods for solving problems with non-ideal unilateral constraints under dynamic loading. As a result, a numerical algorithm has been developed based on the finite element model of contact and the step-by-step analysis method, which allows simultaneous integration of the motion equations and the realization of contact conditions with Coulomb friction. At the same time, to comply with the limitations under the conditions of ultimate friction-sliding, the method of compensating loads is applied. Using the proposed approach, numerical solutions of some problems of contact of a structure with a base have been obtained and analyzed. The reliability of the calculation results is confirmed by comparing them with the solution obtained by the alternative iteration algorithm. It can be concluded that the step-by-step analysis algorithm is more efficient in terms of computation time, showing satisfactory convergence, stability, and accuracy of the solution in a fairly wide range of time integration steps.

1. Introduction

Problems with unilateral constraints and friction between contacting surfaces are often encountered in the calculation of various types of structures. The solution of such problems under the action of static loads and different contact conditions was considered in some works [1–12]. At the same time, it is quite typical when it is necessary to take into account the dynamic loads on the structure [13–15].

When solving constructively nonlinear contact problems, both iterative (successive approximations) and incremental (step-by-step) methods are used. In particular, for the numerical solution of the dynamic contact problem, the variant of the iterative Schwartz method (with the finite-element discretization of the problem) is proposed in works [16, 17]. In works [18, 19] the iterative algorithm of the speeds correction of the Udzawa type is used for the finite-difference discrete model. In works [20–22] different versions of the method of iterations on the ultimate friction forces are applied on the basis of variational formulations of contact problems. As mentioned in these and other works, the application of the method of successive approximations allows one to build effective iterative algorithms that have a computational stability and guarantee the fulfillment of contact conditions for ideal unilateral constraints. However, the fulfillment of friction-sliding conditions on the contact here has certain difficulties and cannot always be realized.

In [23, 24], to fulfill the contact conditions, the weak formulation in the form of an elliptic quasi-variational inequality is used. The numerical solution of the variational problem is based on the finite element method and the implicit time integration scheme. The construction of various schemes for the numerical integration of the equations of motion is considered in works [25, 26]. In the first work the Lagrange multipliers and the minimum work principle are used at each time step, in the second one, the non-convex linear

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complementarity problem is solved at each step. The advantage of the step-by-step methods is that the solution of the contact problem can be obtained at any time point and at any stage of dynamic impact by using them. At the same time, there is an opportunity to satisfy the friction-sliding conditions more precisely, since the solution of the problem taking friction into account depends on the history of structure loading [1, 2]. The constructive nonlinearity in the case of dynamic loading will be manifested in the change of the working schemes of the structure in time – switching on and off unilateral constraints, both in normal and tangential direction. It is assumed that between two consecutive events on the contact, i.e. within the limits of each fixed working scheme, the character of the structure deformation is linear.

This paper is devoted to the development of a numerical model and algorithm for solving contact problems with unilateral constraints and friction under the dynamic load action. The immediate solution of the dynamic contact problem is made on the basis of time discretization using the direct schemes of integration of the equations of motion [27]. After each time step, the boundary conditions on the contact are checked. If within a certain step Δt there is a change of the working scheme, the time point of changing of the contact state (occurrence of the next event) is determined by the means of the step-by-time analysis of the contact state with the use of appropriate approximating expressions for displacements, speeds and accelerations on the time interval Δt. In this case, the integration step size is corrected and the current step is recalculated. As a result, a new state of contact is established at the given time point and, thus, the current working scheme of the structure is changed [7, 10]. Based on the step-by-step analysis of the dynamic loading process, the given approach has a clear physical nature and makes it possible to track the current state of the calculated system at any time point.

2. Methods

Let's consider the dynamic problem of interaction of linearly elastic bodies V^+ and V^-, which may be, for example, the structure and base, with contacting surfaces, S_c^+ and S_c^- correspondingly. To calculate this system we use a discrete computational model FEM, for which the following matrix equation of motion with initial conditions is true [27]:

$$\left[M\right]\left\{\ddot{U}^{\,t+\Delta t}\right\}+\left[C\right]\left\{\dot{U}^{\,t+\Delta t}\right\}+\left[K\right]\left\{U^{\,t+\Delta t}\right\}=\left\{P^{\,t+\Delta t}\right\};\ \ \left\{U\right\}\Big|_{t=0}=\left\{U_0\right\},\ \ \left\{\dot{U}\right\}\Big|_{t=0}=\left\{\dot{U}_0\right\}. \tag{1}$$

Taking into account that the displacement at time point $t + \Delta t$ can be represented as $U^{t+\Delta t}=U^{t}+\Delta U^{t+\Delta t}$, let us convert equation (1) in the form that allows the solution of a constructively nonlinear dynamic contact problem to be reduced to the solution of the sequence of linear dynamic problems on the basis of step-by-step on time analysis of the contact state [28]:

$$\left[M\right]\left\{\ddot{U}^{\,t+\Delta t}\right\}+\left[C\right]\left\{\dot{U}^{\,t+\Delta t}\right\}+\left[K\right]\left\{\Delta U^{\,t+\Delta t}\right\}=\left\{P^{\,t+\Delta t}\right\}-\left[K\right]\left\{U^{\,t}\right\};\ \ \left\{U\right\}\Big|_{t=0}=\left\{U_0\right\},\ \ \left\{\dot{U}\right\}\Big|_{t=0}=\left\{\dot{U}_0\right\}. \tag{2}$$

Here $[M]$, $[C]$ and $[K]$ are the mass, damping, and stiffness matrices of the finite element system respectively, at the same time, the proportional damping model is adopted $\left[C\right]=\alpha\left[M\right]+\beta\left[M\right]$; $\left\{U^{t+\Delta t}\right\}$, $\left\{\dot{U}^{t+\Delta t}\right\}$, $\left\{\ddot{U}^{t+\Delta t}\right\}$ and $\left\{P^{t+\Delta t}\right\}$ are the vectors of nodal displacements, speeds, accelerations, and the external nodal load at time point $t + \Delta t$; $\left\{\Delta U^{t+\Delta t}\right\}$ is the vector of displacements increment at step Δt. In addition, on the part of the outer boundaries $S_g^{\pm}$, for V^+ and V^- correspondingly, boundary conditions on the forces, and on $S_u^{\pm}$ – in the displacements should be given. It is assumed that at the initial time $t = 0$ the vectors of displacements, speeds and accelerations are given and it is necessary to find a solution (2) during the time interval from 0 to some value T.

For the numerical integration of the motion equations (2), the implicit Newmark finite difference scheme is used, which is based on the assumption of a linear change of accelerations in the Δt interval. In this case, the following dependencies between the increments of displacements, speeds and accelerations for the time point $t + \Delta t$ are used:

$$\ddot{U}^{t+\Delta t}=\frac{1}{\alpha\,(\Delta t)^2}\left(\Delta U^{t+\Delta t}-\Delta t\dot{U}^{t}\right)-\left(\frac{1}{2\alpha}-1\right)\ddot{U}^{t};\quad \dot{U}^{t+\Delta t}=\dot{U}^{t}+\left((1-\beta)\ddot{U}^{t}+\beta\ddot{U}^{t+\Delta t}\right)\Delta t. \tag{3}$$

Here α and β are the parameters determining the accuracy and stability of integration [27]. Let us take $\alpha = 1/4$, $\beta = 1/2$, which correspond to the case of the constant average acceleration at each of the intervals Δt. In this case, the Newmark integration scheme for linear problems is unconditionally stable, i.e. the solution does not grow indefinitely at large values of the step Δt.

Substituting the expression (3) in the equation (2), thereby excluding $\ddot{U}^{t+\Delta t}$ and $\dot{U}^{t+\Delta t}$ from the number of unknown ones, after simple transformations, we obtain the following matrix equation to determine $\Delta U^{t+\Delta t}$:

$$\left[\hat{K}\right]\left\{\Delta U^{t+\Delta t}\right\} = \left\{\hat{P}^{t+\Delta t}\right\}, \quad (4)$$

where $\left\{\hat{P}^{t+\Delta t}\right\} = \left\{P^{t+\Delta t}\right\} + \left[M\right]\left(\alpha_2\left\{\dot{U}^t\right\} + \alpha_3\left\{\ddot{U}^t\right\}\right) + \left[C\right]\left(\alpha_4\left\{\dot{U}^t\right\} + \alpha_5\left\{\ddot{U}^t\right\}\right) - \left[K\right]\left\{U^t\right\}$; $\left[\hat{K}\right] = \left[K\right] + \alpha_0\left[M\right] + \alpha_1\left[C\right]$. The coefficients $\alpha_0 - \alpha_5$ depend on the step Δt and the parameters α and β:

$$\alpha_0 = \frac{1}{\alpha(\Delta t)^2};\quad \alpha_1 = \frac{\beta}{\alpha\Delta t};\quad \alpha_2 = \frac{1}{\alpha\Delta t};\quad \alpha_3 = \frac{1}{2\alpha} - 1;\quad \alpha_4 = \frac{\beta}{\alpha} - 1;\quad \alpha_5 = \frac{\beta\Delta t}{2\alpha} - \Delta t.$$

The system of algebraic equations (4) is solved by the LDLT factorization method, taking into account the sparsity of the symmetric matrix $[\hat{K}]$ and its variable profile. After finding $\Delta U^{t+\Delta t}$ and, correspondingly, $U^{t+\Delta t}$ for the calculation of accelerations $\ddot{U}^{t+\Delta t}$ and speeds $\dot{U}^{t+\Delta t}$, equations (3) are used. In their turn, at any time point t' within the interval Δt $(t \le t' \le t + \Delta t)$, the values of accelerations $\ddot{U}(t')$, speeds $\dot{U}(t')$ and displacements $U(t')$ can be calculated with by the following formulas:

$$\ddot{U}(t') = \ddot{U}^t + \frac{(t'-t)}{\Delta t}\left(\ddot{U}^{t+\Delta t} - \ddot{U}^t\right);\quad \dot{U}(t') = \dot{U}^t + \frac{(t'-t)}{2}\left(\ddot{U}^t + \ddot{U}^{t'}\right);$$
$$U(t') = U^t + (t'-t)\dot{U}^t + \frac{(t'-t)^2}{4}\left(\ddot{U}^t + \ddot{U}^{t'}\right). \quad (5)$$

The first of the equations (5), according to the Newmark scheme, expresses the linear law of change of acceleration on the interval Δt, the second and third ones are obtained from the expressions (3) with the value substitutions $\alpha = 1/4$, $\beta = 1/2$, and the replacement of the value $t + \Delta t$ by the value t'.

Furthermore, when taking into account unilateral constraint with Coulomb friction in addition to the initial conditions at $t = 0$ and boundary conditions on $S_g^{\pm}$, $S_u^{\pm}$, the conditions on the contacting surfaces $S_c^{\pm}$ should be satisfied. The contact interaction will be modeled using frame-rod contact finite elements (CFE) [4, 10]. The CFE data provide a discrete contact between the nodes of the finite element grid located on the boundary surfaces of the contacting bodies.

Let us write conditions on the contact in terms of forces and displacements for each discrete unilateral constraint k (i.e. the k CFE), for the time point t:

$$u_{nk}^t \ge 0;\quad N_k^t \le 0;\quad u_{nk}^t N_k^t = 0,\quad k \in S_c; \quad (6)$$

$$\left|T_k^t\right| \le \left|T_{Uk}^t\right|;\quad T_k^t \dot{u}_{\tau k}^t \ge 0;\quad (T_k^t - T_{Uk}^t)\dot{u}_{\tau k}^t = 0,\quad k \in S_c. \quad (7)$$

Here u_{nk}^t, $u_{\tau k}^t$ are mutual displacements of the opposite nodes for unilateral constraint k in the normal and tangential direction; $\dot{u}_{\tau k}^t = \partial u_{\tau k}^t / \partial t$ is the speed of mutual tangential displacement on the contact k; N_k^t, T_k^t are contact forces in the normal and tangential direction (forces in k CFE); $T_{Uk}^t = -f_k N_k^t$ is ultimate Coulomb friction force for the contact k; $f_k \ge 0$ is the coefficient of friction-sliding.

The last of the conditions (6) means that upon contact $u_{nk}^t = 0$, $N_k^t < 0$; upon separation $u_{nk}^t > 0$, $N_k^t = 0$. The last two conditions (7) establish the correspondence between the speeds of the mutual slippage of the opposite nodes on the contact k and the magnitude of the force T_k^t at the time point *t*. Under the conditions $\dot{u}_{\tau k}^t = 0$ and $\left|T_k^t\right| < \left|T_{Uk}^t\right|$ there is a state of clutching (pre-ultimate friction); when $\dot{u}_{\tau k}^t \neq 0$, $\left|T_k^t\right| = \left|T_{Uk}^t\right|$ is the state of slippage, while the direction of the slip rate is in line with the direction of the shear force. Changing the current state, namely the moment of transition from one state to another, is an event – correspondingly, it will be the events of slippage, clutch, separation (switching off unilateral constraint), or contact (switching on it).

Let's briefly discuss the sequence of actions implementing a step-by-step algorithm for solving the dynamic problem with unilateral constraints and Coulomb friction. The General case is considered when the normal forces of interaction and, accordingly, the ultimate friction forces on the contact change in the process of dynamic loading, i.e. in time, as it often occurs in practical tasks.

It is believed that at the current time point *t* the state on the contact is known. The values of mutual displacements u_{nk}^t, $u_{\tau k}^t$, speed $\dot{u}_{\tau k}^t$ and contact forces N_k^t, T_k^t, T_{Uk}^t are determined for each unilateral constraint k. Let part of the constraints $(k \in S_{1c})$ be in the state of clutching, the other part $(k \in S_{2c})$ — in the state of contact with the slip and, finally, the third part $(k \in S_{3c})$ — in the state of separation, $S_c = S_{1c} \cup S_{2c} \cup S_{3c}$. At the beginning of the calculation, at *t* = 0, the displacements, as well as the speeds and accelerations, are assumed to be zero.

1. The current time step Δt is performed, in the process of calculation its value can be changed in accordance with the established moment of occurrence of the next events on the contact. From the solution (4), the increments $\Delta u_{nk}^{t+\Delta t}$, $\Delta u_{\tau k}^{t+\Delta t}$, then the values of displacements $u_{nk}^{t+\Delta t}$, $u_{\tau k}^{t+\Delta t}$, speeds $\dot{u}_{\tau k}^{t+\Delta t}$, and contact forces $N_k^{t+\Delta t}$, $T_k^{t+\Delta t}$, $T_{Uk}^{t+\Delta t}$ for the time point $t + \Delta t$ are determined.

2. The traversal of all discrete constraints is performed, therewith for each constraint k there is (within the current step Δt) the time point $\widehat{t}_k$ of occurrence of the next, i.e. the closest in the time event. The expression for determining the time point of slippage for the constraint k that was previously in the state of the clutching has the following form:

$$\widehat{t}_k = t + \Delta t \left(\frac{T_{Uk}^t - T_k^t}{(T_k^{t+\Delta t} - T_k^t) - (T_{Uk}^{t+\Delta t} - T_{Uk}^t)} \right), \; k \in S_{1c}. \tag{8}$$

The time point of clutch for the constraint k that was previously in the state of the sliding

$$\widehat{t}_k = t + \Delta t \left(\frac{-\dot{u}_{\tau k}^t}{\dot{u}_{\tau k}^{t+\Delta t} - \dot{u}_{\tau k}^t} \right), \; k \in S_{2c}. \tag{9}$$

The time point of separation or contact for the constraint k that, respectively, was previously in the state of the contact or separation

$$\widehat{t}_k = t + \Delta t \left(\frac{-N_k^t}{N_k^{t+\Delta t} - N_k^t} \right), \; k \in S_{1c}, \; S_{2c}; \quad \widehat{t}_k = t + \Delta t \left(\frac{-u_{nk}^t}{u_{nk}^{t+\Delta t} - u_{nk}^t} \right), \; k \in S_{3c}. \tag{10}$$

Since the change of displacements, speeds, and, therefore, forces, within the step Δt does not actually follow the linear law, then, additionally, using the expressions (4), an iterative refinement of the time point $\widehat{t}_k$ can be performed, the time consumption increases slightly in this case [28].

3. Of all the values $\widehat{t}_i$ having been found using the formulas (7) and being in the interval $(t, t + \Delta t)$, the smallest one corresponding to the moment of occurrence of the nearest time event on the contact is selected: $\widehat{t} = \min(\widehat{t}_k), \; k \in S_c$. In case $\widehat{t} > t + \Delta t$ the next basic integration step Δt is executed, i.e. transition to p.1 is performed.

4. In case $t < \widehat{t} < t + \Delta t$ — the recalculation of updated in such a way step with value $\Delta\widehat{t} = \widehat{t} - t$ is performed. Herewith, the method of compensating loads is applied to comply with the conditions of the ultimate

friction [10, 28]. Changing of the ultimate friction forces on the contact is taken into account by the application of compensating forces to the opposite nodes

$$\widehat{F}_{\tau k} = -\Delta\widehat{T}_{Uk} = -\frac{\Delta\widehat{t}}{\Delta t}(T_{Uk}^{t+\Delta t} - T_{Uk}^{t}), \;\; k \in S_{2c}. \tag{11}$$

The value of the transverse force on the contact k is corrected by the same quantity: $T_k^{\widehat{t}} = T_k^t + \Delta\widehat{T}_{Uk}$.

As a result of the step recalculation, the values of displacements, speeds and forces on the contact in the time point $\widehat{t}$ are determined. The conditions of the expected event are checked; in case of slippage it will be a condition $T_k^{\widehat{t}} = T_{Uk}^{\widehat{t}}$; in case of clutching — $\dot{u}_{\tau k}^{\widehat{t}} = 0$, in case of detachment — $N_k^{\widehat{t}} = 0$, in case of contact — $u_{nk}^{\widehat{t}} = 0$. If the corresponding condition does not work, the time point $\widehat{t}$ should be updated once again but in the interval $(t, \widehat{t})$ or $(\widehat{t}, t+\Delta t)$.

5. In case of occurrence of the next on time event on the corresponding support the state of contact changes — thereby the current working scheme of the construction changes too. Therewith the results of the recalculated step are considered final for the time point $\widehat{t}$. Then all the above actions are repeated, but for the next integration step Δt.

The given algorithm is implemented by the author in the computer program [29]. The program is intended for doing numerical research and comparative analysis of various models of contact interaction of the deformable systems, methods and algorithms of their calculation. For the purpose of comparative evaluation of the results obtained, the program also implements the well-known methods for calculating systems with unilateral constraints and friction, in particular, the method of iterations on ultimate friction forces [21, 22].

Let us demonstrate the considered approach using the example of calculation of a plane framed system, which, for example, can simulate a pipeline section with a difference of relief under dynamic loading (Figure 1, a). It is considered that the system is fixed from lateral displacements, i.e. from the xy plane. On the rigid supports k = 1, 2 the conditions of Coulomb friction-sliding with the possibility of separation on the contact operate. The structure is in the state of rest, then a horizontal impulse load $P(t)$ is applied at the left end. The law of change of the pulse has a triangular shape with the duration of 0.1 s, the amplitude of 100 kN (Figure 1, b).

The longitudinal stiffness of the rods EA = 9·10^6 kN, bending stiffness EI = 2·10^6 kN·m^2, the linear mass m = 0.4 t/m. The damping matrix here simplistically computed as $[C] = \alpha \cdot [M]$, the damping coefficients were taken to be the following: $\alpha = 0.2\omega_1$, $\beta = 0$. Contact interaction was modelled with frame-rod CEF [4, 10], connecting the support nodes of the framed system with fixed supports.

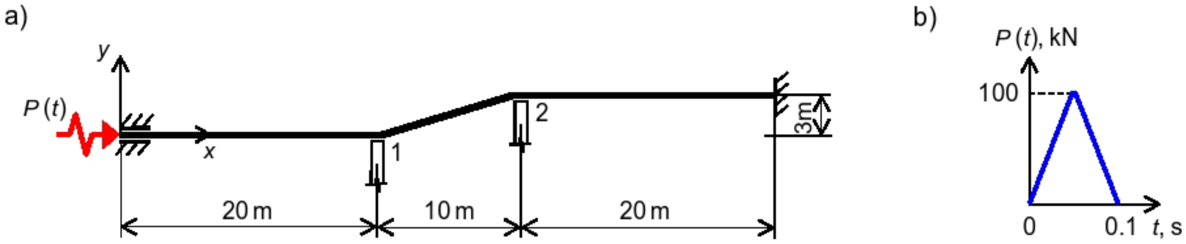

Figure 1. Framed system under the action of pulsed load $P(t)$.

At the initial time point t = 0 on all supports the clutch state was set. The normal forces of interaction N_k^0 on the contact of the structure nodes with the corresponding supports were taken to be equal to the reactions from the own weight of the structure. In the future, as a result of the action of the dynamic load, slippage, and the subsequent clutch on the contact is possible, as well as the switching off (separation) and the switching on of unilateral constraints during the considered time period. At the same time, due to the geometry of the framed system, the normal forces of interaction N_k^0 also change over time.

The next was the problem of interaction of the water-fight slab of the dam with ground base at hydrodynamic effect of the water flow discharged from the headwater of the dam. The calculation scheme of the slab (Figure 2) corresponds to one of the objects of the Volzhsky hydroelectric complex. The purpose of the calculations was to assess the impact of the pulsating component of water pressure in the discharge flow for the contact interaction of the water-fight slab of the dam with the base ground. The criterion condition for determining the ultimate values for the slab thickness in this case is to prevent the slippage or separation of the slab from the ground base.

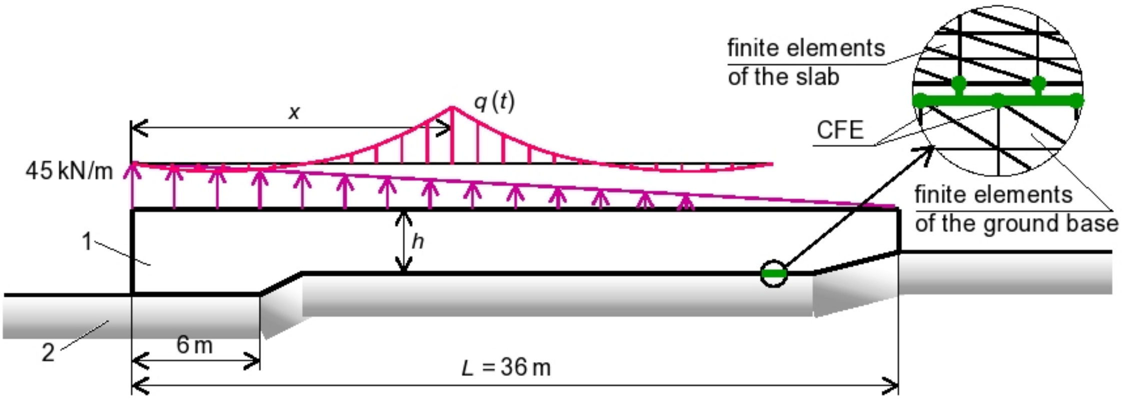

Figure 2. Scheme of the water-fight slab and applied loads.

The calculation took into account loads stipulated by the own weight of the slab, the hydrodynamic acting from the water flow, the filtration backpressure. The following characteristics of the slab material were taken: volume weight 24 kN/m^3, modulus of elasticity E_1 = 40000 MPa, Poisson's ratio ν_1 = 0.2. The considered area of the ground base was 64×20 m, volume weight 18 kN/m^3, modulus of elasticity and Poisson's ratio E_2 = 600 MPa, ν_2 = 0.25, the friction coefficient f = 0.31.

The pulsating component of the water pressure in the discharge flow was taken into account as a dynamic impulse load. The amplitude of the pulsating pressure q and the correlation of its distribution over the slab surface, depending on the position of the pulse center, were taken into account according to the recommendations from [30]. Taking into consideration the demonstration nature of the calculation, the accounting for damping is performed using a simplified scheme — similar to the previous example.

To study the dependence of the solution on the characteristics of the hydrodynamic acting, the behavior of the water-fight slab at different positions of the impulse on the slab (x/l), likewise at its different directions and duration of the action was calculated.

3. Results and Discussion

3.1. The problem of interaction of the framed system with rigid supports

The numerical solution of the considered problem, both by the proposed and, for comparative evaluation, by an alternative method, was carried out using the computer program [29]. The purpose of the calculations was to estimate the effect of the friction coefficient (the value f in the calculations varied from 0 to 1.5) on the behavior of the framed structure under dynamic loading. Figures 3 and 4 shows the horizontal and vertical displacements of the frame on one of the supports depending on the time at different values of the coefficient f, the integration step here was taken 0.0002 s.

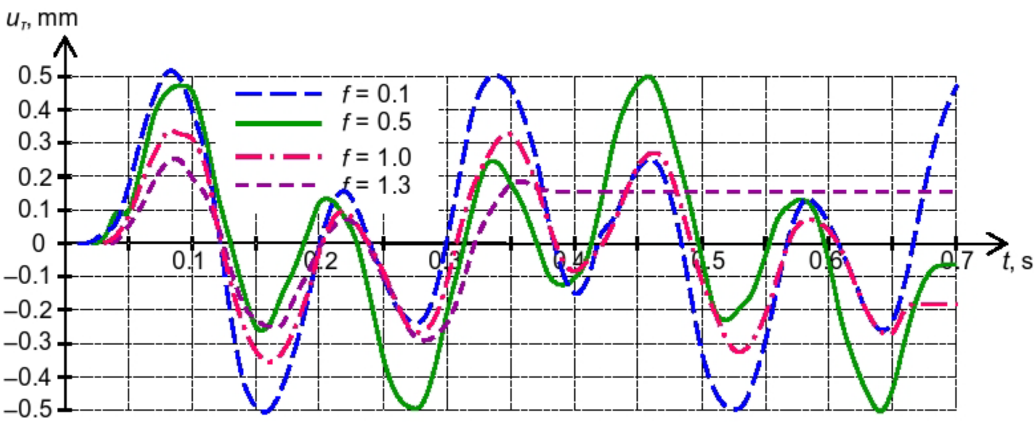

Figure 3. Horizontal displacements (slippage) on support 1 depending on time.

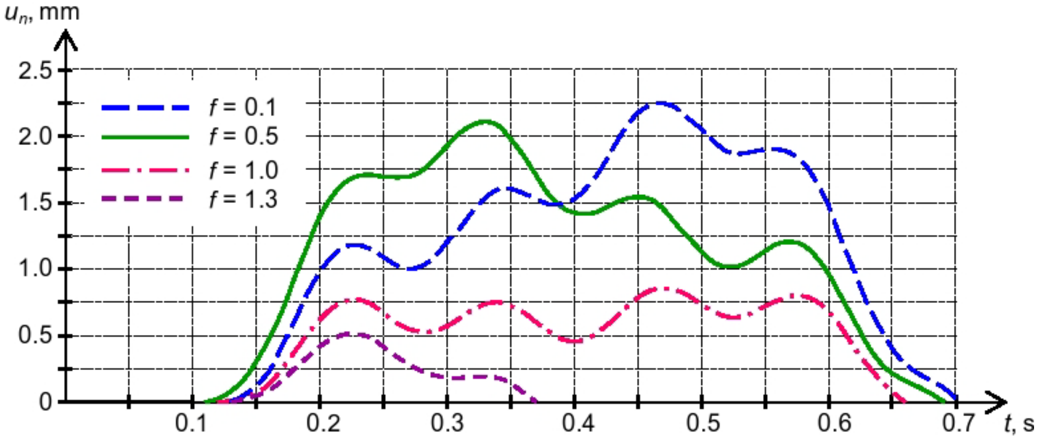

Figure 4. Vertical displacements (separation) on support 1 depending on time.

Figure 5 represents the dependencies between the amplitude values of the horizontal (slippage) u_τ and vertical (separation) u_n displacements on the supports and the specified values of the friction coefficients f on the contact.

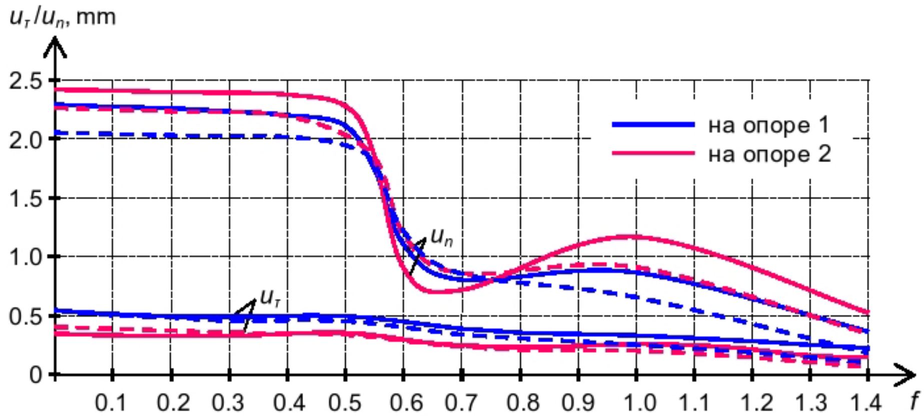

Figure 5. Amplitude values of displacements u_τ and u_n on supports at different values of friction coefficient f.

The reliability of the obtained results is confirmed by the software check of the fulfillment of the equilibrium conditions and the compatibility of the system deformations after each time step. Besides, for the comparative evaluation of the results, the calculation of system was performed using an alternative algorithm where the contact conditions at each time step are realized by means of the method of iterations on ultimate friction forces (Figure 5 shows this solution by dotted line). As it can be seen, the results of the calculations by the proposed and alternative methods correspond each other, however, the algorithm of iterations on the ultimate forces requires much more calculation time (for the considered problem almost twice).

From the given graphs (Figures 3–5) it follows, that with an increase in the coefficient f, the separation of the structure from the supports as a result of the dynamic loading decreases significantly. Moreover, in the example considered here, there is a certain threshold value of the coefficient f (0.6–0.7), where the effect of contact friction on the value of maximum separation is extreme.

In order to study the dependence of the solution on the value of the integration step over time, the behavior of the framed structure for different values of Δt in the range from 0.0001 to 0.0064 s, with the successive doubling of the step length, was calculated. The comparison of the obtained results allows us to conclude that the proposed numerical approach shows satisfactory internal convergence in a rather wide range of integration steps on time. Thus, the values of horizontal and vertical displacements on the supports do not differ much when assigning the basic step in the range from 0.0001 to 0.0008 s. With a further increase in Δt, there is some deterioration in the convergence, especially at large values of the friction coefficient.

Note that, in the general case, the choice of the optimal integration step is a rather complex problem [27]. In this regard, basing on conducted numerical studies, it can be recommended to choose the value of the basic integration step in such a way that when it is increased, for example, twice, the change in the results does not exceed some specified error of calculation.

3.2. The problem of interaction of the water-fight slab with ground

As the results of the calculations show, the possible separation of the water-fight slab from the ground base in all cases occurs only at its edges (first from the left edge). The moment of separation depends on the position and direction of the impulse of pressure. The most dangerous, from the point of view of the separation slab from the base, is the pressure pulsation with the pulse duration of from 0.46 s (at the location of the pulse closer to the edges of the slab) to 0.5 s (for the middle of the slab). Figure 6 shows the change of contact stresses on the left edge of the slab base in time – to the moment of separation of the sole from the base. Here t/T_{imp} is the ratio of the current time t to the duration of the impulse of pressure T_{imp}, x/L is the ratio of the x coordinate that defines the position of the impulse at the water-fight slab, to the slab length L. The slab thickness in these cases was taken to be slightly less than the ultimate values for separation h_{pr}.

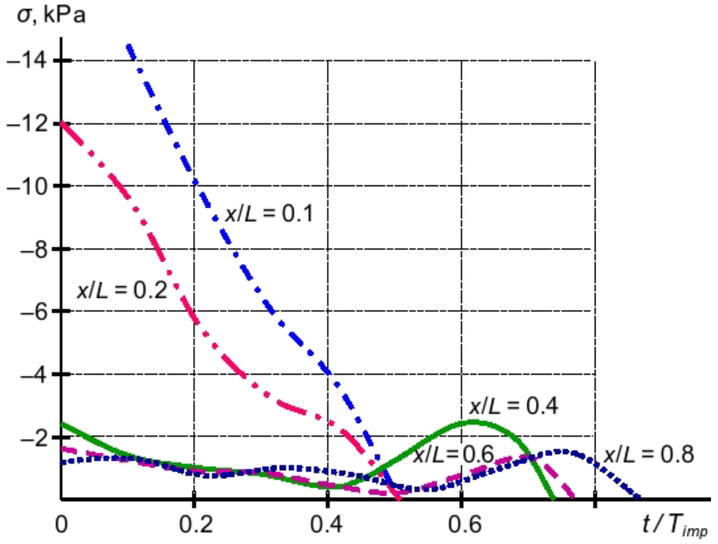

Figure 6. Contact stresses on the left edge of the slab base before the moment of separation.

Figures 7 and 8 demonstrate the deformed pattern of the slab-base system at the moment of maximum slab separation from the ground base. Here the slab thickness h = 2.5 m – less than the ultimate one by separation, pulse position x/L = 0.4. Yellow and light blue points-markers indicate the zones of slippage or separation of the slab from the base.

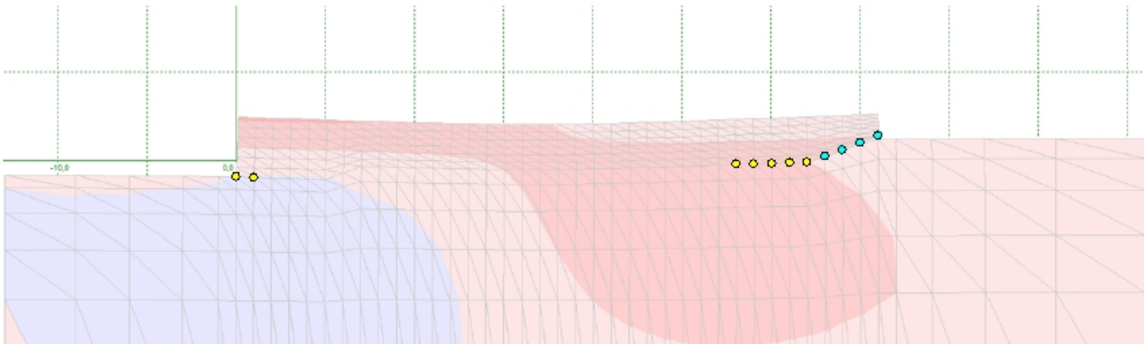

Figure 7. Isopole of horizontal displacements of the slab on the base.

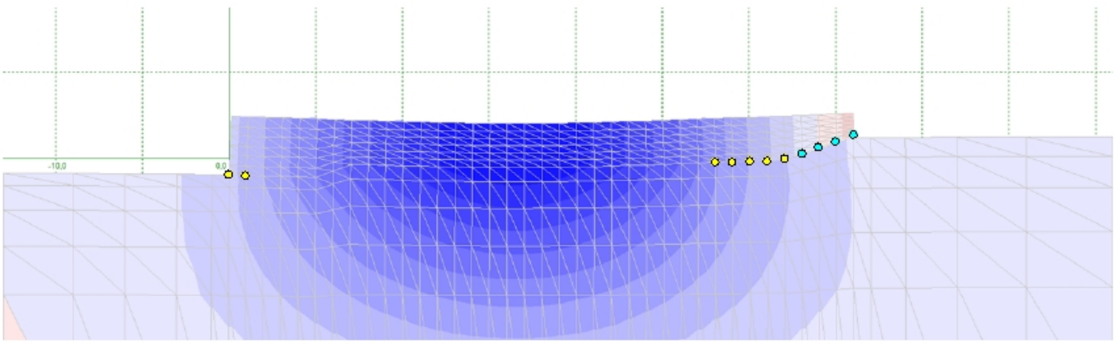

Figure 8. Isopole of vertical displacements of the slab on the base.

Using numerical experiments, dependencies for the ultimate values of the water-fight slab thickness on the pulse position on it have been obtained (Figure 9). A solid line shows the envelope relative values of slab thickness, satisfying the condition its non-separation and shear from the base. Here h_{cr} = 3.48 m is the critical depth corresponding to the design specific discharge of water. The dotted line corresponds to the ultimate slab thickness when only static loads are applied – pressure deficit and own weight of the slab. The given dependence can serve as a guide in the appointment of the thickness of the water-fight slab on the condition of preventing its slippage and separation from the ground base – you can take $h = h_{cr}$.

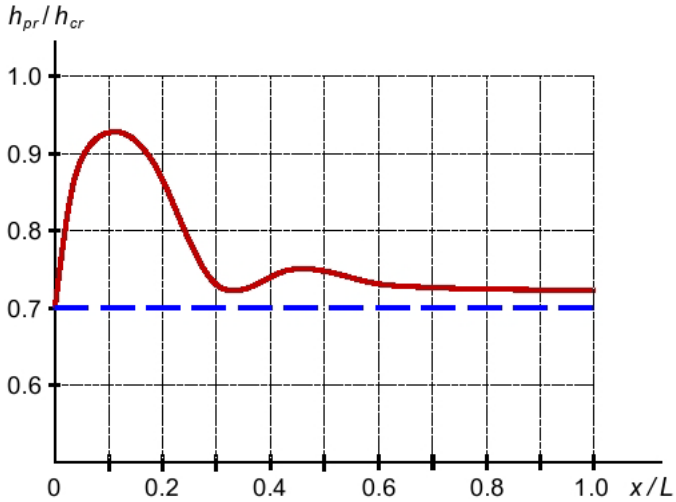

Figure 9. Dependence of the maximum slab thickness h_{pr} on the pulse position x / l.

4. Conclusion

1. The problem of the contact interaction of elastically deformable systems under dynamic loading is considered in the article. To solve it, we propose the numerical algorithm combining in one step-by-step process with the integration of the equations of motion with step-by-time analysis of the contact state. For more accurate compliance with the limitations under the conditions of ultimate friction-sliding, the method of compensating loads has been applied.

2. The basis of the above algorithm is a step-by-step analysis method that has a clear physical interpretation. It is shown that this approach provides the possibility of analyzing the contact interaction of structures with the base under dynamic loading and has the advantage in cases when the solution of the problem depends on the history of loading, in particular, when accounting for friction-sliding in unilateral constraints.

3. The discrete calculation model of the FEM is used, upon that for the modeling of unilateral constraints with Coulomb friction, the contact finite elements in the form of frame-rod system has been used. The CFE data providing a discrete contact between the boundary surfaces of the interacting elastic bodies, allows you to determine the forces and displacements in the contact zone with the same and high accuracy, to apply an inconsistent finite element grids, to take the physical and geometric properties of the contact seam into account.

4. The carried out of test calculations allow us to conclude about the efficiency and reliability of the proposed algorithm, taking into account the complicated contact conditions and dynamic loading, which are essential for solving applied problems of structural mechanics. The comparison with the well-known algorithm of iterations on ultimate forces shows in our case a significant saving of calculation time. Using the analysis of the calculation results of the water-fight slab, proposals concerning the constructive solutions of the considered structure, taking into account the nature and position of the dynamic loads acting on it, were made.

5. In conclusion, we note that the account of complicated conditions of contact interaction contributes to the approximation of the calculation scheme to the real working conditions of structure and, thus, allows us getting more accurate and complete information about its strength and reliability.

References

1. Laursen, T.A. Computational Contact and Impact Mechanics. Berlin-Heidelberg: Springer, 2002. 454 p.
2. Wriggers, P. Computational Contact Mechanics. Berlin-Heidelberg: Springer, 2006. 521 p.
3. Laursen, T., Yang, B. A contact searching algorithm including bounding volume trees applied to finite sliding mortar formulations. Computational Mechanics. 2008. No. 41. Pp. 189–205.

4. Bukhartsev, V.N., Lukashevich, A.A. Stability assessment of a structure subject to asymmetric load application on a rock-free bed. Power Technology and Engineering. Springer New York Consultants Bureau. 2012. Vol. 45. No. 5. Pp. 346–350.
5. Wriggers, P., Schroder, J., Schwarz, A. A finite element method for contact using a third medium. Computational Mechanics. 2013. No. 52(4). Pp. 837–847.
6. Kolosova, G.S., Lalin, V.V., Kolosova, A.V. The effect of construction joints and cracks on the stress-strain state of the arch-gravity dam. Magazine of Civil Engineering. 2013. 40(5). Pp. 76–85. (rus) doi: 10.5862/MCE.40.9
7. Lukashevich A.A. Computational solution of contact problems with unilateral constraints and friction by step-by-step analysis. Advanced Materials Research. 2014. Vol. 941–944. Pp. 2264–2267.
8. Barboteu, M., Danan, D., Sofonea, M. Analysis of a contact problem with normal damped response and unilateral constraint. Journal of Applied Mathematics and Mechanics. 2016. 96(4). Pp. 408–428.
9. Wriggers, P., Rust, W.T., Reddy, B.D. A virtual element method for contact. Computational Mechanics. 2016. 58(6). Pp. 1039–1050.
10. Lukashevich, A.A. Computational modelling of stiffness and strength properties of the contact seam. Magazine of Civil Engineering. 2018. 81(5). Pp. 149–159. doi: 10.18720/MCE.81.15
11. Ignatyev, A.V., Ignatyev, V.A., Gamzatova, E.A. Analysis of bending plates with unilateral constraints through the finite element method in the form the of classical mixed method. Izvestiya vuzov. Stroitelstvo. 2018. No. 8. Pp. 5–14. (rus)
12. Lukashevich, A.A., Lukashevich, N.K., Timohina, E.I. Modelling of contact interaction with allowance for nonlinear. IOP Conference Series: Materials Science and Engineering. 2018. Vol. 463. No. 042054.
13. Albert, Y.U., Dolgaya, A.A. Ivanova, T.V. et al. Seismic input models for tuned mass damper designing. Magazine of Civil Engineering. 2017. 76(8). Pp. 98–105. doi: 10.18720/MCE.76.9.
14. Smirnov, V.N., Shestakova, E.B., Chizhov, S.V. et al. Dynamic interaction of high-speed trains with span structures and flexible support. Magazine of Civil Engineering. 2017. 76(8). Pp. 115–129. doi: 10.18720/MCE.76.11.
15. Tusnin, A. Dynamic factors in case of damaging continuous beam supports. Magazine of Civil Engineering. 2018. 78(2). Pp. 47–64. doi: 10.18720/MCE.78.4
16. Galanin, M.P., Lukin, V.V., Rodin, A.S., Stankevich, I.V. Application of the schwarz alternating method for simulating the contact interaction of a system of bodies. Computational Mathematics and Mathematical Physics. 2015. 55(8). Pp. 1393–1406.
17. Galanin, M.P., Lukin, V.V., Rodin, A.S. Use of various versions of Schwarz method for solving the problem of contact interaction of elastic bodies. IOP Conf. Series: Journal of Physics: Conf. Series. 2018. Vol. 991. No. 012021.
18. Bychek, O.V., Sadovskii, V.M. On the investigation of the dynamic contact interaction of deformable bodies. Journal of Applied Mechanics and Technical Physics. 1998. 39(4). Pp. 628–633.
19. Annin, B.D., Sadovskaya, O.V., Sadovskii, V.M. Dynamic contact problems of elastoplasticity. Voprosy Materialovedeniya. 2003. 33(1). Pp. 426–434.
20. Wriggers, P., Nackenhorst, U. Analysis and Simulation of Contact Problems. Berlin-Heidelberg: Springer, 2006. 394 p.
21. Kravchuk, A.S. Variatsionnyy metod v kontaktnykh zadachakh. Sostoyaniye problemy, napravleniya razvitiya [The variational method in contact problems. The present state of the problem and trends in its development]. Journal of Applied Mathematics and Mechanics. 2009. 73(3). Pp. 492–502. (rus)
22. Sofonea, M., Souleiman, Y. Analysis of a sliding frictional contact problem with unilateral constraint. Mathematics and Mechanics of Solids. 2017. No. 22. Pp. 324–342.
23. Barboteu, M., Danan, D. Analysis of a dynamic viscoelastic contact problem with normal compliance, normal damped response, and nonmonotone slip rate dependent friction. Advances in Mathematical Physics. 2016. No. 1. Pp. 1–15.
24. Barboteu, M., Cheng, X., Sofonea, M. Analysis of a contact problem with unilateral constraint and slip-dependent friction. Mathematics and Mechanics of Solids. 2016. No. 21. Pp. 791–811.
25. Barauskas, R. Dynamic analysis of structures with unilateral constraints: Numerical integration and reduction of structural equations. International Journal for Numerical Methods in Engineering. 1994. 37(2). Pp. 323–342.
26. Liolios A., Liolios K., Iossifidou K. et al. A numerical approach to the dynamic unilateral contact problem of soil-pile interaction under instabilizing and environmental effects // Numerical Methods and Applications, 6th International Conference, NMA 2006, Borovets, Bulgaria, August 20–24. Berlin; New York: Springer, 2007. Pp. 646–651.
27. Bathe, K.-J., Wilson, E.L. Numerical methods in finite element analysis. Englewood Cliffs: Prentice Hall, 1976. 544 p.
28. Lukashevich A.A., Rozin L.A. Numerical decision of problems of structural mechanics with nonideal unilateral constraints. Applied Mechanics and Materials. Advances in Civil and Industrial Engineering IV. 2014. Vol. 580–583. Pp. 2932–2935.
29. Lukashevich, A.A. Programma resheniya zadach kontaktnogo vzaimodejstviya s uchetom odnostoronnih svyazej i treniya metodom konechnyh elementov [The program for solving contact interaction problems taking into account unilateral constraints and friction by the finite element method]. Svidetelstvo o gosudarstvennoj registracii programmy dlya EVM № 2008610113. Zaregistrirovano v Reestre programm dlya EVM. Federalnaya sluzhba po intellektualnoj sobstvennosti, patentam i tovarnym znakam RF, 2008. 1 p. (rus)
30. Lappo, D.D., Veksler, A.B. et al. Gidravlicheskie raschety vodosbrosnyh gidrotekhnicheskih sooruzhenij: Spravochnoe posobie [Hydraulic calculations of spillway hydraulic structures: Reference manual]. Moscow: Energoatomizdat, 1988. 624 p. (rus)

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Моделирование контактного взаимодействия конструкций с основанием при динамическом нагружении

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Ключевые слова: конструкция, основание, контактное взаимодействие, односторонние связи, динамическое нагружение, дискретная модель, метод конечных элементов

Аннотация. Конструктивно нелинейные задачи с односторонними связями часто встречаются при расчете различного рода конструкций и сооружений. При этом определенные трудности в решении представляют задачи при учете трения на контакте, а также при динамическом действии нагрузки. В этих случаях контактная задача усложняется в математическом отношении и усложняется ее численное решение. Настоящая статья посвящена построению расчетных моделей и методов решения задач с неидеальными односторонними связями при динамическом нагружении. На основе конечно-элементной модели контакта и метода пошагового анализа разработан численный алгоритм, позволяющий выполнять одновременно интегрирование уравнений движения и реализацию контактных условий с трением Кулона. Для соблюдения ограничений в условиях по предельному трению-скольжению применен способ компенсирующих нагрузок. С помощью предложенного подхода получены и проанализированы численные решения некоторых задач контакта сооружения с основанием. Достоверность результатов проведенных расчетов подтверждается сопоставлением их с решением, полученным альтернативным итерационным методом. При этом алгоритм пошагового анализа является более эффективным по времени вычислений, показывая удовлетворительную сходимость, устойчивость и точность решения в достаточно широком диапазоне шагов интегрирования по времени.

Литература

1. Laursen T.A. Computational Contact and Impact Mechanics. Berlin-Heidelberg: Springer, 2002. 454 p.
2. Wriggers P. Computational Contact Mechanics. Berlin-Heidelberg: Springer, 2006. 521 p.
3. Laursen T., Yang B. A contact searching algorithm including bounding volume trees applied to finite sliding mortar formulations // Computational Mechanics. 2008. No. 41. Pp. 189–205.
4. Bukhartsev V.N., Lukashevich A.A. Stability assessment of a structure subject to asymmetric load application on a rock-free bed // Power Technology and Engineering. Springer New York Consultants Bureau. 2012. Vol. 45. No. 5. Pp. 346–350.
5. Wriggers P., Schroder J., Schwarz A. A finite element method for contact using a third medium // Computational Mechanics. 2013. No. 52(4). Pp. 837–847.
6. Колосова Г.С., Лалин В.В., Колосова А.В. Влияние строительных швов и трещин на напряженно-деформированное состояние арочно-гравитационной плотины // Инженерно-строительный журнал. 2013. № 5(40). С. 76–85. doi: 10.5862/MCE.40.9
7. Lukashevich A.A. Computational solution of contact problems with unilateral constraints and friction by step-by-step analysis // Advanced Materials Research. 2014. Vol. 941–944. Pp. 2264–2267.
8. Barboteu M., Danan D., Sofonea M. Analysis of a contact problem with normal damped response and unilateral constraint // Journal of Applied Mathematics and Mechanics. 2016. Vol. 96. No. 4. Pp. 408–428.
9. Wriggers P., Rust W.T., Reddy B.D. A virtual element method for contact // Computational Mechanics. 2016. No. 58(6). Pp. 1039–1050.
10. Лукашевич А.А. Численное моделирование жесткостных и прочностных свойств контактного шва // Инженерно-строительный журнал. 2018. № 5(81). С. 149–159. doi: 10.18720/MCE.81.15
11. Игнатьев А.В., Игнатьев В.А., Гамзатова Е.А. Анализ изгибаемых пластинок с односторонними связями по методу конечных элементов в форме классического смешанного метода // Изв. Вузов. Строительство. 2018. № 8. С. 5–14.
12. Lukashevich A.A., Lukashevich N.K., Timohina E.I. Modelling of contact interaction with allowance for nonlinear // IOP Conference Series: Materials Science and Engineering. 2018. Vol. 463. No. 042054.
13. Альберт И.У., Долгая А.А., Иванова Т.В. и др. Расчетное сейсмическое воздействие для сооружения с динамическим гасителем колебаний // Инженерно-строительный журнал. 2017. № 8(76). С. 98–105. doi: 10.18720/MCE.76.9.

14. Смирнов В.Н., Шестакова Е.Б., Чижов С.В. и др. Динамическое взаимодействие высокоскоростных поездов с пролетными строениями и гибкими опорами // Инженерно-строительный журнал. 2017. № 8(76). С. 115–129. doi: 10.18720/MCE.76.11
15. Туснин А.Р. Коэффициенты динамичности при повреждении опор неразрезных балок // Инженерно-строительный журнал. 2018. № 2(78). С. 47–64. doi: 10.18720/MCE.78.4
16. Galanin M.P., Lukin V.V., Rodin A.S., Stankevich I.V. Application of the Schwarz alternating method for simulating the contact interaction of a system of bodies // Computational Mathematics and Mathematical Physics. 2015. Vol. 55. No. 8. Pp. 1393–1406.
17. Galanin M.P., Lukin V.V., Rodin A.S. Use of various versions of Schwarz method for solving the problem of contact interaction of elastic bodies // IOP Conf. Series: Journal of Physics: Conf. Series. 2018. Vol. 991. No. 012021.
18. Bychek O.V., Sadovskii V.M. On the investigation of the dynamic contact interaction of deformable bodies // Journal of Applied Mechanics and Technical Physics. 1998. Vol. 39. No. 4. Pp. 628–633.
19. Аннин Б.Д., Садовская О.В., Садовский В.М. Динамические контактные задачи теории упругости и пластичности // Вопросы материаловедения. 2003. № 1(33). С. 426–434.
20. Wriggers P., Nackenhorst U. Analysis and Simulation of Contact Problems. Berlin-Heidelberg: Springer, 2006. 394 p.
21. Кравчук А.С. Вариационный метод в контактных задачах. Состояние проблемы, направления развития // Прикладная математика и механика. 2009. Т. 73. № 3. С. 492–502.
22. Sofonea M., Souleiman Y. Analysis of a sliding frictional contact problem with unilateral constraint // Mathematics and Mechanics of Solids. 2017. No. 22. Pp. 324–342.
23. Barboteu M., Danan D. Analysis of a dynamic viscoelastic contact problem with normal compliance, normal damped response, and nonmonotone slip rate dependent friction // Advances in Mathematical Physics. 2016. No. 1. Pp. 1–15.
24. Barboteu M., Cheng X., Sofonea M. Analysis of a contact problem with unilateral constraint and slip-dependent friction // Mathematics and Mechanics of Solids. 2016. No. 21. Pp. 791–811.
25. Barauskas R. Dynamic analysis of structures with unilateral constraints: Numerical integration and reduction of structural equations // International Journal for Numerical Methods in Engineering. 1994. No. 37(2). Pp. 323–342.
26. Liolios A., Liolios K., Iossifidou K. et al. A numerical approach to the dynamic unilateral contact problem of soil-pile interaction under instabilizing and environmental effects // Numerical Methods and Applications, 6th International Conference, NMA 2006, Borovets, Bulgaria, August 20–24. Berlin; New York: Springer, 2007. Pp. 646–651.
27. Bathe K.-J., Wilson E.L. Numerical methods in finite element analysis. Englewood Cliffs: Prentice Hall, 1976. 544 p.
28. Lukashevich A.A., Rozin L.A. Numerical decision of problems of structural mechanics with nonideal unilateral constraints // Applied Mechanics and Materials. Advances in Civil and Industrial Engineering IV. 2014. Vol. 580–583. Pp. 2932–2935.
29. Лукашевич А.А. Программа решения задач контактного взаимодействия с учетом односторонних связей и трения методом конечных элементов. Свидетельство о государственной регистрации программы для ЭВМ № 2008610113. Зарегистрировано в Реестре программ для ЭВМ // Федеральная служба по интеллектуальной собственности, патентам и товарным знакам РФ, 2008. 1 с.
30. Лаппо Д.Д., Векслер А.Б. и др. Гидравлические расчеты водосбросных гидротехнических сооружений: Справочное пособие. М.: Энергоатомиздат, 1988. 624 с.

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