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Dynamics of a physically nonlinear viscoelastic cylindrical shell with a concentrated mass

D.A. Khodzhaev[a], R.A. Abdikarimov[b], M.M. Mirsaidov[a]
[a] Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, Tashkent, Uzbekistan
[b] Tashkent Financial Institute, Tashkent, Uzbekistan
** E–mail: rabdikarimov@mail.ru*

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Abstract. It is known that the theory of linear and nonlinear elastic plates and shells is the most developed part of the theory of elasticity. In this area, the necessary equations are obtained and the methods to solve them are developed. At the same time, there are gaps in considering the viscoelastic properties of a material in the problems of thin-walled structures dynamic calculations. It should be noted that in some publications the viscoelastic properties of the material (i.e. the deviation of material test diagram from Hooke's law) were taken into account according to the Voigt model, not confirmed by experiments. Ignoring viscoelastic properties of the material significantly limits practical applicability of results. The first part of the paper presents the statement and method of solution to the problem of axisymmetric vibrations of a physically nonlinear viscoelastic cylindrical shell with concentrated masses. The function characterizing the deviation of stress intensity curve from the Hooke straight line is taken in the form of cubic nonlinearity. A mathematical model, solution method and computational algorithm were developed for the problem of axisymmetric oscillations of a cylindrical shell with a concentrated mass, taking into account physically nonlinear strain of the material under different boundary conditions in the frame of the Kirchhoff-Love hypothesis. For the study of the effect of a concentrated mass the Dirac delta function was introduced. With the Bubnov-Galerkin method, based on a polynomial approximation of deflections, the problem in question is reduced to the solution, in the general case, of non-decay systems of nonlinear integro-differential equations of Volterra type. To solve the resulting system with the Koltunov-Rzhanitsyn weakly singular kernel, a numerical method was applied based on the use of quadrature formulas. A unified computational algorithm has been developed to determine the deflection of a viscoelastic cylindrical shell with concentrated masses.

1. Introduction

In modern technology, construction and other fields of industry, the structures of more and more complex patterns are being used; to ensure their strength, reliability and high efficiency is a problem of great importance. It is impossible to optimally design them without constructing mathematical models that account for the maximum possible number of factors affecting the efficiency of such structures.

In literature there are a number of publications in which linear and nonlinear problems of thin-walled structures dynamics are considered with and without a concentrated mass. An analysis of these publications shows that little attention has been paid to the features of viscoelastic inhomogeneous systems inertial behavior. In these papers, the problems were considered either using the Voigt's differential model or the Boltzmann-Volterra integral model, where the exponential kernels that could not describe the actual processes occurring in shells and plates at initial point of time were taken in calculations as the relaxation

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kernels [1]. The choice of the exponential kernel in calculations was not accidental. The systems of integro-differential equations obtained in calculations were reduced by differentiation to the solution of ordinary differential equations, which, in most cases, were solved by the well-known Runge–Kutta numerical method.

Static studies of viscoelastic materials for creep and relaxation indicate an extremely high intensity of relaxation processes at the initial stage of testing. The process velocity is so high that its direct measurement at the initial moment turns out to be impossible. The processes are considered as dynamic ones and their velocities are conditionally considered equal to infinity [1]. This fact can be described by weakly singular functions, which provide finite strains and stresses, in contrast to strongly singular functions. Such weakly singular functions describe well the velocities of relaxation processes if they contain a sufficient number of parameters. Such kernels include the Koltunov-Rzhanitsyn three-parameter kernel [2].

In practice, the materials are found in which stress σ and strain ε relation is nonlinear at increasing stress in the region of small strains, [3], i.e. the material has a physical nonlinearity. Physical nonlinearity can be with a soft or hard characteristic.

The monograph [4] provides the fundamentals of a physically nonlinear theory of elasticity, in construction of which Hooke's law is replaced by the nonlinear law of elasticity; geometrical linear relations of the classical theory of elasticity are preserved. Along with this, nonlinear static problems of the theory of elasticity under static load and the problems of nonlinear theory of oscillations are described in [4].

As shown by experimental studies, in most materials, namely, in soils, in polymeric materials, etc., physical nonlinearity manifests itself even at low stresses. Recently, the materials with nonlinear viscoelastic properties, have been widely used in practice. To describe such processes, it is desirable that the physically nonlinear viscoelasticity law has a simpler form and more accurately reflects physical properties of the material.

For a wide class of nonlinear problems in the theory of viscoelasticity, an account for real physical properties of a material allows revealing additional reserves of its strength and studying the effect of material properties, size and type of loads on the stress-strain state. Despite the complexity of solving the problem of nonlinear viscoelasticity, an account for the material features in the theory of viscoelasticity allows us to refine the strength calculation and to choose reliable optimal parameters of structures [3, 4]. Physically nonlinear problems relate to the complex problems of the theory of viscoelasticity and structural mechanics.

Various nonlinear viscoelasticity models were proposed by Yu.N. Rabotnov [1], A.A. Ilyushin [5]. However, some materials, depending on size and acting stresses duration, could not be described by one model only.

In mechanical engineering, construction and aviation industries, thin-walled structures such as plates and shells often play the role of a bearing surface, to which longitudinal and transverse ribs, linings and machine units are attached. In theoretical consideration of such problems, the attached elements are interpreted as an additional mass rigidly fixed to the systems and concentrated in points.

There is a number of papers in which linear and nonlinear problems of oscillations and dynamic stability of thin-walled structures such as plates and shells with a concentrated mass are considered. The problems were considered with and without account for inhomogeneous and viscoelastic properties of the material.

Analytical and experimental studies of dynamic instability of hinge-supported plates with an arbitrarily located concentrated mass are considered in [6]. Differential equations obtained with the Karman theory are solved by the Galerkin method. It is shown that a concentrated mass has a significant effect on dynamic instability of a plate.

In [7], nonlinear forced oscillations of rectangular plates carrying a concentrated mass in the center were investigated. It was assumed that the plate had rigidly fixed edges. The Karman non-linear theory of plates is used. The problem is discretized into a system with a multitude of degrees of freedom using the power approach and the Lagrange equations.

The eigenmodes of a rectangular plate, with two adjacent edges fixed, and the other two free (CCFF-plate) are investigated in [8]. The sought for deflection function is selected as the sum of two hyperbolic-trigonometric series. An analysis of the accuracy of calculations and comparison with known results are given.

In [9], orthotropic shallow shells with a double curvature are considered, as well as cylindrical panels reinforced from the side of the concavity by an orthogonal grid of ribbed stiffeners. External transverse load acting on the shell structure is uniformly distributed and linearly dependent on time. Calculations have shown a significant increase in critical load of instability, when the shell is reinforced with ribbed stiffeners.

To study free and forced axisymmetric oscillations of a cylindrical shell, two approaches were proposed in [10], based on three-dimensional theory of elasticity and division of initial cylindrical shell by concentric transverse circles.

A method for calculating natural oscillations of a cylindrical shell of an orthotropic material was proposed in [11]. The shell is reinforced by a set of rather densely arranged transverse-longitudinal ribs. The problem is reduced to a system of homogeneous algebraic equations, the number of which is equal to twice the number of discrete ribs. Comparison of calculated and experimental data is given.

The effect of a small added mass on the frequency and mode of free oscillations of a thin shell is studied in [12] using the theory of shallow shells. In proposed mathematical model it is assumed that the mass asymmetry, even in a linear statement, leads to coupled radial bending oscillations.

The most recent advances in the mechanics of soft and composite shells and their nonlinear vibrations and stability are presented in [13].

In [14], resolving equations were obtained and a calculation procedure was developed with account for nonlinear creep of three-layer plates and shallow shells with lightweight aggregate. The problem was reduced to a system of three differential equations for the stress function, displacement function and deflection.

Stability of rods, plates, and shells was investigated in [15], taking into account physical nonlinearity. Critical state of thin-walled structures is determined using some limit dependencies.

Wide use of personal computers in calculations made it possible to develop and implement numerical analysis methods for solving the problems of the hereditary theory of viscoelasticity and, thus, significantly expand the class of solved problems of the hereditary theory of viscoelasticity [16–21].

Based on the above, the aim of the first part of this study is to build a mathematical model, to develop a solution method and a unified computational algorithm for finding the deflection of the problem of axisymmetric oscillations of a physically nonlinear viscoelastic cylindrical shell with a concentrated mass.

2. Methods

Consider a viscoelastic cylindrical shell carrying concentrated masses M_p at points with coordinates (x_p), p = 1,2,…,*I*, obeying the Kirchhoff-Love hypothesis. The cylindrical shell of radius R_1 is under axisymmetric external pressure q. It is believed that there is no tensile force T_1 along the generatrix of cylindrical shell [5].

Figure 1 shows the shell element and the forces acting on it. The x-axis is directed along the generatrix of the middle surface, the *y* axis along the circumference and z-axis along the radius of the cylinder. The strain of cylindrical shell is characterized by radial displacement w, which is considered positive if it is directed towards positive direction of the z-axis and axial strain of the element of middle surface ε_1. Shell material is taken as equally working in tension and compression. The directing stress and strain tensors coincide.

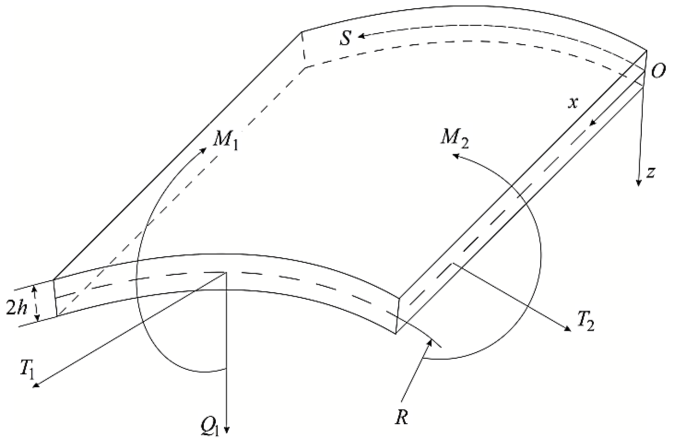

Figure 1. Element of cylindrical shell and the forces acting on it.

In accordance with the accepted assumptions and the axisymmetric nature of strain for any elementary layer located at a distance z from the middle surface, we get [5]:

$$\varepsilon_x = \varepsilon_1 - z\frac{\partial^2 w}{\partial x^2}, \quad \varepsilon_s = \varepsilon_2, \quad \varepsilon_z = -\left(\varepsilon_x + \varepsilon_s\right), \quad \varepsilon_2 = -\frac{w}{R_1}, \quad \varepsilon_{xs} = \varepsilon_{sz} = \varepsilon_{zx} = 0. \qquad (1)$$

Here, relative elongations of the element of the middle surface as a result of shell strain are indicated by: $\varepsilon_1 = \left(\varepsilon_x\right)_{z=0}$, $\varepsilon_2 = \left(\varepsilon_s\right)_{z=0}$.

The condition for the absence of axial force $T_1 = 0$ is satisfied if and only if

$$\varepsilon_1 + \frac{1}{2}\varepsilon_2 = 0, \quad \varepsilon_1 = \frac{w}{2R_1}. \tag{2}$$

According to [17], the initial physical equations are taken as

$$\sigma_x - \frac{1}{2}\sigma_s = \frac{3}{2}\left(1-R^*\right)\varphi(\varepsilon_i)\varepsilon_x, \ \ \sigma_s - \frac{1}{2}\sigma_x = \frac{3}{2}\left(1-R^*\right)\varphi(\varepsilon_i)\varepsilon_s, \tag{3}$$

here $R^* f(t) = \int_0^t R(t-\tau)\, f(\tau)\, d\tau,$ where $R(t-\tau)$ is the relaxation kernel.

It is assumed that the shell strain is small; a nonlinear relationship is assumed between the intensity of stresses σ_i and the intensity of strains ε_i. The nonlinearity function $\varphi(\varepsilon_i)$ is taken as

$$\varphi\left(\varepsilon_i\right) = c + d\varepsilon_i^2, \tag{4}$$

here c, d are the constants, depending on the properties of shell material.

Calculate the strain rate [5, 16], taking into account (1) and (2)

$$\varepsilon_i = \frac{2}{\sqrt{3}}\left[\frac{3}{4}\varepsilon_2^2 + z^2\left(\frac{\partial^2 w}{\partial x^2}\right)^2\right]^{\frac{1}{2}}. \tag{5}$$

Solving (3) relative to stresses, and using (1) and (2), we get

$$\sigma_x = -2z\left(1-R^*\right)\varphi\left(\varepsilon_i\right)\frac{\partial^2 w}{\partial x^2}, \ \ \sigma_s = -\left(1-R^*\right)\varphi\left(\varepsilon_i\right)\left(\frac{3}{2}\frac{w}{R_1} + z\frac{\partial^2 w}{\partial x^2}\right). \tag{6}$$

Using the last formula for stresses (6), calculate the force and moment acting on shell element using formulas [16]

$$M_1 = \int_{-h}^{h}\sigma_x z dz, \ \ T_2 = \int_{-h}^{h}\sigma_s dz. \tag{7}$$

Substituting expressions (4)–(6) in (7) we get

$$M_1 = -D\left(1-R^*\right)\frac{\partial^2 w}{\partial x^2} - \frac{4bh^3}{3R_1^2}\left(1-R^*\right)\left[w^2\frac{\partial^2 w}{\partial x^2} + \frac{4}{5}R_1^2 h^2\left(\frac{\partial^2 w}{\partial x^2}\right)^3\right]; \tag{8}$$

$$T_2 = -\frac{B}{R_1}\left(1-R^*\right)w - \frac{3bh}{R_1^3}\left(1-R^*\right)\left[w^3 + \frac{4}{9}R_1^2 h^2 w\left(\frac{\partial^2 w}{\partial x^2}\right)^2\right], \tag{9}$$

where $D = \dfrac{4ah^3}{3}$; $B = 3ha$; h is the shell half thickness.

Differential equation of element equilibrium (Figure 1) has the form

$$\frac{\partial^2 M_1}{\partial x^2} + \frac{T_2}{R_1} + q = 0. \tag{10}$$

Adding inertial forces to the load q, according to the d'Alembert principle, and substituting (8) and (9) in (10) we get

$$D\left(1-R^*\right)\frac{\partial^4 w}{\partial x^4}+\frac{B}{R_1^2}\left(1-R^*\right)w+\frac{4bh^3}{3R_1^2}\left(1-R^*\right)\left\{2\left[\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial x^2}+w\left(\frac{\partial^2 w}{\partial x^2}\right)^2+2w\frac{\partial w}{\partial x}\frac{\partial^3 w}{\partial x^3}\right]+\right.$$
$$\left.+\frac{12}{5}R_1^2h^2\left[2\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial^3 w}{\partial x^3}\right)^2+\left(\frac{\partial^2 w}{\partial x^2}\right)^2\frac{\partial^4 w}{\partial x^4}\right]+w^2\frac{\partial^4 w}{\partial x^4}\right\}+ \quad (11)$$
$$+\frac{3bh}{R_1^4}\left(1-R^*\right)\left[w^3+\frac{4}{9}R_1^2h^2w\left(\frac{\partial^2 w}{\partial x^2}\right)^2\right]+m\frac{\partial^2 w}{\partial t^2}=q\left(x,t\right).$$

The effect of a concentrated mass on viscoelastic shell is inertial in nature and is accounted in the equation of motion (11) with the Dirac δ-function [22]:

$$m(x)=\rho h+\sum_{p=1}^{I}M_p\delta(x-x_p)$$

where ρ is the density of the shell material.

Thus, the problem of axisymmetric oscillations of viscoelastic cylindrical shells in a physically nonlinear statement is reduced to a system of partial integro-differential equations of the form (11) with appropriate initial and boundary conditions.

Most of dynamic problems of viscoelastic thin-walled structures [17] after applying the Bubnov-Galerkin method are reduced to solving non-decay systems of integro-differential equations of the following form:

$$\sum_{n=1}^{N}(c_{kn}\ddot{w}_n+\omega_{kn}^2w_n)=Z_k\left(t,w_1,...,w_N,\int_0^t\psi_k\left(t,\tau,w_1,...,w_N\right)d\tau\right),$$
$$w_n(0)=w_{0n},\ \dot{w}_n(0)=\dot{w}_{0n},\ n=1,2,...,N, \quad (12)$$

where $w_n=w_n(t)$ are the unknown time functions;

Z_k, ψ_k are the continuous functions in the domain of change of arguments;

c_{kn}, ω^2_{kn} are the given constant numbers.

Many nonlinear dynamic problems of viscoelasticity are reduced to system (12), in particular, problems of oscillations and dynamic stability of viscoelastic structures such as rods, beams and cylindrical shells bearing a concentrated mass.

Integrating system (12) twice over t, it is reduced to integral form. Assuming then that $t=t_i$, $t_i=i\Delta t$, $i=1,\ 2,\ \ldots$ (Δt = const is the interpolation step) and replacing the integrals with quadrature formulas to calculate $w_{in}=w_{in}\left(t_i\right)$, we obtain the following system:

$$\sum_{n=1}^{N}c_{kn}w_{in}=\sum_{n=1}^{N}c_{kn}(w_{0n}+\dot{w}_{0n}t_i)+\sum_{j=0}^{p-1}A_j(t_p-t_j)\times$$
$$\times\left\{Z_k\left(t_j,w_{j1},...,w_{jN},B_s\psi_{1k}\sum_{s=0}^{j}(t_j,t_s,w_{s1},...,w_{sN})\right)-\sum_{n=1}^{N}\omega_{kn}^2w_{jn}\right\}. \quad (13)$$

The next step in numerical method is the regularization of a system of nonlinear integro-differential equations (13) with the singular Koltunov-Rzhanitsyn kernel [2]

$$\Gamma(t)=Ae^{-\beta t}\cdot t^{\alpha-1},\quad A>0,\quad \beta>0,\quad 0<\alpha<1.$$

Using change of variables

$$\frac{t}{\omega}-\frac{\tau}{\omega}=z^{\frac{1}{\alpha}},\ 0\le z\le\left(\frac{t}{\omega}\right)^{\alpha},\ \left(0<\alpha<1\right)$$

the integral at the Koltunov-Rzhanitsyn kernel with a singularity of the following form

$$A\int_0^{\frac{t}{\omega}}\left(\frac{t}{\omega}-\frac{\tau}{\omega}\right)^{\alpha-1}e^{-\beta\left(\frac{t}{\omega}-\frac{\tau}{\omega}\right)}w(\tau)d\tau$$

has the form

$$\frac{A}{\alpha}\int_0^{\left(\frac{t}{\omega}\right)^{\alpha}}e^{-\beta z^{\frac{1}{\alpha}}}w\left(\frac{t}{\omega}-z^{\frac{1}{\alpha}}\right)dz.$$

Note that after the change of variables, the integrand with respect to *z* becomes regular. To numerically solve the system (13), we apply the method of direct replacement of integrals entering the system with a certain sum using some quadrature formula, in particular, using the trapezium formula

$$\frac{A}{\alpha}\sum_{k=0}^{i}B_k e^{-\beta t_k}w_{i-k},$$

where the coefficients are:

$$B_0=\frac{1}{2}\left(\frac{\Delta t}{\omega}\right)^{\alpha};\quad B_i=\frac{1}{2}\left(\frac{\Delta t}{\omega}\right)^{\alpha}(i^{\alpha}-(i-1)^{\alpha});$$

$$B_k=\frac{1}{2}\left(\frac{\Delta t}{\omega}\right)^{\alpha}((k+1)^{\alpha}-(k-1)^{\alpha}),\quad k=\overline{1,i-1}.$$

Thus, due to twice integration of initial system (12) over time *t* and the use of the quadrature formula, system (13) is obtained to find the deflections $w_{in}=w_{in}\left(t_i\right)$. Solution (13) is found by the Gauss method.

3. Results and Discussion

Solution of equation (11) at initial conditions

$$w=\gamma\left(x\right),\ \frac{\partial w\left(x,0\right)}{\partial t}=0 \tag{14}$$

is sought in the following form [23, 24]

$$w\left(x,t\right)=\sum_{n=1}^{N}w_n\left(t\right)\psi_n\left(x\right), \tag{15}$$

where $\psi_n\left(x\right)$ are the known coordinate functions that satisfy all the boundary conditions of the shell.

Substituting (15) into (11) and performing the procedure of the Bubnov-Galerkin method, we obtain

$$\begin{gathered}\sum_{n=1}^{N}a_{kn}\ddot{w}_n+D\left(1-R^*\right)\sum_{n=1}^{N}b_{kn}w_n+2B\left(1-R^*\right)\sum_{n,i,r=1}^{N}c_{knir}w_n w_i w_r=q_k,\\ w_n\left(0\right)=w_{0n},\quad \dot{w}_n\left(0\right)=\dot{w}_{0n},\end{gathered} \tag{16}$$

where $a_{kn}=\int_0^a\left(\rho h+\sum_{p=1}^{I}M_p\delta\left(x-x_p\right)\right)\psi_n\psi_k dx,\quad b_{kn}=\int_0^a\left(\psi_{n,xxxx}^{IV}+2\psi_{n,xxyy}^{IV}+\psi_{n,yyyy}^{IV}\right)\psi_k dx.$

$$c_{knir} = \int_0^a \left(6\psi''_{n,xx}\psi'''_{i,xxx}\psi'''_{r,xxx} + 3\psi''_{n,xx}\psi''_{i,xx}\psi^{IV}_{r,xxxx} + 3\psi'''_{n,xxx}\psi'''_{i,xxx}\psi''_{r,xx} + \right.$$

$$+ \psi'''_{n,xxy}\psi'''_{i,xxy}\psi''_{r,yy} + \psi''_{n,xx}\psi'''_{i,yyy}\psi'''_{r,xxy} + \psi''_{n,xx}\psi''_{i,yy}\psi^{IV}_{r,xxyy} + 6\psi''_{n,xy}\psi'''_{i,xyy}\psi'''_{r,xxy} +$$

$$\left. + 3\psi''_{n,xy}\psi''_{i,xy}\psi^{IV}_{r,xxyy} \right)\psi_k dx, \quad q_k = \int_0^a q\psi_k dx.$$

Integrating the system of resolving equations (16) twice over t, we can write it in integral form. Then, assuming that $t = t_i$, $t_i = i\Delta t$, $i = 1, 2, \ldots$ (Δt is the integration step) and replacing the integrals with the quadrature formulas of the trapezium to compute the unknowns $w_{in} = w_{in}\left(t_i\right)$, we obtain the following recurrence formula

$$\sum_{n=1}^{N} a_{kn} w_{pn} = \sum_{n=1}^{N} a_{kn}\left(w_{0n} + \dot{w}_{pn} t_p\right) - \sum_{q=0}^{p-1} A_q\left(t_p - t_q\right)\left\{ D\sum_{n=1}^{N} b_{kn}\left(w_{qn} - \frac{A}{\alpha}\sum_{z=0}^{q} B_z e^{-\beta t_z} w_{q-z,n}\right) + \right.$$

$$\left. + 2B\sum_{n,i,r=1}^{N} c_{knir}\left(w_{qn}w_{qi}w_{qr} - \frac{A}{\alpha}\sum_{z=0}^{q} B_z e^{-\beta t_z} w_{q-z,n}w_{q-z,i}w_{q-z,r}\right) - q_k \right\}; \quad (17)$$

$$w_n(0) = w_{0n}, \; \dot{w}_n(0) = \dot{w}_{0n},$$

where A_q, B_z are the numerical coefficients that do not depend on the choice of integrands and take on different values depending on the quadrature formulas used.

The dependence obtained makes it possible to study the axisymmetric oscillations of viscoelastic cylindrical shells carrying a concentrated mass with account for physical nonlinearity.

4. Conclusions

In the first part of this study in physically nonlinear and geometrically linear statements the following aspects were stated:

1. A boundary-value problem was formulated for the dynamic calculation of a cylindrical shell carrying concentrated masses based on the cubic theory of viscoelasticity.

2. Using the Bubnov-Galerkin method, the main resolving equations were obtained in the form of a system of non-decay integro-differential equations of the problem for dynamic calculation of a cylindrical shell carrying concentrated masses.

3. A method for solving the obtained systems of non-decay integro-differential equations based on the quadrature formula was proposed.

In the second part of the study, numerical results of the stress-strain state of a cylindrical shell with concentrated masses will be presented in physically nonlinear and geometrically linear statements.

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Contacts:

Dadakhan Khodzhaev, +7(99871)2370981; dhodjaev@mail.ru
Rustamkhan Abdikarimov, +7(99890)9284554; rabdikarimov@mail.ru
Mirziyod Mirsaidov, +7(987)2370981; theormir@mail.ru

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Динамика физически нелинейной вязкоупругой цилиндрической оболочки с сосредоточенными массами

***Д.А. Ходжаев*[a]*, Р.А. Абдикаримов*[b]**, М.М. Мирсаидов*[a]**

[a] *Ташкентский институт инженеров ирригации и механизации сельского хозяйства, г.Ташкент, Узбекистан*

[b] *Ташкентский финансовый институт, г.Ташкент, Узбекистан*

** E–mail: rabdikarimov@mail.ru*

Ключевые слова: тонкостенные конструкции, цилиндрическая оболочка, вязкоупругость, физическая нелинейность, сосредоточенные массы, осесимметричные колебания, нелинейное интегро-дифференциальное уравнение, ядро релаксации, метод Бубнова-Галёркина, численный метод

Аннотация. Известно, что наиболее разработанной частью теории упругости является теория линейных и нелинейных упругих пластин и оболочек. В этой области получены все необходимые уравнения и разработаны методы их решения. В то же время, в области учета вязкоупругих свойств материала в задачах по динамическим расчетам тонкостенных конструкций имеются пробелы. Отметим, что в некоторых работах вязкоупругие свойства материала, т.е. отклонение диаграммы испытаний материала от закона Гука учитывались по модели Фойхта, не подтверждающиеся экспериментами. Не учет вязкоупругих свойств материала существенно ограничивает практическую применимость результатов. В первой части работы рассматриваются постановка и метод решения задачи об осесимметричных колебаниях физически нелинейной вязкоупругой цилиндрической оболочки с сосредоточенными массами. Функция, характеризирующая меру отклонения кривой интенсивности напряжений от прямой Гука, принята в виде кубической нелинейности. Построена математическая модель, предложен метод решения и разработан вычислительный алгоритм задачи об осесимметричных колебаниях цилиндрической оболочки, несущей сосредоточенные массы, с учетом физически нелинейного деформирования материала при различных граничных условиях в рамках гипотезы Кирхгофа-Лява. Эффект действия сосредоточенных масс вводится с использованием дельта-функции Дирака. С помощью метода Бубнова-Галёркина, основанного на многочленной аппроксимации прогибов, рассматриваемая задача сводится к решению, в общем случае, не распадающихся систем нелинейных интегро-дифференциальных уравнений типа Вольтерры. Для решения полученной системы, при слабо-сингулярном ядре Колтунова-Ржаницына, применен численный метод, основанный на использовании квадратурных формул. Разработан единый вычислительный алгоритм для нахождения прогиба вязкоупругой цилиндрической оболочки с сосредоточенными массами.

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Контактные данные:

Дадахан Акмарханович Ходжаев, +7(99871)2370981; dhodjaev@mail.ru

Рустамхан Алимханович Абдикаримов, +7(99890)9284554; rabdikarimov@mail.ru

Мирзиед Мирсаидович Мирсаидов, +7(987)2370981; theormir@mail.ru